Fault-tolerant topology in the wireless sensor networks for energy depletion and random failure*

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Nodes in the wireless sensor networks (WSNs) are prone to failure due to energy depletion and poor environment, which could have a negative impact on the normal operation of the network. In order to solve this problem, in this paper, we build a fault-tolerant topology which can effectively tolerate energy depletion and random failure. Firstly, a comprehensive failure model about energy depletion and random failure is established. Then an improved evolution model is presented to generate a fault-tolerant topology, and the degree distribution of the topology can be adjusted. Finally, the relation between the degree distribution and the topological fault tolerance is analyzed, and the optimal value of evolution model parameter is obtained. Then the target fault-tolerant topology which can effectively tolerate energy depletion and random failure is obtained. The performances of the new fault tolerant topology are verified by simulation experiments. The results show that the new fault tolerant topology effectively prolongs the network lifetime and has strong fault tolerance.

Keywords: wireless sensor networks, fault tolerant topology, energy depletion, random failure

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1. Introduction

Wireless sensor networks (WSNs) are self-organized networks that typically consist of a large number of sensor nodes with finite energy supply.[1,2] As it is well known that the WSNs are usually deployed in remote and hostile surroundings that people cannot access. These unattended nodes in WSNs are prone to failure due to battery energy depletion and poor environment.[3–5] The failure nodes may lead to network partition, thus causing the coverage and the availability of the network to be decreased and even cause network failure.[6–8] Therefore, it is significant to build such a network topology that can effectively tolerate energy depletion and random failure.

Previously, considering the tolerance ability of a multi-connected graph, some literatures[9–12] about the fault-tolerance topology of wireless sensor networks emerged. In these literatures, the problem of developing fault-tolerance topology was treated as looking for a multiply connected graph. The k-connected network with energy-optimization to tolerate $k-1$ failure nodes by power control was produced. However, the energy optimal solution for multi-connected graph problems was proven to be NP-hard.[13] The k-connected network whose approximate optimum achieved solution was so dense that the power consumption increased rapidly.[14] Thus the k-connected topology can only tolerate 1–4 failure nodes.

Recently, some researchers found that the scale free structure can strongly tolerate the random failure,[15,16] and many articles about the scale free topology of WSNs have been appearing. A WSNs clustering topology with scale free characteristics was proposed in Ref. [17] based on the random walk mechanism. In Ref. [18], Zhu *et al*. built the energy efficient fault tolerant topology EAEM (energy–ware evolution model) by combining the residual energy of nodes with the preferential attachment mechanism of the scale free network. The constraints on the node transmission radius, energy efficiency, and fault tolerance were considered in the process of building the topology EAEM. Thus, the topology EAEM can effectively resist the random failure and selective failure on the premise of energy saving. The fitness of nodes was introduced in Ref. [19], and the calculation of fitness is associated with the value of node energy. Then a WSNs scale free topology was proposed according to the fitness topology evolution model. These WSNs scale free topologies could tolerate random failure and optimize the energy of nodes. However, these topologies ignored the node failure caused by energy depletion, which could reduce the fault-tolerance ability of topology and then have a serious effect on the performance of the network.

The energy depletion and random failure are both considered in this paper. Through analyzing the two kinds of failure, the comprehensive failure model about energy depletion and random failure is built. Then an improved evolution model is presented based on the classical preferential attachment mechanism. According to the evolution model, a fault-tolerant topology with adjusted degree distribution is generated. The tolerance of the topology depends on topology evolution model parameter τ. The optimal value of τ can be obtained through quantitative analysis. Then the topology which

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can effectively tolerate energy depletion and random failure is obtained.

The rest of this paper is organized as follows. In Section 2, we build the comprehensive failure model of nodes. In Section 3, we present the evolution model of the DDIA (degree distribution is adjustable) fault-tolerate topology and analyze the degree distribution characteristics. In Section 4, we obtain the optimal value of evolution model parameter τ. The simulation results are given in Section 5. In the end, we summarize this paper in Section 6.

2. Node failure model

In this section, the energy depletion and random failure are analyzed quantificationally. Then the comprehensive failure process model is built.

