PC synchronization of a class of chaotic systems via event-triggered control*

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The PC synchronization of a class of chaotic systems is investigated in this paper. The drive system is assumed to have only one state variable available. By constructing proper observers, some novel criteria for PC synchronization are proposed via event-triggered control scheme. The Lü system and Chen system are taken as examples to demonstrate the efficiency of the proposed approach.

Keywords: chaotic system, state observer, event-triggered control

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1. Introduction

Since the synchronization between two identical chaotic systems with different initial conditions has been proposed by Pecora and Carroll in 1990,[1] synchronization in chaotic systems has received considerable attention in various fields such as in secure communication, biological systems, chemical reactions, information science, etc. After the complete synchronization (PC) proposed in Ref. [1], different types of synchronization have been investigated theoretically and experimentally, such as: the lag synchronization,[2] Cluster synchronization,[3] generalized synchronization,[4] projective synchronization,[5] and modified projective synchronization (MPS),[6] combination synchronization.[7] Nowadays, many control methods have been proposed for the synchronization of chaos systems, for example: linear control,[8] backstepping design,[9] sliding-mode control,[10] LMI approach,[11] adaptive control,[12] impulsive control,[13] etc.

Recently, the event-triggered control has been studied extensively in networked control systems.[14–16] In event-triggered control, the measurement error plays an essential role in the event design. The control signal is kept constant until the error reaches the prescribed threshold. If the error exceeds a threshold, then an event is triggered and the controller is updated. The possibility of reducing the number of re-computations, and thus of transmissions makes event-triggered control very appealing in networked control systems (NCSs). It is well known that the complex network is composed of interconnected nodes, in which each node is a nonlinear dynamical system. It is noted that the chaotic system is nonlinear dynamical system and can been seen as a simplest network, thus a natural question is raised: can we control or synchronize chaotic system via event-triggered control? Unfortunately, to the best of the authors' knowledge, the control and synchronization of chaotic system via event-triggered control have not been reported in the open literatures.

In the literature, there are two types of drive-response synchronization framework. One of them is that the drive system and response system are all chaotic systems. In this paper, we refer to this as chaos-based synchronization. Another synchronization is the so called observer-based synchronization. In the observer-based synchronization scheme, the drive system is a chaotic system and the response system is an observer of the drive system.

There are many results for chaos-based synchronization in Refs. [1]–[13] and [17]–[24]. However, most of the existing literature concerning on the chaos-based synchronization problem have been investigated under the condition that the states of the drive system and the response system are completely known. It is well known that the states both in the drive and response systems are partially or fully unavailable in many practical control problems. Thus, a natural and interesting question is: can we synchronize two different chaotic systems with only partial state variables available?

Motivated by the above discussion, the PC synchronization of a class of chaotic systems is investigated in this paper. The drive system is assumed to have only one state variable available. By constructing proper observers, some novel criteria for PC synchronization are proposed via event-triggered control scheme. The Lü system and Chen system are taken as examples to demonstrate the efficiency of the proposed approach.

The layout of the rest of this paper is organized as follows.

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A brief description of a class of chaotic systems is introduced in Section 2. Section 3 investigates the design of the observer of a class of chaotic systems. The PC synchronization scheme of a class of chaotic systems is proposed in Section 4. Section 5 includes several numerical examples to demonstrate the effectiveness of the proposed approach. Finally, some conclusions are shown in Section 6.

2. System description

Consider the following class of chaotic system:

$$\begin{cases} \dot{x}_1 = a_1 x_2 + f_1(x_1), \\ \dot{x}_2 = b_1 x_1 x_3 + c_1 x_2 + g_1(x_1), \\ \dot{x}_3 = p_1 x_1 x_2 - d_1 x_3 + h_1(x_1), \end{cases} \tag{1}$$

where $x = (x_1, x_2, x_3)^{\mathrm{T}} \in R^{3\times 1}$ is the state vector of system (1), $f_1(x_1), g_1(x_1), h_1(x_1)$ are functions of x_1, $a_1 \neq 0, d_1 > 0$ and b_1, c_1, p_1 are the system's parameters, which are known in advance.

