TOPICAL REVIEW — Statistical physics and complex systems

Statistical physics of human beings in games: Controlled experiments*

Liang Yuan(梁 源)[a)b)] and Huang Ji-Ping(黄吉平)[b)†]

[a)] *Department of Applied Physics, College of Science, Donghua University, Shanghai* 201620, *China*
[b)] *Department of Physics and State Key Laboratory of Surface Physics, Fudan University, Shanghai* 200433, *China*

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It is important to know whether the laws or phenomena in statistical physics for natural systems with non-adaptive agents still hold for social human systems with adaptive agents, because this implies whether it is possible to study or understand social human systems by using statistical physics originating from natural systems. For this purpose, we review the role of human adaptability in four kinds of specific human behaviors, namely, normal behavior, herd behavior, contrarian behavior, and hedge behavior. The approach is based on controlled experiments in the framework of market-directed resource-allocation games. The role of the controlled experiments could be at least two-fold: adopting the real human decision-making process so that the system under consideration could reflect the performance of genuine human beings; making it possible to obtain macroscopic physical properties of a human system by tuning a particular factor of the system, thus directly revealing cause and effect. As a result, both computer simulations and theoretical analyses help to show a few counterparts of some laws or phenomena in statistical physics for social human systems: two-phase phenomena or phase transitions, entropy-related phenomena, and a non-equilibrium steady state. This review highlights the role of human adaptability in these counterparts, and makes it possible to study or understand some particular social human systems by means of statistical physics coming from natural systems.

Keywords: experimental econophysics, computer-aided controlled experiment, phase transition, entropy-like quantity

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1. Introduction

A social human system belongs to the family of complex adaptive systems.[1–9] The systems cannot be simply described by Newtonian mechanics due to strong coupling interactions and complexity, and the agents in these systems often possess intelligence and learning ability, such as investors in the stock markets and species in the ecological systems. Thus, these agents are able to make their own decisions in the process of interaction with the environment (that involves the other agents), and then adapt themselves to the environment accordingly. That is, in the social human system that we are interested in, a human (or an institution made up of humans) is just an agent in the system where everyone can adapt to the environment because he/she carries out social or economic activities according to his/her own strategies.

The research and analysis focusing on complex adaptive systems emerged in several fields, such as physics, chemistry, and mathematics. In the past few decades, interdisciplines formed ceaselessly, which accelerated the development of various disciplines. A further combination between natural science and social science has brought new ideas and concepts to economics.

Nobel physics prize laureate Anderson and Nobel economics prize laureate Arrow organized a workshop on the topic of complex economic systems in September 1987, and they gave some basic thoughts that describe the evolution of economic systems:[10] they imagined that there is a core dynamic mechanism with a few variables and parameters in the economic systems, which dominates the evolution of the systems. Exploring the evolution laws of complex adaptive systems not only helps to understand the nature of the economic system, but also essentially promotes the development of the related disciplines such as physics, mathematics, and computer science. During the last two decades, physicists have also gradually carried out studies of economic systems worldwide and formed some specialized research groups and international academic conferences. In the mid-1990s, Stanley and coworkers first proposed the word econophysics to name the new interdiscipline:[11] econophysics means that physicists survey economic problems in the same way as they do for physics research.[12] After that, econophysics opens up its new age of rapid development; see Refs. [13]–[29] for examples.

It is known that for natural systems handled in the tradi-

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†Corresponding author. E-mail: jphuang@fudan.edu.cn

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tional physics, all kinds of materials have their unique microscopic configurations and interactions. The different configurations and interactions cause different materials to possess different properties (e.g., see Refs. [30]–[32]). Certainly, the basic units (like atoms or molecules) constructing the microscopic configurations in natural systems have neither intelligence nor learning ability. Nevertheless, for social systems, the basic units are human beings (or institutions consisting of humans) with high adaptability, which causes the internal mechanism to be more complex than that for natural systems. Therefore, the analytical methods for social systems seem to be quite different from those for natural systems. Physicists have persistently explored natural systems for a long time and accumulated vast experiences in repeated experiments. They summarized a series of statistical physical laws to describe natural systems. Through actual tests, these laws form the theorems in physics, which are conveniently used to understand the mechanism of natural systems. For example, the famous second law of thermodynamics describes the statistical law of the thermodynamic evolution process for natural systems. We are inspired by these statistical physical laws when researching complex adaptive systems. Accordingly, here we raise a relevant question: do the traditional laws or phenomena of statistical physics for natural systems with non-adaptive units still hold for social human systems with adaptive agents? The answer to this question will help to understand the evolution process and the microscopic mechanism of complex adaptive systems. On the other hand, this answer might also foster the application of traditional statistical physical laws to social human systems.

Each agent in a complex adaptive system interacts with the environment that can be regarded as the sum of all individual behaviors. Thus, the interaction between an agent and the environment is essentially the interaction between this agent and the other agents. If we do research by only relying on the traditional modeling method (where an artificial process has to be assumed to represent the decision-making process of real human beings), the theoretical results of the model will unavoidably deviate from the real facts because of the difficulty in grasping agents' complex interactions during modeling.[33–37] Generally speaking, there are at least two methods for researching the laws of complex adaptive systems composed of humans. One is an empirical analysis that generally denotes the statistics of the unrepeatable historical data. Such an analysis takes the history as a guide for the future. Two key defects of this method are as follows: people are usually unable to verify its conclusion's correctness with respect to a certain or a few factors due to uncontrollability; independent variables become unclear due to uncertain cause–effect. The other method uses controlled experiments. In this method, one extracts a sample that is small enough to be conducted in a laboratory and big enough to contain the key features of the whole system under consideration. Then he/she conducts the experiment by tuning independent variables appropriately, which is similar to what experimenters do in physical laboratories, say, studying the free falling body by changing the height only. As a result, the cause–effect mechanism underlying the human system can be revealed due to controllability. In general, such human experiments cannot be easily carried out for the real economy that contains a huge amount of humans. However, it is feasible to build a small real system (in laboratory) to experiment on. In this review, we shall focus on the second method (controlled experiments) that makes it possible to study complex adaptive systems' evolution mechanisms where statistical physics plays a role as well. Actually, the method of controlled human experiments should be attributed to economists; a pioneer in behavioral economics, V. L. Smith, pointed out long ago the three elements of constructing human experiments: environment, institutions, and behavior,[38–42] which gives the initial state of the system, while the institutions define the game rules. The environment and the institutions are controllable variables. Controlling the variables can affect the final individual behavior. Such a method of experimental economics[42–46] is now gradually adopted by researchers in econophysics[47–55] as to be extensively introduced in this review. The roles of such controlled experiments could be at least two-fold: (i) adopting the real human decision-making process so that the system under consideration could reflect the performance of genuine human beings; (ii) making it possible to obtain macroscopic physical properties of a human system by tuning a particular factor of the system, thus directly revealing cause and effect.

The unique feature of controlled human experiments lies in the fact that introducing behavioral economics experiments allows the researchers in the laboratory to recur to the individual behavior characteristics in the real economic activities that are affected by various factors including psychology.[56] However, controlled human experiments also have some limitations: they offer experimental results that are only valid for a specific population (both the number and the identity of subjects are fixed for one experiment), a specific avenue (where the experiment is conducted), and specific time (when the experiment is conducted). In order to overcome such limitations and make the experimental results more general, agent-based simulations can be used because they are beyond a specific population, a specific avenue, and specific time. Meanwhile, a theoretical analysis can help to further reveal the dynamical mechanism underlying the agent-based simulations and hence the controlled human experiments. In this sense, controlled human experiments, agent-based simulations, and theoretical analyses are three complementary tools among which the controlled human experiments serve as a basis, and they can be

used to efficiently extract the core laws from experimental phenomena. The present review will introduce the development in the relevant field.[47–57] Since 2002, scientists have designed and conducted a series of (computer-aided) controlled human experiments[47–55] on the basis of a resource-allocation system.[58–60]

It is worth mentioning that social psychology has been carrying out the study of human behaviors for a long time.[60–66] However, the analytical process and the emphasis of social psychology are different from what will be presented in this review. Social psychology focuses on the psychological process behind individuals or groups, while this review focuses on the macroscopic statistical physical properties of the complex system made up of humans. Accordingly, this review offers a new view to survey human complex systems. In thermodynamics, one does not know the movements of specific gas molecules, but can obtain the macroscopic statistical properties as a whole, say, the principle of increasing entropy. On the same footing, this review does not concern the reason behind specific human behaviors, but focuses on the macroscopic statistical properties of the human systems under consideration. For this purpose, after introducing some fundamental knowledge related to market-directed resource-allocation games (Section 2), we first clarify the role of human adaptability in four kinds of specific human behaviors, including normal behavior (Section 3.1),[50,52] herd behavior (Section 3.2),[51] contrarian behavior (Section 3.3),[53] and hedge behavior (Section 3.4).[51,53] Then we survey a few human counterparts of some laws or phenomena of the traditional statistical physics under some conditions, namely, two-phase phenomena or phase transitions (Section 4.1),[50,51,53] entropy-related phenomena (Section 4.2),[51,67] and non-equilibrium steady state (Section 4.3).[54] This review ends with a summary and outlook in Section 5.

2. Fundamentals of market-directed resource-allocation games

2.1. El Farol bar problem

The essence of the formation of human social activities lies in the acquisitiveness for resources. In many social and biological systems, the agents always spontaneously adaptively compete for limited resources, and thus change their environments. In order to effectively describe the complexity of the system, scientists have made a series of attempts. Such a resource competition system is just a kind of complex adaptive system.

For economic systems, the basic issue appears as well. Generally, in an economic market, if the resources are rationally allocated, the market is full of vitality. Otherwise, the development will be impeded, at least to some extent. Thus, the allocation of resources is the most fundamental economic problem. As one knows, most popular economics theories are related to deductive reasoning. According to those economic theories, as long as all individuals are almost smart, everyone will choose the best action, and then each individual can reason his/her best action.

However, people gradually find that in real life, individuals often have no complete rationality and superb deductive reasoning ability when making decisions. Instead, it is very common for them to simply use the feasible method of trial and error. Therefore, it looks like inductive generalization and continuous learning when real individuals make decisions (namely, inductive reasoning).

In the game theory, researchers often use evolutionary games to study the similar dynamic process. However, when using evolutionary game models, economists usually do not take into account the character of limited rationality. Therefore, they cannot convincingly yield interesting phenomena and critical phase transition behaviors. A social human system contains a large number of agents who have the limited ability of inductive reasoning. Even so, the microscopic simplicity can still lead to the complexity of the macroscopic system. Obviously, from a physical point of view, this system has a variety of statistical physical phenomena.

In the past, there were some studies about the allocation of resources. For example, in 1994, economist Arthur put forward a very representative resource allocation problem, the El Farol Bar problem, when he studied the inductive reasoning and bounded rationality.[68] It can be described as follows.

There is an El Farol bar in Santa Fe (a city in New Mexico of U.S.) which offers Irish music on every Thursday night. Each Thursday, 100 persons need to decide independently whether to go to this bar for fun or stay at home because there are only 60 seats in the bar. If more than 60 persons are present, the bar is so crowded that the customers get a worse experience than staying at home. If most people choose to stay at home at that day, then the people who go to the bar enjoy the elegant environment and make a wise choice.

In this problem, Arthur assumed no communication in advance among the 100 persons. They only know the historical numbers in the past weeks and have to make decisions independently. In order to make a wise choice, each person needs to possess his own strategies which are used to predict the attendance in the bar this week. People cannot obtain the perfect equilibrium solutions at the initial time when making decisions. They must consider others' decisions, and keep learning according to the limited historical experience in their mind. The elements of inductive reasoning and limited rationality in the El Farol bar problem lay a foundation for the further development of econophysics modeling, as shown in Section 2.2 below.

2.2. Unbiased distribution of resources

Inspired by the El Farol bar problem, physicists Challet and Zhang in 1997 proposed a minority game to quantitatively describe this problem and statistically analyzed the emerging collective phenomena in complex adaptive systems.[58] In the following years, scientists have done extensive researches about the minority game and its applications in different fields, which have significantly promoted the development of econophysics.[18,19,49,69–94] We introduce the minority game model as follows.

There are two rooms (indicated as Room A and Room B) and N agents, where N is an odd number. Each agent chooses independently to enter one of the two rooms. If one room contains fewer agents than the other, then the agents in this room win. That is to say, the minority wins. The two rooms in the minority game actually correspond to the case of an unbiased distribution of two resources. This game is repeated. Each agent can only make a decision next time according to the historical information. As a matter of fact, in daily life, people often face similar choices. Examples include choosing which road to avoid a traffic jam during rushing hour and choosing a less crowded emergency exit to escape. Although each of us can keep learning from limited historical experiences, it cannot guarantee that we make the correct choice every time.

In the minority game, the decision-making process, which is based on historical information, is modeled to form strategy tables. The minority game assumes that agents' memory length of the historical information is limited. Each agent can only remember the latest m rounds. If $m = 2$, it can form a strategy as shown in Table 1. The historical information in the left column records the attendance in the past two rounds, which is filled with a string of bits of 0 and 1. For example, a string of "10" represents the past two winning rooms, Room A and Room B. The right column is the prediction, which is filled with bits of 0 or 1. Bit 1 is linked to the choice of entering Room A, while bit 0 to that of Room B. So one can obtain a strategy pool with a size of 2^{2^m}. As m increases, the total number of strategy tables increases rapidly. In the original minority game model, the designers let each agent randomly select strategy tables. That is, the right column of each strategy table is randomly filled with 0 or 1. These agents are likely to repeat the same selected strategy (namely, the right columns of the strategy tables are the same). However, appropriately increasing the memory length can significantly reduce the repetition probability. Here it is worth noting a special case: if the right column of a strategy table is all 1 (or 0), this strategy means that the agents are always locked into Room A (or B) no matter what happens.

According to these results, it is not hard to find that the minority game model with such a strategy structure is closely related to memory length m, and the historical information can only increase with 2^m.

In econophysics, minority game models have been widely used to simulate a special kind of complex adaptive system, the stock market.[76,116,118] Researchers always hope to generate similar stock market data through the minority game model. The stand or fall of this similarity often needs to be tested to see whether model data have the same stylized facts as the real market data. Besides, the minority game can also be used to study competition problems about an unbiased distribution of resources.[48,90,92,117]

Table 1. A model strategy table in the minority game with memory length $m = 2$.

Information	Choice
00	1
01	0
10	1
11	0

2.3. Biased distribution of resources

In the real world, however, agents competing for resources often face a biased distribution of resources in different locations. This phenomenon has many instances in our daily life. For example, a company chooses to enter into or exit out of several industries with different capacities,[118,119] a driver chooses different traffic routes,[120,121] and people decide to go to a small bar or a large one.[122]

From an overall view, the ideal evolution process of resource allocation systems should be like this: although each agent carries the selfish purposes to compete for limited resources with other agents, the system as a whole can eventually reach a harmonious balance state in which the resource allocation is efficient, stable, and arbitrage-free. In the process of approaching the harmonious state, each agent in the system cannot know the real quantity of resources at a certain location, cannot communicate with others, and cannot be told the correct choice. That is, there exists an "invisible hand" (as coined by Smith[123]) to guide the whole group to cooperate with each other. Then, does the invisible hand always effectively play a role? In daily practice, a lot of evidence indicates that the invisible hand in some systems like the financial markets has a very strong guiding force, but sometimes it fails. The temporary failure implies that if the invisible hand can work, there should be some necessary basic conditions.

Based on the minority game, Wang *et al.* proposed a market-directed resource-allocation game (MDRAG) in 2009,[50] which handles a kind of biased distribution of resources. It is an extension of the minority game that deals with an unbiased distribution of resources. A further development is that the strategy adopted in the MDRAG gives agents preference heterogeneity and decision-making ability which

match the environmental complexity. We summarize several differences of the MDRAG model as follows.

1) Resources in two rooms can continuously change from the unbiased distribution to an extremely biased distribution.

2) The possible economic situations P replace the historical information based on the memory length in the minority game. In the strategy table, compared to 2^m, P can continuously change with the smallest unit 1. The number of rows is not requested to be 2^m, but can be any positive integer P. See Table 2.

3) Each strategy is given its own preference L, $L \in [0, P]$. There is a probability of L/P to be 1 in the right column of the strategy table and a probability of $(P-L)/P$ to be 0. At each time step, agents choose to enter a room according to the right column of the strategy tables directed by the given situation P_i, $P_i \in [1, P]$.

Table 2. A model strategy table in the MDRAG.

Economic situation	Choice
1	0
2	1
3	1
...	...
$P-1$	0
P	1

Each normal agent will be randomly given S strategy tables to form his/her own strategy book, which is shown in Fig. 1. From the start of the game, each normal agent will score all the strategies in his/her strategy book so as to evaluate how successful they are to predict the winning room. Following the hitherto best performing strategy in their strategy books, normal agents are enabled to make decisions for the next round.

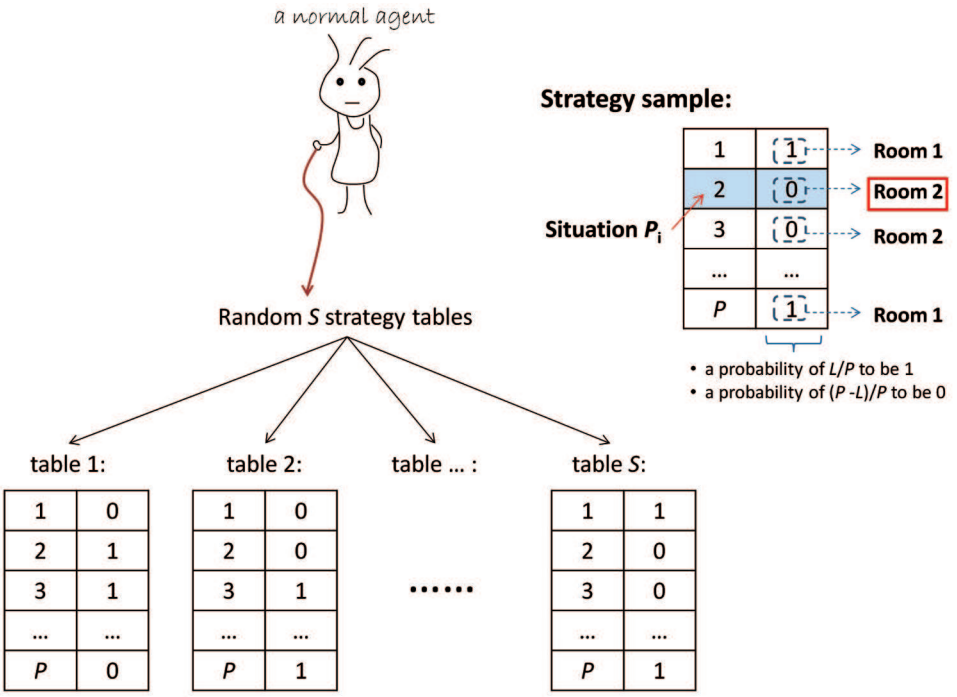


Fig. 1. Schematic graph showing strategy tables of a normal agent. Each normal agent is allocated S strategy tables. After scoring at the end of every simulating round, the best strategy is used for the next round. In the strategy sample, the current situation is $P_i = 2$, whose right column is 0, thus this agent will choose to enter room 2 in this round.[66]

3. Four kinds of specific human behaviors

In order to survive, all kinds of agents in complex adaptive systems need to maximize self-benefits and have to compete against others for the limited resources with a biased or unbiased distribution by adopting different strategies. The competition mechanism causes agents' high adaptability to external environments. In order to observe statistical physical properties of social human systems, we have to clarify the role of human adaptability in various kinds of human behaviors. For this purpose, we first classify human behaviors into normal behavior, herd behavior, contrarian behavior, and hedge behavior. Here the normal behavior is regarded as the behavior of ordinary humans who have independent decision-making ability. Both herd behavior and contrarian behavior are a kind of collective behavior. Though ordinary humans' decisions are basically made according to individual thinking, there are always some people who tend to pay heed to what others are doing and thus follow the current trend or anti-trend to form groups. If the herd behavior and the contrarian behavior exist in a market simultaneously, they can form a special kind of be-

havior, the hedge behavior. Next we shall experimentally observe the performance of a specific social human system with these four kinds of human behaviors. All the experiments surveyed in this review are essentially online anonymous games; the researchers obtained verbal consent from all the subjects who were informed that they could remove themselves from the experiment at any time.

