# Transversal reverse transformation of anomalous hollow beams in strongly isotropic nonlocal media*

Dai Zhi-Ping(戴志平)[a], Yang Zhen-Jun(杨振军)[b]†, Zhang Shu-Min(张书敏)[b], Pang Zhao-Guang(庞兆广)[b], and You Kai-Ming(游开明)[a]

[a] *Department of Physics and Electronic Information Science, Hengyang Normal University, Hengyang* 421002, *China*
[b] *Department of Physics and Hebei Advanced Thin Film Laboratory, Hebei Normal University, Shijiazhuang* 050024, *China*

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Based on the nonlocal nonlinear Schrödinger equation, the propagation properties of anomalous hollow beams in strongly isotropic nonlocal media are investigated. The analytical expressions of the beam propagation, the on-axis intensity and the beam width are obtained. The results show that the evolution of the beam is periodical and the input power is the most important parameter. The input power determines the variation of the period. Furthermore, it is found that there exists a critical input power in the $x$ direction and in the $y$ direction separately when the initial beam widths in the two transversal directions are unequal. The beam width remains invariant in the corresponding transversal direction when the input power equals the critical power in one of the transversal directions. Selecting a proper input power, the beam can be broadened or compressed in the two transversal directions at the same time. In particular, the beam can be broadened (compressed) in one transversal direction, whereas in the other transversal direction, it is compressed (broadened), i.e., the transversal reverse transformation.

**Keywords:** spatial nonlocality, beam propagation, transversal reverse transformation

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## 1. Introduction

In the past decade, nonlocal nonlinearity has attracted considerable interest. The nonlocality allows the refractive index of a material at a particular point to be related to the beam intensity at all other material points, which is distinctly different from the conventional local nonlinearity. If the characteristic length of the nonlocal response is much larger than the beam width, these media are called strongly nonlocal nonlinear media. In reality, many physical systems exhibit the nonlocal nonlinearity, for instance nematic liquid crystals,[1] lead glasses,[2] atomic vapors,[3] Bose–Einstein condensates,[4] photorefractive crystals,[5] etc. Nonlocality can support many novel solitons, such as Gaussian and higher-order Gaussian solitons,[6–8] elliptic solitons,[2,9,10] vortex solitons,[11,12] dipole or multipole solitons,[13,14] dark solitons,[15,16] surface solitons,[17,18] solitons in nonlocal optical lattices,[19–21] etc. Up to now, various shape-invariant beams, i.e., solitons, have been investigated in the nonlocal domain. However, due to the complexity of the shape-variant beams, there are relatively few investigations on the nonlocal propagation of shape-variant beams which include the anomalous hollow beam.

On the other hand, because the dark hollow beams have wide applications in atomic optics, binary optics, medical sciences, optical trapping of particles, etc., these beams have been widely studied. Experimentally, various techniques have been used to generate the dark hollow beams. In an experiment, Wu *et al.* observed an anomalous hollow beam which can be used as a powerful tool to study the linear and nonlinear particle dynamics in the storage ring.[22] Therefore, the anomalous hollow beam is a kind of novel and important beam. In order to study this beam theoretically, Cai proposed a mathematical model of the anomalous hollow beam.[23] Later, the anomalous hollow beam was investigated in various optical systems, for example a turbulent atmosphere,[24] uniaxial crystals,[25] fractional Fourier system,[26] a misaligned first-order optical system,[27] etc. In addition, Cai *et al.* also proposed an alternative theoretical model for an anomalous hollow beam.[28] Most recently, Wang *et al.* studied the propagation properties of a radial phased-locked partially coherent anomalous hollow beam array in turbulent atmosphere.[29] To the best of our knowledge, the propagation of the anomalous hollow beam in nonlocal nonlinear media remains unexploited. In this paper, we investigate the propagation of anomalous hollow beams in strongly isotropic nonlocal nonlinear media. The on-axis intensity, the beam width, and the evolution of the transversal intensity are discussed. We find that the input power is a very important parameter. Selecting a proper input power, the beam can be broadened or compressed

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†Corresponding author. E-mail: zjyang@vip.163.com

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in the two transversal directions at the same time. In particular, the beam can be broadened (compressed) in one transversal direction, whereas in the other transversal direction it is compressed (broadened), i.e. the transversal reverse transformation.

## 2. Analytical expression

In Cartesian coordinates, the propagation of a paraxial optical beam in nonlocal nonlinear media is governed by the nonlocal nonlinear Schrödinger equation

$$2\mathrm{i}kn_0\frac{\partial \Phi}{\partial z}+n_0\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi + 2k^2n_2\Phi\iint R(\boldsymbol{r}-\boldsymbol{r}')|\Phi|^2\mathrm{d}^2\boldsymbol{r}'=0, \tag{1}$$

where $\Phi$ is the complex amplitude, $\boldsymbol{r}=(x,y)$ denotes the transversal coordinates, $k$ is the wave number without nonlinearity, $n_0$ is the linear refractive index of the media, $n_2$ is the nonlinear index coefficient, and $R$ is the normalized symmetrical real spatial response function of the media. The last term on the left in Eq. (1) represents the nonlocal nonlinearity, if this term is ignored, equation (1) reduces to the paraxial wave equation in free space, which has been thoroughly investigated in the past decades.