Remark 1 Many chaotic systems can be written as in the form of (1), such as the Lü system,[25] the Chen system,[26] the Lorenz system,[27] the unified system,[28] the Liü system,[29] etc.

Suppose system (1) is the drive system, then the controlled response system is

$$\begin{cases} \dot{y}_1 = a_2 y_2 + f_2(y_1) + u_1(t_k), \\ \dot{y}_2 = b_2 y_1 y_3 + c_2 y_2 + g_2(y_1) + u_2(t_k), \\ \dot{y}_3 = p_2 y_1 y_2 - d_2 y_3 + h_2(y_1) + u_3(t_k), \\ t \in [t_k, t_{k+1}), \quad k = 0, 1, 2, \ldots, \end{cases} \tag{2}$$

where $x = (y_1, y_2, y_3)^{\mathrm{T}} \in R^{3\times 1}$ is the state vector of system (2), $f_2(y_1)$, $g_2(y_1)$, $h_2(y_1)$ are functions of y_1, a_2, d_2 and b_2, c_2, p_2 are system's parameters, which are known in advance. $u_i(t_k)$, $i = 1, 2, 3$ are controllers to be designed.

Before going any further, we make the following assumptions.

Assumption 1 Suppose that the available state variables of drive system is x_1, which is the only variable that can be transmitted to the response system.

Assumption 2 Suppose that f_i, g_i, h_i are Lipschitz functions, i.e., there exist nonnegative constants $L_{f_i}, L_{g_i}, L_{h_i}$ such that $|f_i(x) - f_i(y)| \leq L_{f_i}|x - y|, |g_i(x) - g_i(y)| \leq L_{g_i}|x - y|, |h_i(x) - h_i(y)| \leq L_{h_i}|x - y|, i = 1, 2$.

Assumption 3 There exists nonnegative constants M such that $|x_i| \leq M, |y_i| \leq M, |\hat{x}_2| \leq M, |\hat{x}_3| \leq M, i = 1, 2, 3$.

Assumption 1 means that the controllers u_1, u_2, and u_3 in system (2) are functions of x_1, y_1, y_2, y_3. Without loss of generality, we can assume that $u_i = u_i(x_1, \hat{x}_2, \hat{x}_3, y_1, y_2, y_3)$, where $\hat{x}_2, \hat{x}_3$ are the estimated values of x_2, x_3,, respectively.

The purpose of this paper is to design a suitable controller such that systems (1) and (2) can achieve PC synchronization. In order to synchronize systems (1) and (2), we shall estimate x_2, x_3. For this purpose, we propose the scheme of constructing observers for system (1).

3. Design of observer for a class of chaotic system

In this section, the observer for system (1) is given. For the purpose of giving the observer of system (1), two cases are discussed in this section.

Case 1 $b_1 + p_1 = 0$. In this case, we obtain the following observer for system (1):

$$\begin{cases} \hat{x}_2 = w + \dfrac{c_1 + 1}{a_1} x_1, \\ \dot{w} = -w + g_1(x_1) + b_1 x_1 \hat{x}_3 - \dfrac{c_1 + 1}{a_1}(x_1 + f_1(x_1)), \\ \dot{\hat{x}}_3 = p_1 x_1 \hat{x}_2 - d_1 \hat{x}_3 + h_1(x_1), \end{cases} \tag{3}$$

where $\hat{x}_2$ and $\hat{x}_3$ are the estimated values of x_2 and x_3, respectively.

If we define $e_{x_2} = \hat{x}_2 - x_2, e_{x_3} = \hat{x}_3 - x_3$, then from systems (1) and (3) we can obtain

$$\begin{aligned} \dot{e}_{x_2} &= \dot{\hat{x}}_2 - \dot{x}_2 = \dot{w} + \frac{c_1 + 1}{a_1}\dot{x}_1 - \dot{x}_2 \\ &= -w + g_1(x_1) + b_1 x_1 \hat{x}_3 - \frac{c_1 + 1}{a_1}(x_1 + f_1(x_1)) \\ &\quad + \frac{c_1 + 1}{a_1}(a_1 x_2 + f_1(x_1)) - \dot{x}_2 \\ &= -(\hat{x}_2 - \frac{c_1 + 1}{a_1} x_1) + g_1(x_1) + b_1 x_1 \hat{x}_3 \\ &\quad - \frac{c_1 + 1}{a_1} x_1 + (c_1 + 1) x_2 - \dot{x}_2 \\ &= -\hat{x}_2 + g_1(x_1) + b_1 x_1 \hat{x}_3 + (c_1 + 1) x_2 \\ &\quad - (b_1 x_1 x_3 + c_1 x_2 + g_1(x_1)) \\ &= -e_{x_2} + b_1 x_1 e_{x_3}, \end{aligned}$$