The node failure caused by energy depletion is analyzed firstly. As we all know, the energy of the network node is mainly consumed in the process of communication. The average energy consumed by the communication unit of one node is expressed as follows:

$$\begin{aligned} E_{\mathrm{c}} &= M_{\mathrm{tx}}\left[P_{\mathrm{tx}}(T_{\mathrm{tx}}+T_{\mathrm{so}})+P_{\mathrm{T}}T_{\mathrm{tx}}\right] \\ &\quad + M_{\mathrm{rx}}\left[P_{\mathrm{rx}}\left(T_{\mathrm{rx}}+T_{\mathrm{so}}\right)\right], \end{aligned} \tag{1}$$

where $M_{\mathrm{tx/rx}}$ is the working frequency of the sending/receiving unit, $P_{\mathrm{tx/rx}}$ is the average power consumption of the sending/receiving unit, and $T_{\mathrm{tx/rx}}$ is the operating time of the sending/receiving unit, which depends on the packet length L and the transmission rate R, i.e., $T_{\mathrm{tx/rx}} = L/R$. The T_{so} is the switch time of the node-state, and its value is generally in the range of several hundred microseconds, P_{T} is the transmission power of the node, which depends on the channel condition.

According to the free space channel attenuation mode, P_{T} can be expressed as

$$P_{\mathrm{T}} = P_{\mathrm{R}}(4\pi d_0/\lambda)^2(d/d_0)^n, \tag{2}$$

where P_{R} is the receiving power of the node, λ is the wave length of the carrier, d is the distance between the sending node and the receiving node, d_0 is the reference distance, its value generally is 1 meter, n is the path loss exponent and its value generally is 2∼4, here $n = 2$. Let P_0 be the minimum receiving power, based on which, the receiving node can complete the monitoring task and decode the signal successfully. Then substituting $P_{\mathrm{R}} = P_0$ into Eq. (2), the minimum transmission power of the node can be obtained, i.e., $P_{\mathrm{T}} = P_0(4\pi d/\lambda)^2$.

Suppose that the sensor nodes are uniformly distributed in the monitoring circle domain with radius r. It is easy to obtain that the probability density function is $f(x) = 1/\pi r^2$, then $(d/r)^2$ denotes the probability that each node joins into the circle domain with radius d. Therefore, $k = N(d/r)^2$, where k denotes the number of nodes in the communication circle domain with radius d. Then we can obtain

$$P_{\mathrm{T}} = \left[P_0(4\pi r)^2/N\lambda^2\right]k = \rho k, \tag{3}$$

where $\rho = P_0(4\pi r)^2/N\lambda^2$.

Assume that each node sends or receives one packet per second, i.e., $M_{\mathrm{tx/rx}} = 1$, and the switch time T_{so} of the node-state is ignored. Let E_{c}^t denote the energy consumption of the node i at time t, then based on Eqs. (1) and (3), it can be expressed as follows:

$$E_{\mathrm{c}}^t = \int_0^t E_{\mathrm{c}}\,\mathrm{d}t = (W + Sk_i)t, \tag{4}$$

where $W = (P_{\mathrm{tx}} + P_{\mathrm{rx}})L/R$, $S = \rho L/R = P_0(4\pi r)^2 L/N\lambda^2 R$, and k_i is the degree of i.

Let E_0 denote the initial energy of the nodes, t_{c} is the current moment of the network operating, and k_{c} is the reference value of k_i of the node without energy at time t_{c}. When $E_{\mathrm{c}}^t = E_0$, according to Eq. (4), k_{c} can be expressed by

$$k_{\mathrm{c}} = (E_0/t_{\mathrm{c}} - W)/S = Z_1(t_{\mathrm{c}}). \tag{5}$$

Equation (4) shows that E_{c}^t is proportional to k_i. Then based on Eq. (5), the node failure behavior caused by energy depletion at time t_{c} can be regarded as a selective failure process. During this process, the nodes are removed in accordance with the variation of node degrees from big to small, until the nodes whose degrees are greater than k_{c} are all removed.

The node failure behavior caused by the poor environment is usually regarded as the random failure process. Suppose that such failure follows a uniform distribution in the time domain T, and the total number of such failure nodes is Ω. Then the number of such failure nodes at time t_{c} can be calculated by

$$f_{\mathrm{c}} = \int_0^{t_{\mathrm{c}}} \frac{\Omega}{T}\,\mathrm{d}t = \frac{\Omega}{T}t_{\mathrm{c}} = Z_2(t_{\mathrm{c}}). \tag{6}$$

That is to say, at time t_{c}, f_{c} nodes will fail due to the poor environment and be removed in the network.

