Scintillation of partially coherent Gaussian–Schell model beam propagation in slant atmospheric turbulence considering inner- and outer-scale effects*

Li Ya-Qing(李亚清)[a], Wu Zhen-Sen(吴振森)[a]†, Zhang Yuan-Yuan(张元元)[a], and Wang Ming-Jun(王明军)[b]

[a] School of Physics and Optoelectronic Engineering, Xidian University, Xi'an 710071, China
[b] School of Physics and Electronic Engineering, Xianyang Normal College, Xianyang 712000, China

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Based on the modified Rytov theory and the international telecommunication union-radio (ITU-R) slant atmospheric structure constant model, the uniform scintillation index of partially coherent Gaussian–Schell model (GSM) beam propagation in the slant path is derived from weak- to strong-turbulence regions considering inner- and outer-scale effects. The effects of wavelength of beams and inner- and outer-scale of turbulence on scintillation are analyzed numerically. Comparison between the scintillation of GSM beams under the von Karman spectrum and that of beams under the modified Hill spectrum is made. The results obtained show that the scintillation index obtained under the von Karman spectrum is smaller than that under the modified Hill spectrum. This study can find theory bases for the experiments of the partially coherent GSM beam propagation through atmospheric turbulence.

Keywords: atmospheric turbulence, partially coherent beam, scintillation, atmospheric propagation

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1. Introduction

During electromagnetic wave propagation in atmospheric turbulence, random fluctuations in the atmospheric refractive index give rise to random phase perturbations across the wave front. After propagating some distance from the point of introduction, these phase perturbations develop into random intensity fluctuations (scintillation) that have lognormal statistics in the turbulence region. Atmospheric optical communication, tracking, remote sensing, and imaging system performance may be significantly affected by this turbulence-induced scintillation.

Tatarskii[1] developed an expression for the scintillation index of a plane wave under weak irradiance fluctuations based on the Rytov approximation, but Gracheva[2] demonstrated the limitation of the weak fluctuation theory by the experimental data in 1965. Several qualitative models of scintillation were subsequently developed,[3,4] along with a theory of saturation proposed by Gochelashvili[5] and later modified by others.[6,7] Hill studied the theories of saturation of optical scintillation by strong turbulence for arbitrary refractive-index spectra[8] and has given the plane-wave variance and covariance and spherical-wave covariance based on the above theories.[9] Unfortunately, numerical results from these asymptotic theories do not match experimental and simulation data. Based on the assumption that small-scale irradiance fluctuations are modulated by large-scale fluctuations,[3,4,10] it was applicable for the case of weak-to-strong irradiance fluctuations. Then the standard Rytov theory was formally extended from weak irradiance fluctuations into the saturation regime by the introduction of large- and small-scale filter functions. Importantly, modern turbulence theory suggests that the atmospheric turbulence effect is determined by both the atmospheric structure constant and inner-scale effects. Experimental results[11] show that theoretical results considering inner-scale effects are closer to measurerment data. Thus, inner-scale effect theories were proposed.[12,13]

Recently, some researchers studied scintillation problems of beams propagating in atmospheric turbulence considering inner or outer scales. A numerical simulation of the propagation of an initially spherical wave through atmospheric turbulence considering the inner scale effects was used by Hill and Frehlich[14] to calculate irradiance variance (scintillation) and variance of log irradiance. By applying a modified Rytov method,[15] Andrews *et al.*[10,16] proposed expressions for the scintillation index of a plane wave, a spherical wave, and a fully coherent Gaussian beam considering inner and outer scales that are valid under moderate-to-strong irradiance fluctuations.

Propagation characteristics of the partially coherent beam and wave scintillation effects in the atmosphere have been studied extensively.[17–21] Researchers from our group[22] have studied a uniform model of scintillation index considering inner and outer scales from weak- to strong-fluctuation regions with fully coherent Gaussian beam propagation in the slant path. In this paper, we focus on discussing the scin-

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†Corresponding author. E-mail: wuzhs@mail.xidian.edu.cn
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tillation index of partially coherent GSM beams propagating in slant atmospheric turbulence considering inner and outer scales by applying a modified Rytov method. We compare the scintillation of partially coherent GSM beams and that of Gaussian beams, as well as the scintillation of GSM beams under the von Karman and the modified Hill spectra, respectively.

2. Theoretical formulas of scintillation index

2.1. Modified Rytov theories in slant path

Using an approximation of the modified Rytov method[16,22] and considering the inner- and outer-scale effects of atmospheric turbulence, Andrews dealt approximately with the Hill spectrum under the Kolmogorov spectrum model and provided the following fluctuation spectrum of spatial refractive-index:[15,22]

$$\Phi_{nG}(\kappa, l_0, L_0) = \Phi_n(\kappa) G(\kappa, l_0, L_0), \tag{1}$$

where $\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3}$ is the Kolmogorov turbulence spectrum, C_n^2 is the atmospheric structure constant, $k = 2\pi/\lambda$ is the optical wave number in free space, and λ is the wavelength of the beam. The spatial filter function is given by

$$\begin{aligned} G(\kappa, l_0, L_0) &= G_x(\kappa, l_0, L_0) + G_y(\kappa) \\ &= f(\kappa, l_0) g(\kappa, L_0) \exp\left(-\frac{\kappa^2}{\kappa_x^2}\right) + \frac{\kappa^{11/3}}{(\kappa^2 + \kappa_y^2)^{11/6}}, \end{aligned} \tag{2}$$

where the subscripts x and y are respectively the parameters of large- and small-scale turbulence, κ_x and κ_y are the corresponding spatial cutoff frequencies, $G_y(\kappa)$ is the high-pass filter function of small-scale turbulence without considering inner and outer scales, while $G_x(\kappa, l_0, L_0)$ is the low-pass filter function of large-scale turbulence.