In the case of strong nonlocality, where the beam width is much narrower than the width of the response function (i.e., the characteristic length of the material), if the response function is twice differentiable at the original point, it can be expanded in a Taylor's series. If the response function is expanded to the second order, the nonlocal nonlinear Schrödinger equation reduces to the following model:[6]

$$2\mathrm{i}k\frac{\partial \Phi}{\partial z}+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi-k^2\gamma^2P_0(x^2+y^2)\Phi=0, \tag{2}$$

where $\gamma$ is a material constant associated with the response function, and $P_0=\iint|\Phi|^2\mathrm{d}x\mathrm{d}y$ is the input power of the beam. In theory, one can use Eq. (2) to deal with the propagation of an arbitrary beam in strongly isotropic nonlocal nonlinear media.

The anomalous hollow beam at the initial position $z=0$ can be expressed as[23]

$$\Phi(x_0,y_0,0) = C_0\exp\left(-\frac{x^2}{w_{0x}^2}-\frac{y^2}{w_{0y}^2}\right) \times H_2\left(\sqrt{\frac{2x^2}{w_{0x}^2}+\frac{2y^2}{w_{0y}^2}}\right), \tag{3}$$

where $C_0=\sqrt{P_0/10\pi w_{0x}w_{0y}}$ is a normalized constant which ensures the input power equals $P_0$, $w_{0x}$ and $w_{0y}$ are the beam waist width of an astigmatic Gaussian beam in the $x$ and $y$ directions, respectively, $H_2$ is the second-order Hermite polynomial. The above equation can also be expressed as superposition of astigmatic Gaussian modes and astigmatic doughnut modes as follows:

$$\Phi(x_0,y_0,0) = C_0\left(-2+\frac{8x^2}{w_{0x}^2}+\frac{8y^2}{w_{0y}^2}\right) \times\exp\left(-\frac{x^2}{w_{0x}^2}-\frac{y^2}{w_{0y}^2}\right). \tag{4}$$

Only based on Eqs. (2) and (4), it is almost impossible to study the beam propagation in strongly isotropic nonlocal media due to the mathematical complication. Fortunately, equation (2) is a linear equation, therefore a strongly nonlocal medium can be regarded as a linear optical system, and the propagation expression of paraxial beams in such media can be obtained by using the following integral formula as shown in our previous work in Refs. [30] and [31]:

$$\Phi(x,y,z) = -\frac{\mathrm{i}k\sqrt{\gamma^2P_0}}{2\pi\sin\alpha}\exp\left[\frac{\mathrm{i}k\sqrt{\gamma^2P_0}(x^2+y^2)}{2\tan(\sqrt{\gamma^2P_0}z)}\right] \times\iint\Phi(x_0,y_0,0)\exp\left[\frac{\mathrm{i}k\sqrt{\gamma^2P_0}(x_0^2+y_0^2)}{2\tan(\sqrt{\gamma^2P_0}z)} -\frac{\mathrm{i}k\sqrt{\gamma^2P_0}(xx_0+yy_0)}{\sin(\sqrt{\gamma^2P_0}z)}\right]\mathrm{d}x_0\mathrm{d}y_0. \tag{5}$$

Substituting Eq. (4) into Eq. (5), after tedious but straightforward integration, one can obtain

$$\Phi(x,y,z) = \frac{\mathrm{i}C_0}{D}(\rho_1+\rho_2x^2+\rho_3y^2) \times\exp\left[-\frac{\mathrm{i}z_\mathrm{R}\cos\alpha}{z_\mathrm{p}\sin\alpha}\left(\frac{x^2}{w_{0x}^2}+\frac{y^2}{w_{0y}^2}\right)\right] \times\exp\left[-\frac{z_\mathrm{R}^2}{z_\mathrm{p}^2\sin^2\alpha+\mathrm{i}z_\mathrm{p}z_\mathrm{R}\cos\alpha\sin\alpha}\cdot\frac{x^2}{w_{0x}^2} -\frac{p^2z_\mathrm{R}^2}{z_\mathrm{p}^2\sin^2\alpha+\mathrm{i}p^2z_\mathrm{p}z_\mathrm{R}\cos\alpha\sin\alpha}\cdot\frac{y^2}{w_{0x}^2}\right], \tag{6}$$