To sum up, k_{c} is obtained based on Eq. (5) and f_{c} is obtained based on Eq. (6). In WSNs, at time t_{c}, k_{c} is the node degree threshold of the failure node caused by energy depletion, and f_{c} is the number of failure nodes caused by the poor environment. In the specific application scenario, k_{c} and f_{c} only depend on the network running time t_{c}, and they can be expressed by the functions $Z_1(t_{\mathrm{c}})$ and $Z_2(t_{\mathrm{c}})$, respectively. During the initial period of network operation, environment damage is the main reason for node failure. During the later period of network running, energy depletion is the main reason for node failure. Thereby, based on Eqs. (5) and (6), the comprehensive failure behavior caused by both energy depletion and environment damage can be regarded as two independent

processes of nodes being removed. Then the comprehensive failure process model is described as follows. In the process of network operation, first, there are f_c nodes being removed randomly, then some nodes are removed selectively in accordance with the variation of node degrees from big to small, until the nodes whose degrees are greater than k_c are all removed.

3. DDIA topology evolution model and dynamic characteristics analysis

In this section, we propose an improved evolution model according to the preferential attachment mechanism. Based on the evolution model, a fault tolerance topology whose degree distribution can be adjusted (DDIA topology) is generated. This topology can tolerate the comprehensive failure by adjusting the parameter τ, and the degree distribution characteristic of DDIA topology is studied to quantify the topological structure properties, which helps us to better study the fault tolerance of the WSNs topology.

3.1. The evolution model of DDIA topology

The preferential attachment mechanism was first applied to the BA model proposed by Barabási and Albert. According to the classical preferential attachment mechanism, we can obtain a stationary network topology with a power-law probability distribution for the degree $p(k) \sim k^{-\lambda}$. Compared with the network topology with the exponential probability distribution for the degree $p(k) \sim \mathrm{e}^{-k}$, this topology has better fault tolerance for random node failure, but poor fault tolerance for selective failure.

During the process of topology evolution, the restriction on transmission radius should be considered, simultaneously, the requirements on fault tolerance for random failure and selective failure should be taken into consideration. Therefore, we improve the preferential attachment mechanism, and the improved evolution model of DDIA topology is described as follows.

1) The network grows incrementally, starting with a small clique of m_0 nodes. At each successive time step, a new node is added such that its $m \leq m_0$ edges link it to m nodes already in the graph.

2) The probability Π with which a new node will be connected to node i depends on the current degree k_i of i and the preferential measure factor τ. The preferential attachment probability $\Pi(k_i)$ of node i is expressed as

$$\Pi(k_i) = \frac{k_i^\tau}{\sum\limits_{j\in A} k_j^\tau}, \quad \tau \in [0,+\infty) \cap \tau \neq 1, \tag{7}$$

where A is the neighbor node set of the new node.

The connection probability between nodes changes dynamically. Therefore, through adjusting the evolution model parameter τ, the DDIA topology can approximately follow a power-law distribution and also can follow an exponential distribution. Then it is known that the DDIA topology can tolerate the random failure and selective failure by adjusting the evolution model parameter τ.

3.2. Analysis of DDIA topology evolution model

Suppose that the new node Q joins the network at time t. Let r_0 be the initial radius of the network, then r_t is the radius of the network at time t, and r_Q is the transmission radius of the new node Q.

In order to utilize the mean field theory to analyze the performance of the evolving topology, we assume that the network undergoes a long time evolution and the node degree k_i is a continuous change parameter with t, so the growth rate of k_i can be described as follows:

$$\frac{\partial k_i}{\partial t} = m\Pi(k_i). \tag{8}$$

Note that the nodes in the monitoring area follow a uniform distribution, the probability that the neighbor node set of the new node Q is selected can be expressed by the area ratio $(r_Q^2/2)/r_t^2$, so $\Pi(k_i)$ can be shown as follows:

$$\Pi(k_i) = \frac{k_i^\tau}{\sum\limits_{j\in A} k_j^\tau} = \frac{r_Q^2/2}{r_t^2}\frac{k_i^\tau}{N_t(r_Q^2/2r_t^2)\langle k\rangle_t^\tau} = \frac{k_i^\tau}{N_t\langle k\rangle_t^\tau}, \tag{9}$$

where N_t is the number of network nodes at time t, and $\langle k\rangle_t$ is the average degree of network nodes at time t. According to the improved preferential attachment, at time t, a topology with m_0+t nodes and mt edges is generated, i.e., $\langle k\rangle_t = 2mt/(m_0+t)$.