3.1. Normal behavior

3.1.1. Heterogeneous preferences and decision-making capacity

To illustrate the system behavior, we designed and conducted a series of controlled human experiments in a computer lab, collaborating with the university students (who serve as the subjects); see Fig. 2 for an example. In the experiments, 89 students from different departments (mainly physics, mathematics, and economics) of Fudan University were recruited and randomly divided into 7 groups (Groups A–G, see Tables 3–6). The number of students in each group was just set for convenience and denoted by N in Tables 3–6. In the games played in the experiments, the students were told that they have to make a choice among a number of rooms in each round of a session for sharing the different amounts of virtual money in different rooms. The students who obtain more than the global average, namely, those belonging to the relative minority, would win the payoff. At the beginning of a session, participants were told the number of rooms (2 or 3), and in some cases the different but fixed amount of virtual money in each room. In the following, M_i is used to denote the amount of virtual money in room i ($i = 1,2,3$). A piece of global information about the payoff in the preceding round in all rooms was announced before a new round started. In each round, the students must make their own choices without any kinds of communication. The payoff per round for a student in room i is 2 points if $M_i/N_i > \sum M_i/N$, and -1 point otherwise. Here N_i is the number of the students choosing room i. The total payoff of a student is the sum of payoffs of all rounds, which will be converted to money payoffs in Chinese Yuan with a fixed exchange rate: 10:1 (10 points equals to 1 Chinese Yuan). Since the organizational and the statistic procedures were done by humans, one session of 10 rounds took roughly 20 minutes.[50]

Three kinds of games, GAME-I, GAME-II, and GAME-III, have been investigated. GAME-II differed from GAME-I in the global information being announced. In GAME-I, both the resource distribution M_i and the current population N_i in room i were announced, while only payoffs (2 or -1) in each room of the current round were conveyed to the players in GAME-II. Note that the environmental complexity was increased in GAME-II, since in order to win the game, players would have to predict other players' decision, and in the meantime infer the actual amounts of virtual money in different rooms. In GAME-III, the global information was the same as that in GAME-II, except an abrupt change of the amount of virtual money was introduced during the game without an announcement. On the contrary, all the participants have already been told that each M_i is unchanged. No further information was given to the participants.



Fig. 2. A photo of a representative experimental field, which was taken on October 13, 2012. Each subject plays the online game independently through his/her own computer in the computer lab of Fudan University.

Table 3. Results of GAME-I.[50]

Session	Group	Round	M_1	M_2	M_3	$\langle N_1\rangle$	$\langle N_2\rangle$	$\langle N_3\rangle$
1	A($N=12$)	1–10	3	2	1	5.3	4.6	2.1
2	A($N=12$)	1–10	3	2	1	5.5	3.8	2.7
3	B($N=12$)	1–10	3	2	1	5.5	4	2.5
4	C($N=24$)	1–20	3	2	1	12.2	7.4	4.4
5	D($N=10$)	1–10	5	3	–	6.1	3.9	–
6	D($N=10$)	1–10	3	1	–	7.4	2.6	–

Table 4. Results of GAME-II.[50]

Session	Group	Round	M_1	M_2	$\langle N_1\rangle$	$\langle N_2\rangle$
1	D($N=10$)	1–10	2	1	6.2	3.8
2	E($N=10$)	1–10	1	3	3.3	6.7
3	F($N=11$)	1–10	3	1	7.2	3.8
3	F($N=11$)	11–20	3	1	8.3	2.7
4	C($N=24$)	1–15	7	1	17.8	6.2
4	C($N=24$)	16–30	7	1	21.1	2.9

Table 5. Track of group F ($N=11$) converging to $M_1/M_2=3$.[50]

Round	N_1	N_2	Round	N_1	N_2
1	5	6	11	8	3
2	9	2	12	10	1
3	4	7	13	9	2
4	6	5	14	7	4
5	6	5	15	9	2
6	7	4	16	7	4
7	7	4	17	7	4
8	8	3	18	9	2
9	10	1	19	8	3
10	10	1	20	9	2

Table 6. Results of GAME-III.[50]

Session	Group	Round	M_1	M_2	$\langle N_1\rangle$	$\langle N_2\rangle$
1	G($N=10$)	1–5	3	1	5.4	4.6
1	G($N=10$)	6–10	3	1	8.2	1.8
1	G($N=10$)	11–15	3	1	7	3
1	G($N=10$)	16–20	3	1	7	3
1	G($N=10$)	21–25	1	3	7.8	2.2
1	G($N=10$)	26–30	1	3	4.2	5.8
1	G($N=10$)	31–35	1	3	2.8	7.2
1	G($N=10$)	36–40	1	3	2.6	7.4
1	G($N=10$)	41–45	1	3	2.4	7.6

The results of six sessions of GAME-I, four of GAME-II, and one of GAME-III are given in Tables 3–6. In Table 3, the results of GAME-I are listed, where the time average of the player number in room i is represented as $\langle N_i\rangle$. As the data shows, a kind of cooperation seems to emerge in the game within 10 rounds. In particular, the ratios of $\langle N_i\rangle$ converge to those of M_i, implying that the system becomes efficient in delivering the resources, even if they were distributed in a biased way. To the players, no room is better or worse in the long run, there is also no evidence that any of them could systematically beat the resource allocation "market". One might naively think that the system could evolve to this state only because the participants knew the resource distribution (prior to playing the games) and the population in each room (during the game). However, the results of GAME-II show that this explanation could not be correct. As shown in Table 4, although the players who know neither the resource distribution nor the current populations in different rooms seem not to be able to adapt to the unknown environment during the first 10 or 15 rounds, eventually, the relation $\langle N_1\rangle/\langle N_2\rangle \approx M_1/M_2$ is achieved again in groups C and F. For instance, Table 5 shows the track through which group F gradually find the balanced state under the environmental complexity $M_1/M_2=3$. Furthermore, the results of GAME-III support the conclusion of GAME-II, in which the system can reach this state even with an abrupt change of the unknown resource distribution when playing the game; see the results of 21 to 45 rounds played by the G group in Table 6. It is surprising that the players can "cooperate" even without direct communications as well as no information of the resource distribution. We can define the source of a force which drives the players to get their quota evenly as the invisible hand of the resource allocation market. In the sequel, however, we shall show that the effectiveness of this invisible hand relates to the heterogeneous preference and the adequate decision-making capacity of the participants of the game.

To find out the mechanism behind this adaptive system of resource distribution, we carried out a series of agent-based simulations to compare with the experimental results. In the agent-based model, normal agents correspond to the human subjects in the experiment. Before the game starts, each normal agent will take S strategies shown in Section 2.3 from the full strategy space and use the strategy tables to achieve a decision-making process like human subjects. Since we defined a preference index L in the MDRAG, it means that the normal agents have heterogeneous preferences because of the heterogeneous strategies. That is the essential difference from the minority game. The ratio M_1/M_2 represents the environmental complexity of the games. The results of the experiments are compared with the simulation results of the original minority game and the MDRAG in Fig. 3. With a certain set of parameters ($S=8$ and $P=16$), the MDRAG's results agree with the experimental data under a higher degree of environmental complexity. In other words, the agents in both

experiments and the MDRAG can be directed by the market to cooperate with each other, so an efficient allocation of the biased distributed resource can be realized even without giving the agents full information or instructions. On the other hand, the original minority game fails to reproduce the experimental results unless the distribution of resources is biased very weakly up to $M_1/M_2 = 3$. Hence, one may infer that the heterogeneity of agents' preferences is a significant factor in making the invisible hand play a role.

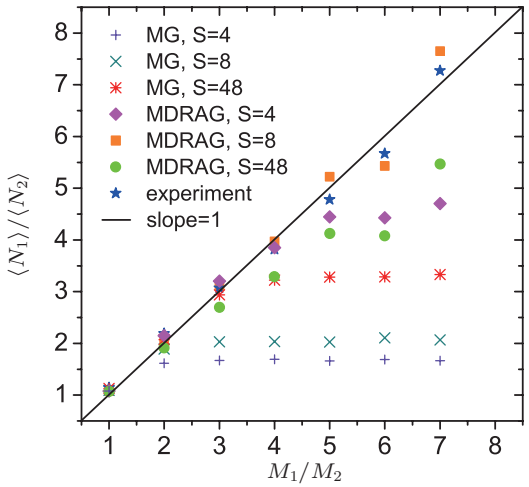


Fig. 3. The $\langle N_1\rangle/\langle N_2\rangle$ as a function of M_1/M_2, $P = 16$ in the minority game and the MDRAG, and $N = 24$ for all the simulations in the experiment. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). The line with slope = 1 indicates the efficient states $\langle N_1\rangle/\langle N_2\rangle = M_1/M_2$.[50]

Figure 3 also shows that the decision-making capacity, in particular the deliberation of choices (the parameter S), would be another factor having an influence on the effectiveness of the invisible hand. Typically, as the environmental complexity (M_1/M_2) increases, both the minority game and the MDRAG will deviate from the experimental results. Nevertheless, the problem of the minority game is very severe. As shown in the figure, even the minority game with an extremely large S ($S = 48$, a situation which seems to be inconsistent with the real system and will drastically increase the computational cost) can only work at a very low level of environmental complexity. At the same time, the result of the MDRAG provides a convincing fit with the experimental data when S is large enough, but not too large for a given P value. Basically, the minority game does not provide a good fit even for a large S, while the MDRAG can fit the data and is less demanding in terms of the computational cost.

Comparing the minority game and the MDRAG, we observe that the performances of the MDRAG are superior to the minority game, especially in the case of an extremely biased distribution of resources. It indicates that once the agents are equipped with a heterogeneous preference and decision-making capacity which match the environmental complexity, the system can achieve spontaneously the harmonious state as mentioned above. Thus, in this review, we choose the MDRAG model as the basis for researching resource allocation problems.

3.1.2. Risk-return relationship

In Section 3.1.1, we only introduced the case of resource allocation without the income of agents in repeated games. Here we make some changes to the above experimental design and treat the human subjects as investors to consider the risk-return relationship.[50]

In the experiments, we recruited 24 students and teachers from Fudan University as the subjects. The reward rules were the same as those listed in Section 3.1.1. There were two virtual rooms, room 1 and room 2, for the subjects to invest in. The subjects acted as fund managers who were responsible for implementing a fund's investing strategy and managing its trading activities. We told the subjects the requirement of a total of 30 rounds for each M_1/M_2, and offered every subject 1000 points (the amount of virtual money constructing the fund managed by the subject) as his/her initial wealth for each M_1/M_2. At the beginning of the 1st round of each M_1/M_2, we told the subjects the actual ratio of M_1/M_2 and asked each subject to decide his/her investing weight (denoted as $x(i)$ for subject i). Note that the investing weight $x(i)$ is the percentage of his/her investing wealth (investment capital) to his/her total wealth, and it would keep fixed within the 30 rounds for a certain M_1/M_2. At every round, each subject could only independently invest in one of the two rooms. After all the subjects made their own decisions, with the help of a computer program, we immediately knew the total investments in each room (denoted as W_1 and W_2 for Room 1 and Room 2, respectively) at this round. While keeping the total wealth conserved, we redistributed the total investment $W_1 + W_2$ according to the following two rules.

(i) We divided the total investment, $W_1 + W_2$, by the ratio of M_1/M_2, yielding $(W_1 + W_2)\frac{M_1}{M_1+M_2}$ and $(W_1 + W_2)\frac{M_2}{M_1+M_2}$ as the payoffs for Room 1 and Room 2, respectively.

(ii) We redistributed the payoff of Room k ($k = 1, 2$) by the investment of the the subjects. Namely, for each round, the payoff for subject i choosing room k to invest in, $w_{\text{payoff}}(i)$, is determined by $w_{\text{payoff}}(i) = (W_1 + W_2)\frac{M_k}{M_1+M_2} \times \frac{w_{\text{in}}(i)}{W_k}$, where $w_{\text{in}}(i)$ is the investing wealth of subject i, $w_{\text{in}}(i) = x(i)w(i)$. Here $w(i)$ is the total wealth possessed by subject i at the end of the previous round.

Before the experiments, we told the subjects the above

two rules for wealth re-allocation. After each round, every subject knew his/her payoff $w_{\text{payoff}}(i)$. If there is $w_{\text{payoff}}(i) > w_{\text{in}}(i)$, that is, subject i obtains more than the amount he/she has invested, we consider subject i as a winner at this round. Equivalently, if $W_1/M_1 < W_2/M_2$, the subjects choosing room 1 to invest win at this round. Clearly, when $W_1/W_2 = M_1/M_2$, every subject obtains the payoff which equals to his/her investing wealth. Namely, the arbitrage opportunity has been used up. Accordingly, we define the $W_1/W_2 = M_1/M_2$ state as a balanced state. This state may have some practical significance because global arbitrage opportunities for investing in human society always tend to shrink or even disappear once known and used by more and more investors. As shown in Fig. 4, our experimental system can indeed achieve $\langle W_1/W_2\rangle \approx M_1/M_2$ at which the system automatically produces the balanced allocation of investing wealth; this system thus reaches a statistical equilibrium. In other words, the invisible hand plays a full role. That is, all subjects are pursuing self-interest and we run the present system under three conditions: with sufficient information (namely, the wealth change for each round has reflected the possible information), with free competition (i.e., no subjects dominate the system and there are zero transaction costs), and without externalities (the wealth change of a subject has reflected the influence of his/her behavior on the others).

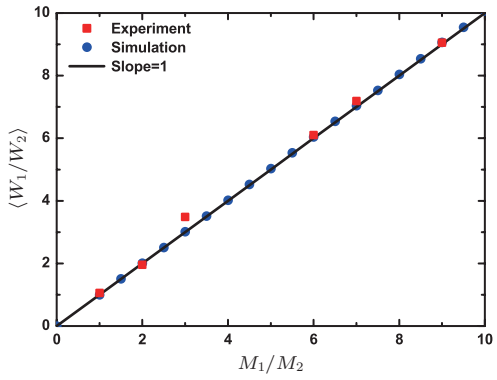


Fig. 4. Averaged ratio $\langle W_1/W_2\rangle$ versus M_1/M_2 for the human experiments with 24 subjects (red squares) and agent-based computer simulations with 1000 agents (blue dots). Here $\langle\cdots\rangle$ denotes the average over the total 30 experimental rounds (experimental data of W_1/W_2 for each round are shown in Table 1) or over the 800 simulation rounds (the additional 200 rounds are performed at the beginning of the simulation for each M_1/M_2; during the 200 rounds, we train all of the strategies by scoring them whereas the wealth of each agent remains unchanged). All the experimental and the simulation points lie in or beside the diagonal line (slope = 1), which indicates $\langle W_1/W_2\rangle \approx M_1/M_2$. Parameters for the simulations are $S = 4$ and $P = 16$.[52]

If a subject (namely, a fund manager in the experiment) chooses a larger investing weight, he/she will invest more virtual money in a room. According to the rules of our experiment, the room he/she chooses will then be more likely to be the losing one. Besides, the initial wealth is the same for every subject and he/she knows nothing but himself/herself. From this point of view, the larger investing weight he/she chooses, the higher risk (or uncertainty) he/she will take for the fund (i.e., the initial 1000 points). Therefore, throughout this work, we simply set the investing weight, $x(i)$, to equal the risk he/she is willing to take. Here we should remark that the present definition of risk appears to be different from that in the finance theory. For the latter, one often defines the risk according to variance. Nevertheless, the two kinds of risks are essentially the same because they both describe the uncertainty of funds and have a positive association with each other. On the other hand, we should mention that the risk we define for each subject does not change with the evolution of the time. This is a simplification which makes it possible to discuss the pure effect of a fixed value of risk. Nevertheless, if we choose to let the risk change with the time, for the same purpose, we may take an average of the risk over the full range of time. Figures 5(a)–5(f) show the risk-return relationship for the investments in designing a complex adaptive system. From the statistical point of view, we find that investments with a negative risk-return relationship are dominant over those with a positive risk-return relationship in the whole system.

Next, we resort to the computer simulation to extend the experimental results. As shown by Fig. 4, our agent-based computer simulations also give $\langle W_1/W_2\rangle \approx M_1/M_2$, that is, the system under simulation also exhibits the market efficiency. Further, according to the simulations, we achieve the same qualitative conclusion: investments with a negative risk-return relationship are statistically dominant over those with a positive risk-return relationship in the whole system; see Figs. 5(g)–5(l). Nevertheless, when we scrutinize Figs. 5(j)–5(l), we find that some particular data seem to be located on a smooth upward line. We plot these data in blue, and further find that they are just corresponding to all the agents with preference = 1. Encouraged by this finding, we turn blue all the data of preference = 1 in the other 9 panels of Fig. 5, and observe that a similar upward line also appears in the experimental results (see the blue dots in Figs. 5(a)–5(f); note the blue dots in Figs. 5(c) and 5(e) are also, on average, in an upward line even though they appear to be not so evident).

For the upward lines themselves, they are clearly indicative of investments with a positive risk-return relationship. Hence, to distinctly understand our main conclusion about the dominance of investments with a negative risk-return relationship in the whole system, one has to overcome the puzzle, namely, the strange appearance of these upward lines (constructed by the blue dots in Fig. 5), as already analyzed above.