where

$$\begin{aligned}
D &= 8z_\mathrm{p}z_\mathrm{R}^3\sin\alpha\left(\cos\alpha-\mathrm{i}\frac{z_\mathrm{p}}{z_\mathrm{R}}\sin\alpha\right)^2 \times\left(\mathrm{i}p^2\cos\alpha+\frac{z_\mathrm{p}}{z_\mathrm{R}}\sin\alpha\right)^2 \times\sqrt{\left(\frac{\mathrm{i}z_\mathrm{R}\cos\alpha}{z_\mathrm{p}\sin\alpha}+1\right)\left(\frac{\mathrm{i}z_\mathrm{R}\cos\alpha}{z_\mathrm{p}\sin\alpha}+\frac{1}{p^2}\right)},\\
\rho_1 &= -48z_\mathrm{p}^4\sin^4\alpha-\mathrm{i}64z_\mathrm{p}^3z_\mathrm{R}(1+p^2)\cos\alpha\sin^3\alpha +16p^4z_\mathrm{R}^4\cos^4\alpha+16z_\mathrm{p}^2z_\mathrm{R}^2(1+4p^2+p^4)\cos^2\alpha\sin^2\alpha,\\
\rho_2 &= 64\frac{z_\mathrm{R}^4}{w_{0x}^2}\left(\frac{z_\mathrm{p}}{z_\mathrm{R}}\sin\alpha+\mathrm{i}p^2\cos\alpha\right)^2,\\
\rho_3 &= 64\frac{p^2z_\mathrm{R}^4}{w_{0x}^2}\left(\frac{z_\mathrm{p}}{z_\mathrm{R}}\sin\alpha+\mathrm{i}\cos\alpha\right)^2.
\end{aligned}$$

For the convenience of discussion in the following, we have used parameters: $p = w_{0y}/w_{0x}$ describes the ellipticity of the beam, $z_{\rm R} = kw_{0x}^2/2$ is the Rayleigh distance of a Gaussian beam defined by the beam waist width in the $x$ direction, $z_{\rm p} = 1/\sqrt{\gamma^2 P_0}$ is a parameter related to the input power, and $\alpha = z/z_{\rm p}$ is a normalized propagation distance. Equation (6) is the analytical expression of the anomalous hollow beam propagating in strongly isotropic nonlocal nonlinear media, and one can study the propagation properties using it.

Based on Eq. (6), one can obtain the on-axis intensity as

$$\begin{aligned} I(z) = {} & \frac{8p^2 z_{\rm p}}{\csc^2\alpha}\cdot\{z_{\rm R}^4\cos^2\alpha\left[(1+8p^2+p^4)z_{\rm p}^2+p^4 z_{\rm R}^2\cot^2\alpha\right] \\ & + 9z_{\rm p}^4 z_{\rm R}^2\sin^2\alpha\}[(z_{\rm R}^2\cos^2\alpha+z_{\rm p}^2\sin^2\alpha) \\ & \times (p^4 z_{\rm R}^2\cos^2\alpha+z_{\rm p}^2\sin^2\alpha)]^{-1} \\ & \times\{\left[z_{\rm p}^2+z_{\rm R}^2-(z_{\rm p}^2-z_{\rm R}^2)\cos(2\alpha)\right] \\ & \times\left[z_{\rm p}^2+p^4 z_{\rm R}^2+(p^4 z_{\rm R}^2-z_{\rm p}^2)\cos(2\alpha)\right]\}^{-1/2}, \end{aligned} \tag{7}$$

where $\csc(\cdot)$ is the cosecant function, and $\cot(\cdot)$ is the cotangent function.

Now we discuss the beam width. Because of the complexity of the intensity distribution of the anomalous hollow beam during propagation, we take the second-order moment beam width to describe the beam width. The second-order moment beam width gives good gross information on the beam width, and it is widely applied in practice. The second-order moment beam widths in the $x$ and $y$ directions are respectively defined as follows:

$$w_{\rm rx}^2(z) = \frac{4\iint x^2|\Phi(x,y,z)|^2\,{\rm d}x{\rm d}y}{\iint|\Phi(x,y,z)|^2\,{\rm d}x{\rm d}y}, \tag{8}$$

$$w_{\rm ry}^2(z) = \frac{4\iint y^2|\Phi(x,y,z)|^2\,{\rm d}x{\rm d}y}{\iint|\Phi(x,y,z)|^2\,{\rm d}x{\rm d}y}. \tag{9}$$

Evidently, it is difficult to calculate the second-order moment beam width of the anomalous hollow beam through integrating Eqs. (8) and (9) directly. However, with the aid of the method in Ref. [32] and the results in our previous work,[30] one can obtain the beam width of the anomalous hollow beam

$$w_{\rm rx}^2(z) = \frac{w_{0x}^2}{5}\left(17\cos^2\alpha+\frac{9z_{\rm p}^2}{z_{\rm R}^2}\sin^2\alpha\right), \tag{10}$$

$$w_{\rm ry}^2(z) = \frac{w_{0x}^2}{5p^2}\left(17p^4\cos^2\alpha+\frac{9z_{\rm p}^2}{z_{\rm R}^2}\sin^2\alpha\right). \tag{11}$$