In Eq. (2), $g(\kappa, L_0)$ is the outer-scale factor with large-scale effects

$$g(\kappa, L_0) = 1 - \exp\left(-\frac{\kappa^2}{\kappa_0^2}\right), \quad \kappa_0 = 8\pi/L_0. \tag{3}$$

Substituting Eq. (3) into Eq. (2), the low-pass filter function of large-scale turbulence can be expressed by two items: the low-pass filter function of the inner-scale effect and the low-pass filter function of the outer-scale effect

$$G_x(\kappa, l_0, L_0) = G_x(\kappa, l_0) + G_x(\kappa, L_0), \tag{4}$$

where

$$G_x(\kappa, l_0) = f(\kappa, l_0) \exp\left(-\frac{\kappa^2}{\kappa_x^2}\right), \tag{5}$$

$$G_x(\kappa, L_0) = f(\kappa, l_0) \exp\left(-\frac{\kappa^2}{\kappa_x^2}\right) \exp\left(-\frac{\kappa^2}{\kappa_0^2}\right), \tag{6}$$

and

$$\begin{aligned} f(\kappa, l_0) &= \exp(-\kappa^2/\kappa_l^2) \\ &\quad \times [1 + 1.802(\kappa/\kappa_l) - 0.254(\kappa/\kappa_l)^{7/6}], \\ \kappa_l &= 3.3/l_0, \end{aligned} \tag{7}$$

where $f(\kappa, l_0)$ is the filter function of the modified Hill spectrum.

The modified Hill spectrum is written as follows:[22]

$$\begin{aligned} \Phi_{n\mathrm{H}}(\kappa) &= 0.033 C_n^2 [1 + a_1(\kappa/\kappa_l) - a_2(\kappa/\kappa_l)^{7/6}] \\ &\quad \times \exp(-\kappa^2/\kappa_l^2)/(\kappa^2 + \kappa_0^2)^{11/6}, \end{aligned} \tag{8}$$

where $a_1 = 1.802$, $a_2 = 0.254$, and $\kappa_l = 3.3/l_0$. Let $a_1 = a_2 = 0$ and then replace κ_l by κ_m, this spectrum model can be degenerated to the von Karman spectrum model as follows:[23]

$$\begin{aligned} \Phi_{n\mathrm{V}}(\kappa) &= 0.033 C_n^2 \exp(-\kappa^2/\kappa_m^2)/(\kappa^2 + \kappa_0^2)^{11/6}, \\ \kappa_m &= 5.92/l_0. \end{aligned} \tag{9}$$

Under the von Karman spectrum

$$f_{\mathrm{V}}(\kappa, l_0) = \exp(-\kappa^2/\kappa_m^2), \tag{10}$$

$$\begin{aligned} G_{\mathrm{V}x}(\kappa, l_0) &= f_{\mathrm{V}}(\kappa, l_0) \exp\left(-\frac{\kappa^2}{\kappa_x^2}\right) \\ &= \exp\left[-\left(\frac{1}{\kappa_m^2} + \frac{1}{\kappa_x^2}\right)\kappa^2\right], \end{aligned} \tag{11}$$

$$\begin{aligned} G_{\mathrm{V}x}(\kappa, L_0) &= f_{\mathrm{V}}(\kappa, l_0) \exp\left(-\frac{\kappa^2}{\kappa_x^2}\right) \exp\left(-\frac{\kappa^2}{\kappa_0^2}\right) \\ &= \exp\left[-\left(\frac{1}{\kappa_m^2} + \frac{1}{\kappa_0^2} + \frac{1}{\kappa_x^2}\right)\kappa^2\right]. \end{aligned} \tag{12}$$

A diagram of the slant propagation is shown in Fig. 1.

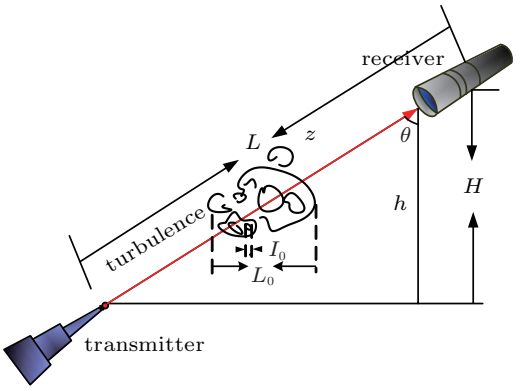


Fig. 1. (color online) Diagram of the laser beam propagation in the slant path.

In Fig. 1, θ is the path zenith angle, L is the slant length from the transmitter to the receiver, H is the vertical height of the receiver, h and z ($h \le H$, $z \le L$) are the vertical propagation height and the slant propagation distance of the beam, respectively, both of which are variables. The l_0 and L_0 are the inner scale and outer scale of the turbulence, respectively. The altitude-dependent model of the ITU-R structure constant $C_n^2(h)$ in the slant atmospheric turbulence is used.[24] Let

$0 < \xi = h/H = z/L \leq 1$, the atmospheric structure constant $C_n^2(h)$ can be written as $C_n^2(\xi H)$.

