Nonlinear impedances of thermoacoustic stacks with ordered and disordered structures*

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Nonlinear impedances of two thermoacoustic stacks with ordered structures (plate-type and pipe-type) and one with a disordered structure (copper mesh) are studied. The linear resistances, nonlinear coefficients and effective acoustic masses of the stacks are extracted from the experimental results based on an analogical model of nonlinear impedances of porous materials. The resistance and nonlinear coefficient of the disordered stack are found to be much larger than those of the ordered stacks, which have similar volume porosities. In the ordered stacks, the resistance is only marginally influenced by the length of the stack, while in the disordered stack, the resistance increases significantly with the length. These characteristics of the impedances of ordered and disordered stacks are explained with the minor loss theory and the tortuosity of a stack.

Keywords: thermoacoustic stack, impedance, nonlinearity

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1. Introduction

In 1777, Byron Higgins was the first to study thermoacoustic oscillation.[1] He observed acoustical oscillation when a tube with both ends open was heated by a hydrogen flame, and it was named the "singing flame" phenomenon. More than fifty years later, Rijke used a heated wire screen to produce strong acoustic oscillation and Sondhauss studied the frequency and intensity of the oscillation.[2] A breakthrough in the study and modeling of thermoacoustic phenomena was made by Rott by developing a successful linear theory[3] that led to systematic studies of thermoacoustic effects and devices.[4,5] Thermoacoustic devices work based on the energy conversion between the acoustic energy and thermal flux.[5,6] These devices can be divided into two types according to the direction of energy conversion: thermoacoustic engines which convert heat into acoustic energy and thermoacoustic refrigerators which convert acoustic energy into thermal flux. Among various refrigeration technologies developed in recent years,[7,8] thermoacoustic refrigerators have been considered to be an important refrigeration technology and show great research and application prospects by virtue of the advantages of no pollution and long service life.

With the development of thermoacoustic technology, the thermoacoustic refrigerator driven by a loudspeaker was first presented by Hofler[9] and was considered to be a major step toward the practical application of thermoacoustic refrigeration. In order to increase the efficiency and power of this type of thermoacoustic refrigerator, systematical analysis,[10] acoustic coupling between the acoustic source and the thermoacoustic resonant pipe[11–13] and natural convection in the pipe[14] have all been widely studied. Furthermore, because of the high intensities of acoustic fields in thermoacoustic resonant pipes, nonlinear effects cannot be avoided in thermoacoustic refrigerators. In fact, the research into the nonlinearities in thermoacoustic refrigerators has focused on nonlinear standing waves[15,16] and the energy conversions from electric energy to acoustic energy[17,18] and from acoustic energy to thermal flux.[19,20] In a thermoacoustic refrigerator, the thermoacoustic stack is the most important part, because the conversion between acoustic energy and thermal flux takes place in it. Therefore, an acoustic model of the thermoacoustic stack was set up in order to improve the acoustic coupling in the thermoacoustic refrigerator,[21] and recently, the research was extended to the nonlinear scope, in which the nonlinear impedance of a type of thermoacoustic stack made of the copper mesh was studied.[22]

For the optimization of the performance of thermoacoustic refrigerators, thermoacoustic stacks with different structures have been used in order to achieve the energy conversions with high efficiency.[6] Thermoacoustic stacks were considered as porous materials when the acoustic characteristics were studied.[23] According to the theory of acoustic transmission in porous material, we divide various thermoacoustic stacks with different structures into two types: (i) the or-

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dered stack with tortuosity q as 1,[24] in which each of the main longitudinal pores in the stack is straight; (ii) the disordered stack with $q > 1$, which indicates the possibility of a gentle longitudinal curvature of each of the main pores, i.e., the twisted degree of each acoustical path in the stack. In this paper, in order to evaluate the acoustic characteristics of both types of thermoacoustic stacks, two ordered stacks with different structures (plate-type and pipe-type stacks) and one disordered stack (copper mesh stack) are studied. Based on the experimental results, three characteristic parameters describing the acoustic properties of the stacks are extracted—the linear resistance, nonlinear coefficient, and effective acoustic mass. Next, the linear and nonlinear resistances of the ordered and disordered thermoacoustic stacks are compared and discussed based on the minor loss theory and their different transmission paths. The results can be used to formulate more accurate nonlinear models of thermoacoustic refrigerators that incorporate different thermoacoustic stacks.