In Eqs. (10) and (11), if we select a proper input power, the beam width in the $x$ or $y$ direction can be invariant on propagation, i.e., it always keeps the initial beam width as $w_{\rm rx}(z) \equiv w_{\rm rx}(0) = \sqrt{17/5}w_{0x}$ or $w_{\rm ry}(z) \equiv w_{\rm ry}(0) = \sqrt{17/5}w_{0y} = \sqrt{17/5}pw_{0x}$. We define this input power as the critical power. Then the critical power in the $x$ or $y$ direction is, respectively,

$$P_{cx} = \frac{9}{17}P_{\rm cg}, \tag{12}$$

$$P_{cy} = \frac{9}{17p^4}P_{\rm cg}, \tag{13}$$

where $P_{\rm cg} = 1/\gamma^2 z_{\rm R}^2$ is the critical power of a Gaussian beam (i.e., the soliton power).[6–8] As a special case of $p = 1$, the beam widths both in the $x$ and $y$ directions can keep invariant on propagation at the same time. Otherwise, the beam widths only keep invariant in one direction.

## 3. Propagation properties

We first discuss the intensity evolution for the special case of $p = 1.0$, $P_0 = P_{cx}$ in Fig. 1. Figure 1 shows the propagation is periodical and the period is $\pi z_{\rm p}$. When $w_{0x} = w_{0y}$ (i.e., $p = 1.0$), the anomalous hollow beam reduces to a circular anomalous hollow beam, which has a circular symmetry with a circular solid core at the initial position. With the propagation distance increasing, the beam energy converges to the beam center gradually and the intensity of the solid core arrives at its maximal at $z = \pi z_{\rm p}/2$. Then the intensity evolution for $\pi z_{\rm p}/2 < z < \pi z_{\rm p}$ is an inverse process of $0 < z < \pi z_{\rm p}/2$, and completes a period. In the whole process, there always exists a bright ring around the solid core, although the intensities of the ring and the core are unequal. These properties above are similar to those discussed in Refs. [7], [8], [33], and [34]. Namely, for $w_{0x} = w_{0y}$, the beams here have the same propagation properties as those discussed in Refs. [7], [8], [33], and [34]. In the following, we focus on the general case, i.e. $p \neq 1.0$.

Figure 2 gives the beam width in the $x$ direction with different input powers. It shows that the beam width keeps invariant on propagation for $P_0 = P_{\rm cx}$ (note that the distribution of transversal intensity is variant, see Fig. 1). If $P_0 \neq P_{cx}$, the beam width varies periodically. The longer the period is, the lower the input power is. It can be found from Eq. (10) that the period is $\pi z_{\rm p}$. In Eqs. (10) and (11), if the derivatives of $w_{0x}^2(z)$ and $w_{0y}^2(z)$ with respect to the propagation distance $z$ are equal to zero, the positions where the beam width arrives at its maximum or minimum can be obtained. One can easily obtain these positions $z_{\rm m} = n\pi z_{\rm p} \pm \pi z_{\rm p}/2, n = 0,1,2,\ldots$ Because of $z_{\rm p}$ related to $P_0$ (i.e., $z_{\rm p} = 1/\sqrt{\gamma^2 P_0} = \sqrt{P_{\rm cg}/P_0}z_{\rm R}$), the positions with maximal or minimal beam width and the period of the beam width evolution are both related to the input power (the red dashed and the blue dotted curves in Fig. 2). These properties are similar to the Gaussian beam[6] and the higher-order Gaussian beams[7,8] in strongly nonlocal media, but the Gaussian beam and the higher-order Gaussian beams keep their transversal shapes invariant on propagation, while the transversal shapes of the anomalous hollow beam are variant (see Fig. 1). It should be noted that the discussion above is in the $x$ direction, it is independent of the $y$ direction, i.e., the results are invariable whatever $p$ is, and vice versa. This is very different from the Gaussian and higher Gaussian beams whose widths in the $x$ and $y$ directions are always correlative and synchronous.