Based on Eqs. (8) and (9), the growth rate of k_i can be expressed as follows:

$$\begin{aligned}\frac{\mathrm{d}k_i}{\mathrm{d}t} &= m\frac{k_i^\tau}{(m_0+t)[2mt/(m_0+t)]^\tau} \\ &= 2^{-\tau}m^{1-\tau}\frac{k_i^\tau}{t}.\end{aligned} \tag{10}$$

By using the separation variable method, the solution of the differential equation above can be obtained

$$k_i(t) = [(2^{-\tau}m^{1-\tau}\ln t + C)(1-\tau)]^{\frac{1}{1-\tau}}, \tag{11}$$

and based on the initial conditions $k_i(t_i) = m$, we can obtain

$$k_i(t) = m\left[2^{-\tau}(1-\tau)\ln\frac{t}{t_i} + 1\right]^{\frac{1}{1-\tau}}, \tag{12}$$

then the probability $p\,(k_i(t) < k)$ can be expressed as follows:

$$p(k_i(t) < k) = p(t_i > \mathrm{e}^{\frac{1-(k/m)^{1-\tau}}{2^{-\tau}(1-\tau)}}t). \tag{13}$$

During the evolution process of the DDIA fault tolerant topology, only one new node is added to the network at each

time step, so t_i follows a uniform distribution, and its probability density function is $p(t_i) = 1/(m_0 + t)$. Substituting $p(t_i) = 1/(m_0 + t)$ into Eq. (13), we can obtain the degree distribution expression of DDIA fault tolerant topology as follows:

$$p(k) = \frac{\partial p(k_i(t) < k)}{\partial k} = \frac{\partial (1 - p(t_i \le \mathrm{e}^{\frac{1-(k/m)^{1-\tau}}{2^{-\tau}(1-\tau)}} t))}{\partial k} = \frac{2^{\tau}}{m}\left(\frac{k}{m}\right)^{-\tau} \mathrm{e}^{\frac{1-(k/m)^{1-\tau}}{2^{-\tau}(1-\tau)}} = Z_3(\tau). \quad (14)$$

It is shown that, in Eq. (14), when the edge growth scale m is given, the degree distribution of DDIA fault tolerant topology depends on the evolution model parameter τ. When τ is infinite, the network has a preference, which leads to that the new node prefers to connect to the node with a bigger degree. Therefore, the degree distribution follows the power-law distribution $p(k) \sim k^{-\lambda}$ approximately, and the DDIA topology has a strong fault tolerance for the random failure. When $\tau = 0$, the network has no preference, the degree distribution follows the exponential distribution $p(k) \sim \mathrm{e}^{-k}$ approximately, and the DDIA topology can effectively tolerate selective failure. All in all, the DDIA fault tolerant topology can tolerate the random failure and selective failure by adjusting the evolution model parameter τ.

4. Analysis of evolution model parameter τ

In this section, a measurement index of the WSNs network topological fault tolerance is proposed to measure the fault tolerance of the DDIA topology. Then we give the process of how to acquire the optimal value of the evolution model parameter τ, and the strong fault tolerant topology with an optimal value of τ is obtained.

4.1. Fault tolerance measurement of WSNs topology

When the network topology is divided, only the largest connected component is available, and its scale can reflect the performance of the network. Therefore, the ratio of the total number of nodes in the largest component to the total number of nodes in the initial network is usually used to measure the fault tolerance of the topology structure. However, this measurement index cannot reflect the energy efficiency of the network topology. Therefore, a new measurement index of the WSNs topology fault-tolerance is defined. The network survival time can reflect the network energy efficiency to a certain extent. When the network is divided, the network scale begins to decrease until reducing to the minimum scale of the largest connected component that meets the network application requirements, and this period of time is defined as the network survival time. Based on this network survival time, the new measurement index of the WSNs topological fault-tolerance is given as follows.