2. Theory and experimental apparatus

According to the definition of acoustic fluid impedance and Euler's motion equation, the acoustic fluid impedance of a stack set up in a resonant pipe can be expressed as $Z = R + \mathrm{i}X = -(\partial P/\partial x)/U$ when a sinusoidal signal is used as excitation,[22–26] where R and X indicate the resistance and reactance of the stack, respectively; P and U are the amplitudes of the complex acoustic pressure and particle velocity of the fluid in the resonant pipe; x is the traveling direction of the acoustic wave. Based on this theory, an experimental system can be built, as shown in Fig. 1(a).[22] The working gas employed in the experiment is air at room temperature with a filling pressure of 1 atm. Acoustic fields with high sound pressure levels (SPL) up to more than 140 dB are established in the resonant pipe by a high power loudspeaker, and the acoustic pressures on both sides of the stack (P_1, P_2) and at the rigid terminal of the pipe (P_t) are measured by three mini-type microphones 1–3. The particle velocity in the stack can be seen as a constant and is indicated by U_2 in the experiments, as the length of the stack d is much shorter than the acoustic wave length λ at the measuring frequency, as is always the case in thermoacoustic refrigerators. Then, the approximate equation for the impedance can be expressed as $Z_m = (P_1 - P_2)/U_2 d$. Using the plane wave theory, U_2 can be expressed as $U_2 = \mathrm{i}P_t \sin(k'L)/\rho_0 c'$, where L is the distance between the stack and the rigid terminal of the resonant pipe; ρ_0 is the density of the fluid in the resonant pipe; $c' = \omega/k'$ is the complex acoustic velocity and $k' = k + \mathrm{i}\alpha\,(\alpha < 0)$ is the complex wave number, with α as the attenuation coefficient in the resonant pipe. According to the relation $P_2 = P_t \cos(k'L)$ between the acoustic pressures obtained with microphones 2 and 3, the attenuation coefficient α can be determined by $\alpha = \ln(\sqrt{\zeta^2+1} - \zeta)/L$, where $\zeta = |P_2/P_t|$. Although the equation is available for an arbitrary L, the stack must be fixed at the antinode of the particle velocity in the experiment, which leads to the most obvious nonlinear effect and the least error with the approximation of the short stack in the theory.[22] Then, the distance between the stack and the rigid terminal is determined by the operating frequency as $L = (2n-1)\lambda/4$, where n is a positive integer and is usually taken as 1. In this case, the approximate equation for the impedance can be obtained[22]

$$Z_m = \rho_0 c' \left((-1)^n \mathrm{i}P_1/P_t + \alpha L\right)/d. \tag{1}$$

Then, the resistance $R_m = \mathrm{Re}\,[Z_m]$ and reactance $X_m = \mathrm{Im}\,[Z_m]$ of the stack, as well as its equivalent acoustic mass $M_m = X_m/\omega$, can be obtained correspondingly.

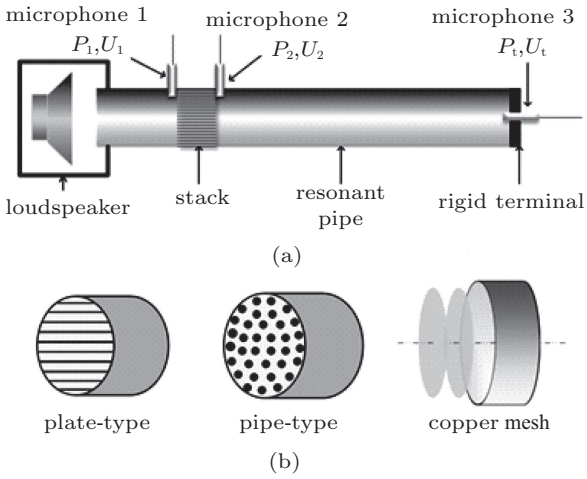


Fig. 1. (a) Experimental system for measuring acoustic fluid impedances of thermoacoustic stacks in thermoacoustic resonant pipes. The $P_1(U_1)$, $P_2(U_2)$, and $P_t(U_t)$ are acoustic pressures (particle velocities) on both sides of the stack and at the rigid terminal of the resonant pipe; (b) three types of thermoacoustic stacks.