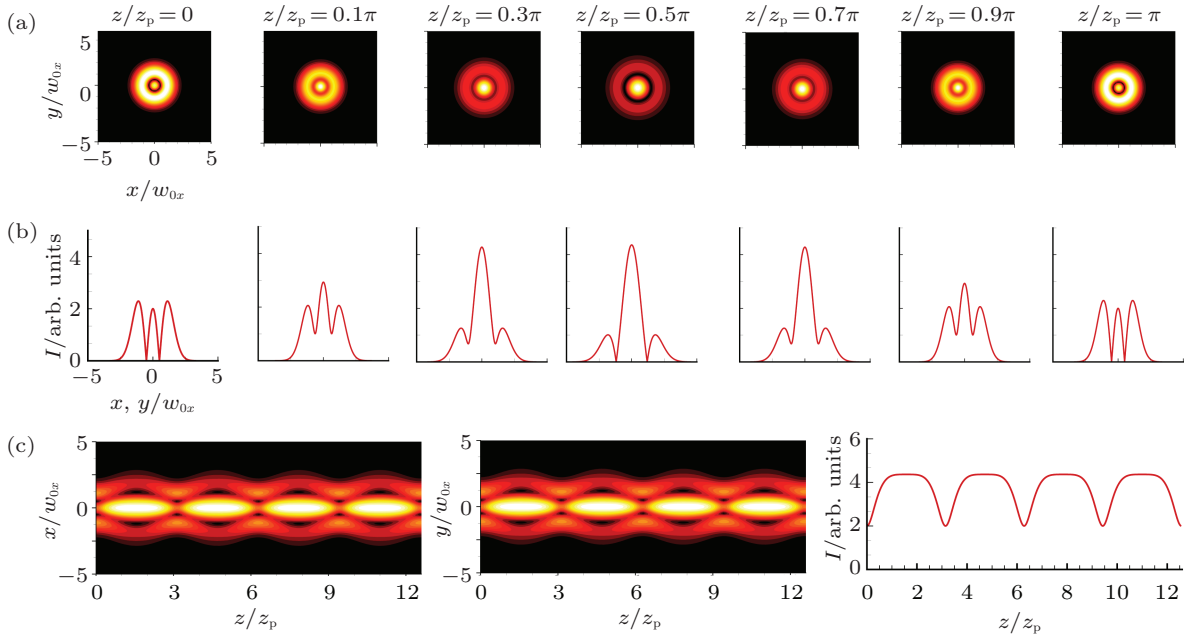



**Fig. 1.** (color online) Intensity evolution of the anomalous hollow beam on propagation. (a) The transversal intensity distributions at different propagation distances given on the top; (b) the transversal intensity distributions in the $x$ direction corresponding to panel (a); (c) the intensity evolutions of the anomalous hollow beam in the $x$ direction (left), the $y$ direction (center), and the on-axis direction (right). Parameters: $p = 1.0, P_0 = P_{cx}$.

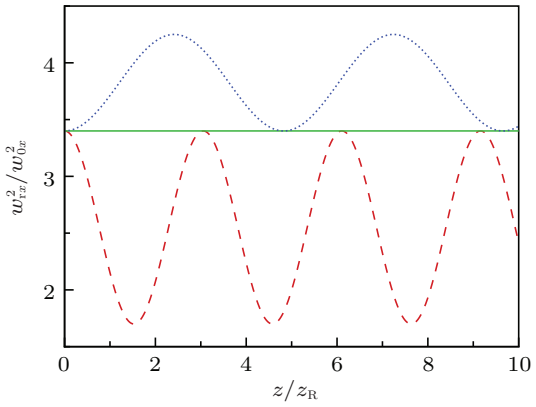



**Fig. 2.** (color online) Evolution of the beam width in the $x$ direction with different input powers. The blue dotted curve, the green solid curve, and the red dashed curve correspond to the input power $P_0 = 0.8P_{cx}$, $P_0 = 1.0P_{cx}$, and $P_0 = 2.0P_{cx}$, respectively.

Figure 3 gives the beam width evolutions with different $p$ and $P_0$. Fig. 3(a) shows that when $P_0 = P_{cx}$, $w_{rx}$ keeps invariant on propagation (the purple solid curve in Fig. 3(a)), but $w_{ry}$ can become wider or shorter for different $p$. For $p < 1$, $w_{ry}$ is broadened firstly, and then compressed and varies periodically (the green dotted and the blue dashed-dotted curves in Fig. 3(a)). For $p > 1$, $w_{ry}$ is compressed firstly, and then broadened and varies periodically (the red dashed curve in Fig. 3(a)). Figure 3(b) shows that when $P_0 = 2.0P_{cx}$, $w_{rx}$ is compressed firstly, while $w_{ry}$ can be broadened ($p = 0.6$) or be compressed ($p = 1.2$), even keep invariant ($p = 1/\sqrt[4]{2}$). The reason can be explained as follows. When the input power equals the critical power, the self-focusing counteracts the diffraction, and the beam width keeps invariant. If the input power does not equal the critical power, the beam can be broadened by a strong diffraction for a low input power or be compressed by a strong self-focusing for a high input power. A higher input power produces a stronger self-focusing, and a narrower beam produces a stronger diffraction. Because $w_{0x} \neq w_{0y}$, the diffraction and the self-focusing are also unequal in the $x$ and $y$ directions. The variety of beam width is the homeostasis between the diffraction and the self-focusing. Therefore, the input power is very important. For an anomalous hollow beam ($p$ is fixed), by setting a proper input power, the beam can be broadened or compressed in two directions at the same time. In particular, the beam can be broadened (compressed) in one transversal direction, whereas in the other transversal direction, it is compressed (broadened) at the same time, i.e., the transversal reverse transformation. In other words, with the beam width decreasing (increasing) in one transversal direction, it increases (decreases) in the other transversal direction. For example, for $P_0 = 2.0P_{cx}$ and $p = 0.6$, in the $x$ direction, the beam width is compressed (the purple solid curve in Fig. 3(b)), while in the $y$ direction, it is broadened (the green dotted curve in Fig. 3(b)).