Definition For a network $G(V,E)$, suppose that the network monitoring requirement is θ $(0 < \theta \le 1)$. At time t_c, when the failure nodes in the network $G(V,E)$ have been removed, the residual network is composed of w connected components, i.e., $G_1 = (V_1,E_1)$, ..., $G_w = (V_w,E_w)$. Then the number of available nodes in the network is $A(N) = \max\{|V_1|,|V_2|,\ldots,|V_w|\}$. At this moment, the scale of the largest connected component is $C(N)_{t_c} = A(N)/N$, and at time t_{c+1}, the scale of the largest connected component is $C(N)_{t_c+1}$. When $C(N)_{t_c} \ge \theta \cap C(N)_{t_c+1} < \theta$, t_c is the measurement index of the fault tolerance of the network $G(V,E)$.

The definition above shows that the fault tolerance of the WSNs topology is denoted by the longest time of the network maintaining the application requirements. Namely, t_c is bigger, the lifetime of the topology is longer, and the ability of the topology to tolerate the comprehensive failure behavior is better.

4.2. Acquisition of the optimal value of τ

Based on the new measurement index of the WSNs network topology, the process of acquiring the optimal value of τ is described as follows.

Input $k_c = Z_1(t_c)$, $f_c = Z_2(t_c)$, $p(k) = Z_3(\tau)$, and θ, k_c is the node degree threshold of the failure node caused by the energy depletion at time t_c, f_c is the number of failure nodes caused by poor environment at time t_c, $p(k)$ is the degree distribution of the DDIA topology, and θ is the network application requirement.

Output the optimal value of τ. At this moment, the fault tolerance of the DDIA topology is strongest under comprehensive failure, i.e., the value of t_c is the biggest.

1) The degree distribution $p(k_{II}) = Z_5(\tau,t_c)$ of the DDIA topology under comprehensive failure at time t_c is deduced based on $p(k) = Z_3(\tau)$.

2) The scale $C(N)_{t_c} = Z_6(\tau,t_c)$ of the largest connected component is obtained according to $p(k_{II}) = Z_5(\tau,t_c)$.

3) When the value of t_c is maximum, the corresponding τ is obtained based on $C(N)_{t_c} \ge \theta \cap C(N)_{t_c+1} < \theta$.

4.2.1. Degree distribution $p(k_{II})$

Based on the comprehensive failure model, suppose p_I is the probability that the node is removed randomly. According to Eq. (6), we can obtain $p_I = f_c/N = \Omega t_c/NT$. After the DDIA topology suffers environmental damage, some nodes in the network fail. Therefore, the degree of the survival nodes become

$$\binom{k}{k_I}(1-p_I)^{k_I} p_I^{k-k_I},$$

where k is the original degree of the survival nodes. Then the degree distribution of the DDIA topology under random failure becomes

$$p(k_I) = \sum_{k=k_I}^{\infty} p(k)\binom{k}{k_I}(1-p_I)^{k_I} p_I^{k-k_I} = Z_4(\tau,t_c). \quad (15)$$

At time t_c, after some nodes fail due to the energy depletion, all the nodes whose degree is greater than k_c in the DDIA topology are removed. Then the maximum degree of the nodes and the degree distribution $p(k_I)$ change. The process of removing the network nodes selectively can be regarded as the process of removing the links randomly. Let $\tilde{p}$ be the probability that the links are removed randomly. It can be expressed by the ratio of the number of links connected to the nodes without energy to the number of the total links in the network

$$\tilde{p} = \sum_{k_I=k_c}^{\infty} \frac{k_I p(k_I)}{\langle k_I \rangle}, \tag{16}$$

where $\langle k_I \rangle$ is the average node degree of the topology with degree distribution $p(k_I)$. The $\langle k_I \rangle$ can be expressed by $\langle k_I \rangle = \langle k \rangle (1-p_I)$ based on the average node degree $\langle k \rangle$ of the DDIA topology. Then at time t_c, after some nodes fail due to the energy depletion, the degree distribution $p(k_{II})$ of the DDIA topology under comprehensive failure is expressed as follows:

$$p(k_{II}) = \sum_{k_I=m}^{k_c} p(k_I) \begin{pmatrix} k_I \\ k_{II} \end{pmatrix} (1-\tilde{p})^{k_{II}} \tilde{p}^{k_I - k_{II}} = Z_5(\tau, t_c), \tag{17}$$

where m is the minimum degree of the nodes.