and

$$\begin{aligned} \dot{e}_{x_3} &= \dot{\hat{x}}_3 - \dot{x}_3 = p_1 x_1 \hat{x}_2 - d_1 \hat{x}_3 + h_1(x_1) \\ &\quad - (p_1 x_1 x_2 - d_1 x_3 + h_1(x_1)) \\ &= p_1 x_1 e_{x_2} - d_1 e_{x_3}. \end{aligned}$$

Thus, we get the error dynamic system as follows:

$$\begin{cases} \dot{e}_{x_2} = -e_{x_2} + b_1 x_1 e_{x_3}, \\ \dot{e}_{x_3} = p_1 x_1 e_{x_2} - d_1 e_{x_3}. \end{cases} \tag{4}$$

For system (4) we can prove that

$$\lim_{t\to\infty} e_{x_2} = \lim_{t\to\infty} e_{x_3} = 0.$$

In fact, we choose the Lyapunov candidate as

$$V = \frac{1}{2}(e_{x_2}^2 + e_{x_3}^2).$$

Its derivative is

$$\dot{V} = e_{x_2}\dot{e}_{x_2} + e_{x_3}\dot{e}_{x_3}$$

$$\begin{aligned}&= e_{x_2}(-e_{x_2}+b_1x_1e_{x_3})+e_{x_3}(p_1x_1e_{x_2}-d_1e_{x_3})\\&= -e_{x_2}^2-d_1e_{x_3}^2+(b_1+p_1)x_1e_{x_2}e_{x_3}\\&= -e_{x_2}^2-d_1e_{x_3}^2\le 0.\end{aligned}$$

Since $d_1>0$, based on the Lyapunov stability theory the error system (4) is globally stable about the origin, we have

$$\lim_{t\to\infty}e_{x_2}=\lim_{t\to\infty}e_{x_3}=0,$$

which implies that $\hat{x}_2$, $\hat{x}_3$ can recover the signals of x_2, x_3, respectively. Thus, system (4) is an observer of system (1).

Case 2 $b_1+p_1\neq 0$. In this case, we obtain the following observer for system (1):

$$\begin{cases}\hat{x}_2=w_1+\dfrac{c_1+1}{a_1}x_1,\\ \dot{w}_1=-w_1+g_1(x_1)+b_1x_1\hat{x}_3-\dfrac{c_1+1}{a_1}(x_1+f_1(x_1)),\\ \hat{x}_3=w_2+\dfrac{p_1}{2a_1}x_1^2,\\ \dot{w}_2=-d_1w_2-\dfrac{p_1d_1}{2a_1}x_1^2-\dfrac{p_1x_1}{a_1}f_1(x_1)+h_1(x_1),\end{cases}\tag{5}$$

where $\hat{x}_2$ and $\hat{x}_3$ are the estimated values of x_2 and x_3, respectively.

By defining $e_{x_2}=\hat{x}_2-x_2, e_{x_3}=\hat{x}_3-x_3$, then from systems (1) and (5) we have

$$\dot{e}_{x_2}=\dot{\hat{x}}_2-\dot{x}_2=-e_{x_2}+b_1x_1e_{x_3},$$

and

$$\begin{aligned}\dot{e}_{x_3}&=\dot{\hat{x}}_3-\dot{x}_3=\dot{w}_2+\frac{p_1x_1\dot{x}_1}{a_1}-\dot{x}_3\\&=-d_1w_2-\frac{p_1d_1}{2a_1}x_1^2-\frac{p_1x_1}{a_1}f_1(x_1)+h_1(x_1)\\&\quad+\frac{p_1x_1}{a_1}(a_1x_2+f_1(x_1))-\dot{x}_3\\&=-d_1(\hat{x}_3-\frac{p_1}{2a_1}x_1^2)-\frac{p_1d_1}{2a_1}x_1^2\\&\quad+h_1(x_1)+p_1x_1x_2-\dot{x}_3\\&=-d_1\hat{x}_3+h_1(x_1)+p_1x_1x_2\\&\quad-(p_1x_1x_2-d_1x_3+h_1(x_1))\\&=-d_1e_{x_3}.\end{aligned}$$