In addition, we notice that although $w_{rx}$ and $w_{ry}$ vary on propagation in Fig. 3, the two beam widths can be equal at some positions. If set $w_{rx}(z) = w_{ry}(z)$, solving the equation, we can get the positions

$$z_e = n\pi z_p + z_p \arccos\left(\frac{\pm 3 z_p}{\sqrt{9z_p^2 + 17p^2 z_R^2}}\right),$$
$$n = 1, 2, 3, \ldots \tag{14}$$

It is noted that equation (14) is unsuitable for $p = 1.0$. For the case of $p = 1.0$, it is evident that the beam widths in the $x$ and $y$ directions are always equal on propagation.

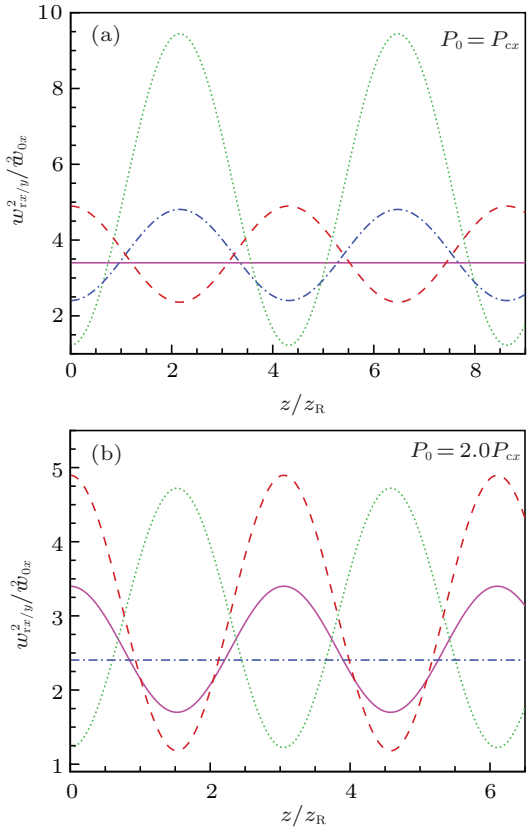



**Fig. 3.** (color online) Evolution of the beam width with different $p$ under different input powers. The input power is (a) $P_0 = P_{cx}$, (b) $P_0 = 2.0P_{cx}$. The purple solid curve represents $w_{rx}$ with a random value of $p$. The green dotted curve, the blue dashed-dotted curve, and the red dashed curve represent $w_{ry}$ with $p = 0.6$, $p = 1/\sqrt[4]{2}$, and $p = 1.2$, respectively.

Figure 4 shows the intensity evolution for the case of $P_0 = P_{cx}$, $p = 0.6$ corresponding to Fig. 3(a). It is found that in the $x$ direction, $P_0 = P_{cx}$, $w_{rx}$ keeps invariant; however, in the $y$ direction, $P_0 = P_{cx} < P_{cy}$, $w_{ry}$ is broadened. At $z = n\pi z_p/2, n = 0, 1, 2, \ldots$, the beam keeps the elliptical symmetry, and at other positions, the beam shapes change and is not elliptical symmetric. The long and short axis of the beam interchanges at $z = n_e\pi z_p/2$ and $z = n_o\pi z_p/2$, $(n_e = 0, 2, 4, \ldots; n_o = 1, 3, 5, \ldots)$ Figure 5 gives two cases of $P_0 = 2P_{cx}$, $p = 0.6$ and $P_0 = 2P_{cx}$, $p = 1.2$ corresponding to Fig. 3(b). For the case of $P_0 = 2P_{cx}$, $p = 0.6$, it shows in Figs. 5(a) and 5(b) that the evolution is periodical, but the evolutions in the $x$ direction and in the $y$ direction are distinctly different. As $w_{0y}$ is smaller than $w_{0x}$, although the input power is higher than $P_{cx}$, it is still weaker than $P_{cy}$. Therefore, at the initial position, the self-focusing is stronger than the diffraction in the $x$ direction, but the opposite is the case in the $y$ direction. As a result, the beam is compressed in the $x$ direction, and broadened in the $y$ direction firstly, the transversal reverse transformation appears. Especially at $z = 0$ and $z = \pi z_p/2$, the anomalous hollow beam still retains its elliptical symmetry and there exists a bright elliptical ring around the strong elliptical solid core, but the long axis and the short axis of the elliptical beam have interchanged their positions. For the case of $P_0 = 2P_{cx}, p = 1.2$, the evolutions can be discussed in a similar way, therefore, we do not discuss it here.