Figure 1(b) shows the stacks with different structures used in the experiments. The plate-type and pipe-type stacks have regular longitudinal main slots or pores, which are ordered stacks. In order to achieve the efficient thermal exchange in the thermoacoustic stack, the sizes of slots or pores in the stack must be on the same order of thickness δ_h as the thermal boundary layer, which is about 0.3 mm in the experiments. Therefore, the width of the slot in the plate-type stack and the radius of the pore in the pipe-type stack are designed to be 0.3 mm and 0.5 mm, respectively. The copper mesh stack is made of copper mesh with a mesh number of 30 and wire diameter of 0.15 mm. The piled pieces of copper mesh are fixed in a metal frame located in the resonant pipe, so the porosity of the copper mesh stack can be easily changed by changing the number of the piled pieces. Obviously, there is no straight longitudinal path in the copper mesh stack and so it is considered to be a disordered stack.

3. Results and discussion

Figure 2 shows the nonlinear impedances versus the SPL of P_t measured in the three types of thermoacoustic stacks with the same length of 1 cm, in which the measuring frequency is 130 Hz. It is found that the acoustic impedances of the ordered and disordered stacks show similar behaviors versus the SPL, i.e., the resistances keep constant at low SPLs and begin to increase exponentially after a critical SPL, while the effective acoustic masses (or reactances) are independent of the SPL. Therefore, the nonlinear impedances of ordered and disordered thermoacoustic stacks can be described by the acoustic models analogous to the theory of acoustic nonlinearities in porous materials, as follows:[22]

$$R_m \approx \begin{cases} r_0, & \mathrm{spl_t} < \mathrm{spl_c}, \\ r_0' + P_{ref} r_1 10^{\mathrm{spl_t}/20}, & \mathrm{spl_t} \geq \mathrm{spl_c}, \end{cases} \tag{2a}$$

$$M_m \approx m_0, \tag{2b}$$

where r_0 indicates the linear resistance at low SPLs and r_1 is defined as the nonlinear coefficient indicating the degree of the nonlinear effect; $P_{ref} = 2 \times 10^{-5}$ Pa is the reference sound pressure; m_0 is the effective acoustic mass, which is a constant versus the SPL; $\mathrm{spl_t}$ is the sound pressure of P_t, and $\mathrm{spl_c}$ is a critical SPL above which the nonlinear effect arises. Then, r_0, r_1, and m_0 are three characteristic parameters describing the acoustic impedance of the stack, providing both linear and nonlinear information.

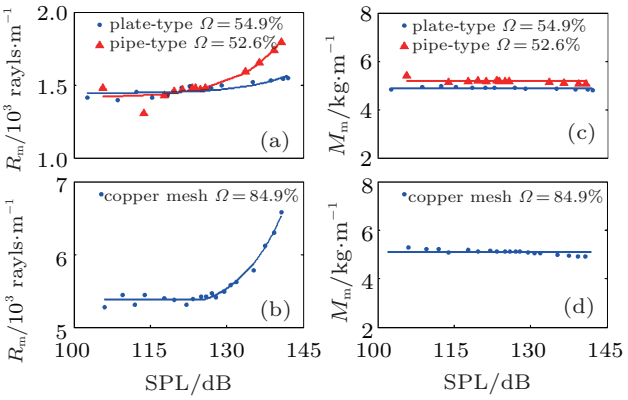


Fig. 2. (color online) Experimental acoustic impedances versus the SPL for thermoacoustic stacks with the length of 1 cm at the measuring frequency of 130 Hz: (a)–(b) resistances, (c)–(d) effective acoustic masses of the plate-type, pipe-type, and copper mesh stacks. Solid lines indicate the fitted results.

In order to obtain the three characteristic parameters, experimental results are fitted with the least squares method, as the solid lines show in Fig. 2, and the extracted parameters are listed in Table 1. It can be seen that unlike the resistance, the effective mass is much smaller and independent of the SPL; in addition, it is marginally influenced by the structure of the stack. Therefore, the resistance is considered to play a more important part in the performance of the thermoacoustic stack and must be given more consideration in research.[24] The linear part r_0 of the resistance can be obtained according to the linear model of the thermoacoustic stack. In the ordered stacks, the linear resistances are related to the characteristic wavelength $\lambda_c = \sqrt{2} l_c / \delta_\upsilon$,[25,26] where l_c is the characteristic dimension of the slot or pore in the plate-type or pipe-type stack and δ_υ is the thickness of the viscous boundary layer. On the other hand, the linear resistance of the disordered mesh stack can be derived from the momentum equation as $R_m = \mu\left(\gamma_1(\Omega)/8 + \gamma_2(\Omega) Re/(3\pi)\right)/r_h^2$,[27] with μ the dynamic viscosity of air as 18.1 μPa·s and r_h the hydraulic radius as 0.2 mm. The $Re = 4u_0 r_h \rho_0/\mu$ is the Reynolds number, which is 0 in our experiment because the static flow speed u_0 in the resonant pipe is 0. Here, $\gamma_1(\Omega) = 1268 - 3545\Omega + 2544\Omega^2$ and $\gamma_2(\Omega) = -2.82 + 10.7\Omega - 8.6\Omega^2$ are functions showing the relation between the friction factor and the volume porosity Ω.[28] Then, according to the structural parameters of the stacks used in the experiments, the theoretical resistances are calculated to be 1260 rayls/m for the plate-type stack, 1312 rayls/m for the pipe-type stack and 5376 rayls/m for the copper mesh stack. Actually, the resistance of the ordered stack is sensitive to the sizes of the pores or slots in the stack. As calculated with the theory, a fabrication error of 0.01 mm in the width of the slot can induce a deviation of over 7% in the resistance of the plate-type stack, which also exists in the pipe-type stack. Because the sizes of the slots and pores in the ordered stacks are all sub-millimeter, fabrication errors in geometry and sizes are almost inevitable. In this case, deviations of 14.9% and 6.95% between the experimental and theoretical results can be found for the plate-type and pipe-type stacks, respectively.