To obtain partially coherent GSM beams, a phase diffuser is placed over the laser transmitter aperture. Considering that the ensemble average of the spatially dependent random phases is Gaussian, the beam transmitted from the transmitter can be expressed in the form of a GSM beam. We can characterize the effective diffuser GSM beam at the receiver by a set of effective beam parameters Θ_e and Λ_e,[25,26]

$$\Theta_e = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2 \zeta_S}, \quad \bar{\Theta}_e = 1 - \Theta_e, \tag{13}$$

$$\Lambda_e = \frac{\Lambda_0 \zeta_S}{\Theta_0^2 + \Lambda_0^2 \zeta_S}, \tag{14}$$

where $\Theta_0 = 1 - L/F_0$, $\Lambda_0 = 2L/kw_0^2$ are the effective Gaussian wave parameters at the receiver, and w_0 and F_0 are the initial beam radius and phase front radius of curvature, respectively. $\zeta_S = 1 + 4q_c/\Lambda_0 = 1 + w_0^2/\sigma_\mu^2$ is the source coherence parameter of the beam emitted by the transmitter (if $\zeta_S = 1$, the beam is fully coherent, if $\zeta_S > 1$, the beam is partially coherent) and $q_c = L/2k\sigma_\mu^2$. σ_μ is the partial coherence length, which describes the partial coherence properties of the transmitter source.

Additional diffraction caused by the atmosphere and diffuser leads to an effective spot radius given by[27]

$$\Lambda_{rec} = \frac{2L}{kW_{rec}^2} = \frac{\Lambda_1}{1 + 4\Lambda_1 q_c + 1.63\sigma_1^{12/5}\Lambda_1}, \tag{15}$$

$$W_{rec} = W_1 \left(1 + 4q_c\Lambda_1 + 1.63\sigma_1^2\Lambda_1^{5/6}\right)^{1/2}, \tag{16}$$

where $\Lambda_1 = \Lambda_0/\left(\Theta_0^2 + \Lambda_0^2\right)$ is the effective wave parameter of the Gaussian beam after propagating for distance L through the free space at the receiver, and $W_1 = W_0\left(\Theta_0^2 + \zeta_S\Lambda_0^2\right)^{1/2}$.

Suppose small-scale fluctuations are multiplicatively modulated by large-scale fluctuations that can be applied in all of the atmospheric turbulence conditions from weak to strong fluctuations. The scintillation variance of a beam is composed of radial and longitudinal components and is written as

$$\sigma_I^2(r, L) = \sigma_{I,r}^2(r, L) + \sigma_{I,l}^2(0, L), \tag{17}$$

where r is the off-axis distance of the beam.

The radial component of the scintillation index can be closely approximated by the radial log-variance[26,28]

$$\sigma_{I,r}^2(r, L) \cong \sigma_{\ln I,r}^2(r, L) \cong 4.42\sigma_0^2\Lambda_{rec}^{5/6}\frac{r^2}{W_{rec}^2}, \quad r \leq W_{rec}, \tag{18}$$

where $\sigma_0 = (1.23C_0^2k^{7/6}L^{11/6})^{1/2}$ and C_0 is the atmospheric structure constant at zero vertical height ($h = 0$), Λ_{rec} and W_{rec} are given by Eqs. (15) and (16), and r/W_{rec} is the Fresnel ratio (concentration ratio) which represents the focusing characteristics of the beam.

For the longitudinal (axis) component of the scintillation index, the outer-scale effects on small-scale fluctuation scintillation are small, similar to the conditions of the plane and spherical wave. Therefore, the outer-scale effect is taken into account only in large-scale fluctuations. The longitudinal component of the scintillation index of a beam considering inner- and outer-scale effects and large- and small-scale turbulence fluctuations is expressed as[10]

$$\sigma_{I,l}^2(l, L) = \exp(\sigma_{\ln x, l_0}^2 - \sigma_{\ln x, L_0}^2 + \sigma_{\ln y}^2) - 1, \tag{19}$$

where $\sigma_{\ln x, l_0}^2$ and $\sigma_{\ln x, L_0}^2$ are large-scale log irradiance fluctuation variances considering inner- and outer-scale effects, respectively, and $\sigma_{\ln y}^2$ is the small-scale log irradiance fluctuation variance.

2.2. Theoretical calculation of scintillation index

The longitudinal component of the scintillation index of a GSM beam considering inner- and outer-scale effects is expressed as[16]

$$\begin{aligned}\sigma_{\ln x, l_0, L_0}^2 = {} & 8\pi^2k^2L\int_0^1\int_0^\infty \kappa\Phi_n(\kappa)G_x(\kappa, l_0, L_0) \\ & \times \exp[-(\Lambda_e L\xi^2/k)] \\ & \times \left\{1 - \cos\left[\frac{L\kappa^2}{k}\xi(1 - \bar{\Theta}_e\xi)\right]\right\} \mathrm{d}\kappa\mathrm{d}\xi,\end{aligned} \tag{20}$$

where Λ_e and $\bar{\Theta}_e$ are given by Eqs. (13) and (14).

Substituting the Kolmogorov turbulence spectrum into Eq. (20), the Rytov variance of a GSM beam propagation in the slant path is expressed as follows:[6]

$$\sigma_{I,B}^2 = 1.83\sigma_0^2\int_0^1 \frac{C_n^2(\xi H)}{C_{n0}^2}\xi^{5/6}\left(1 - \bar{\Theta}_e\xi\right)^{5/6}\mathrm{d}\xi. \tag{21}$$

2.2.1. Small-scale log irradiance fluctuation variance

Based on the modified Rytov method and optical scintillation theory of Gaussian beams,[16,22] the small-scale log amplitude fluctuation variance is written as

$$\begin{aligned}\sigma_{\ln y}^2 = {} & 8\pi^2k^2L\int_0^1\int_0^\infty \kappa\Phi_n(\kappa)G_y(\kappa)\exp(-\Lambda_e L\kappa^2\xi^2/k) \\ & \times \left\{1 - \cos\left[\frac{L\kappa^2}{k}\xi(1 - \bar{\Theta}_e\xi)\right]\right\} \mathrm{d}\kappa\mathrm{d}\xi.\end{aligned} \tag{22}$$