4.2.2. The optimal value of τ

The degree distribution $p(k_{II})$ has been obtained from the section above. According to the probability generating function, we can deduce the scale $C(N)_{t_c}$ of the largest connected component of the DDIA topology under comprehensive failure. Then the optimal value of τ can be obtained. The probability generating function of the degree distribution $p(k_{II})$ can be expressed as follows:

$$g_0(x) = \sum_{k_{II}=m}^{k_c} p(k_{II}) x^{k_{II}}. \tag{18}$$

Let $p_L(k_{II})$ be the probability of arriving at the node with degree k_{II} by following any direction of a randomly selected link, and $p_L(k_{II})$ is proportional to not only the number of nodes with degree k_{II} but also the value of k_{II}, then $p_L(k_{II})$ can be expressed by

$$p_L(k_{II}) = k_{II} p(k_{II}) \Big/ \sum_{k_{II}=m}^{\infty} k_{II} p(k_{II}). \tag{19}$$

Then the probability generating function of arriving at a node with residual degree k_{II} by following any direction of a randomly selected link can be expressed by

$$\begin{aligned} g_1(x) &= \sum_{k_{II}=m}^{k_c} p_L(k_{II}) x^{k_{II}-1} = \frac{\sum_{k_{II}=m}^{k_c} k_{II} p(k_{II}) x^{k_{II}-1}}{\sum_{k_{II}=m}^{k_c} k_{II} p(k_{II})} \\ &= \frac{\sum_{k_{II}=m}^{k_c} k_{II} p(k_{II}) x^{k_{II}-1}}{\langle k_{II} \rangle}. \end{aligned} \tag{20}$$

Suppose $h_1(\psi)$ is the probability of arriving at a connected component s with scale ψ by following any direction of a randomly selected link. Then its probability generating function is

$$H_1(x) = \sum_{k_{II}=0}^{\infty} h_1(\psi) x^{k_{II}}. \tag{21}$$

Suppose that the connected component s cannot be the largest connected component. The $H_1(x)$ can be expressed by the recursive form as follows:

$$\begin{aligned} H_1(x) &= x p_L(0) + x p_L(1) H_1(x) + x p_L(2) [H_1(x)]^2 + \cdots \\ &= x g_1(H_1(x)). \end{aligned} \tag{22}$$

Similarly, suppose that $h_0(k_{II})$ is the probability that a randomly selected node belongs to the connected component s. Then its probability generating function is

$$H_0(x) = \sum_{k_{II}=0}^{\infty} h_0(\psi) x^{k_{II}}. \tag{23}$$

Being similar to Eq. (22), we can obtain

$$H_0(x) = x g_0(H_1(x)). \tag{24}$$

Then according to Eqs. (18) and (24), the largest connected component scale $C(N)_{t_c}$ can be expressed by

$$\begin{aligned} C(N)_{t_c} &= 1 - H_0(1) = 1 - g_0(H_1(1)) \\ &= 1 - \sum_{k_{II}=m}^{k_c} p(k_{II}) H_1(1)^{k_{II}} = Z_6(\tau, t_c), \end{aligned} \tag{25}$$

where $H_1(1)$ is the smallest nonnegative real number solution of the equation $H_1(1)\langle k_{II} \rangle = \sum_{k_{II}=m}^{k_c} k_{II} p(k_{II}) H_1(1)^{k_{II}-1}$ obtained by Eqs. (20) and (22).

The network under comprehensive failure is just in the critical state of collapse when $t = t_c$. According to $C(N)_{t_c} \geq \theta \cap C(N)_{t_c+1} < \theta$, when θ is given, we can obtain the mathematical relationship between τ and t_c based on the theoretical derivation method above, and when the fault tolerance measurement index t_c of the WSNs topology is maximum, the fault tolerance of the DDIA topology is the best. At this moment, the value of τ is optimal.

5. Simulation results

Extensive simulation tests were carried out using the MATLAB tool. In order to ensure the practical significance of our topology, the deviation of the degree distribution of the DDIA topology is analyzed for different values of parameters τ in the specific application scenario, and the range of τ is determined. Then the performances between the DDIA topology and the classical topology EAEM are compared.