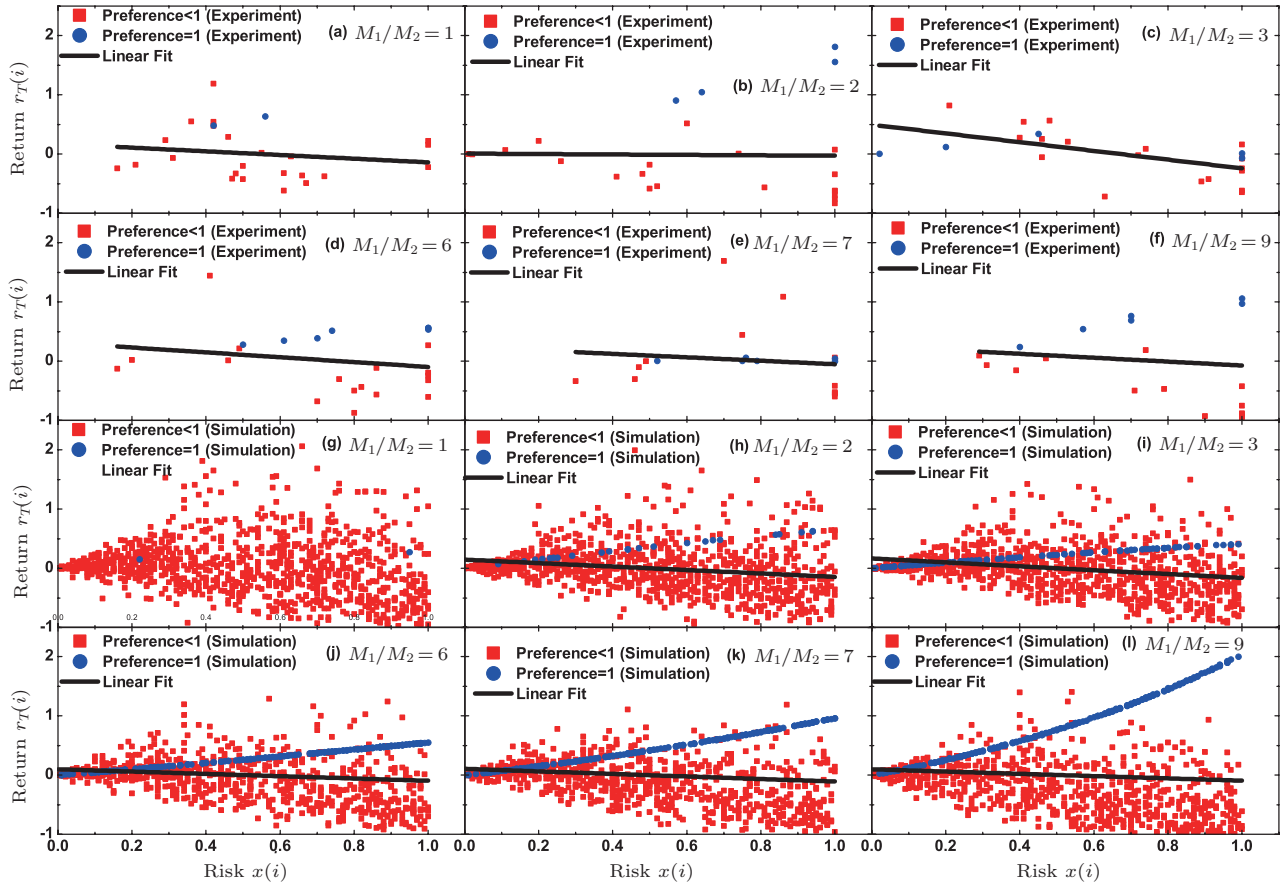


Fig. 5. Relationship between the risk, $x(i)$, and the return, $r_T(i) = [w_T(i) - w_0(i)]/w_0(i)$ for (a)–(f) 24 subjects and (g)–(l) 1000 agents at various M_1/M_2's. (a)–(f) Data of the human experiments (total of 30 rounds for each M_1/M_2); (g)–(l) data of the agent-based computer simulations (total of 800 rounds for each M_1/M_2, with an additional 200 rounds performed at the beginning of the simulations; during the 200 rounds, we train all of the strategies by scoring them whereas the wealth of each agent remains unchanged). Here $w_T(i)$ is agent i's wealth at the end of the T-th round (the total number of rounds, T, is $T = 30$ and 800 for the experiments and the simulations, respectively), and $w_0(i)$ is agent i's initial wealth. All of the subjects or agents are divided into two groups with preference < 1 (red squares) and preference $= 1$ (blue dots). Here, the preference is given by C_1/T, where C_1 is the number of times for the subjects or agents to choose room 1 within the total of T rounds. Here, the linear fit denotes the line fitting the data in each panel using the least square method, which serves as a guide for the eye. All of the lines are downward, which indicates a statistically negative relationship between risk and return. The present negative relationship just reflects the dominance of investments with a negative risk-return relationship in the whole system, in spite of a relatively small number of investments with a positive risk-return relationship. Other parameters in panels (g)–(l) are $S = 4$ and $P = 16$.[52]

3.2. Herd behavior

According to the above controlled experiments, the allocation of the resources in a complex adaptive system could reach a balanced state due to the preferences and the decision making ability of normal agents. Besides normal behavior, here we also consider a special kind of collective behavior, herd behavior. In fact, herding extensively exists in the collective behavior of many species in complex adaptive systems, including human beings. Though human decisions are basically made according to individual thinking, people tend to pay attention to what others are doing, emulate successful persons, or those of a higher status, and thus follow the current trend. For example, young girls often copy the clothing style of some famous stars, called trendsetters in the fashion world. Similarly, researchers would rather choose to work on a topic that is currently hot in the scientific society. As a result, large numbers of people may act in concert, and this unplanned formation of crowds is called herd behavior.[124] Locally, for an individual agent, the herd behavior may suggest either irrationality[125] or rationality,[126,127] with an implication that herding can ruin the balance of the whole resource-allocation system by causing excess volatility. Accordingly, herd behavior is commonly seen as a tailor-made cause for explaining bubbles and crashes in a complex adaptive system with the existence of extremely high volatility. Is this common sense always right? Here we focus on the effect of herding on the resource allocation system and conduct a series of computer-aided human experiments, on the basis of the MDRAG, to study the necessary conditions for a complex adaptive system to reach the ideal balanced state.[51]

Human subjects of the resource-allocation experiments were students recruited from several departments of Fudan University (the reward rules were the same as those listed in Section 3.1.1). There were two rooms (room 1 and room 2) and the amounts of resources in the two rooms were M_1 and M_2, respectively. In the experiment, M_1 and M_2 were kept

fixed and unknown to all the participants. For each experiment round, each participant had to choose one of the two rooms to share alike the virtual resource. Apart from human participants, there were also imitating agents joining the experiment. All the imitating agents were generated by a computer program, their decisions were simply made by mimicking human participants' behaviors. In particular, each imitating agent would randomly select a new group (of size 5) of human participants at every experiment round, and then follow the choice of the best participant (who has the highest score) in the group for the next round. In each round of the experiment, the number of human participants and imitating agents in room 1 is denoted as N_1, and the number in room 2 as N_2. Therefore, the total number of human participants and imitating agents can be counted as $N = N_1 + N_2$. The human participants or imitating agents who earned more than the global average $(M_1 + M_2)/N$ are regarded as winners of the round, and the room in which the winners have entered as the winning room. The total number of human participants or imitating agents can also be expressed as $N = N_{\rm n} + N_{\rm m}$. Here $N_{\rm n}$ is the total number of human participants who make decisions on their own, and $N_{\rm m}$ is the total number of imitating agents who do not have their own ideas. The ratio between imitating agents and human participants is defined as $\beta = N_{\rm m}/N_{\rm n}$. The game panel for human subjects and the control panel for parameter adjustment are shown in Fig. 6.

As shown in Section 3.1, the heterogeneity of preferences is an indispensable factor for the whole system to reach a balanced state. Hence the preferences of the human participants need to be checked under the influence of imitating agents. Preferences of the 44 participants are plotted in Fig. 7 with different M_1/M_2 and/or β. Figures 7(a)–7(c) show the preferences of the human participants and the imitating agents are absent. It can be found that the heterogeneity of preferences remains even for the cases in which $N_{\rm m} = N_{\rm n}/2$ imitating agents are involved; see Figs. 7(g)–7(i). Despite this heterogeneity, the average of participants' preferences changes along with M_1/M_2. In other words, the human participants have the ability to adapt themselves to fit the environment no matter whether the imitators exist or not. Similar results appear in the distributions of human participants' preferences, which are shown in Figs. 7(d)–7(f) and 7(j)–7(l). Even in an extreme biased case, a few subjects with preference to room 2 still exist. Here the involvement of the imitating agents does not bring much change to the preference distribution of the human participants. One may say that, in this case, the herd behavior has no harmful effect on the analyzing ability of the human participants.

To evaluate the performance of the whole system, we have calculated efficiency (which, herein, only describes the degree of balance of resource allocation), stability, and predictability of the resource-allocation system. The efficiency of the whole system can be defined as $e = |\langle N_1\rangle/\langle N_2\rangle - M_1/M_2|/(M_1/M_2)$. A smaller e means a higher efficiency in the allocation of resources. The stability of the resource-allocation system can be described as $\sigma^2/N \equiv \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \tilde{N}_i)^2\rangle$, where $\langle A\rangle$ denotes the average of time series A. This definition describes the fluctuation in the room population away from the balanced state where the optimal room populations $\tilde{N}_i = M_i N/\sum M_i$ can be realized. The predictability of the system is measured by the uniformity of the winning rates in different rooms. The winning rate in room 1 is denoted as w_1. It is obvious that if w_1 is close to 0.5, the choices of the two rooms are symmetrical and the system is unpredictable. If the winning rate were too biased, smart participants should be able to predict the next winning room in the experiment. As shown in Fig. 8, when M_1/M_2 is small ($M_1/M_2 = 1$ or 3), adding some imitating agents will lower the efficiency and cause large fluctuations. On the other hand, when M_1/M_2 becomes even larger ($M_1/M_2 = 20$), the formation of a herd can improve the efficiency, the stability, and the unpredictability of the resource-allocation system.



Fig. 6. The panels of the experiment-control computer program used in the computer-aided human experiments: (a) the control panel for the organizer, and (b) the panel for the human subjects.[51]

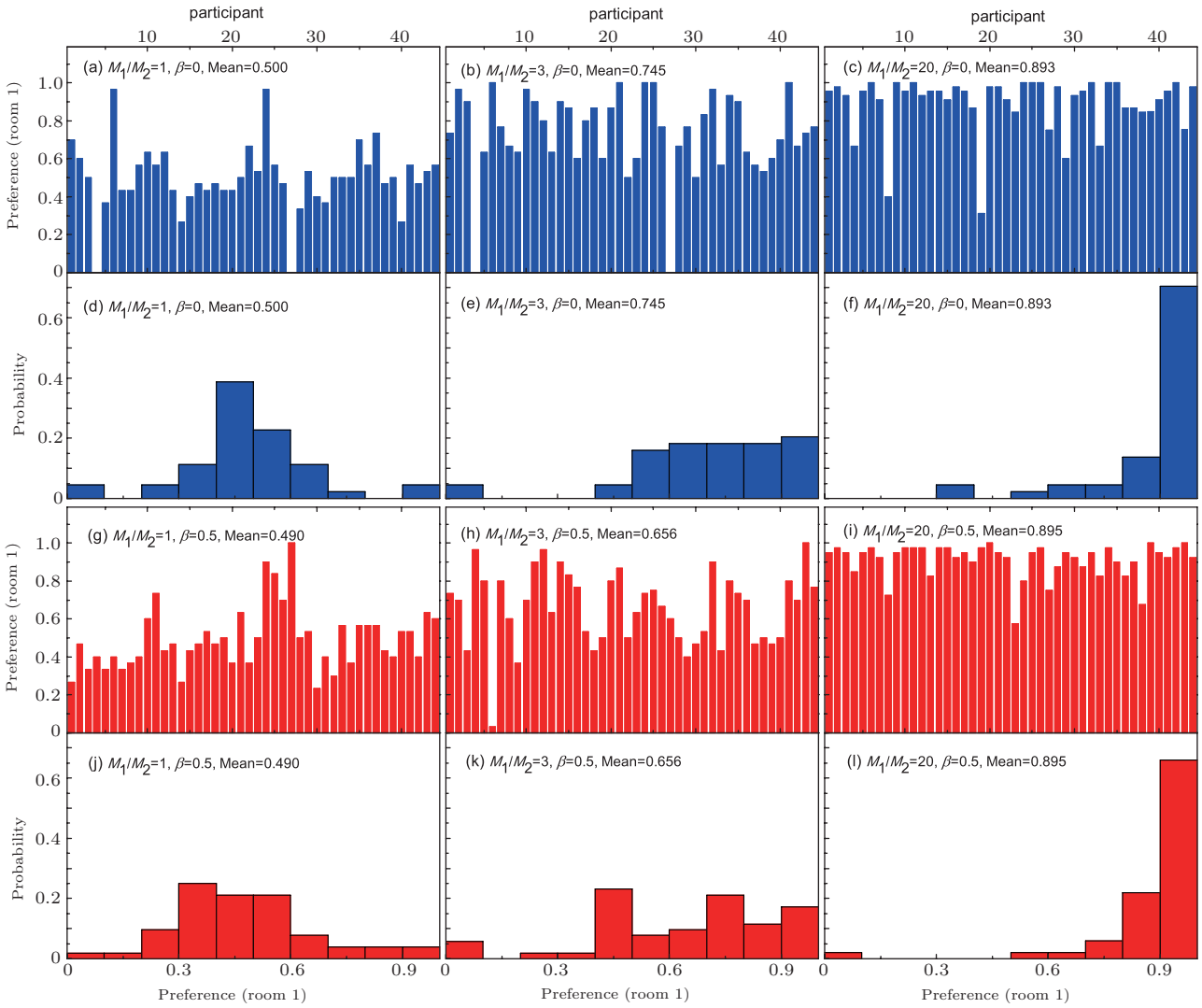


Fig. 7. Data obtained from the human experiment. (a)–(c) and (g)–(i) Preferences of the 44 participants in sequence to room 1 for the cases (a)–(c) without and (g)–(i) with imitating agents for different resource distributions M_1/M_2. The mean denotes the average of the preferences of the 44 participants. (d)–(f) and (j)–(l) Distributions of the 44 participants' preferences.[51]

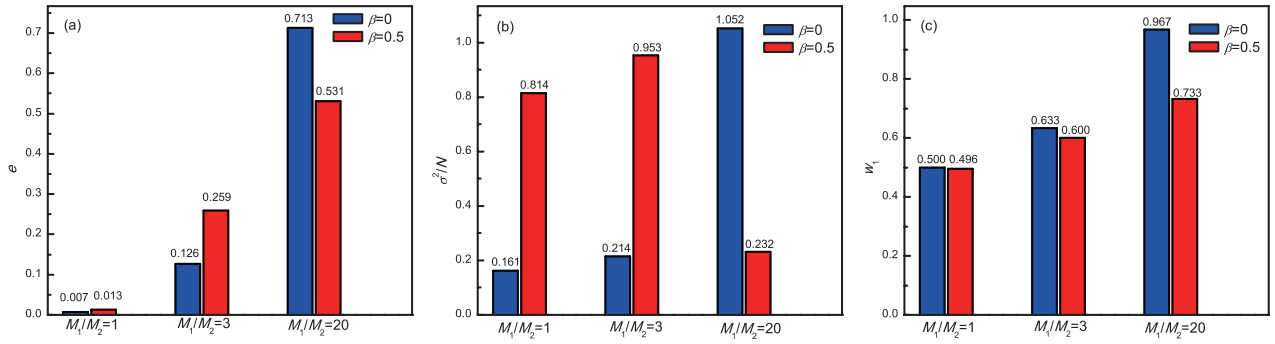


Fig. 8. Experimental results for (a) efficiency e, (b) stability σ^2/N, and (c) predictability w_1 of the modeled resource-allocation system, with human participants $N_n = 50$. The $\beta = 0$ and 0.5 correspond to imitating agents $N_m = 0$ and 25, respectively. Each experiment lasts for 30 rounds.[51]

Clearly, the above experiment has some unavoidable limitations: specific time, a specific experiment avenue (a computer room in Fudan University), specific subjects (students and teachers of Fudan University), and the limited number of subjects. Now we are obliged to extend the experimental results beyond such limitations through computer simulations. The agent-based model is the MDRAG with imitators. Normal agents correspond to the subjects in the preceding experiments not knowing the ratio of the resource distribution. Imitators in the model do not have their own strategy tables and behave in a different way. Before each round of play starts, each imitating agent will randomly select a group of k ($1 \leq k \leq N_n$) normal

agents. Within this group, the imitating agent will find the normal agent who has the best performance so far and imitate its behavior in the following experiment round. It is assumed that the imitating agents know neither the historical record of the winning room nor the details of the strategy books of other group members. The only information for them to access is the performance of the normal agents, that is, the virtual money that these normal agents have earned from the beginning of the experiment. If the number of imitating agents $N_{\rm m}$ kept increasing, there would be more and more positive correlations among agents' decisions, which would trigger the formation of a herd in the system.

Following the analysis of the experimental results, we first investigate the simulation results for the preferences of the normal agents. Figure 9 shows the distributions of the preferences similar to those shown in Fig. 7. The qualitative agreement indicates that our agent-based modeling has taken into account the heterogeneity of preferences with a reasonable modeling of the decision making process for the human participants. We have also investigated the preferences of the normal agents in an alternative way by analyzing the Shannon information entropy (see Section 4.2.1).

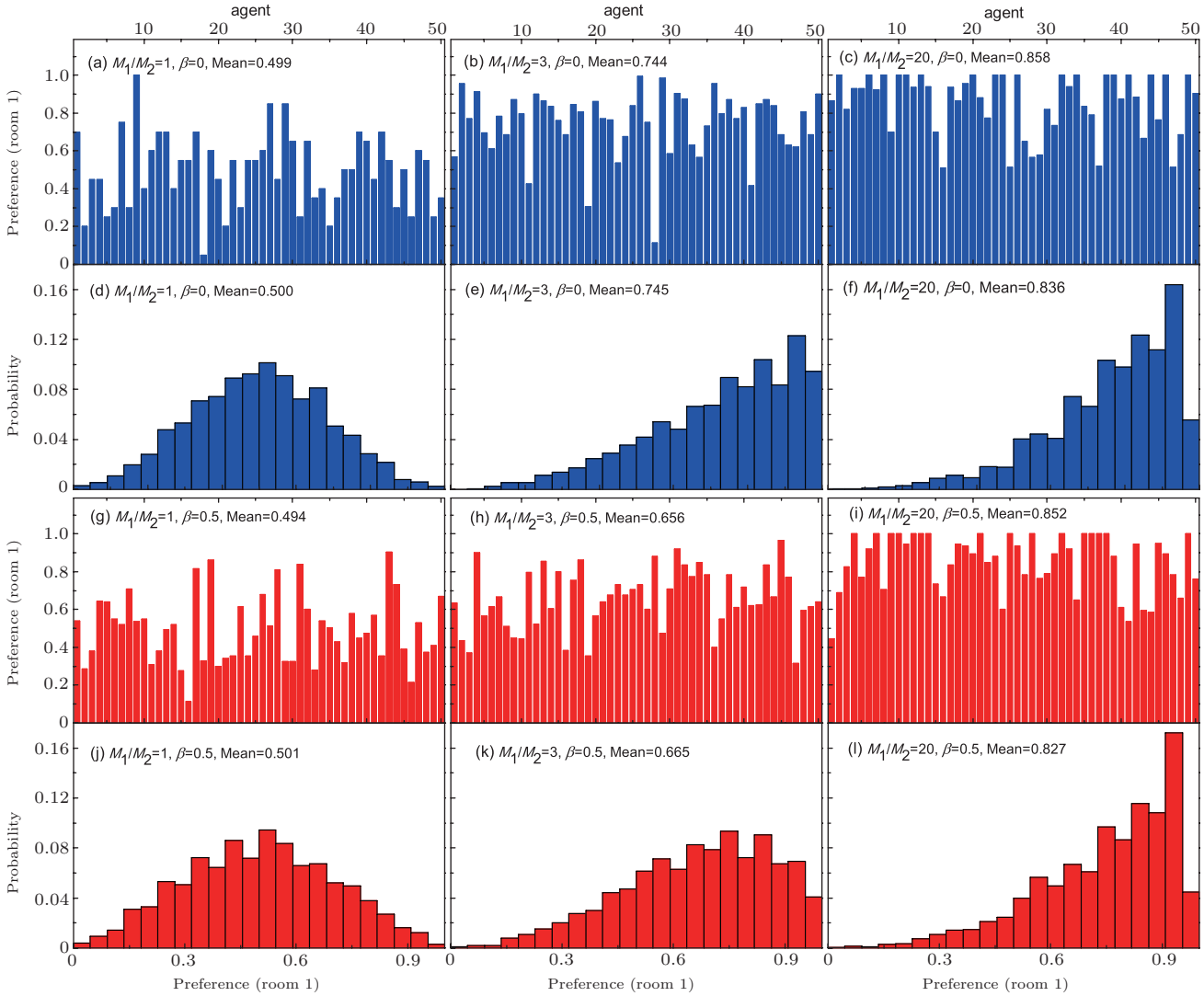


Fig. 9. Simulation results obtained from the agent-based simulations. (a)–(c) and (g)–(i) Preferences of the 50 normal agents to room 1 for the cases (a)–(c) without ($\beta = 0$) and (g)–(i) with (β=0.5) imitating agents for different resource distributions M_1/M_2. We have run the simulations 200 times, each with over 400 time steps (first half for equilibration, the remaining half for statistics). The plots show the typical results of one of the 200 runs, and the mean denotes the mean value of the preferences of the 50 normal agents. (d)–(f) and (j)–(l) Distributions of the 50 normal agents' preferences. Note that the results in panels (d)–(f) and (j)–(l) are obtained from the average over the 200 runs, and the mean also denotes this average. Simulation parameters are $S = 4$, $P = 16$, and $N_{\rm n} = 50$.[51]

Next, efficiency, stability, and predictability of the whole modeled system are calculated according to the definitions made in the experimental study. The change of the system behavior along with the variation of the resource ratio M_1/M_2 is shown in Fig. 10. As shown in Fig. 10(a), when the resource distribution is almost unbiased ($M_1/M_2 \approx 1$), the averaged population ratio $\langle N_1\rangle/\langle N_2\rangle$ can always be in concert with M_1/M_2 no matter whether the imitating agents are involved

or not. On the other hand, as the resource distribution becomes more and more biased (M_1/M_2 increases), surprisingly, the whole system tends to reach the balanced state only if more imitating agents (larger β) join the system. Figure 10(b) shows the change of the efficiency of the resource-allocation system. The tendency is that when the resource ratio gets more biased, a larger size of herd is needed to realize a higher efficiency of the resource distribution. As shown in Fig. 10(c), the increase of the number of imitating agents will cause larger fluctuations in the low M_1/M_2 region. However, as M_1/M_2 increases, more imitating agents can yield higher stability of the resource-allocation systems. Finally, the effect of the herd behavior on the predictability of the resource-allocation system is shown in Fig. 10(d). When more imitating agents are introduced to the system for large M_1/M_2, the prediction of the next winning room becomes more difficult as the winning rates for the two rooms are more symmetric. Notice that the system behavior under various conditions found herein by the agent-based simulations echoes with the observations in the experiment.