We can substitute the expressions of atmospheric refractive index spectrum $\Phi_n(\kappa)$ and small-scale filter function $G_y(\kappa)$ into Eq. (22), and then obtain

$$\begin{aligned}\sigma_{\ln y}^2 & \approx 1.06\sigma_0^2\int_0^1 \frac{C_n^2(\xi H)}{C_{n0}^2}\int_0^\infty (\eta + \eta_y)^{-11/6}\mathrm{d}\eta\,\mathrm{d}\xi \\ & \approx 1.27\sigma_G^2\eta_y^{-5/6},\end{aligned} \tag{23}$$

where $\eta = L\kappa^2/k$, η_y is the cutoff frequency of the filter function without dimensions,

$$\eta_y = 3 + 2.07\sigma_G^{12/5}, \tag{24}$$

σ_G^2 is the log-amplitude fluctuation variance under weak fluctuation in the slant path, which is expressed as

$$\begin{aligned}\sigma_G^2 &= 1.302k^{7/6}L^{11/6}\int_0^1 C_n^2(\xi H)\\ &\times\left\{\Gamma(-5/6)\left[a_1^{5/6}-(a_1^2+b_1^2)^{5/12}\cos\left(\frac{5}{6}\phi_1\right)\right]\right.\\ &+\frac{1.802}{Q_l^{1/2}}\Gamma(-1/3)\left[a_1^{1/3}-(a_1^2+b_1^2)^{1/6}\cos\left(\frac{1}{3}\phi_1\right)\right]\\ &+\frac{0.254}{Q_l^{7/12}}\Gamma(-1/4)\\ &\times\left.\left[a_1^{1/4}-(a_1^2+b_1^2)^{1/8}\cos\left(\frac{1}{4}\phi_1\right)\right]\mathrm{d}\xi\right\},\end{aligned}\tag{25}$$

where $Q_l=10.89L/kl_0^2$, $a_1=1/Q_l+\Lambda_e\xi^2$, $b_1=\xi(1-\bar{\Theta}_e\xi)$, and $\phi_1=\tan^{-1}(b_1/a_1)$.

Based on Eqs. (23) and (24), the small-scale log-amplitude fluctuation variance considering the inner-scale effect is given by

$$\sigma_{\ln y}^2(l_0,L)=\frac{0.509\sigma_G^2}{[1+0.69(\sigma_G^2)^{6/5}]^{5/6}}.\tag{26}$$

When the modified Hill spectrum is simplified as the von Karman spectrum, σ_G^2 is expressed as

$$\begin{aligned}\sigma_{GV}^2 &= 8\pi^2k^2L\int_0^1\int_0^\infty\kappa\Phi_n(\kappa)f_V(\kappa,l_0)\exp(-\Lambda_eL\kappa^2\xi^2/k)\\ &\times\left\{1-\cos\left[\frac{L\kappa^2}{k}\xi(1-\bar{\Theta}_e\xi)\right]\right\}\mathrm{d}\kappa\mathrm{d}\xi.\end{aligned}\tag{27}$$

Using the same derivation method as Eq. (25), σ_{GV}^2 is derived as follows:

$$\begin{aligned}\sigma_{GV}^2 &= 1.302k^{7/6}L^{11/6}\int_0^1 C_n^2(\xi H)\Gamma(-5/6)\\ &\times\left[a_m^{5/6}-(a_m^2+b_m^2)^{5/12}\cos\left(\frac{5}{6}\phi_m\right)\right]\mathrm{d}\xi,\end{aligned}\tag{28}$$

where $Q_m=35.05L/kl_0^2$, $a_m=1/Q_m+\Lambda_e\xi^2$, $b_m=\xi(1-\bar{\Theta}_e\xi)$, and $\phi_m=\tan^{-1}(b_m/a_m)$.

The expression form of small-scale log-amplitude fluctuation variance considering the inner-scale effect under the von Karman spectrum is the same as Eq. (26).

2.2.2. Large-scale scintillation index

The large-scale log-amplitude fluctuation variance with inner-scale effects under the modified Hill spectrum is given by

$$\begin{aligned}\sigma_{\ln x,l_0}^2 &= 8\pi^2k^2L\int_0^1\int_0^\infty\kappa\Phi_n(\kappa)G_x(\kappa,l_0)\\ &\times\exp[-(\Lambda_eL\xi^2/k)]\\ &\times\left\{1-\cos\left[\frac{L\kappa^2}{k}\xi(1-\bar{\Theta}_e\xi)\right]\right\}\mathrm{d}\kappa\mathrm{d}\xi,\end{aligned}\tag{29}$$

For long-distance propagation and the strong-turbulence saturation region, the geometrical optics method can be applied to large-scale intensity fluctuations. The geometrical optics approximation is applied in Eq. (29). Substituting Eq. (5) into Eq. (29), the large-scale log intensity fluctuation of the longitudinal component with finite inner-scale has the form

$$\begin{aligned}\sigma_{\ln x,l_0}^2 &= 0.652k^{7/6}L^{11/6}\int_0^1 C_n^2\xi^2(1-\bar{\Theta}_e\xi)^2\int_0^\infty\eta^{1/6}\\ &\times\exp\left[-\left(\frac{1}{Q_l}+\frac{1}{\eta_x}\right)\eta\right]\\ &\times\left[1+1.802\left(\frac{\eta}{Q_l}\right)^{1/2}-0.254\left(\frac{\eta}{Q_l}\right)^{7/12}\right]\mathrm{d}\eta\,\mathrm{d}\xi\\ &\cong 0.697k^{7/6}L^{11/6}\eta_x^{7/6}\int_0^1 C_n^2\xi^2(1-\bar{\Theta}_e\xi)^2\mathrm{d}\xi,\\ &Q_l\sigma_0^2>>100.\end{aligned}\tag{30}$$