The nonlinear resistance of the stacks can be explained by the minor loss and tortuosity of the stack. The excess loss occurs at the face of the stack and can be modeled with the minor loss theory.[29,30] The tortuosity indicates the shape of the transmission path of the acoustic wave in the stack,[24,27] which also influences the resistance of the stack. In an acoustic

Table 1. Extracted parameters for acoustic impedances of the stacks shown in Fig. 2.

Type of stack	Linear resistance r_0/rayls·m^{-1}	Fitting coefficient r_0'/rayls·m^{-1}	Nonlinear coefficients r_1/rayls·m^{-1}·Pa^{-1}	Acoustic mass m_0/kg·m^{-1}	Critical SPL $\mathrm{spl_c}$/dB
Plate-type	1480	1478	0.2955	4.879	~ 123
Pipe-type	1410	1399	1.816	5.166	~ 123
Copper mesh	5379	5188	6.486	5.099	~ 125

field with high intensity, the excess resistance of the stack increases linearly with the particle velocity, corresponding to a minor loss coefficient $K = \alpha - 2\beta\sigma + \sigma^2$, in which α and β are the kinetic energy coefficient and momentum coefficient, respectively, σ is the ratio of the areas before and after the expansion and can be considered to be the same value as Ω.[30] According to the theory and the experiment, the minor loss coefficient K for the pipe-type transmission path is larger than that for the slot-type transmission path,[30] which results in a larger nonlinear coefficient for the pipe-type stack than for the plate-type stack. In a thermoacoustic stack, the excess pressure drop between both sides caused by the minor loss can be expressed as $\Delta P = K\rho_0 \langle U\rangle^2/2$,[29,30] in which $\langle U\rangle = qU/\Omega$ is the velocity averaged over a single acoustical path in the stack.[30,31] Then, according to the definition of the stack impedance $Z = \Delta P/Ud$, we can define a coefficient $K_e = \mathrm{d}R/\mathrm{d}U = \rho_0 q^2 K/2d\Omega^2$ to demonstrate the nonlinear behavior of the resistance R versus the particle velocity U. This effective coefficient K_e is determined by both the minor loss coefficient K and the tortuosity q, which can be used to describe the nonlinear resistances of both ordered and disordered stacks. Therefore, the nonlinear coefficient r_1 in Eq. (2a), which we extract from the experimental data, can be expressed by the coefficient K_e as follows: $r_1 = \mathrm{d}R/\mathrm{d}P_t = \mathrm{d}R/\mathrm{d}(Z_{t2}U_2) = K_e/Z_{t2}$, where $Z_{t2} = P_t/U_2 = \rho_0 c'/\mathrm{i}\sin(k'L)$. In this case, the nonlinear coefficient r_1 describing the nonlinear effects in both the ordered and disordered thermoacoustic stacks can be directly determined by

$$r_1 = \frac{\rho_0 q^2}{2d\Omega^2 Z_{t2}} K. \tag{3}$$

By comparing the performances of the ordered and disordered thermoacoustic stacks, it can be seen that both the linear resistance and nonlinear coefficient of the disordered copper mesh stack are much larger than those in the ordered stacks, even with much smaller volume porosities, although both parameters increase with the decrease of the volume porosity.[22] The phenomenon results from different transmission paths in the stacks. In the ordered stacks, the tortuosity $q = 1$ and the slots or pores in the stack are straight, so the acoustic wave can transmit through the stack easily. However, in the disordered stack, the transmission paths in the stack are twisted, which results in a larger q. Therefore, in the disordered type of copper mesh stack, the additional resistance caused by the twisted transmission paths increases the linear resistance, and the nonlinear coefficient r_1 describing the nonlinear effects, as indicated by Eq. (3), is also increased by a large tortuosity q.