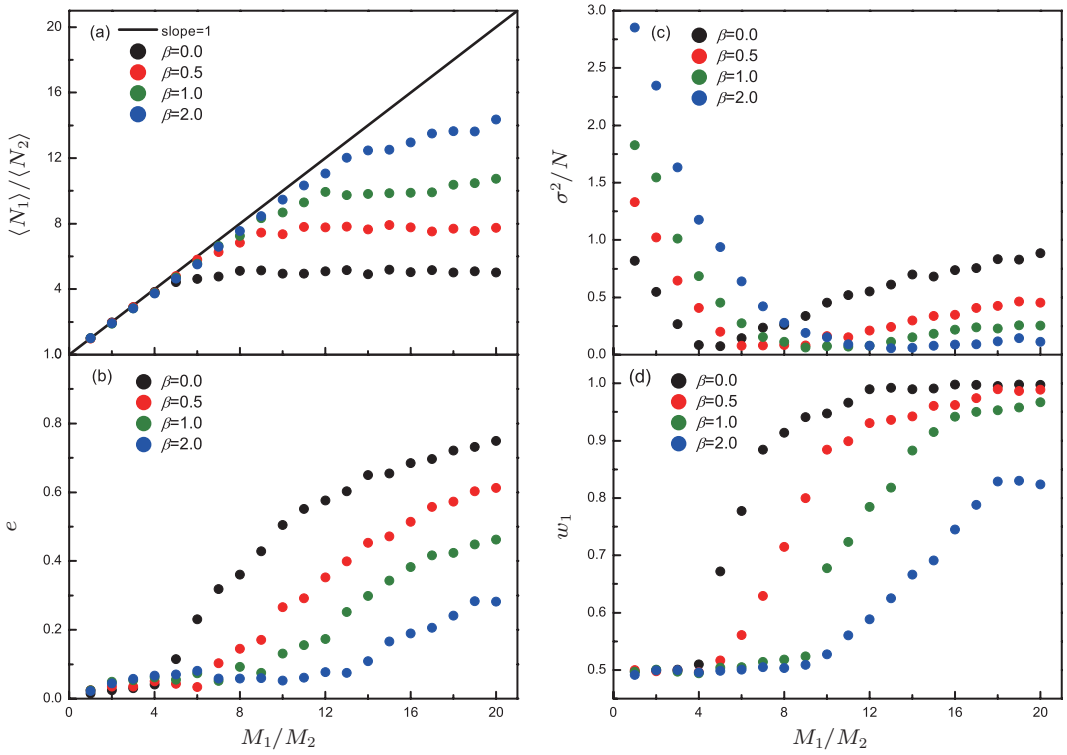


Fig. 10. The (a) $\langle N_1\rangle/\langle N_2\rangle$, (b) e, (c) σ^2/N, and (d) w_1 as a function of M_1/M_2 for an open system in the agent-based simulations. Parameters are $N_n = 50$, $S = 4$, $P = 16$, $k = 5$, and $\beta = 0$, 0.5, 1.0, and 2.0. For each parameter set, the simulations are run for 200 times, each over 400 time steps (the first half for equilibration, the remaining half for statistics). In panel (a), slope = 1 denotes the straight line with its slope being 1.[51]

We have revealed that, if the bias between the two resources M_1/M_2 is large and unknown to the participants, a herd of a typical size could help the overall system to reach the optimal state, namely, the state with a minimal fluctuation, a high efficiency, and a relatively low predictability. When the resource distribution is not biased so much, the normal agents can play pretty well so that the resource-allocation system behaves in a healthy manner (efficient or balanced, stable and unpredictable). In such situations, adding imitating agents will only bring about a "crowded system" in which larger fluctuations turn up. In this respect, our study shares some common features with the binary-agent-resource model.[128,129] When a large bias exists in the distribution of resources, the richer room will offer more arbitrage opportunities so that it deserves to be chosen without too much deliberation. Since the imitating agents learn from the best local human participant or normal agent, the herd formed by these agents will certainly be more oriented to the richer room. Based on these results, we argue that the herd behavior should not be labeled the killer of balance and stability all the time.

3.3. Contrarian behavior

In the minority game, the minority can win more resources. One may consider following the minority to increase the probability of winning. Besides the herd behavior, the contrarian behavior is also a specialized kind of self-organization in complex adaptive systems. Determining the nature of the contrarian behavior is also of practical importance when one faces the relevant problems of resource allocation. Thus, the contrarian behavior has been an active subject of studies in

various fields like finance and economics,[130,131] complexity science,[132] and social science.[133–136] In social fields, the previous contrarian studies using a Galam model of two-state opinion dynamics[133–135] aimed at the effect of contrarian choices on the dynamics of opinion forming, which shed a significant light on hung elections. Then, how does the contrarian behavior affect the social resource allocation? It is a common belief that the contrarian behavior always stabilizes the resource allocation by shrinking the redundancy or the lack of resources (positive role). However, is this common belief true? Here we specially raise this question because unbiased or biased distributions of resources are everywhere in nature where contrarians are often needed. In other words, to comply with the real world, we need to investigate the role of contrarian behavior. To proceed, based on the extensively adopted methods of both statistical analysis[137–139] and agent-based modeling,[140,141] we also conduct a series of computer-aided human experiments.[53]

As revealed in the MDRAG, the system can reach a macroscopic dynamic balance state that corresponds to the most stable state where the resources are allocated most efficiently and the total utilities of the system are maximal due to the absence of macroscopic arbitrage opportunities. Here we add a proportion of contrarians in order to observe how the contrarian behavior affects the macroscopic properties of the resource-allocation system. For the experiments, we recruited 171 subjects, all of whom were students and teachers from several departments of Fudan University (the reward rules were the same as those listed in Section 3.1.1). The basic setup of the computer-aided online experiment is analogous to that in Section 3.2. The difference lies in that the computer program secretly adds contrarians into the system instead of the imitators. The contrarian behaviors are controlled by the following settings. In every round of the experiment, each contrarian randomly chooses five subjects as his or her group. Then the contrarian will choose to enter the less-entered room according to the group. For example, if most of the subjects in a contrarian's group choose to enter room 1, the contrarian will choose to enter room 2. Let us denote the number of subjects as $N_{\rm n}$ and the number of contrarians as $N_{\rm c}$, thus yielding $\beta_{\rm c} = N_{\rm c}/N_{\rm n}$. In addition, the total number of all the subjects and contrarians is $N = N_{\rm n} + N_{\rm c} = N_1 + N_2$. The experiment was conducted over two successive days: 88 subjects on the first day and 83 on the second day. The different number or different subjects showed no influence on the results of the experiment. The experimental results are shown in Fig. 11, where $\langle N_1\rangle/\langle N_2\rangle$ is plotted as a function of M_1/M_2. When the distribution of resources is weakly biased up to $M_1/M_2 = 3$, the experimental results of $\langle N_1\rangle/\langle N_2\rangle$ are approximately located on the line with slope $= 1$ for the three values of $\beta_{\rm c}$. In such cases, the system reaches the dynamic balance at which the total utilities of the system are maximal due to the elimination of the macroscopic arbitrage opportunities. Nevertheless, for the strongly biased resource ratios, say $M_1/M_2 = 10$, the balance is broken as shown by the three experimental values that deviate far from the slope $= 1$ line. In other words, when the resource ratio is unbiased or weakly biased, adding a small proportion of contrarians does not hurt the system balance. In contrast, when the resource ratio is srongly biased, the contrarians of the same proportion break the balance instead.

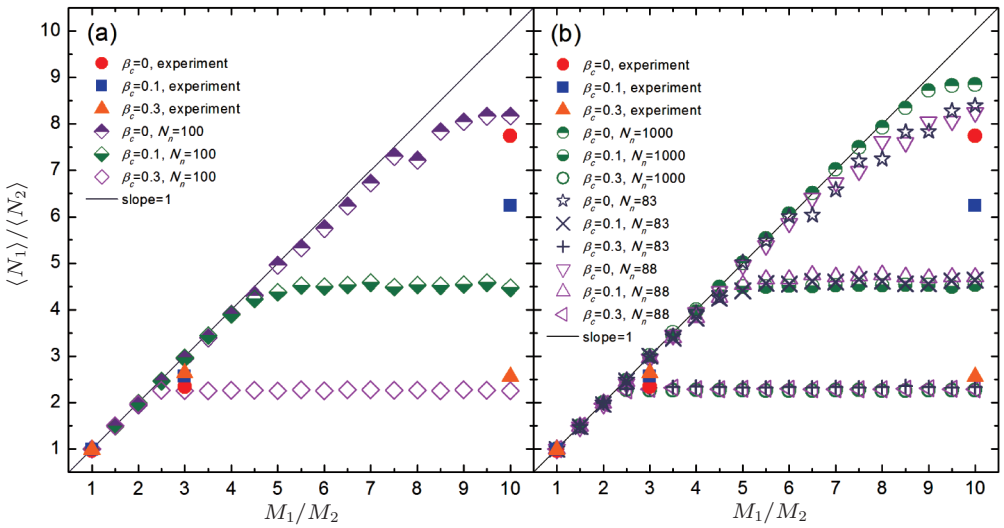


Fig. 11. Population ratio $\langle N_1\rangle/\langle N_2\rangle$, as a function of resource ratio M_1/M_2. The line with slope $= 1$ indicates the balance state where $\langle N_1\rangle/\langle N_2\rangle = M_1/M_2$. Each experiment lasts for 30 rounds (the first 6 rounds for equilibration and the last 24 rounds for statistics). The simulations are run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration). In panel (a), the number of normal agents in simulations is 100. In panel (b), the numbers of normal agents are respectively 1000, 83, and 88. Here $\langle\cdots\rangle$ denotes the average over the last 24 rounds for the experiments or the last 200 time steps for the simulations. The three experimental data at $M_1/M_2 = 1$ are overlapped. Parameters for the simulations are $S = 8$ and $P = 64$.[53]

Then, we analyze the experimental results from both individual and overall aspects of human subjects' preference. The statistical results are shown in Fig. 12. The preferences of the subjects are different albeit of the unbiased distribution of the two resources, $M_1/M_2 = 1$. Such heterogeneity of preferences remains after introducing contrarians in Figs. 12(b) and 12(c). As for the larger resource ratios in Figs. 12(d)–12(f) and 12(g)–12(i), the subjects still have different preferences. However, the average preference of all the subjects varies with M_1/M_2, which illustrates the environmental adaptability of the subjects.

Next, in order to clearly observe the influence of the contrarians on the macroscopic system, we also calculate the above-mentioned stability of the system. Figure 13(a) displays that, for a small M_1/M_2, the fluctuations of the system decrease after introducing the contrarians. Namely, the system becomes more stable. However, for a large M_1/M_2, adding the contrarians makes the system more unstable. Thus, we generally conclude that M_1/M_2 has a threshold, which distinguishes the different role of contrarians in the stability of the system. To further evaluate the performance of the overall system, we have also calculated the efficiency and the predictability of the resource-allocation system. Figure 13(b) shows the change of e when adding contrarians into the experiment. When M_1/M_2 is 1 or 3, the adding of contrarians makes the resource-allocation system more efficient. However, for $M_1/M_2 = 10$, the presence of contrarians reduces the efficiency. Figure 13(c) shows the predictability of the system, which is represented by the winning rate of room 1, w_1. According to Fig. 13(c), when $M_1/M_2 = 1$, the winning rate w_1 fluctuates around 0.5, which means that it is hard to do the prediction. However, if M_1/M_2 becomes larger, the subjects easily predict the winning room for the next round, especially when enough contrarians are added.

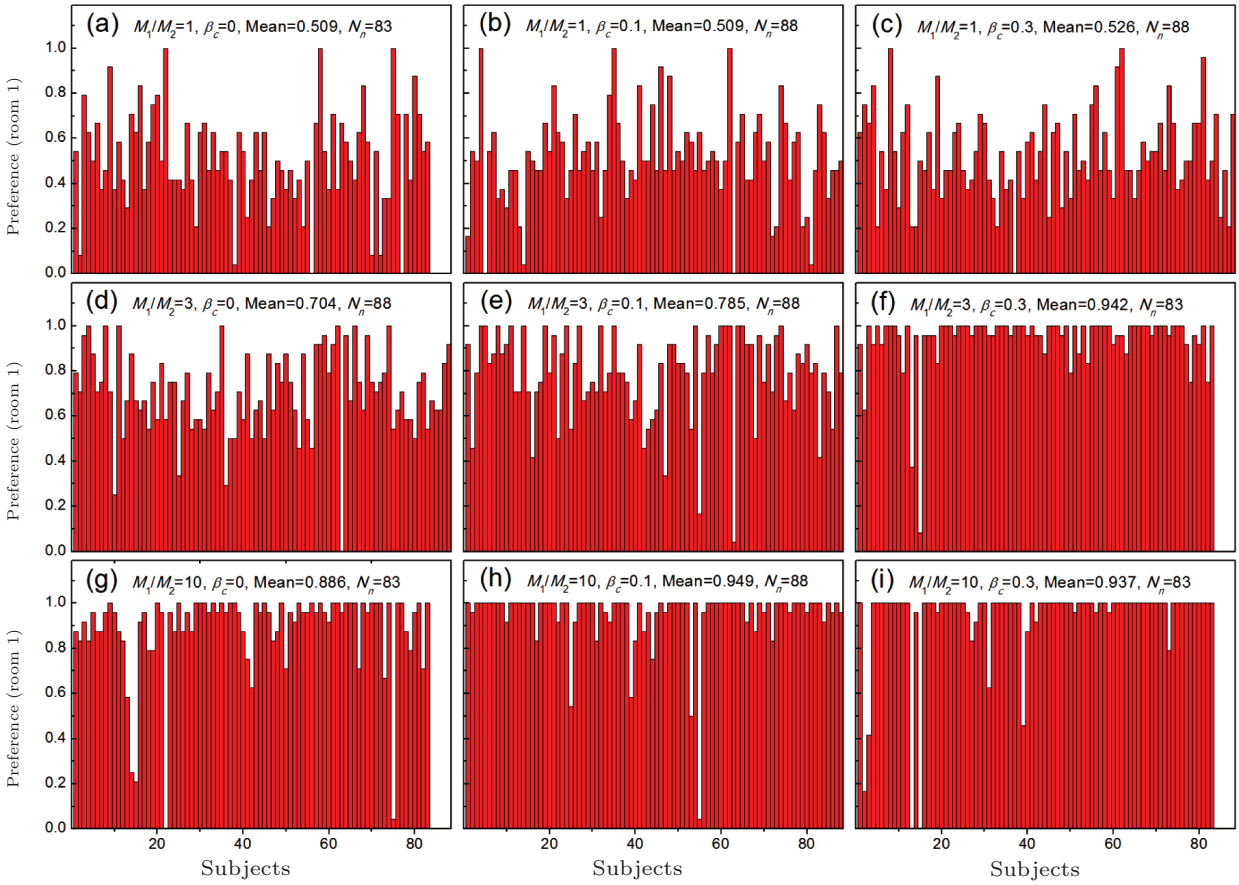


Fig. 12. (a)–(i) Experimental data of the preference of each subject to room 1 for the nine parameter sets with different M_1/M_2 and different β_c. For each parameter set, the experiment lasts for 30 rounds (the first 6 rounds for equilibration and the last 24 rounds for statistics). In the figure, the mean denotes the average preference of all the subjects.[53]

Then we extend the experimental results using agent-based simulations. For the computer simulations, we use 100 normal agents and set $S = 8$ and $P = 64$. The result of $\langle N_1 \rangle / \langle N_2 \rangle$ versus M_1/M_2 is shown in Fig. 11(a). Clearly, a qualitative agreement between experiments and simulations is displayed. In order to confirm this result, we conduct more simulations with different numbers of normal agents to compare with the experimental results, which are shown in

Fig. 11(b). We choose to use 83 and 88 normal agents, which are consistent with the experiments, and 1000 normal agents, which represent the case of a remarkably different size. Comparing the different simulations in Figs. 11(a) and 11(b), the results show no qualitative differences though the number of normal agents varies. Therefore, we can conclude that the number of agents has no influence on our simulation results. This means that the experimental results reported in Fig. 11 are general. For convenience, we use 100 normal agents in the remainder.

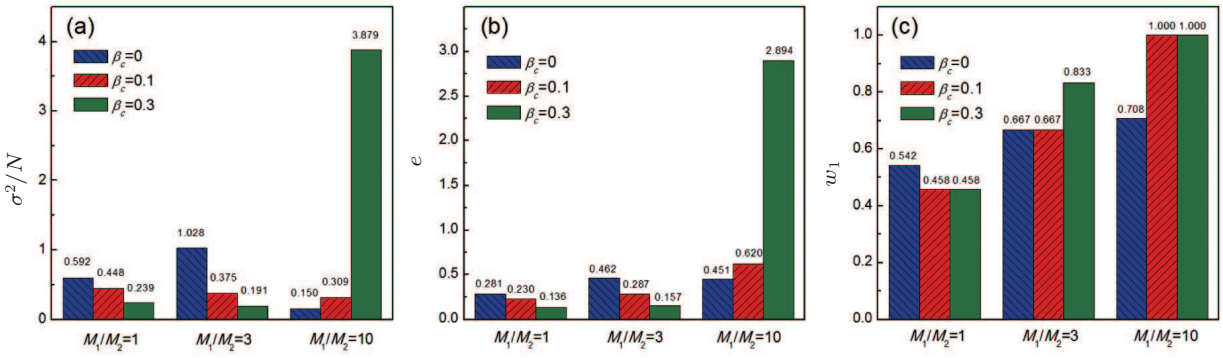


Fig. 13. Experimental data for (a) stability σ^2/N, (b) efficiency e, and (c) predictability w_1 at $M_1/M_2 = 1$, 3, and 10. Each experiment lasts for 30 rounds (the first 6 rounds for equilibration and the last 24 rounds for statistics).[53]