Parameter η_x has the following form:

$$\begin{aligned}\frac{1}{\eta_x} &= \frac{0.38}{1-3.21\bar{\Theta}_e+5.29\bar{\Theta}_e^2}\\ &+0.629Q_l^{1/6}\left(\frac{\sigma_{1,B}^{1/3}\sigma_2^2}{1+2.20\bar{\Theta}_e}\right)^{6/7},\end{aligned}\tag{31}$$

where $Q_l=10.89L/kl_0^2$ and

$$\sigma_2^2=k^{7/6}L^{11/6}\int_0^1 C_n^2\xi^2(1-\bar{\Theta}_e\xi)^2\mathrm{d}\xi.$$

The large-scale log-amplitude fluctuation variance with outer-scale effects is given by

$$\begin{aligned}\sigma_{\ln x,L_0}^2 &= 8\pi^2k^2L\int_0^1\int_0^\infty\kappa\Phi_n(\kappa)G_x(\kappa,L_0)\\ &\times\exp[-(\Lambda_eL\xi^2/k)]\\ &\times\left\{1-\cos\left[\frac{L\kappa^2}{k}\xi(1-\bar{\Theta}_e\xi)\right]\right\}\mathrm{d}\kappa\mathrm{d}\xi.\end{aligned}\tag{32}$$

Substituting Eq. (6) into Eq. (32) and using the same derivation method as Eq. (30), we obtain

$$\sigma_{\ln x,L_0}^2=0.697k^{7/6}L^{11/6}\eta_{x0}^{7/6}\int_0^1 C_n^2\xi^2(1-\bar{\Theta}_e\xi)^2\mathrm{d}\xi,\tag{33}$$

where $\eta_{x0}=L\kappa_{x0}^2/k=\eta_xQ_0/(\eta_x+Q_0)$, $Q_0=64\pi^2L/(kL_0^2)$, and η_x is given by Eq. (31).

For the long-distance propagation and the strong turbulence saturation region, substituting Eq. (11) into Eq. (32), we obtain the expression of large-scale log-irradiance variance with a finite inner-scale under the von Karman spectrum, which is simplified by the modified Hill spectrum

$$\begin{aligned}\sigma_{V\ln x,l_0}^2 &= 0.652k^{7/6}L^{11/6}\int_0^1 C_n^2\xi^2(1-\bar{\Theta}_e\xi)^2\int_0^\infty\eta^{1/6}\\ &\times\exp\left[-\left(\frac{1}{Q_m}+\frac{1}{\eta_x}\right)\eta\right]\mathrm{d}\eta\,\mathrm{d}\xi\\ &= 0.652k^{7/6}L^{11/6}\Gamma\left(\frac{7}{6}\right)\left(\frac{1}{Q_m}+\frac{1}{\eta_x}\right)^{-7/6}\end{aligned}$$

$$\times \int_0^1 C_n^2 \xi^2 (1-\bar{\Theta}_e \xi)^2 \mathrm{d}\xi. \quad (34)$$

Substituting Eq. (12) into Eq. (32), the large-scale log-amplitude fluctuation variance with outer scale effects under the von Karman spectrum is given by

$$\begin{aligned}\sigma^2_{\mathrm{V}\ln x,L_0} &= 0.652k^{7/6}L^{11/6}\int_0^1 C_n^2\xi^2(1-\bar{\Theta}_e\xi)^2\int_0^\infty \eta^{1/6} \\ &\quad\times \exp\left[-\left(\frac{1}{Q_m}+\frac{1}{Q_0}+\frac{1}{\eta_x}\right)\eta\right]\mathrm{d}\eta\,\mathrm{d}\xi \\ &= 0.652k^{7/6}L^{11/6}\Gamma\left(\frac{7}{6}\right)\left(\frac{1}{Q_m}+\frac{1}{Q_0}+\frac{1}{\eta_x}\right)^{-7/6} \\ &\quad\times \int_0^1 C_n^2\xi^2(1-\bar{\Theta}_e\xi)^2\mathrm{d}\xi, \quad (35)\end{aligned}$$

where $Q_m = 35.05L/kl_0^2$, $Q_0 = 64\pi^2 L/(kL_0^2)$, and η_x is given by Eq. (31).

3. Beam scintillation index and numerical analysis

The radial component of the scintillation index with inner- and outer-scale effects is described as follows:[28]

$$\begin{aligned}&\sigma^2_{I,r}(r,l_0,L_0) \\ &\cong \sigma^2_{\ln I,r}(r,L) \\ &= 4.42\sigma^2_{1,\mathrm{B}}\Lambda_{\mathrm{rec}}^{5/6} \\ &\quad\times\left[1-1.15\left(\frac{\Lambda_{\mathrm{rec}}L}{kL_0^2}\right)^{1/6}\right]\frac{r^2}{W_{\mathrm{rec}}^2},\quad r\le W. \quad (36)\end{aligned}$$

Based on Eqs. (26), (30), and (33), the total scintillation index of partially coherent GSM beams under the modified Hill spectrum is represented as

$$\begin{aligned}\sigma_I^2(r,L) &= \sigma^2_{\ln I,r}+\exp\left[\sigma^2_{\ln x,l_0}-\sigma^2_{\ln x,L_0}\right. \\ &\quad\left.+\frac{0.509\sigma_G^2}{[1+0.69(\sigma_G^2)^{6/5}]^{5/6}}\right]-1. \quad (37)\end{aligned}$$

Using Eqs. (28), (34), and (35), the scintillation index of partially coherent GSM beams under the von Karman spectrum has the same form as Eq. (37).