The influence of stack length on resistance also differs between ordered and disordered stacks. As shown in Fig. 3(a), two plate-type stacks with the lengths of 1 cm and 2 cm exhibit similar resistances, differing less than 100 rayls/m. It must be noted that the volume porosities of the ordered stacks with different lengths are slightly different. Because the sizes of the slots and pores in the ordered stacks are both in the sub-millimeter scale and there are many slots and pores in one stack, tiny fabrication errors in several slots or pores in a stack can change the porosity, especially in a longer stack. This explains why the porosity of the stack with the length of 2 cm is 5% lower than that of the one with the length of 1 cm, giving the longer stack greater resistance. Then, it can be predicted that the resistances of ordered stacks with the same volume porosity and different lengths are almost the same. In the ordered stack, since the tortuosity remains one regardless of the length, it is reasonable that the resistance of the ordered stack is independent of its length. However, as shown in Fig. 3(b), for the disordered stacks with the same volume porosity, the resistance increases significantly with the length of the stack, unlike the ordered stacks. The phenomenon can also be explained by the tortuosity of the disordered copper mesh stack. The tortuosity of the copper mesh stack can be expressed with an empirical equation $\ln(q) = -7.83\ln(\Omega) - 0.54\ln(D_w) + 0.188\ln(d) + 4.85\Omega^2 - 4.49$,[32] in which D_w is the wire diameter of the copper mesh. It can be seen that the tortuosity increases with the stack length d because it influences the topological characteristics of the stack. Therefore, the longer disordered stack has more twisted transmission paths, which results in a larger tortuosity q as well as a larger resistance R_m.

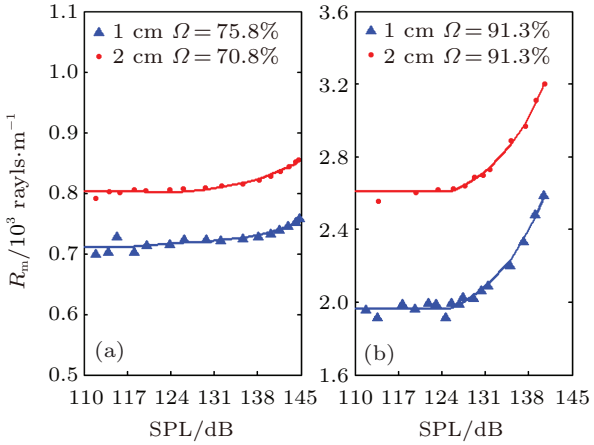


Fig. 3. (color online) Experimental acoustic resistances versus the SPL for thermoacoustic stacks with different lengths at the measuring frequency of 130 Hz: (a) plate-type stacks and (b) copper mesh stacks. Solid lines are the fitted results.

4. Conclusions

In conclusion, the nonlinear impedances of ordered (plate-type and pipe-type) and disordered (copper mesh) thermoacoustic stacks are studied and their different characteristics are presented—characteristics that can be used to describe the nonlinear models of thermoacoustic refrigerators that employ different types of thermoacoustic stacks. The influences of the SPL on the nonlinear impedances of ordered and disordered thermoacoustic stacks are similar. However, both the

linear resistance and nonlinear coefficient of the disordered stack are much larger than those in the ordered stacks. Additionally, the resistance of an ordered stack is only marginally influenced by the stack length, while the resistance of the disordered stack increases with the length. These phenomena can be explained based on the minor loss theory and the tortuosity in the three types of stacks. Obviously, the linear and nonlinear impedances of ordered and disordered stacks show different characteristics. It seems that the disordered stack has a larger resistance than the ordered one because of the large linear resistance and nonlinear coefficient, which must be considered in the impedance match in thermoacoustic engines. On the other hand, the disordered stack fabricated with the mesh screen shows the heat exchange with high efficiency, which is often used in pulse tube refrigerators based on the thermoacoustic effects.[33–35] Therefore, both impedance matching and heat exchange efficiency must be considered simultaneously in order to achieve the high working efficiency of a thermoacoustic refrigerator by choosing the proper thermoacoustic stack.

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