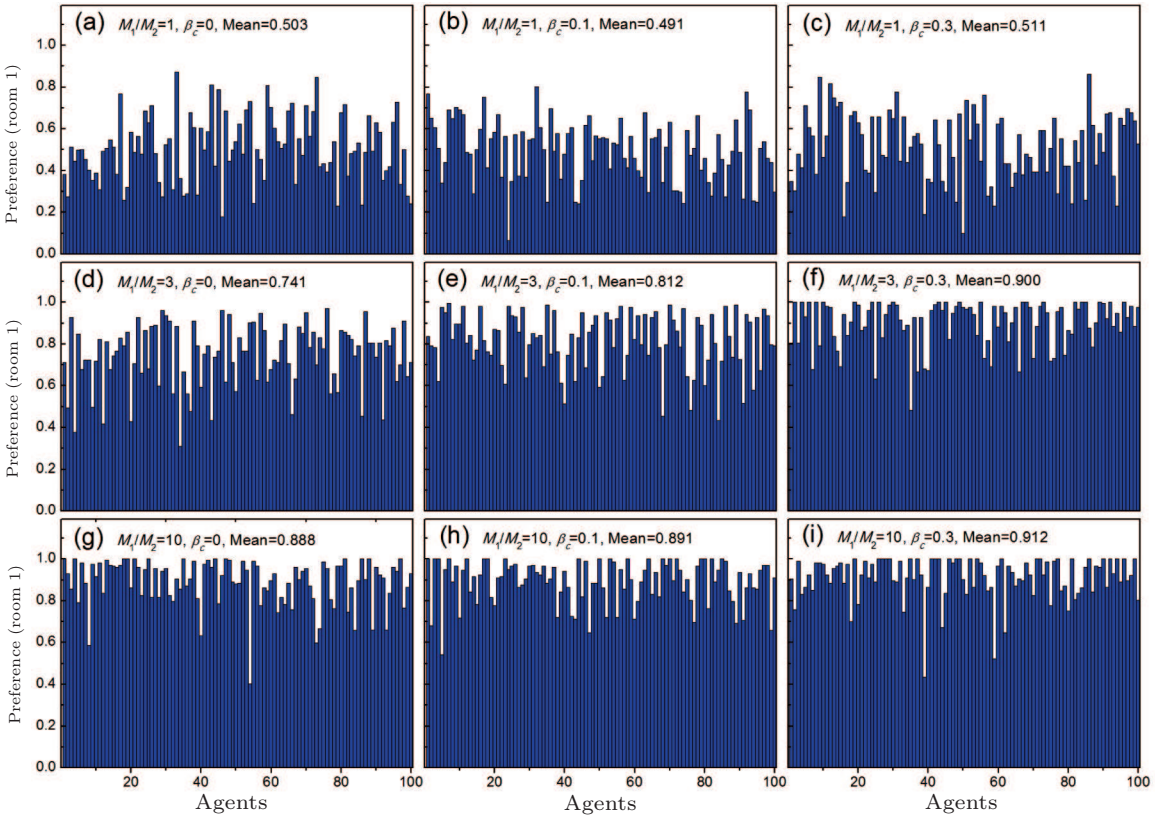


Fig. 14. (a)–(i) Simulation data of the preference of each normal agent to room 1 for nine parameter sets with different M_1/M_2 and different β_c. For each parameter set, the simulations run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration). In the figure, the mean denotes the average preference of the 100 normal agents.[53]

In order to compare with the experiment, the preferences of 100 normal agents are also calculated; see Fig. 14. The simulation results are very similar to the experimental results in Fig. 12. Then we are in a position to scrutinize the role of contrarians. To compare with the experimental result, we also calculate stability, efficiency, and predictability; see Fig. 15. In detail, for a small β_c, increasing M_1/M_2 will increase the system stability until the fluctuation has the minimum value, which corresponds to the most stable state of the system. Once the minimum is passed, the stability of the system will worsen

for larger M_1/M_2. The former (or the latter) is the positive (or negative) role of the contrarians. As for a large β_c, increasing M_1/M_2 will always make the system more unstable (negative role). Figure 15(b) shows the simulation results for the change of system efficiency, e. When M_1/M_2 is small, increasing the contrarians can make the system more efficient at a certain range. In contrast, for a large M_1/M_2, adding the contrarians always reduces the efficiency. Such simulation results echo those experimental results as shown in Fig. 13(b). Figure 15(c) displays the predictability of room 1. Similarly, we can see from Fig. 15(c) that when M_1/M_2 is very small (close to 1), the winning rates of the two rooms remain almost unchanged at 0.5 or so, even though β_c varies. That is, in this case, the system is unpredictable. When M_1/M_2 is gradually increased, adding more contrarians will cause w_1 to increase from the value for $\beta_c = 0$; namely, it becomes more easy for the agents to predict the winning room. Again, these simulation results agree with the experimental results in Fig. 13(c).

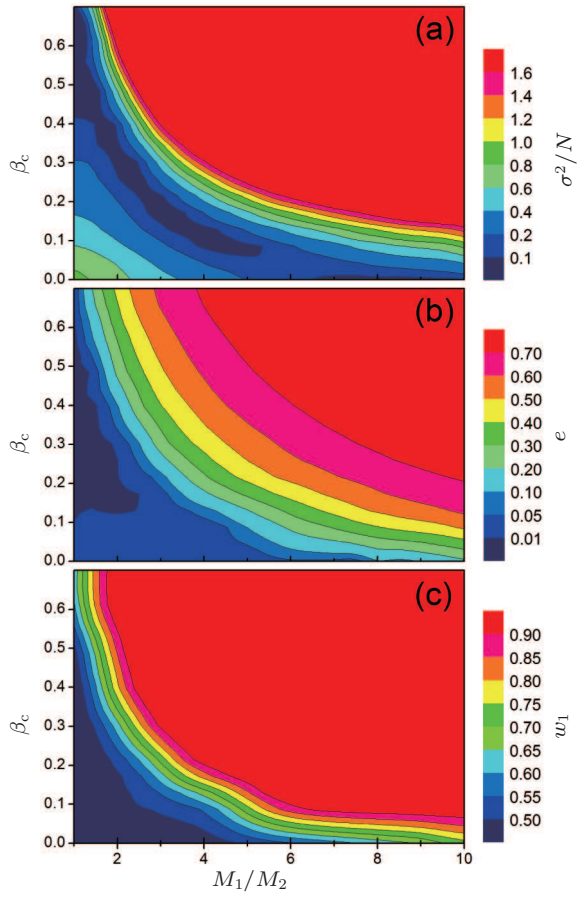


Fig. 15. The β_c–M_1/M_2 contour plots for (a) stability σ^2/N , (b) efficiency e, and (c) predictability w_1. For each parameter set, the simulations run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration).[53]

Now, we can understand the role of contrarians in the resource-allocation system. In contrast to the common belief that the contrarian behavior always plays a positive role in the resource allocation (say, stabilizes the resource allocation by shrinking the redundancy or the lack of resources), on one hand, the contrarians have positive roles when M_1/M_2 is small. Namely, adding contrarians can help to not only improve the system stability, but also increase the system efficiency while keeping the system unpredictable. On the other hand, the contrarians have negative roles when M_1/M_2 becomes large enough. That is, adding contrarians can hurt the system stability and efficiency while making the system more predictable.

3.4. Hedge behavior

In the real market, the behaviors are likely to be more complicated. Considering the opposite effects of herd behavior and contrarian behavior on the resource allocation system from unbiased to biased distributions, we attempt to treat these two behaviors as a kind of hedge behavior when the two behaviors exist in a complex adaptive system simultaneously. The hedge behavior is also a kind of self-organization in complex adaptive systems. The analysis of the effects of the hedge behavior on the whole system will be introduced in Section 4.

4. Human counterparts of some laws or phenomena in statistical physics

4.1. Two-phase phenomena and phase transitions

4.1.1. Phase transition with normal behavior

To confirm the results obtained from the controlled experiments, we have conducted a large number of agent-based simulations.[50,51] As a result, we have found the variation of the system behavior along with the change of environmental complexity M_1/M_2. In Fig. 8, the stability of the system presents a minimum from the unbiased distribution to a highly biased distribution. In order to clarify this performance, we draw the stability and the efficiency with only normal agents in Fig. 16. As shown in Fig. 16(a), the system changes from an efficient state into an inefficient state at some critical value $(M_1/M_2)_c \sim S$. For other values of P, the system behavior stays the same as long as P is larger than M_1/M_2. Meanwhile in Fig. 16(b), around the same critical value of M_1/M_2, σ^2/N changes from a decreasing function to an increasing function, giving the smallest fluctuation in the population distribution at the critical point, suggesting that a phase transition, named the M_1/M_2 phase transition, occurs at this critical point. To be more illustrative, when the environmental complexity is much smaller than the critical value, the system could reside in an efficient but relatively unstable state. Getting closer to the phase transition point, the stability of the system will be improved until the most stable state is reached. Then after crossing the critical point, the decision-making capability of the whole system has been exhausted and it will fall into an inefficient and unstable state. At the vicinity of the critical point, as if the subjects of the game worry about being eliminated from the competition, the market inspires all of its guiding potential and

leads the system to the ideal state for the resource allocation, a state which is both efficient and stable and where no unfair arbitrage chance can exist.

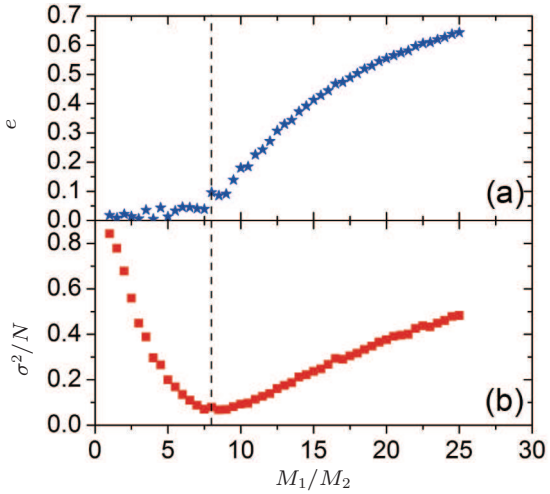


Fig. 16. The M_1/M_2 phase transition in the MDRAG, for $N = 100$, $P = 64$, and $S = 8$. The simulations run 300 times, each over 400 time steps (the first half for equilibration, the remaining half for statistics). The dashed line denotes $M_1/M_2 = 8$: (a) e and (b) σ^2/N as a function of M_1/M_2.[50]

4.1.2. Critical point movement driven by herd behavior

Back to Fig. 8, comparing the system behaviors for the cases of $\beta = 0$ and $\beta \neq 0$, the M_1/M_2 phase transition also indicates the change of role for the herd behavior, namely, from a ruinous herd into a helpful herd. It is clear that the critical point of the M_1/M_2 phase transition becomes larger when the number of imitating agents increases. Denoted as $(M_1/M_2)_c$ hereafter, the critical point refers to the M_1/M_2 value where the minimum of σ^2/N is achieved. To further understand the underlying mechanism for these phenomena, we conduct a theoretical analysis by deriving the critical point $(M_1/M_2)_c$ for the M_1/M_2 phase transition as identified in the agent-based simulations.

It is reasonable to assume that, if P is not too small, the right column of a strategy filled in by 1 with probability L/P is equal to the one filled in 1 with the number of L. Hence strategies with the same preference number L can be regarded as the same. It is worth noting that if the situations vary in a random manner, the probability is L/P for a normal agent to choose room 1 using a strategy with preference number L. Next, we assume that the preference number of the best strategy held by normal agent i at time T is L_i. Denote the choice of room as x_i so that $x_i = 1$ if room 1 is chosen and $x_i = 0$ otherwise. At the same time, let imitating agent j choose to follow the normal agent μ, the best agent (who has the highest score) in a group of size k ($1 \leq k \leq N_n$). For the imitating agent, its choice of room is $y_j = x_\mu$, and its preference number becomes $L_j = L_\mu$. With these definitions, the total number of agents in room 1 at time T can be written as

$$N_1 = \sum_{i=1}^{N_n} x_i + \sum_{i=1}^{N_m} y_j. \tag{1}$$

It is obvious that $\langle x_i \rangle = L_i/P$, which can be used to respectively derive the expectation and the variance of the population in room 1 as

$$\langle N_1 \rangle = \frac{1}{P}\left(\sum_{i=1}^{N_n} L_i + \sum_{i=1}^{N_m} L_j\right), \tag{2}$$

$$\sigma_{N_1}^2 = \sum_{i=1}^{N_n} \sigma_{x_i}^2 + \sum_{j=1}^{N_m} \sigma_{y_j}^2 + \sum_{i=1}^{N_n}\sum_{j=1}^{N_m} (\langle x_i y_j \rangle - \langle x_i \rangle \langle y_j \rangle) + \sum_{p,q=1, p\neq q}^{N_m} (\langle y_p y_q \rangle - \langle y_p \rangle \langle y_q \rangle). \tag{3}$$

Owing to the specific method for the construction of strategies in the resource-allocation model, the covariance between the choices of different normal agents can be neglected. On the right-hand side of Eq. (3), the third item is the correlation between the choices of the normal agents and those of the imitating agents who follow them. The fourth item is the correlation between the choices of different imitating agents who follow the same normal agent. Both terms should always be positive, which means that adding the imitating agents could cause large fluctuations (volatility) in the resource-allocation system. It should be emphasized here that the stability defined herein is different from the traditional definition of variance. The former characterizes both the deviation and the fluctuation to the idealized room population in the balanced state, while the latter only represents the fluctuation to the mean value of the time series. When the resource distribution is comparable ($M_1/M_2 \approx 1$), since the normal agents are able to produce the idealized population or $\langle N_1 \rangle / \langle N_2 \rangle \approx M_1/M_2$, these two kinds of definitions are approximately equal. This explains why the stability can be destroyed when the imitating agents are involved in situations with a nearly unbiased resource distribution. However, when the system environment becomes difficult for the normal agents to adapt to, the difference between the variance and the stability cannot be neglected. If no imitating agents are involved, the normal agents alone cannot make the system reach the balanced state. In that case, even if the fluctuation of N_1/N_2 to its average value could be made small, the deviation to the idealized population ratio can still be very large. This would make the system suffer from a higher dissipation. If an appropriate portion of imitating agents is added, the deviation of N_1/N_2 to the idealized room population diminishes, leaving only some fluctuations around M_1/M_2, which could result in a reduction of waste in the resource allocation.

Then, we study the performance of different strategies (namely, the strategies with different preference numbers). We also consider the condition of $M_1/M_2 \geq 1$. Assume that at time

T, the winning rate of room 1 is $\alpha(T)$. The expectation of the increment of score for the strategy with preference number L should be $1-L/P+(2L/P-1)\alpha(T)$. Then the expectation of the cumulative score for this strategy from $t=1$ to $t=T$ can be expressed as

$$f(L,T)=\left(1-\frac{L}{P}\right)T+\left(\frac{2L}{P}-1\right)\sum_{t=1}^{T}\alpha(t). \tag{4}$$

From this expression, we can calculate the dependence of the cumulative score on the preference number

$$\frac{\Delta f}{\Delta L}=\frac{2}{P}\sum_{t=1}^{T}[\alpha(t)-0.5]. \tag{5}$$

It is easy to derive from Eq. (3) that if $\sum_{t=1}^{T}[\alpha(t)-0.5]>0$, f should be a monotonically increasing function with L. Now we assume that $\alpha(T)-0.5$ is always positive, which is not too stringent a condition as long as M_1 is large enough. As the system evolves under this assumption, the gap among different strategies of different preference numbers will become larger and larger. Eventually, the best performed strategy owned by a normal agent will be the one with the largest L in its strategy book. As a consequence, the imitating agents will choose to follow those who own the strategy with the largest preference number $L_{\max}$. From Eq. (2), it is obvious that $\langle N_1\rangle$ will also reach its maximum value $\langle N_1\rangle_{\max}$ when both L_i and L_j reach their maximum values. With this maximum value of the expected population in room 1, we can propose the following two conditions:

If $\langle N_1\rangle_{\max}<\frac{M_1}{M_1+M_2}N$, the system can never reach the balanced state.

If $\langle N_1\rangle_{\max}>\frac{M_1}{M_1+M_2}N$, the system can fluctuate around the balanced state.

Denoting the population ratio as $\langle R_1\rangle=\langle N_1\rangle/N$, we need to calculate $\langle R_1\rangle_{\max}=\langle N_1\rangle_{\max}/N$ to evaluate the conditions above. As the normal agents construct their strategies in a random way, a strategy with an arbitrary preference number may be picked up with a uniform probability $1/(P+1)$. Thus, among the S strategies of a normal agent, the probability to have $L_{\max}=\tilde{L}$ is $p(\tilde{L})=\left(\frac{\tilde{L}+1}{P+1}\right)^S-\left(\frac{\tilde{L}}{P+1}\right)^S$. Since an imitating agent chooses the best normal agent among the k group members, the probability to have $(L_{\max})_{ks}=\tilde{L}$ should be $p'(\tilde{L})=\left(\frac{\tilde{L}+1}{P+1}\right)^{kS}-\left(\frac{\tilde{L}}{P+1}\right)^{kS}$. With these probabilities, we obtain the population ratio as

$$\begin{aligned}\langle R_1\rangle &= \frac{1}{NP}\left(\sum_{i=1}^{N_n}L_i+\sum_{j=1}^{N_m}L_j\right)\\ &= \frac{1}{NP}\left[N_n\sum_{\tilde{L}=1}^{P}\tilde{L}p(\tilde{L})+N_m\sum_{\tilde{L}=1}^{P}\tilde{L}p'(\tilde{L})\right]\\ &= 1-\frac{1}{(\beta+1)P}\sum_{\tilde{L}=1}^{P}\left[\left(\frac{\tilde{L}}{P+1}\right)^S+\beta\left(\frac{\tilde{L}}{P+1}\right)^{kS}\right],\end{aligned}$$

where $\tilde{L}$ stands for the preference of a normal agent's strategy. If $\langle R_1\rangle_{\max}=M_1/(M_1+M_2)$, the system can fluctuate around the balanced state. Otherwise, the system can never reach the balanced state. Then some insightful comments can be added.

1) The state of the resource-allocation system depends only on M_1/M_2, β, k, P, and S. It has no concern with N_n or N_m.

2) An optimized value of β may be calculated by setting $\langle R_1\rangle_{\max}=M_1/(M_1+M_2)$, which could make the system most stable. After substituting this expression into the equation for $\langle R_1\rangle_{\max}$, we can obtain the numerical solutions for the critical points $(M_1/M_2)_c$ of the phase transitions. Figure 17 shows a good agreement between the simulation results and those of the theoretical derivation for the critical points.

3) It is easy to prove that $\partial\langle R_1\rangle_{\max}/\partial\beta>0$, which means that β and $\langle R_1\rangle_{\max}$ are positively related. When $\beta\to\infty$, the population ratio will converge to $\langle R_1\rangle_{\max}\to 1-\frac{1}{P}\sum_{\tilde{L}=1}^{P}\left(\frac{\tilde{L}}{P+1}\right)^{kS}$. At this limit, the model suggested here will be equivalent to the original resource-allocation model without the imitating agents,[50] except that in this case, each agent would have kS (instead of S) strategies.

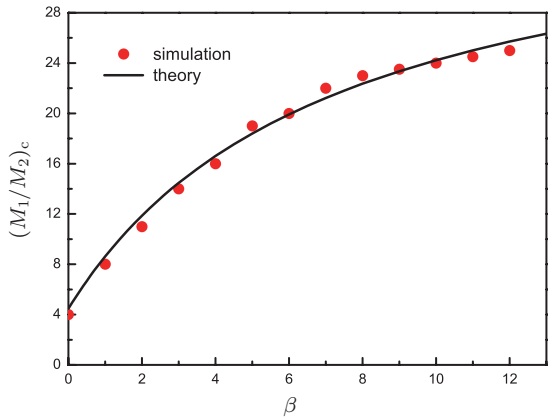


Fig. 17. Critical points of the M_1/M_2 phase transition, $(M_1/M_2)_c$, varying with different population ratio β: simulation results (symbols) versus theoretical results (line). The simulation results are obtained from the data in Figs. 10(a) and 10(c).[51]

4.1.3. Critical point movement driven by contrarian behavior

For the resource-allocation system with the contrarian behavior, from Fig. 15, we find that the system also clearly exhibits a critical point when taking M_1/M_2 and w_1 as the tuning parameter and the order parameter, respectively. For a small β_c, increasing M_1/M_2 will increase the system stability until the stability has the minimum value at $(M_1/M_2)_c$. Once the minimum value is passed, the stability of the system will worsen for larger M_1/M_2. As for a large β_c, increasing M_1/M_2 will always make the system more unstable. In addition, as β_c increases, $(M_1/M_2)_c$ moves along the direction of decreasing M_1/M_2. We will discuss the movement of $(M_1/M_2)_c$ in the following theoretical analysis. When S and P are fixed,

the system of our interest could reach the most stable state only at the transition point, i.e., a particular ratio between the two resources, $(M_1/M_2)_c$. If we adjust β_c, the transition point $(M_1/M_2)_c$ will change accordingly.