From Figs. 2–5, we can find that for $p \neq 1.0$, the properties are much different from those discussed in Refs. [6], [7], [33], and [34]. When $w_{0x} = w_{0y}$ (i.e., $p = 1.0$), the critical powers in the $x$ and $y$ directions are equal in Refs. [6], [7], [33], and [34], therefore, the beam evolutions in the $x$ and $y$ directions are always synchronous, so the beam symmetry is always the same. However, for the anomalous hollow beam with $p \neq 1.0$, the critical

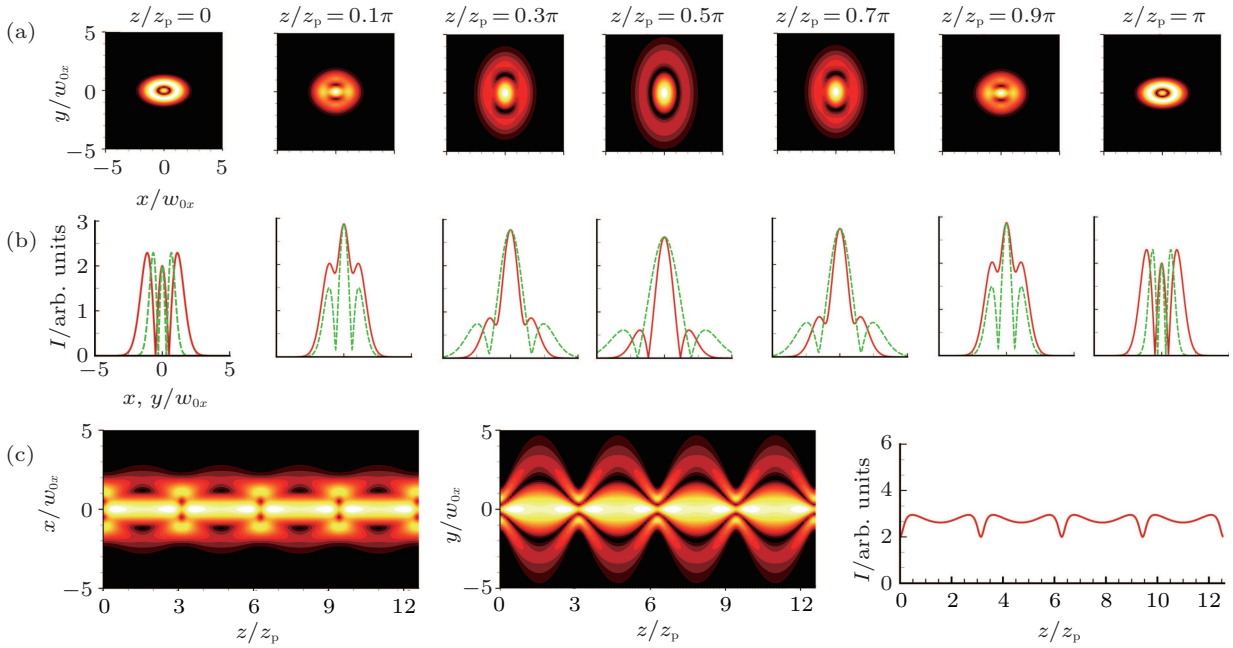



**Fig. 4.** (color online) Intensity evolution of the anomalous hollow beam on propagation. (a) The transversal intensity distributions at different propagation distances given on the top; (b) the transversal intensity distributions in the $x$ direction (red solid curve) and in the $y$ direction (green dashed curve) corresponding to panel (a); (c) the intensity evolutions of the anomalous hollow beam in the $x$ direction (left), the $y$ direction (center), and the on-axis direction (right). Parameters: $p = 0.6, P_0 = P_{cx}$.