If $\zeta_S = 1$ (fully coherent beam), the scintillation index model of partially coherent GSM beams in Eq. (37) can be degenerated to the result of fully coherent Gaussian beams.

In Fig. 2, open circles represent scintillation experimental data taken from Ref. [29]. For validating the results in this paper, figure 2 shows the comparisons of the results by considering the effects of inner- and outer-scale with the experimental data. It is shown that the numerical results of the scintillation with zero inner-scale are smaller than the experimental data, and the results considering inner- and outer-scales are close to the experimental data. This is consistent with the theoretical analysis in Ref. [12]. Figure 2 illustrates the theoretical results of laser beam propagation in a slant path from the weak fluctuation area to the saturated area agree well with the experimental data when considering the effects of inner- and outer-scales.

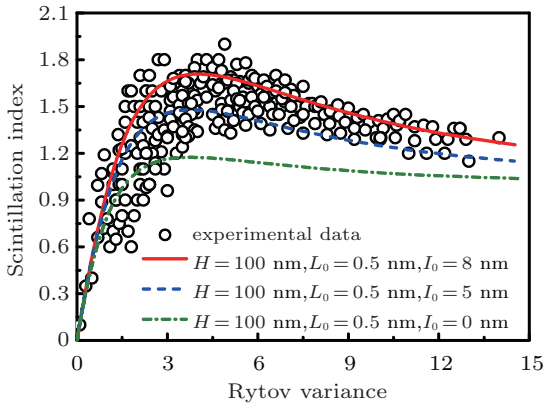


Fig. 2. (color online) Comparison of the results of modified Rytov theory considering the effects of inner- and outer-scale with the experimental data ($L_0 = 0.5$ m, $H = 100$ m, $\lambda = 0.488$ μm).

We take $C_0 = 1.7\times10^{-14}$, $\sigma_\mu = 1$ mm, $H = 500$ m, $\Theta_0 = 1$ (collimated beam), inner scales ($l_0 = 1$, 10 mm), and outer scales ($L_0 = 1$, 10 m). Based on Eq. (37), we can calculate the scintillation indices of partially coherent GSM beams with different wavelengths $\lambda = 1.06$ μm, 1.315 μm, 3.8 μm, and 10.6 μm. Scintillation indices as a function of the turbulence strength parameter $\sigma_0 = (1.23C_n^2k^{7/6}L^{11/6})^{1/2}$ are given in Figs. 3 and 4.

For the partially coherent GSM beam and the fully coherent Gaussian beam in Fig. 3, if the wavelengths are 1.06 μm and 1.315 μm, the inner-scale effects on the scintillation index are larger than the outer-scale effects in moderate-fluctuation regions. However, under the same condition, the outer-scale effects on beam scintillation are greater than the inner-scale effects at strong turbulence levels. Compared with the inner-scale effects, the outer-scale effects have a greater impact on the scintillation index of beams with wavelengths 3.8 μm and 10.6 μm over the entire turbulence range. It shows that both inner- and outer-scale effects on scintillation index should be considered when a beam goes through atmospheric turbulence.

The comparison of the scintillation index of partially coherent GSM beams with that of fully coherent Gaussian beams under different wavelengths and inner- and outer-scales is shown in Fig. 3. Theoretical studies have shown that compared with fully coherent beams, the scintillation index of partially coherent GSM beams is smaller. This is a well-understood effect that holds for the "fast-fluctuating" partially coherent waves when the high-frequency internal source fluctuations are averaged by the detector, but slow turbulence scintillations are not. Take four points (inner scales are all equal 1 cm) in Fig. 3(a): $A = (10, 3.078)$, $B = (10, 2.545)$, $C = (10, 2.201)$,