4.1.3.1. The properties of the transition point $(M_1/M_2)_c$

Without contrarians It can be proven that for the agent-based model, the transition point has two properties: 1) every normal agent uses the strategy with the largest preference $(L_i)_{\max}$ in his/her hand; 2) the system is in the balance state, which means that the ratio between the numbers of agents in the two rooms is equal to that between the two resources. We first define

$$N_1 = \sum x_i,$$

where the choice of agent i is denoted as $x_i = 1$ (room 1) or 0 (room 2). Then, at the transition point, the expected ratio of the normal agents who choose to enter room 1 is

$$\frac{\langle N_1\rangle}{N_n} = \frac{\sum\langle x_i\rangle}{N_n} = \frac{\sum_i^{N_n}(L_i)_{\max}}{PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_c, \quad (6)$$

where $\langle\cdots\rangle$ denotes the averaged value. Equation (6) shows that when $\frac{M_1}{M_1+M_2} > \left(\frac{M_1}{M_1+M_2}\right)_c$, room 1 will become unsaturated. This means that the system does not stay at the balance state.

With contrarians From the properties of the transition point and the behavior of the contrarians, it can be shown that, all the normal agents still use the largest-preference strategy $(L_i)_{\max}$ at the transition point when the contrarians are added. Every contrarian follows the minority in his/her group to make a choice denoted as x_c. Then the expected ratio of agents (both normal agents and contrarians) who choose to enter room 1 at the transition point becomes

$$\frac{\langle N_1\rangle}{N} = \frac{\sum_i^{N_n}(L_i)_{\max} + P\sum_c^{N_c}\langle x_c\rangle}{(1+\beta_c)PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_{c'}, \quad (7)$$

where $\beta_c = N_c/N_n$, and $\left(\frac{M_1}{M_1+M_2}\right)_{c'}$ stands for the new transition point with the contrarians added.

4.1.3.2. Finding the expressions of $\sum_i^{N_n}(L_i)_{\max}$ and $\sum_c^{N_c}\langle x_c\rangle$

Without contrarians The probability that L_i takes a certain integer from the range 0 to P is $1/(P+1)$. Then, the probability of $(L_i)_{\max}$ being a certain value L is

$$p(L) = \left(\frac{L+1}{P+1}\right)^S - \left(\frac{L}{P+1}\right)^S.$$

If N_n is large enough, we have

$$\sum_i^{N_n}(L_i)_{\max} = \sum_{L=0}^{P} N_n p(L) L = PN_n\left[1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S\right]. \quad (8)$$

In the absence of contrarians, the substitution of Eq. (8) into Eq. (6) leads to

$$\frac{\langle N_1\rangle}{N_n} = 1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_c \equiv m_n, \quad (9)$$

where m_n represents the transition point for the system with only normal agents.

With contrarians Since the normal agents still use their strategy with $(L_i)_{\max}$ at the transition point after adding contrarians into the resource-allocation system, therefore, for the normal agents, we have

$$\frac{\langle N_{n1}\rangle}{N_n} = 1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n.$$

When contrarian c chooses k normal agents as his/her group, the probability to get a normal agent who chooses room 1 can be expressed approximately as $\langle N_{n1}\rangle/N_n = (M_1/M_1+M_2)_c \equiv m_n$. Then, the probability for $x_c = 1$ (or 0) is $\sum_{q=0}^{y} C_k^q [(M_1/M_1+M_2)_c]^q \left[\left(\frac{M_2}{M_1+M_2}\right)_c\right]^{k-q} \equiv m_c$ (or $1-m_c$), where $\left(\frac{M_2}{M_1+M_2}\right)_c = 1-m_n$, $y = (k-1)/2$, and k is odd. Thus, we have the average of x_c as

$$\langle x_c\rangle = m_c = \sum_{q=0}^{y} C_k^q (m_n)^q (1-m_n)^{k-q}. \quad (10)$$

Substituting Eq. (10) into Eq. (7) yields

$$\frac{\langle N_1\rangle}{N} = \frac{PN_n m_n + PN_c m_c}{(1+\beta_c)PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_{t'},$$

and then we have

$$\frac{\langle N_1\rangle}{N} = \frac{m_n + \beta_c m_c}{1+\beta_c} = \left(\frac{M_1}{M_1+M_2}\right)_{c'}. \quad (11)$$

Clearly, by adjusting β_c, we can change the transition point of the resource-allocation system. Figure 18 shows the monotonically decreasing trend of $(M_1/M_2)_c$ for increasing β_c, which displays an excellent agreement between theoretical and simulation results.

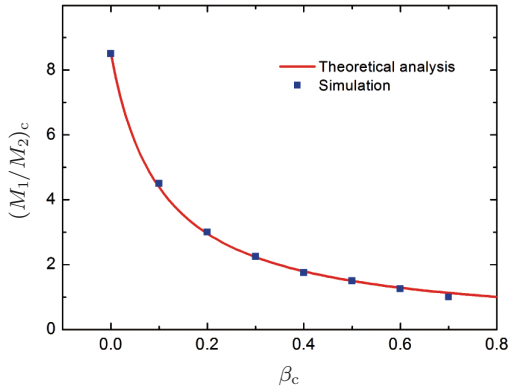


Fig. 18. Critical point $(M_1/M_2)_c$ versus β_c as a result of theoretical analysis and simulation. Parameters are $S = 8$ and $P = 64$.[53]

In both experiments and computer simulations, we have found that when the system is in the balance state ($M_1/M_2 < (M_1/M_2)_c$), the fluctuations of the system decrease after introducing a small number of contrarians. Because in both experiments and simulations, the behavior of the contrarians is set to follow the same rule, it is necessary to further analyze the influence of this behavior on the stability of the whole system. Equation (10) describes the probability of the contrarians choosing to enter room 1 when the system reaches the balance. It is known that at this balanced state, the number of subjects in the experiments (or the normal agents in the simulations) choosing to enter each room still varies at every time step due to fluctuations. Hence we replace m_n in Eq. (10) with N_{n1}/N_n and obtain $\langle x_c \rangle = \sum_{q=0}^{y} C_k^q (N_{n1}/N_n)^q (1 - N_{n1}/N_n)^{k-q}$, where N_{n1} is the number of subjects or normal agents who choose to enter room 1, and the average of x_c, $\langle x_c \rangle$, represents the expected probability of the contrarians choosing room 1. Note that $\langle x_c \rangle$ is a random variable due to the fluctuations of N_{n1}. Then, according to Eq. (11), we have

$$\frac{N_1}{N} = \frac{N_{n1}/N_n + \beta_c \langle x_c \rangle}{1 + \beta_c}. \tag{12}$$

By drawing N_1/N versus N_{n1}/N_n, we obtain Fig. 19, which shows the influence of the deviations of N_{n1}/N_n on N_1/N under different values of β_c. For $M_1/M_2 = 1$, it is shown that the balance point of the system lies at A_0 (0.5,0.5) when $\beta_c = 0, 0.1, 0.3, 0.5$. The deviations of N_{n1}/N_n can cause the system to vibrate around A_0 along a certain line in Fig. 19, which is determined by β_c. Figure 19 also shows that, under the same range of deviations of N_{n1}/N_n, by increasing β_c, we can bring down the vibration of N_1/N around $N_1/N = 0.5$. In addition, we can see from Fig. 19 that, when β_c becomes too large, such as $\beta_c = 1$ or 2, A_0 is no longer a stable point. The state of the system tends to move to the right end of the associated line because now more subjects or normal agents choosing to enter room 1 will make room 1 easier to win. That is, when β_c is too large, adding more contrarians will lead the system to a more unstable state. For a biased distribution of resources, say, $M_1/M_2 = 3$, figure 19 shows that the balance point of the system lies at different points for different values of β_c, i.e., B_0 (0.75,0.75), B_1 (0.82,0.75), and B_2 (0.97,0.75) for $\beta_c = 0$, 0.1, and 0.3, respectively. It can be shown that adding a small number of contrarians makes the system with a biased distribution of resources more stable due to the following two reasons: 1) under the same deviations of N_{n1}/N_n, the vibration of N_1/N (say, around B_0, B_1, or B_2 for $M_1/M_2 = 3$) decreases slightly when more contrarians are added; 2) with more contrarians, the values of N_{n1}/N_n at the balance points (e. g., B_0, B_1, and B_2 for $M_1/M_2 = 3$) increase; in this case, the subjects or normal agents will be more certain to choose room 1, which reduces the deviation range of N_{n1}/N_n, thus decreasing the vibration of N_1/N.

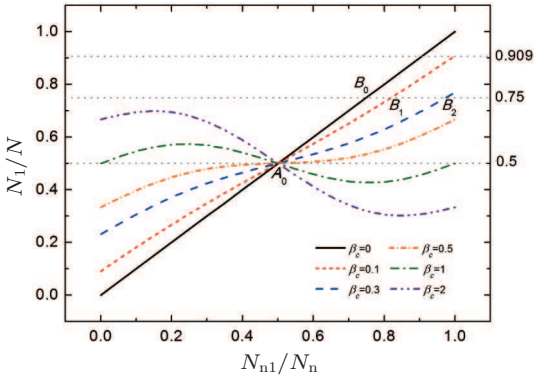


Fig. 19. The N_1/N versus N_{n1}/N_n according to Eq. (12). The three horizontal gray dotted lines are given by $N_1/N = 0.5$, 0.75, and 0.909, which are respectively related to the balance states of three resource ratios, $M_1/M_2 = 1$, 3, and 10.[53]

Comparing our work with the contrarian study by Galam,[133] which also showed the transition point at a critical value of the contrarian proportion to identify opinion group forming, here the transition points in this work help us to reveal that the allocation of resources can be optimized at the transition point by adding an appropriate size of contrarians.

4.1.4. Critical point robustness under hedge behavior

Here we introduce the hedge behavior into the MDRAG to analyze the effects of the hedge behavior on the resource allocation system's fluctuation.[57] In our agent-based model, there are N_n normal agents, N_m imitators, and N_c contrarians. We define $\beta_1 = N_m/N_n$ and $\beta_2 = N_c/N_n$. The total number of all agents is $N = N_n + N_m + N_c = N_1 + N_2$. Based on this model, computer simulations are carried out with 100 normal agents and the simulation parameters of $S = 11$ and $P = 121$. In the process, we change the parameters including the ratio of resource distribution between the two rooms and the ratio of contrarians and imitators. A total of 400 time steps of simulation is performed under each parameter set. In order to scrutinize the effect of the hedge behavior on the whole resource allocation system, we calculate the stability of the systems; see Fig. 20. The systems reach the most stable state at $M_1/M_2 = 11$ without imitators and contrarians. In this case, the critical point is $(M_1/M_2)_c = 11$. The stability of the system changes when the herd behavior is introduced. Likewise, the contrarian behavior can also affect the system's fluctuations. However, if we introduce herd and contrarian behaviors simultaneously, we find that adjusting simultaneously the strengths of the two different behaviors can make the critical point unchanged.

From Fig. 20, we infer that for any strength of one behavior, a proper hedge proportion exists to make the critical point of the system robust. By adjusting the hedge proportion, the resource allocation system could reach the most stable state in any satisfied resource ratio. It is very helpful to optimize the resource allocation under any resource distributions. There-

fore, we carried out more comprehensive simulations. The simulation results are shown in Fig. 21(a).

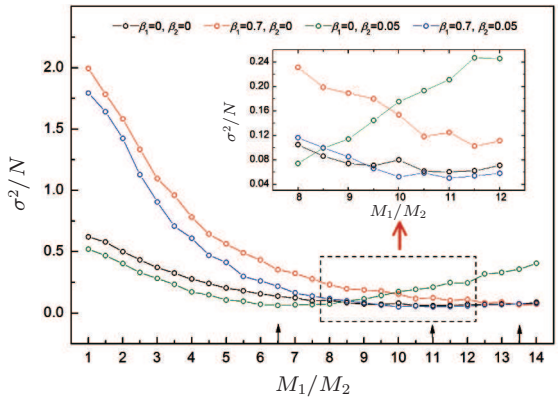


Fig. 20. Simulation results of σ^2/N for different M_1/M_2 from 1 to 14 with normal agents (black), herd behavior (red), contrarian behavior (green), and hedge behavior (blue). The circles in the dashed frame are magnified above the red arrow because the lapped circles are hard to distinguish. Three black arrows on the horizontal axis indicate the four critical points of the most stable states of the four curves. The arrow at $M_1/M_2 = 6.5$ indicates the most stable state of the green curve, while the arrow at $M_1/M_2 = 13.5$ indicates the most stable state of the red curve and the arrow at $M_1/M_2 = 11$ indicates the most stable state of the black and the blue curves. Note that the two critical points of the most stable states of the black and the blue curves are well overlapped, and thus denoted by one arrow only, which clearly show that a suitable hedge behavior could let the system (with contrarian behavior and herd behavior) have the same most stable state as the original system (without contrarian behavior and herd behavior). For each parameter set, the simulations run 100 times, each over 400 time steps (the last 200 time steps for statistics).[57]

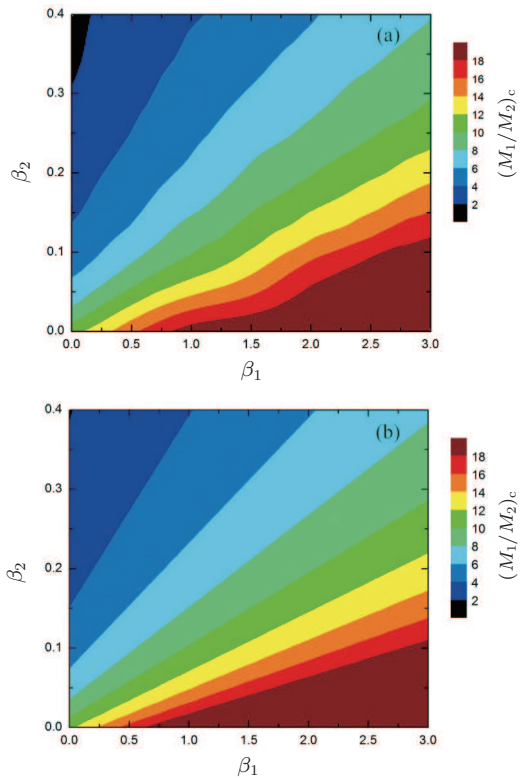


Fig. 21. Critical point $(M_1/M_2)_c$ as functions of β_1 and β_2 as a result of (a) the computer simulations with 100 normal agents and (b) the theoretical analysis. For each parameter set, the simulations run 100 times, each over 400 time steps (the last 200 time steps for statistics).[57]

In Fig. 21(a), each color represents the most stable equilibrium state (i.e., the critical point $(M_1/M_2)_c$ corresponds to the minimum value of σ^2/N) in a certain combination of β_1 and β_2. We can see that, for the case without imitators and contrarians, the system can reach the equilibrium at critical point $(M_1/M_2)_c = 11$. The herd behavior makes the critical point shift to larger M_1/M_2, while the contrarian behavior makes the critical point shift to smaller M_1/M_2, which means only herd behavior or contrarian behavior always change the fluctuation of the system. However, if one adjusts the two behaviors simultaneously, then this kind of hedge behavior can make the critical point robust no matter how many agents exist. Besides, the system can achieve the optimal state with any resource ratio by adjusting β_1 and β_2.

Such an adjustment of the hedge behavior can surely provide a reference to complex adaptive systems. We find that the critical point of the system is robust and independent of the total number of agents composed of normal agents, imitators, and contrarians under some conditions. The resource allocation system could also be controlled to reach the most stable state (critical point) in any resource bias through adjusting the proportion of the two kinds of agents (β_1 and β_2).

Further understanding of critical point $(M_1/M_2)_c$ versus β_1 and β_2 is necessary for understanding the microscopic mechanism in the system. For this purpose, we also conduct a theoretical analysis following the above analysis. Then we obtain the formula

$$\frac{\langle N_1 \rangle}{N} = \frac{m_n + \beta_1(1-m_c) + \beta_2 m_c}{1+\beta_1+\beta_2} = \left(\frac{M_1}{M_1+M_2}\right)_{t'}.$$

By adjusting β_1 and β_2 , we could obtain a robust critical point of the system. Besides, the system could reach the most stable state under any resource ratio M_1/M_2. Figure 21(b) shows $(M_1/M_2)_c$ versus β_1 and β_2 as a result of the theoretical analysis with the same parameters in the simulation, which almost meets the simulation results shown in Fig. 21(a).

4.2. Entropy-related phenomena

4.2.1. Shannon information entropy

In 1948, Shannon established the foundations of information theory by suggesting a mathematical measure of information on the basis of information entropy.[142] Later it was developed by Brillouin in 1956[143] and applied to statistical mechanics by Jaynes in 1957.[144] A comprehensive modern viewpoint on the Shannon information entropy and its connection to statistical thermodynamics can be found in Chapter 20 of Carter's book.[145]

We have indicated that the heterogeneous preference is an indispensable factor to balance the above system and firstly employed the preference analysis of normal agents in Section 3.2 for studying herd behavior. In order to study the

normal agents' preferences and their estimation of the system, Shannon information entropy[142,146] may be introduced to our agent-based modeling. The information entropy S_I of a discrete random variable X with possible values $\{x_1,\ldots,x_n\}$ is defined as $S_I(X) = -\sum_{i=1}^{n} P(x_i)\ln P(x_i)$, in which $P(x_i)$ denotes the probability mass function of x_i. In the agent-based model, the information entropy for a normal agent is $S_{Ii} = -L_i/P\ln L_i/P - P - L_i/p\ln(P-L_i)P$, where L_i stands for the preference of the current strategy. If the normal agent would choose the two rooms with an equal probability, this information entropy could reach the maximum value of $\ln 2$. On the other hand, the information entropy S_{Ij} for imitating agent j will be the same as that of the normal agent he/she followed in the local group. Thus the averaged information entropy of all the agents (i.e., normal agents and imitating agents) can be calculated as

$$S_I = \frac{1}{N}\left(\sum_{i=1}^{N_n} S_{Ii} + \sum_{j=1}^{N_m} S_{Ij}\right),$$

and the results are shown in Fig. 22(a). The averaged information entropy decreases as M_1/M_2 becomes larger, a clear-cut average preference of the agents emerges as the distribution of resources becomes more biased. This agrees with the analysis of participants' preferences in the human experiments; see Fig. 7. Furthermore, the information content of agent i can be defined as $I_i = (\ln 2 - S_{Ii})/\ln 2$. Note that a larger I_i indicates that the agent has more confidence in a certain room. The averaged information content for all the normal agents (I_n) and the imitating agents (I_m) are shown in Fig. 22(b). In this figure, I_n decreases with the increase of the population of imitating agents when M_1/M_2 is small. This means that the normal agents can be confused by the actions of the imitating agents in a rather uniform distribution of the resource. When M_1/M_2 becomes larger, I_n is nearly a constant, implying that the imitating agents will no longer affect the estimation of the normal agents. All of these go well with the analysis of the experimental results in Fig. 7. The averaged information content of the imitating agents has a rather drastic change as the environment varies. When $M_1/M_2 = 1$, I_m is pretty low, even lower than that of the normal agents, a fact indicating that the imitating agents have almost unbiased preferences when the resource distribution is uniform. As M_1/M_2 increases, the imitating agents are apt to flood into a specific room and thus form a herd in the modeled system.