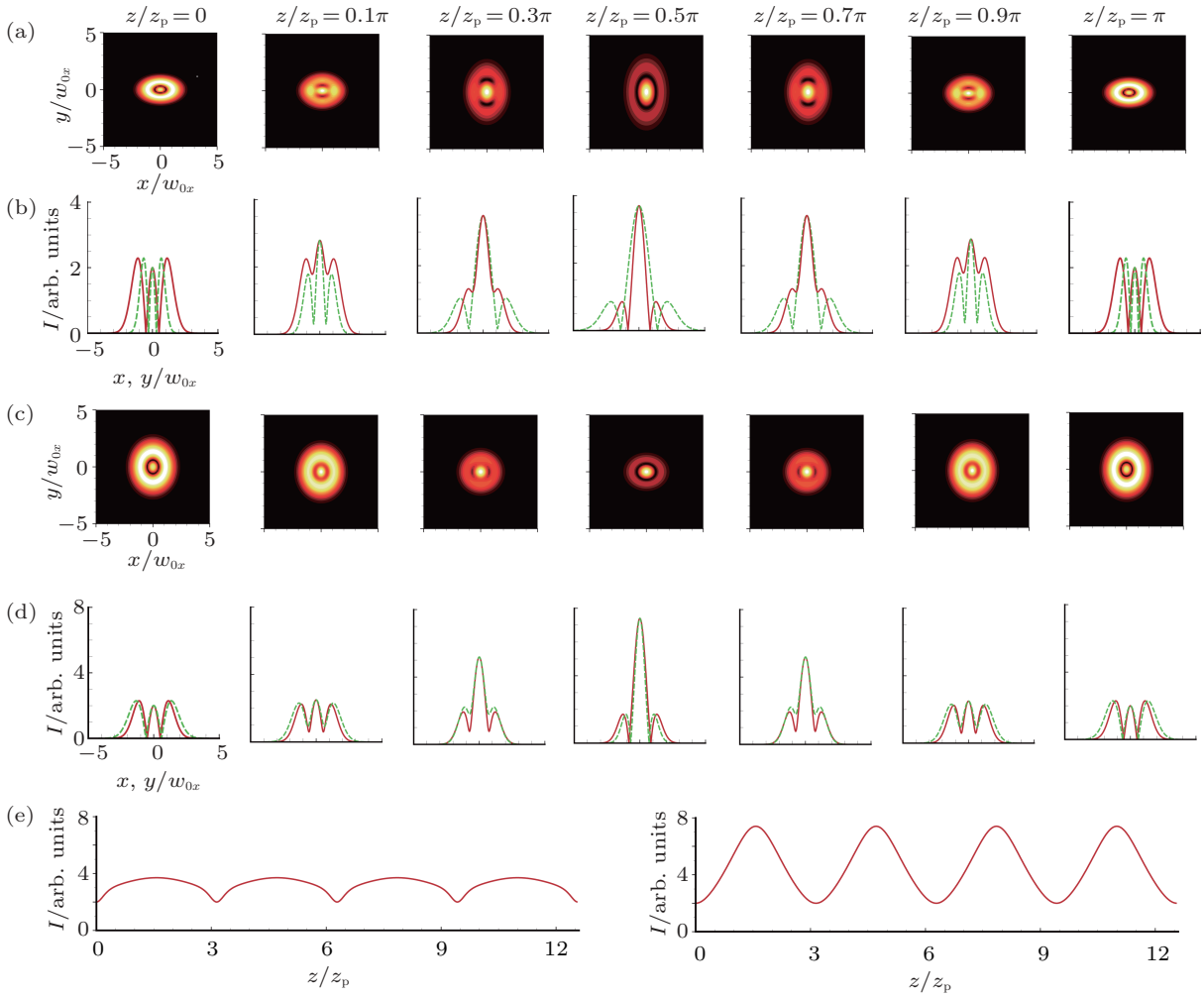



**Fig. 5.** (color online) Intensity evolutions of anomalous hollow beams on propagation. (a) The transversal intensity distributions at different propagation distances with $p = 0.6$; (b) the transversal intensity distributions in the $x$ direction (red solid curve) and in the $y$ direction (green dashed curve) corresponding to panel (a); (c) the same as panel (a) except $p = 1.2$; (d) the same as panel (b) expect $p = 1.2$; (e) the evolution of the on-axis intensity corresponding to panel (a) (left) and the evolution of the on-axis intensity corresponding to panel (c) (right). The input power is $P_0 = 2.0P_{cx}$.

powers in the $x$ and $y$ directions are not equal, as a result many new properties appear on propagation. For instance, when the beam width in one transversal direction keeps invariant, it can be broadened or compressed in the other transversal direction. The long axis and the short axis of the elliptical anomalous hollow beam can be interchanged on propagation. Even the transversal reverse transformation can appear for a proper input power.

It is noted again that $z_p$ is related to the input power, $z_p$ with different input powers denotes different real spatial distances. For the convenience of discussion of the period, in Figs. 4 and 5, the propagation distance is normalized by $z_p$. However, the propagation distance in Figs. 2 and 3 is normalized by $z_R$, which denotes a real spatial distance for a beam with a fixed beam waist width.

## 4. Conclusion

In conclusion, the propagation of the anomalous hollow beam in strongly isotropic nonlocal media is investigated. The expressions of the beam propagation, the on-axis intensity, and the beam width are obtained. The evolution of the anomalous hollow beam is discussed in detail. It is found that the evolution of the transversal intensity is always periodical, and the period depends on the input power. The longer the period is, the weaker the input power is. Furthermore we find that there exits a critical power for each transversal direction. When the input power equals the critical power, the beam width keeps invariant on propagation, otherwise it varies periodically. Therefore, the input power is very important for the beam propagation. For an anomalous hollow beam with a fixed $p$, by setting a proper input power, the beam can be broadened or be compressed in two transversal directions at the same time. In particular, the beam can be broadened (compressed) in one transversal direction, whereas in the other transversal direction, it is compressed (broadened), i.e., the transversal reverse transformation. Note that these properties of the anomalous hollow beam propagating in strongly isotropic nonlocal media are not a special case that only applies to the anomalous hollow beam. As long as the beam owns the elliptical symmetry, i.e., $w_{0x} \neq w_{0y}$, it should exhibit similar properties to the

ones discussed in this paper. Therefore, the above results can be extended to other elliptical symmetric beams. In addition, by using the coordinate transform, $x \to x\cos\theta + y\sin\theta, y \to y\cos\theta - x\sin\theta$, equation (3) describes an elliptical anomalous hollow beam with an elliptical solid core, where $\theta$ is the orientation angle between the long axis of the elliptical beam spot and the $x$ axis.[23] Using the coordinate transform and the method in this paper, the propagation of the elliptical anomalous hollow beam in strongly isotropic nonlocal media can also be investigated.

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