In the simulation, suppose that the initial network scale is m_0 and the initial node energy in the two topology algorithms are the same. All the following simulation results are the average of 50 repeated tests. The simulation parameters are shown in Table 1.

Table 1. The experimental parameters.

Parameter name	Value
Node number N	100
Monitoring circle domain radius r	500 m
Nodes initial energy E_0	1 J
Node maximum transmission radius r_{max}	150 m
The minimum receiving power P_0	−94 dBm
Preferential measure factor τ	0–∞
Packet length L	4000 bits
Date transmission rate R	250 kbps
The carrier wave length λ	69 cm
The sending/receiving unit power consumption $P_{tx/rx}$	50 nJ/bit
Random failure nodes Ω	100
Random failure time domain T	1.0×10^8

5.1. The range of τ

Table 2 shows the deviation of the theory degree distribution and the actual degree distribution of the DDIA topology for the different values of parameters θ ($\theta = 0$, 0.4, 0.8, 1.2, 1.6, 2, 40). From Table 2, we observe that the deviation between the theory degree distribution and the actual degree distribution of the DDIA topology increases with τ. In order to ensure that the fault tolerant topology built above is valid in the practical application, all the simulation experiments are done under $\tau \in [0, 1.6] \cap \tau \neq 1$.

Table 2. The deviation of the theory degree distribution and the actual degree distribution of the DDIA topology.

τ	Average value of deviation
0	0.0801
0.4	0.1358
0.8	0.1674
1.2	0.2188
1.6	0.3104
2.0	0.3789
40	1.831×10^{11}

5.2. Topology performance contract

The optimal value of τ for the different task requirements parameters θ can be obtained according to the theoretical derivation method above and the MATLAB tool. In the following simulations, we choose the DDIA topology ($\theta = 0.5$, $\tau = 1.5$) to study the topological performance.

5.2.1. The topology fault tolerance contrast

In the experiments, random failure is regarded as the process that the nodes are removed randomly in the network, and selective failure is regarded as the process that the nodes are removed selectively according to the node degree descending.

In Fig. 1, the random failure tolerance of the DDIA topology and the EAEM topology is contrasted, and the corresponding relationship between the largest connected component scale and the random failure node radio is shown. From Fig. 1, we can observe that the network scale of the DDIA topology is larger than the EAEM topology when the same number of random failure nodes are removed, which indicates that the DDIA topology can tolerate more random failure nodes than the EAEM topology. Hence, the random failure tolerance of the DDIA topology is better than the EAEM topology.

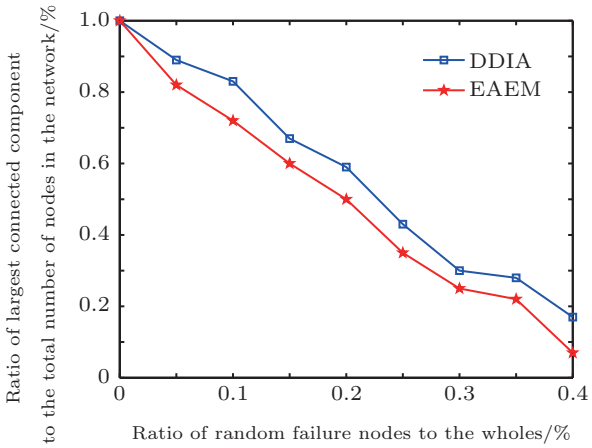


Fig. 1. (color online) The topology random failure tolerance contrast.

Figure 2 shows the changes of the largest connected component scale with the selective failure node ratio for the DDIA topology and the EAEM topology. As shown in Fig. 2, when the same number of nodes are removed selectively, the network scale of the DDIA topology is larger than in the EAEM topology, which indicates that the DDIA topology can complete the monitoring task better than the EAEM topology under selective failure. Therefore, the DDIA topology has a stronger ability to tolerate selective failure nodes than the EAEM topology.