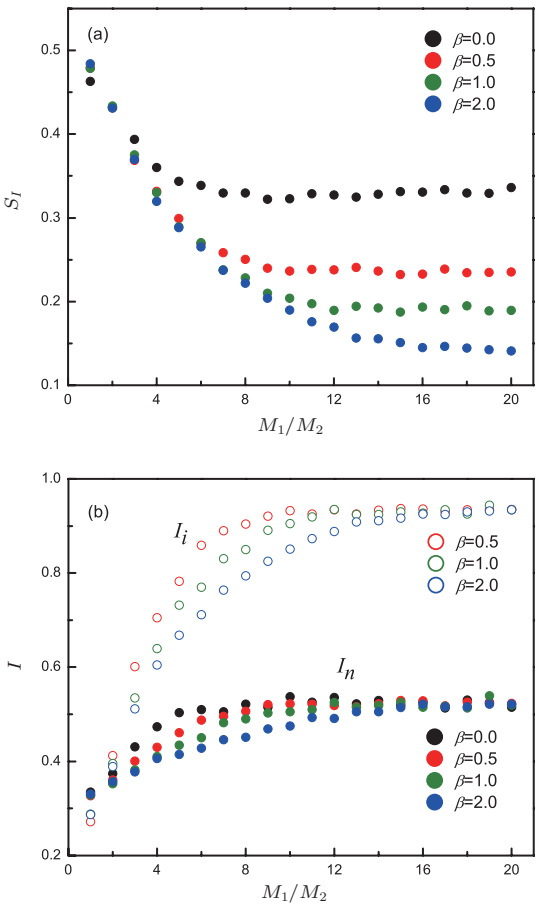


Fig. 22. The changes of (a) the averaged information entropy (S_I) for all the agents including the normal agents and the imitating agents and (b) the averaged information content for the normal agents (I_n) and the imitating agents (I_m). The simulations run 200 times, each over 400 time steps (the first half for equilibration, the remaining half for statistics). Parameters are $N_n = 50$, $S = 4$, $P = 16$, and $k = 5$.[51]

4.2.2. Entropy-like quantity describing the degree of uncertainty

Except the information entropy focusing on the preference analysis, researchers defined another type of entropy to study the evolution process of the whole resource allocation system, inspired by the principle of increasing entropy in thermodynamics systems.[145,147] It is called an entropy-like quantity; the quantity can be used to describe the degree of uncertainty as shown below, but essentially differs from the thermodynamic entropy.

It is well known that the principle of increasing entropy[145,147] holds for isolated natural systems that contain non-adaptive molecules. It may be expressed in the way that the thermodynamic entropy (s_t) of an isolated natural system should gradually increase until it reaches equilibrium with the maximum thermodynamic entropy. It is well justified that the principle of increasing entropy holds for isolated natural systems that involve non-adaptive molecules. This is because the molecules have higher probabilities to change from a regular motion to a thermal one according to the theory of molecular motion. It is Boltzmann who bridges s_t and the thermodynamic probability (characterized by the number of microstates, Ω) as

$$s_t = k_B \ln \Omega,$$

where k_B denotes the Boltzmann constant.

Apparently, s_t is by no means valid only for natural systems with physical interactions among molecules because Ω can also be used to represent the number of microstates in social systems having interactions that differ from those among molecules. However, it is difficult to find the human counterpart in social systems. The challenge lies at least in two aspects: 1) many social systems are open systems that exchange information with external environments, thus violating the conservation of energy required by the principle of increasing entropy; 2) social systems contain adaptive humans with many kinds of human behaviors, which implies that the interactions among humans are quite different from those among molecules in natural systems. So far, to our knowledge, direct experimental evidences have never been shown for the human counterpart, despite many attempts to apply the thermodynamic laws in social systems.[148,149] Here we report one possible experimental evidence in a social human system. In the previous resource allocation systems, the entropy, which is presented with the detailed historical information, indicates the relation between information and predictability, whereas in our system, the entropy-like quantity is quite different, which denotes the degree of uncertainty associated with the macroscopic arbitrage opportunities. Accordingly, our results cannot be obtained from the previous systems including the minority game systems as well as their superb extensions by other scholars.[150] To model the abundant human behaviors in real social systems, we take into account the four kinds of specific behaviors, namely, normal behavior, herd behavior, contrarian behavior, and hedge behavior.

To measure the degree of uncertainty associated with the macroscopic arbitrage opportunities in our human system, we follow the definition of Shannon entropy in the information theory, and define the entropy-like quantity (s),[54,67] a state function of our system, as

$$s = -(f_1 \ln f_1 + f_2 \ln f_2),$$

where $f_1 = \frac{\langle N_1\rangle/M_1}{\langle N_1\rangle/M_1+\langle N_2\rangle/M_2}$ and $f_2 = 1 - f_1$. Here $\langle N_1\rangle$ ($\langle N_2\rangle$) represents the round average of the numbers of the subjects that choose to enter room 1 (room 2). As a feature of our system with repeated actions, averaging more rounds makes the resource allocation more efficient because of the increasing degree of human adaptability. We define the state of a macro system as the macro state and take $M_1/M_2 = 3$ as an example:

1) $M_1/M_2 = 3:1$ (play 1 round) $\rightarrow$ macro-state 1,
2) $M_1/M_2 = 3:1$ (play 2 rounds) $\rightarrow$ macro-state 2,
3) $M_1/M_2 = 3:1$ (play 3 rounds) $\rightarrow$ macro-state 3,
4) $M_1/M_2 = 3:1$ (play 4 rounds) $\rightarrow$ macro-state 4,
$\cdots$

As time goes on, when our system shifts from macro-state 1 to macro-state 2 and then to macro-state 3, etc, the experimental results indicate that the efficiency of resource allocation increases, which is a unique feature in repeated-decision human experiments and provides a more effective pattern for the real resource allocation. In other words, in view of the particularity of this system, we need to define the state of the system as a round average form, which is slightly different from the definition of state in thermodynamics systems. Accordingly, f_1 and f_2 denote the probabilities of two microstates (in a state of the system characterized by s) due to the existence of two rooms because $\langle N_1\rangle/M_1$ ($\langle N_2\rangle/M_2$) represents the averaged number of subjects per amount of resources in room 1 (room 2). It is worth noting that, when the number of rounds is large enough, $\langle N_1\rangle/M_1 = \langle N_2\rangle/M_2$ (or $f_1 = f_2 = 1/2$ herein) can appear as a special emergent result of the current system, e. g., for $M_1/M_2 = 3$ and $\beta_1 = \beta_2 = 0$. In this case, the whole system reaches equilibrium or balance and s has the maximum value of $s_{\max} = 0.693$. Generally speaking, the macro state of the system with $s_{\max} = 0.693$ corresponds to the situation in which the macroscopic arbitrage opportunities are statistically exhausted in the whole system due to $\langle N_1\rangle/M_1 = \langle N_2\rangle/M_2$. Or, equivalently, the degree of uncertainty associated with the macroscopic arbitrage opportunities reaches the maximum as $s_{\max} = 0.693$. In addition, the first experimental round always tends to yield $N_1 = N_2$ (say, for $M_1/M_2 = 3$ and $\beta_1 = \beta_2 = 0$) because human adaptability has not started to play a role due to the lack of chances for learning. As a result, we have $\langle N_1\rangle/M_1 \neq \langle N_2\rangle/M_2$ or $f_1 \neq f_2$ when $M_1/M_2 > 1$. In this case, the macroscopic arbitrage opportunities are evidently the largest, and the degree of uncertainty associated with them is the lowest accordingly. According to the definition of the entropy-like quantity, s is naturally smaller than 0.693 due to $f_1 \neq f_2$. Hence, we may conclude that the defined s describes the degree of uncertainty associated with the macroscopic arbitrage opportunities as time elapses.

Figure 23(a) shows the experimental results for the time evolution of entropy-like quantity s from the 1st macro state (s_{1-1}) to the 30th macro state (s_{1-30}). We can see that for $M_1/M_2 = 3$, s gradually increases to its maximum no matter what kinds of human behaviors are considered. However, one may argue whether this trend is generally true because it is only the result of one repeated experiment (with 30 rounds). Frankly speaking, to make the results more solid, we should, in principle, follow either of the following two ways. The first way is to let the same group perform the repeated experiment for R ($R \rightarrow \infty$) times, each yielding two values of N_1 and N_2 at the j-th round. As a result, the desired value of the entropy-like quantity, $s^{(1)}$, is calculated according to the definition where $\langle N_1\rangle$ or $\langle N_2\rangle$ should further be averaged over R times. The second way is to let Q ($Q \rightarrow \infty$) groups conduct the repeated experiment separately, each also offering two values of N_1 and N_2 at the j-th round. Similarly,

the desired value of the entropy-like quantity, $s^{(2)}$, is calculated where $\langle N_1\rangle$ or $\langle N_2\rangle$ should further be averaged over Q groups. However, the two ways are practically impossible. The first way easily makes the subjects feel boring, thus causing data distortion, and the second way costs too much (say, human resources and money). Fortunately, we have an alternative method to show the generality of the increasing trend as already depicted in Fig. 23(a). Let us take the case of the normal behavior ($\beta_1 = 0$ and $\beta_2 = 0$) as an example. At the first round of the repeated experiment, statistically speaking, subjects can only make decisions randomly, or, the probability for them to choose to enter Room 1 (Room 2) is 50% (50%) in spite of $M_1/M_2 = 3$. This is because these subjects have no information on how to choose the room or they have not yet had the chance to learn. In this case, human adaptability does not play a role in s_{1-1}. After taking the above-mentioned averages over N_1's and N_2's for either R times or Q groups (if any), there must exist $s^{(1)}|_{1\text{st round}} = s^{(2)}|_{1\text{st round}} = 0.562$ where $\langle N_1\rangle/\langle N_2\rangle = 1$ and $M_1/M_2 = 3$. On the other hand, if the number of rounds is infinite, there is $s^{(1)}|_\infty = s^{(2)}|_\infty = 0.693 \equiv s_{\max}$ according to the definition where $\langle N_1\rangle/\langle N_2\rangle = M_1/M_2 = 3$ due to the full role of human adaptability. In other words, the single curve for $\beta_1 = \beta_2 = 0$ in Fig. 23(a) stands for one realistic trajectory from $s^{(1)}|_{1\text{st round}} = s^{(2)}|_{1\text{st round}} = 0.562$ to $s^{(1)}|_\infty = s^{(2)}|_\infty = 0.693$. This trajectory overall increases, as we expected. The arguments hold the same for the other three cases (herd behavior, contrarian behavior, and hedge behavior) as shown in Fig. 23(a). Thus, we may conclude that the increasing trend displayed in Fig. 23(a) is a general one. To further understand its mechanism, we suggest playing a coin game as a non-adaptive counterpart to compare with the human experiments. For this purpose, we design a coin game program (Fig. 24) in which all the subjects are imagined to make decisions according to tossing a coin.

The 50% on the toss panel shown in Fig. 24 means that either the head (H) or the tail (T) of the coin has a probability of 50% to appear after running the program. If H (T) means requesting a subject to choose to enter room 1 (room 2), the model series of HHHTHTTHHTTTHT as a result of 14 rounds shown on the panel let the subject choose to enter room 1 or room 2 accordingly.

Because the coin game has no human adaptability at all, the probability for the head (or the tail) of the coin to appear is set to be 50% in view of the existence of two rooms in our human experiment. As a result, each subject has the probability of 50% to enter room 1 or room 2 although $M_1/M_2 = 3$ and four different kinds of human behaviors have been adopted in our experiment. Eventually, there must be $s^{(1)}|_{1\text{st round}} = s^{(2)}|_{1\text{st round}} = s^{(1)}|_\infty = s^{(2)}|_\infty = 0.562$ for the coin game due to $\langle N_1\rangle/\langle N_2\rangle = 1$ and $M_1/M_2 = 3$. The entropy-like quantity corresponding to the coin game is shown in Fig. 23(a). So far, it becomes convinced that the trend of entropy-like quantity s for different behaviors (Fig. 23(a)) generally increases as time evolves, which originates from human adaptability. This increasing trend evidently echoes with the statement of the principle of increasing entropy. Nevertheless, we also observe that the increasing trend of s in Fig. 23(a) is not always monotonic. This can be qualitatively understood in view of the fluctuation theorem.[151] The theorem[151] conveys the fact that the thermodynamic entropy has a probability to flow in a direction opposite to that dictated by the principle of increasing entropy. This theorem may also have a counterpart in our system, as implied by Fig. 23(a).

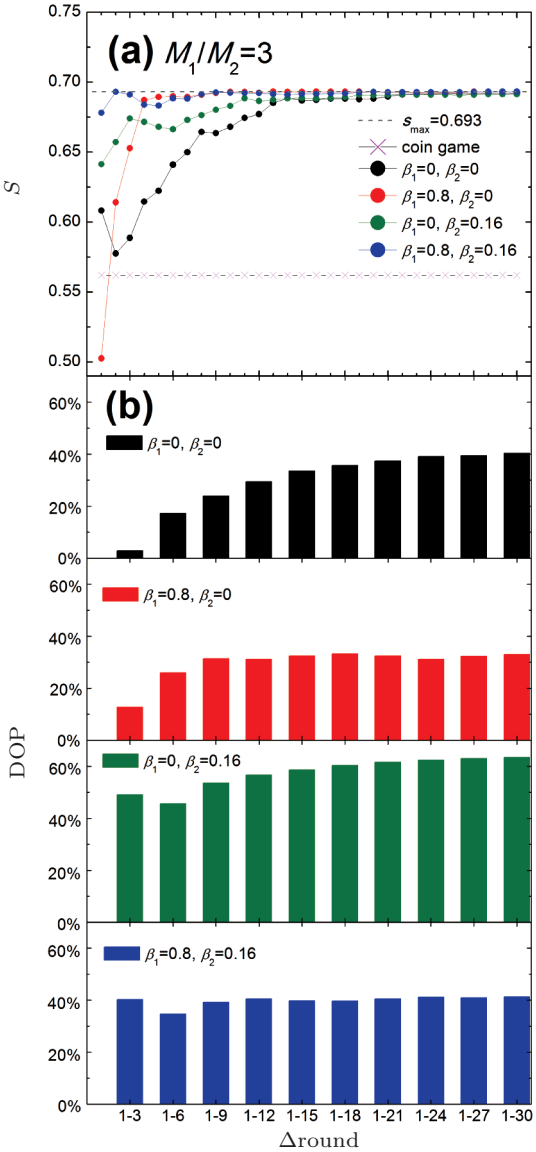


Fig. 23. Experimental results of (a) entropy-like quantity s and (b) degree of preferences for $M_1/M_2 = 3$ with normal behavior ($\beta_1 = 0$ and $\beta_2 = 0$), herd behavior ($\beta_1 = 0.8$ and $\beta_2 = 0$), contrarian behavior ($\beta_1 = 0$ and $\beta_2 = 0.16$), and hedge behavior ($\beta_1 = 0.8$ and $\beta_2 = 0.16$) as a function of round intervals, Δround, which characterize the passage of time. For each parameter set (M_1/M_2, β_1, and β_2), human experiments with 68 subjects are repeatedly conducted for 30 rounds. The dashed line in panel (a) denotes the maximum value of the entropy-like quantity, $s_{\max} = 0.693$. The exact result of the coin game that has no human adaptability is also depicted in panel (a).[67]



Fig. 24. Working panel for tossing a coin.[67]

So far, we have investigated the case of a biased distribution of resources ($M_1/M_2 = 3$). In fact, we can also study the unbiased distribution of resources ($M_1/M_2 = 1$) on the same footing, which is exactly the original minority game system. Nevertheless, it can be readily concluded that this case ($M_1/M_2 = 1$) also yields $s^{(1)}|_{1\text{st round}} = s^{(2)}|_{1\text{st round}} = s^{(1)}|_{\infty} = s^{(2)}|_{\infty} = 0.693 \equiv s_{\max}$ for the normal behavior. This is because, for $M_1/M_2 = 1$, the subjects choose to enter room 1 (or room 2) with a probability of 50%. In this case, the entropy-like quantity stays not only unchanged but also maximum ($s_{\max}$) as time evolves. Similar conclusions are also found for the other three cases (herd behavior, contrarian behavior, and hedge behavior). In other words, the original minority game system has no counterpart of the principle of increasing entropy.

The above analysis has qualitatively shown the influence of human adaptability. To quantitatively understand the presence of human adaptability, we need to define another quantity, namely, the average degree of preferences (DOP), $\text{DOP} = |1-2\rho| \times 100\%$, where ρ denotes the mean preference to room 1 of all the 68 subjects. Thus, the DOP means whether the subjects have preferences to choose to enter one of the two rooms. For example, DOP= 0% means that the subjects have no preference to a certain room, or their chance to choose room 1 equals that to choose room 2. The DOP= 100% means that the subjects completely prefer one of the two rooms. Figure 23(b) shows two evidences for the presence of human adaptability. One is that the DOP has quite different frameworks when different behaviors are adopted. The other is that, for a specific behavior, the DOP changes as time evolves. The two evidences just result from the evolution of heterogeneous preferences of individual subjects according to the definition of DOP.

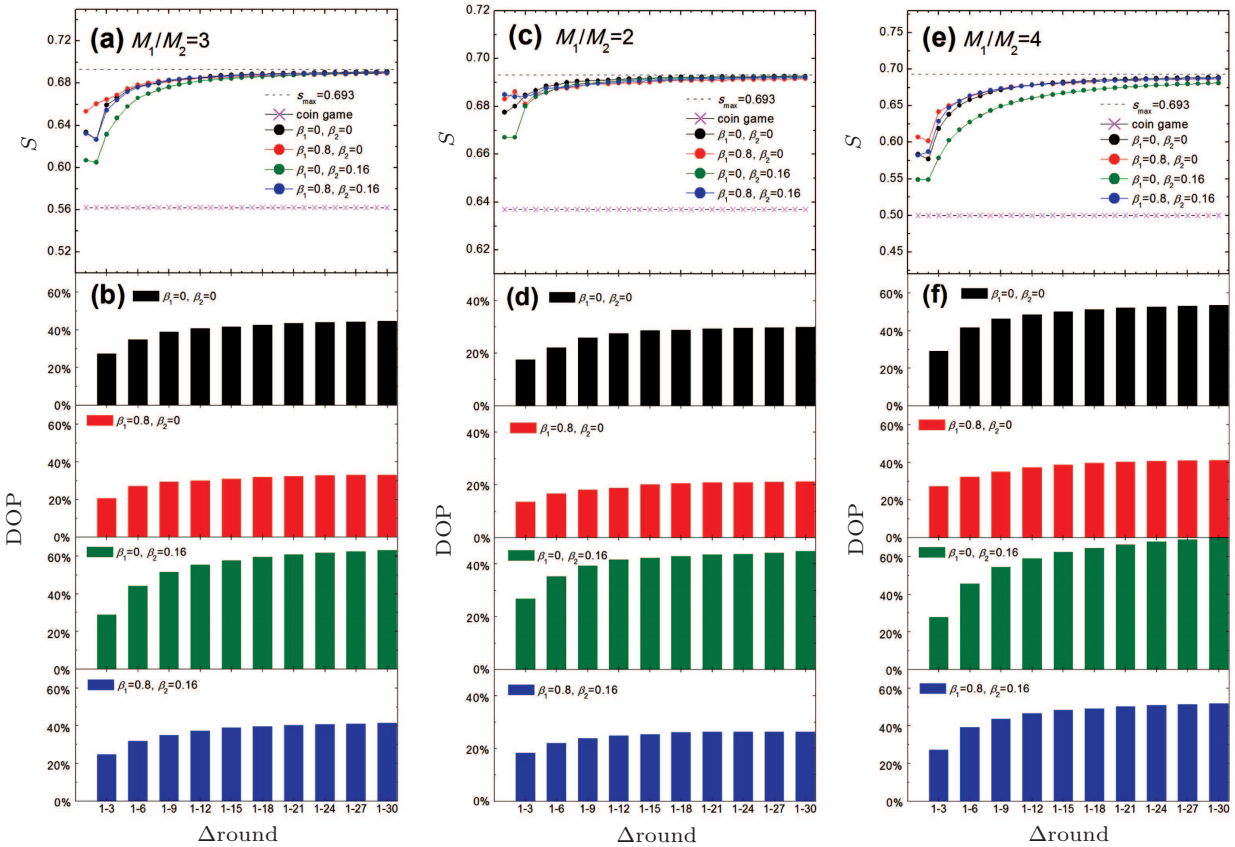


Fig. 25. Entropy-like quantity s and degree of preferences for (a), (b) $M_1/M_2 = 3$; (c), (d) $M_1/M_2 = 2$, and (e), (f) $M_1/M_2 = 4$ obtained from the agent-based simulations. For each parameter set (M_1/M_2, β_1, and β_2), the simulations with 68 normal agents run for 30 time steps, in order to echo with the human experiments (68 subjects and 30 rounds). Parameters are $S = 10$ and $P = 100$. Both the maximum entropy-like quantity line ($s_{\max} = 0.693$) and the coin game result are also depicted in panels (a), (c), and (e).[67]

Then, we conduct agent-based simulations to verify and extend the experimental results. Figures 25(a) and 25(b) show the simulation results for $M_1/M_2 = 3$. Evidently, the entropy-like quantity also gradually increases up to its maximum no matter what kinds of behaviors are taken into consideration, which echoes with Fig. 23(a). Meanwhile, the DOP in Fig. 25(b) also varies accordingly, and has a framework similar to that in Fig. 23(b). Thus, figures 25(a) and 25(b) not only confirm the above experimental findings, but also extend them to a more general case beyond the experimental restrictions on the specific subjects, time, and avenue. Moreover, besides $M_1/M_2 = 3$, figures 25(c)–25(f) also show the simulation results for $M_1/M_2 = 2$ and $M_1/M_2 = 4$, in order to extend the case of a specific biased distribution of resources ($M_1/M_2 = 3$) to more relevant systems. According to the analysis in the previous section, for $M_1/M_2 = 2$ (or $M_1/M_2 = 4$), we have $s^{(1)}|_{1\text{st round}} = s^{(2)}|_{1\text{st round}} = 0.637$ (or 0.500) and $s^{(1)}|_{\infty} = s^{(2)}|_{\infty} = 0.693 \equiv s_{\max}$. All the results obtained from Figs. 23(a) and 23(b) and Figs. 25(a) and 25(b) qualitatively hold for Figs. 25(c)–25(f) as expected.