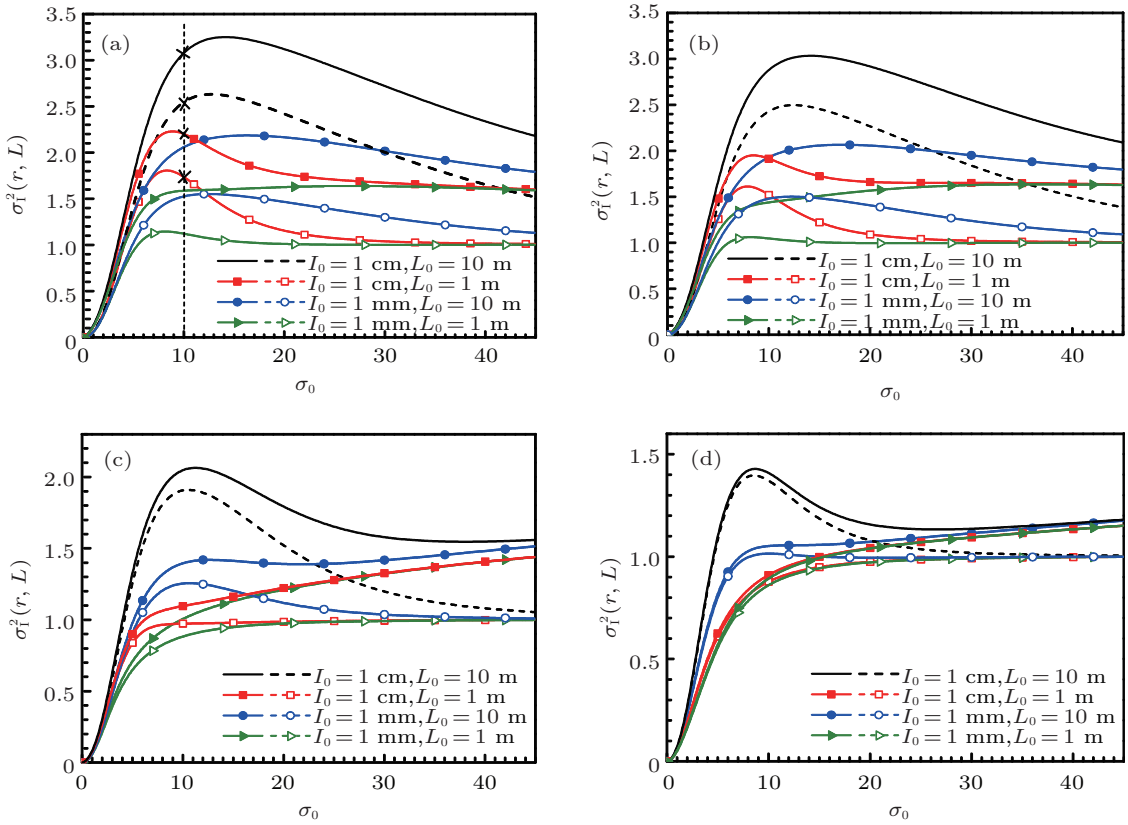


Fig. 3. (color online) Scintillation index of partially coherent GSM beams and fully coherent Gaussian beams considering inner- and outer-scale effects of turbulence under the modified Hill spectrum with $w_0 = 1$ cm for different wavelengths: (a) $\lambda = 1.06$ μm, (b) $\lambda = 1.315$ μm, (c) $\lambda = 3.8$ μm, and (d) $\lambda = 10.6$ μm.

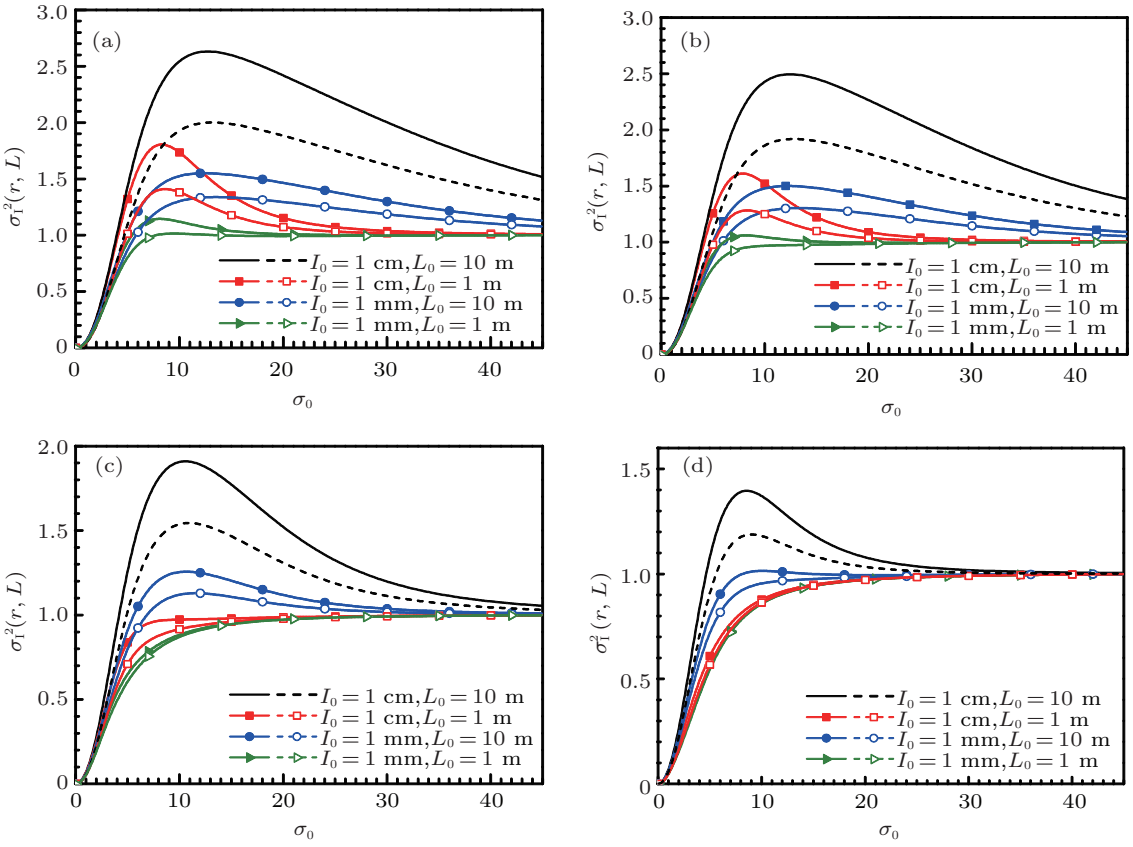


Fig. 4. (color online) Scintillation index of partially coherent GSM beams considering inner- and outer-scale effects of turbulence under von Karman (dashed line) and modified Hill (solid line) spectra for (a) $\lambda = 1.06$ μm, (b) $\lambda = 1.315$ μm, (c) $\lambda = 3.8$ μm, and (d) $\lambda = 10.6$ μm.

$D = (10, 1.735)$, in which the outer scale is 10 m at points A and B, and the outer scale is 1 m at points C and D. $|A-C|$ shows the magnitude of the effect of the outer scale on the scintillation of the fully coherent Gaussian beam, and $|B-D|$ reflects that of the outer scale on the scintillation of the partially coherent GSM beam. $|A-C| = 0.533 > |B-D| = 0.466$; that is, outer scales impose a smaller effect on the scintillation of the partially coherent GSM beams than on that of the fully coherent Gaussian beams. Similarly, inner scales impose a smaller effect on the scintillation of the partially coherent GSM beams compared with the fully coherent Gaussian beams. Figure 3 shows that in order to reduce the scintillation effect, the partially coherent beam can be in place of the fully coherent beam; even the inner- and outer-scale effects of turbulence are considered. Since the 1 cm beam width ($w_0 = 1$ cm) is close to one of the values of the inner scale ($l_0 = 1$ cm), the outer-scale effect on scintillation is stronger. It is much weaker for the 1 mm inner scale ($l_0 = 1$ mm).