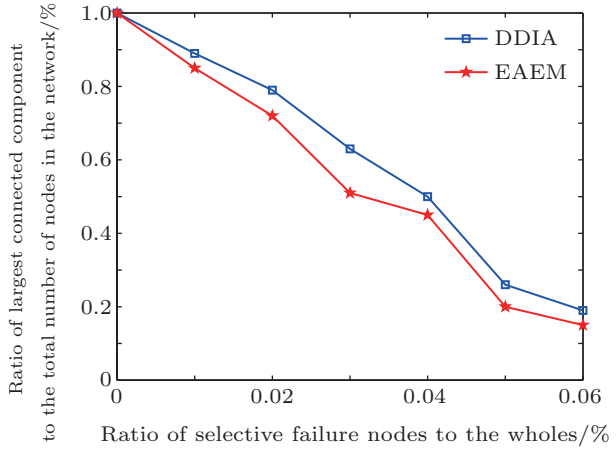


Fig. 2. (color online) The topology selective failure tolerance contrast.

5.2.2. The network lifetime and energy efficiency contrast

The network lifetime is usually regarded as the time when the first nodes in the network fail. In Fig. 3, we show that the network lifetime versus the survival nodes number for the

DDIA topology and the EAEM topology. Figure 3 presents that the network lifetime of the DDIA topology is always longer than the EAEM topology, which shows that the DDIA topology can effectively prolong the network lifetime.

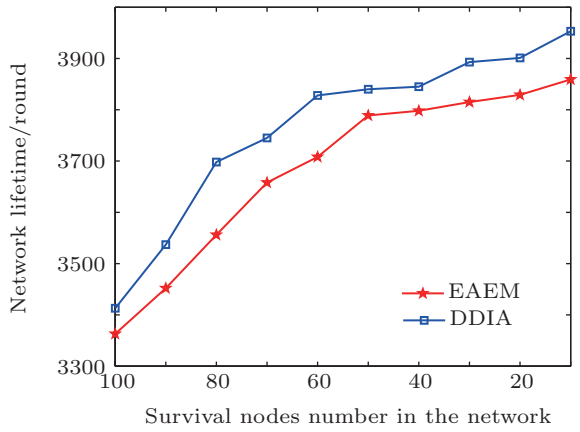


Fig. 3. (color online) The network lifetime contrast.

The energy efficiency is an important part of a topology structure performance. The survival time of the energy exhausting node reflects the network energy efficiency to a certain extent, namely, the time at which some nodes fail owing to the energy depletion is longer, the network utilization is higher. Figure 4 shows the average survival time and the minimum survival time of nodes whose energy is exhausted in the DDIA topology and the EAEM topology. From Fig. 4, we observe that the average survival time and the minimum survival time of the energy exhausting node in the DDIA topology are longer than those in the EAEM topology. Compared with the EAEM topology, the DDIA topology has a higher energy efficiency.

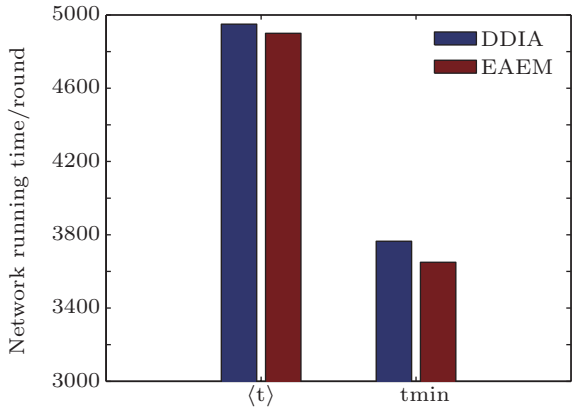


Fig. 4. (color online) The network energy efficiency contrast.

6. Conclusions

In this paper, considering energy depletion and random failure, a comprehensive failure model about the two kinds of failure behaviors is built, and an improved evolution model of the DDIA fault-tolerant topology is proposed. Moreover, we analyze the relationship between the comprehensive failure model and the dynamics characteristics of the DDIA topology. Then the degree distribution of the DDIA topology under the comprehensive failure is obtained. Furthermore, according to a new fault tolerance measurement index, the corresponding optimal value of the evolution model parameter is acquired, and the corresponding optimal DDIA topology that can effectively tolerate both energy depletion and random failure is obtained. Experimental tests demonstrate that the DDIA topology has a higher energy efficiency and stronger fault tolerance than the EAEM topology, and it also can effectively prolong the network lifetime.

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