4.3. Non-equilibrium steady state

The above analysis concerned only human behaviors without networks. However, the real cases may be diversified. Let us imagine that there are many strangers in a place. As time elapses, due to communications, small communities or clusters (each having a small number of strangers) of the strangers are naturally formed in which they get to know each other. Eventually, all of them in the whole place know each other as a result of various kinds of human activities like introductions, communications, etc. Such a process is called snowballing behavior, which apparently arises from human adaptability. Snowballing behavior is an intrinsic property due to the adaptability of human beings, and it actually describes the dynamic evolution of interactions in the system from no interactions (no one knows each other), to local interactions (a few of them know each other), and to global interactions (all of them know each other). As an initial work on the effect of snowballing behavior, for convenience, Zheng *et al.* investigated how three separate sizes of clusters (small, middle, and large) would influence the macro state of the system; this is called the clustering effect. They have also experimentally investigated the social human system with the clustering effect and revealed the trend of the entropy-like quantity based on the MDRAG.[54]

For our purpose, the clustering effect is described by defining a probability, q ($0 \le q \le 1$), for each human to get to know the others. The increase of q can characterize the degree of the clustering effect. In the experiment, firstly, a random network is generated to indicate the relationship between the subjects. Each subject is represented by one node and there is a probability of q ($0 \le q \le 1$) for each node to connect to the others in the network. Therefore, each node and its connected nodes will form its group. However, they know neither the values of M_1 and M_2 nor the numbers and identities of the subjects in their group. The only information feedback to the subject is if she/he earns more resources than her/his group average in the last round. For a given parameter set of q and M_1/M_2, each experiment was performed for 15 rounds. We recruited 25 human subjects. During the experiment, each subject is considered to win or lose according to her/his performance within her/his own group. This should be reasonable because in real life, individuals often make decisions on the basis of their own neighbors or partial information. As a result, the whole society runs healthily or unhealthily. Hence, here we focus on the macroscopic behavior of the whole system and study the evolution of the entropy-like quantity and the DOP.

Figure 26 shows the entropy-like quantity s and the DOP evolve as a function of the experimental period extension. This figure displays three values of q, namely, $q = 0.2$, 0.52, and 0.76, representing the cases of local interactions from lower to higher strengths. Figures 26(a)–26(c) show that the different settings influence both the macroscopic states (denoted by s) and the microscopic behaviors (reflected by the DOP). While the s of all the systems show an increasing trend as the experiment progresses, they (almost) stop increasing when hitting a certain threshold. However, only in the cases of $q = 0.76$ (Fig. 26(c)), which means that the reference groups have spread over most parts of the system (thus having a high strength of local interactions), the entropy-like quantity increases up to the maximum value of $s = 0.693$ no matter what degree of system complexity (indicated by M_1/M_2) the subjects face. When k is smaller ($q = 0.2$ and 0.52), the systems tend to stay at different non-equilibrium steady states (NESSs) (namely, s is almost independent of time, but has a separation from the maximum value of 0.693) except for the relatively simple case of $M_1/M_2 = 3$ in Fig. 26(b). In these states, the macroscopic arbitrage opportunities exist but our subjects are unable to eliminate them because the sizes of the reference groups are small (or, the local interactions among the subjects are relatively weak). On the other hand, the ratio of the resources, M_1/M_2, also contributes to the systems' evolutions to the equilibrium (balance) state and the NESS. In Fig. 26(c), the systems of different M_1/M_2's have different trajectories towards the equilibrium state. In general, the system of a small M_1/M_2 (i.e., a low degree of system complexity) can reach the equilibrium state faster than that of a large M_1/M_2. This means that the subjects can adapt to the system faster for a small M_1/M_2 than that for a large M_1/M_2 by eliminating the macroscopic arbitrage opportunities. In Fig. 26(a), the

three cases of $M_1/M_2 = 3,4,9$ approach to different NESSs with different s's. A similar behavior is found in Fig. 26(b) ($M_1/M_2 = 4,9$). Particularly, in either Fig. 26(a) or Fig. 26(b), while the system of $M_1/M_2 = 4$ can evolve to the NESS that has a small gap with respect to the equilibrium state, there is always a big gap between the NESS of $M_1/M_2 = 9$ and the equilibrium state. This means that the system with a lower degree of system complexity (smaller M_1/M_2) reduces the requirement of the size of the reference group (or the strength of local interactions) to reach the equilibrium.

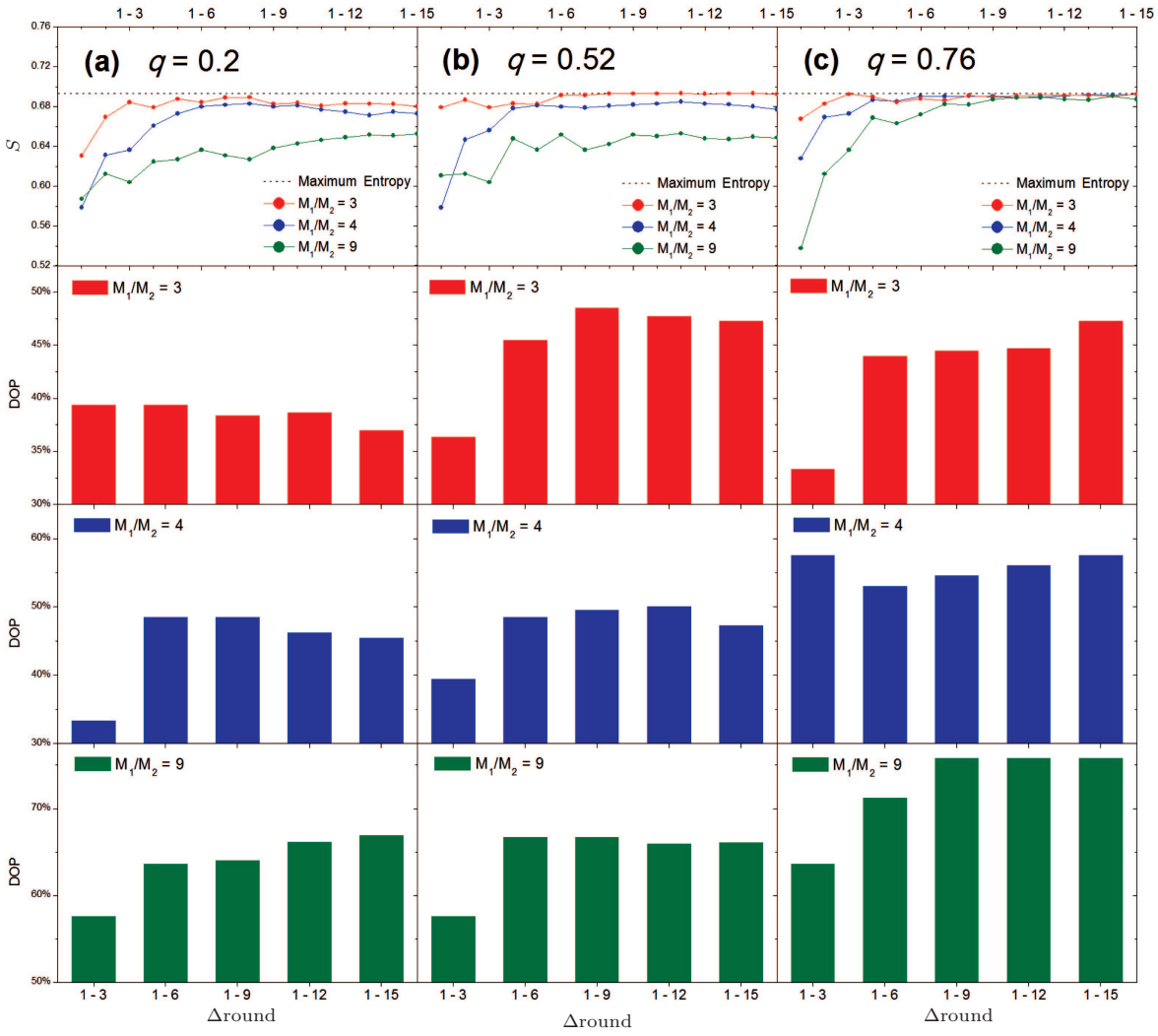


Fig. 26. Experimental results of entropy-like quantity s and DOP for 25 subjects and three M_1/M_2's, as a function of the average rounds (1–3, 1–6, 1–9, 1–12, and 1–15): (a) $q = 0.2$, (b) $q = 0.52$, and (c) $q = 0.76$. For each parameter set of fixed M_1/M_2 and q, the repeated experiment is conducted for 15 rounds; here 1–3, 1–6, 1–9, 1–12, and 1–15 represent the first three, six, nine, twelve, and fifteen rounds, respectively. The horizontal dashed lines denote the equilibrium state with the maximum entropy of $s = 0.693$.[54]

Now let us turn to the DOP displayed in Fig. 26. The preferences evolving over time display how the subjects are trained to adapt the system. We find that the microscopic behaviors of the subjects may provide an explanation for the above-mentioned macro phenomena. Comparing the systems that finally evolve to the equilibrium state ($M_1/M_2 = 3$ in Fig. 26(b) and all M_1/M_2's in Fig. 26(c)) with those that approach to the NESS (all M_1/M_2's in Fig. 26(a) and $M_1/M_2 = 4,9$ in Fig. 26(b)), the subjects in the systems of a larger q generally show a higher degree of preference than those of a lower q when M_1/M_2 is given. That is, the emergence of the adaptability of preferences that matches with the system complexity causes the whole system to reach the equilibrium state and thus to obey the principle of increasing entropy-like quantity. However, when q is relatively small, the weak strength of local interactions inhibits the subjects from developing enough adaptability and then the inadaptability of the subjects leads the whole system to the NESS. Accordingly, the principle of increasing entropy-like quantity becomes invalid. In other words, for a given M_1/M_2, the probability q for each human

to get to know the others has a threshold q_c, above or below which the principle of an increasing entropy-like quantity holds or fails. According to Fig. 26, it is clear that a smaller M_1/M_2 has a smaller q_c. Further, it is easy to conclude that the different degrees of inadaptability make systems of different M_1/M_2's reach different NESSs once q is given.

We also conducted supplementary simulations (Fig. 27). Figure 27 has a framework like Fig. 26. Accordingly, all the qualitative analysis and conclusions (appearing in the experimental results) remain valid for the simulation results.

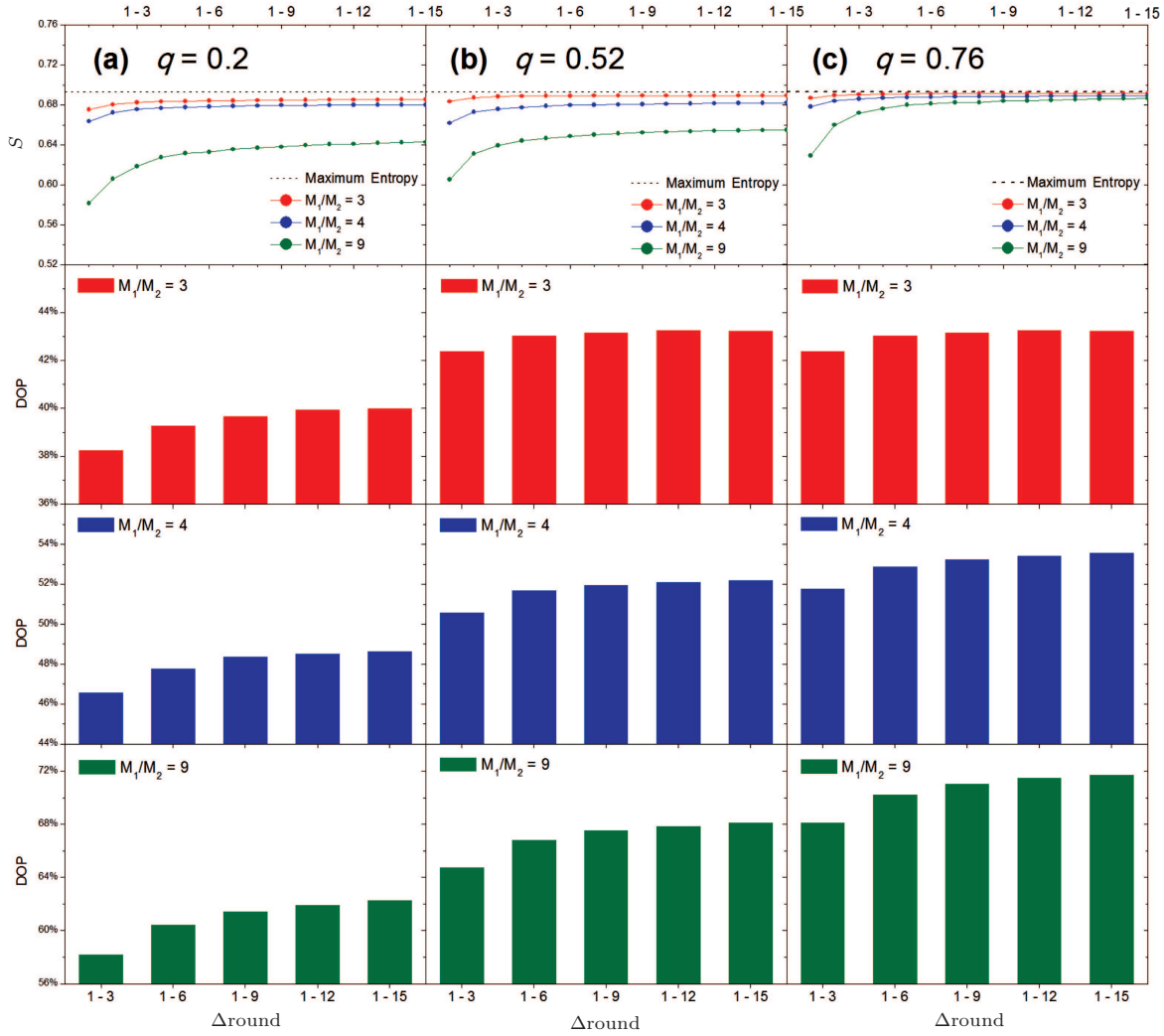


Fig. 27. Simulation results of entropy-like quantity s and DOP for 25 agents and three M_1/M_2's. Others are the same as those in Fig. 26. Parameters are $S = 18$ and $P = 72$.[54]

5. Summary and outlook

5.1. Summary

This review has systematically answered the question: do the traditional laws or phenomena of statistical physics for natural systems with non-adaptive agents still hold for social human systems with adaptive agents? Our answer is yes: under some conditions, some statistical physical laws for natural systems still hold for social human systems although the interaction among the adaptive agents in social human systems (belonging to the family of complex adaptive systems) is essentially different from that among non-adaptive agents in natural systems. To obtain this answer, we have modeled a realistic human social system for resource allocation by designing and conducting a series of controlled experiments under unbiased or biased distributions of resources. Then we clarified the role of human adaptability in four kinds of specific human behaviors, say, normal behavior, herd behavior, contrarian behavior, and hedge behavior. Based on the controlled human experiments with the help of computer simulations and theoretical analyses, we have revealed some human counterparts of the traditional laws or phenomena of statistical physics, namely, two-phase phenomena or phase transitions, entropy-related phenomena, and the non-equilibrium steady state. This review highlights the role of human adaptability

in these human counterparts, and makes it possible to study or understand social human systems by using some traditional statistical physics.

5.2. Outlook

Here we attempt to give three directions for future research (Section 5.2.1). In addition, it is also worth mentioning that modern social psychology might be useful to help develop econophysics on the basis of controlled experiments (section 5.2.2).

5.2.1. Three directions for future research

Regarding the further development of controlled human experiments, here we would like to address the following three directions.

(i) It is necessary to develop controlled experiments to study and predict the movement of real financial/economic markets that are often out-of-equilibrium,[152] which should be helpful for making suggestions for policy makers. For this purpose, it is necessary to take into account the development of complex networks[153–157] that can be used to describe the relations among humans, thus yielding emergent features originating from the interplay between the structure and the corresponding function of markets.

(ii) With the deep development of economic and financial systems in the global scope, economic crises have more catastrophic effects than ever. However, mathematical statistics is unable to dig the essential mechanism underlying the crises. This points to a truth: people urgently need a more efficient method to model the social human systems (say, the resource-allocation systems as discussed in this review). The controlled human experiment, combined with agent-based simulations and theoretical analyses, might be a promising candidate. As a result, statistical physics might play a more important role in crisis prediction, as impled by this review.

(iii) Extending controlled experiments from human beings to other animals like ants[158] or fish[159] could be of value to study statistical physics of other kinds of adaptive agents beyond human beings. By doing so, people could understand those animals more by using concepts or tools originating from the traditional statistical physics.

5.2.2. Inspirations from modern social psychology

Social psychology that started in the early 20th century has experienced vigorous development, and has obtained abundant research methods and achievements over the past one hundred years.[60–65]

Social psychologists are interested in socially psychological phenomena of individuals and groups, including behaviors, thoughts, feelings, attribution, social cognition, the self-concept, group cohesiveness, social facilitation and inhibition, etc. In particular, the content of social psychology has constructed deep and layering concepts for the individual. For example, McAdams divided personality conceptualization into three layers: dispositional traits, characteristic adaptations, and integrative life stories.[64,65,65,66] The first layer means the self as a social actor who corresponds to early childhood and encompasses performance traits and social roles. The second layer corresponds to the self at middle to late childhood who becomes a motivated agent with the main features of personal goals, motives, values, and envisioned projects for the future. After that, the self as an autobiographical author is the third layer which begins in adolescence and emerges in adulthood. In this period, the autobiographical author formulates a live narrative, synthesizing the past and the future to explain the self and then form an integrative life story. In the controlled experiments surveyed in the present review, the human subjects have not been divided into different levels because in traditional statistical physics, the particles/molecules/atoms under consideration are generally identical, but yielding abundant statistical phenomena. In this regard, social psychology sheds light on the role of the above-mentioned three personality layers in the human counterparts of the laws or phenomena in traditional statistical physics.

In addition, social psychology has formed a series of mature research approaches including observational methods, laboratory experiments, surveys, etc. Regarding the observational methods, they are similar to the empirical research in econophysics (also called empirical econophysics), namely, by statistically analyzing existing economic or financial data like stock prices.[12] In laboratory experiments, one or more variables are altered to measure the effect on another dependent variable; they are similar to the controlled experiments in econophysics (also called experimental econophysics) as surveyed in this review. The method of surveys in social psychology involves both oral communications with the subjects and questionnaire surveys to collect information, which has the potential to help develop econophysics in the future. Certainly, there are many other methods adopted in social psychology that might be useful for developing econophysics as well, for example, the method of twin studies that is utilized to reveal the absolute and the relative importance of environmental impacts upon individuals in a given sample.

In summary, because social psychologists have already studied human beings for a long time, their results and experiences are undoubtedly not only useful but also important to help eonophysicists to develop econophysics where studying human beings is still at the early stage.

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