Figure 4 compares the scintillation of beams with different wavelengths under Hill and von Karman spectra, respectively. The scintillation index obtained under the von Karman spectrum is smaller than that under the modified Hill spectrum (Hill spectrum adds energy in the turbulent spectrum at the inner scale range). As the beam wavelength increases, outer- and inner-scale effects on the scintillation index in both spectra gradually become small.

Results obtained show that the influence of magnitude of the inner- and outer-scale on the scintillation is hardly noticeable under weak turbulence ($\sigma_0 < 1$), but it varies notably in the moderate turbulence region while it has small variations at the strong turbulence level for GSM beams or Gaussian beams. It is due to the fact that random fluctuation of the refractive index is obvious in the moderate turbulence region, the strength of turbulence can be negligible and the variations in scintillation caused by fluctuation of the refractive index can be overcome in sufficiently weak and strong turbulence regions. Figures 3 and 4 show that increasing the wavelength of the beam seems to cause an increase in the scintillation index, which reaches saturation in the moderate turbulence region. Equations (36) and (37) show that under these two spectral models, the radial components of the scintillation index are identical, the differences of the total scintillation index are mainly reflected in the longitudinal component.

Figure 5 shows that the scintillation index increases as the slant path length increases, but the scintillation index cannot be so large by reason of the occurrence of the phenomenon of the scintillation saturation. When $L > 680$ m, the variation in the scintillation index is gentle with the slant path length. However, when $L < 680$ m, the variation visibly changes with L, and the greater the vertical height of the receiver H, the larger the scintillation index.

We can see from Eqs. (36) and (37), $r/W_{\rm rec}$ is only related to the radial component of the scintillation index $\sigma^2_{I,r}(r,L)$. The variation curves of the radial component of the scintillation index with $r/W_{\rm rec}$ are given by Fig. 6. Comparing Fig. 6 with Figs. 1–5 shows that the value of the radial component of the scintillation index is much smaller than that of the total scintillation index. Hence, the effect of $r/W_{\rm rec}$ on the total scintillation index is very small. In Fig. 6, we only discuss the effect of $r/W_{\rm rec}$ on the radial component of the scintillation index. A larger concentration ratio $r/W_{\rm rec}$ represents a worse performance of focusing. The radial component of the scintillation index increases with increasing $r/W_{\rm rec}$ (i.e., the weaker the performance of focusing of the beam, the greater the radial component of the scintillation index), and the larger the vertical heights of the receiver, the smaller the radial component of the scintillation index.

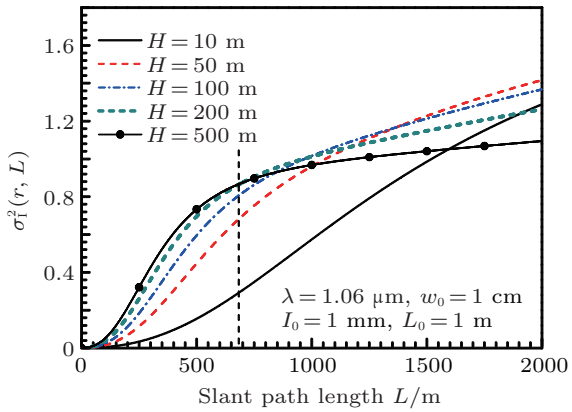


Fig. 5. (color online) Scintillation index of partially coherent GSM beams under the modified Hill spectrum versus slant path length L for different vertical heights of the receiver.

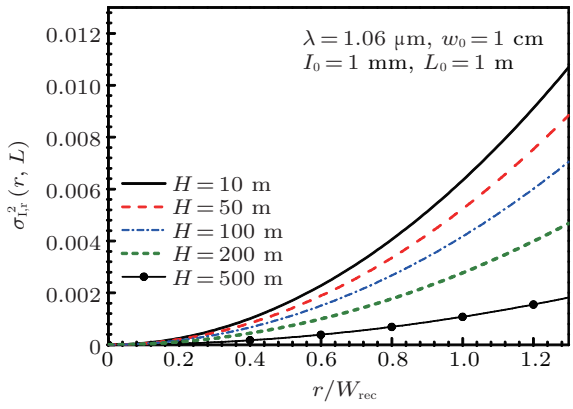


Fig. 6. (color online) Radial component of the scintillation index of partially coherent GSM beams under the modified Hill spectrum versus $r/W_{\rm rec}$ for different vertical heights of the receiver.

4. Conclusions

Based on the studies above, we obtain the following conclusions. (i) The scintillation of the partially coherent beams is smaller than that of the fully coherent Gaussian beams. (ii)

The inner and outer scales impose a smaller effect on the scintillation of the partially coherent GSM beams than on that of the fully coherent Gaussian beams. (iii) The scintillation index obtained under the von Karman spectrum is smaller than that obtained under the modified Hill spectrum. (iv) The influence of the magnitude of inner- and outer-scale on the scintillation is very small under weak turbulence, has a notable variation in the moderate turbulence region, and has little variation at the strong turbulence level. Thus, inner- and outer-scale effects on the scintillation should be considered when beams go through atmospheric turbulence. (v) The weaker the performance of focusing of the beam, the greater the radial component of the scintillation index. These assumptions may serve as theory bases for the experiments of the partially coherent GSM beam propagation through atmospheric turbulence and provide theoretical bases for future applications in the atmospheric optical communication field.

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