Therefore, we have the following error dynamic system:

$$\begin{cases}\dot{e}_{x_2}=-e_{x_2}+b_1x_1e_{x_3},\\ \dot{e}_{x_3}=-d_1e_{x_3}.\end{cases}\tag{6}$$

For system (6) we can show that

$$\lim_{t\to\infty}e_{x_2}=\lim_{t\to\infty}e_{x_3}=0.$$

In fact, since $d_1>0$, it is easy to derive from the second equation of system (6) that $\lim_{t\to\infty}e_{x_3}=0$. Note that x_1 is bounded and $\lim_{t\to\infty}e_{x_3}=0$, by Lemma 1 (given in the following section) we conclude that $\lim_{t\to\infty}e_{x_2}=0$. Thus, we have

$$\lim_{t\to\infty}e_{x_2}=\lim_{t\to\infty}e_{x_3}=0,$$

which means that $\hat{x}_2$, $\hat{x}_3$ can recover the signals of x_2, x_3, respectively. Therefore, system (5) is an observer of system (1).

In the following, we give two examples to illustrate the use of observers (3) and (5)

Example 1 Consider a class of generalized Lorenz chaotic systems, which are described by the following set of differential equations[30]

$$\begin{cases}\dot{x}_1=\left(10+\dfrac{25}{29}k\right)(x_2-x_1),\\ \dot{x}_2=\left(28-\dfrac{35}{29}k\right)x_1+(k-1)x_2-x_1x_3,\\ \dot{x}_3=\left(-\dfrac{8}{3}-\frac{1}{87}k\right)x_3+x_1x_2,\end{cases}\tag{7}$$

where $x=(x_1,x_2,x_3)^{\mathrm{T}}\in R^{3\times 1}$ is the state vector of system (7), and the system parameter k satisfies $k\in\{k|-232<k<-11.6\}\bigcup\{k|k>-11.6\}$. It is observed that when $k\in[0,29]$ system (7) becomes the unified chaotic system,[28] which includes the Lü system,[25] the Chen system,[26] and the classical Lorenz system[27] as special cases.

By comparing system (7) with system (1), it yields that $a_1=10+[25/29]k$, $f_1(x_1)=-[10+(25/29)k]x_1$, $b_1=-1$, $c_1=k-1$, $g_1(x_1)=[28-(35/29)k]x_1$, $h_1(x_1)=0$, $p_1=1$, $d_1=[(8/3)+(1/87)k]$, and $b_1+p_1=0$. Thus, by system (3) we have the following observer:

$$\begin{cases}\hat{x}_2=w+\dfrac{k}{10+\dfrac{25}{29}k}x_1,\\ \dot{w}=-w+\left(28-\dfrac{35}{29}k\right)x_1-x_1\hat{x}_3\\ \qquad+\dfrac{k}{10+\dfrac{25}{29}k}\left(9+\dfrac{25}{29}k\right)x_1,\\ \hat{x}_3=\left(-\dfrac{8}{3}-\dfrac{1}{87}k\right)\hat{x}_3+x_1\hat{x}_2,\end{cases}\tag{8}$$

where $\hat{x}_2$ and $\hat{x}_3$ are the estimated values of x_2 and x_3, respectively.

Remark 2 The observers of system (7) have also been investigated in Refs. [30] and [31]. In Ref. [30], not only the first state x_1 itself but also its first-order derivative $\dot{x}_1$ were employed. It is well known that the derivative signal of the system output may not be always available in practice, hence, it is possible that $\dot{x}_1$ cannot be used for the state estimation. In Ref. [31], although the first-order derivative $\dot{x}_1$ was not employed, its observer is more complex than system (8) proposed in this paper. Since it is well known that the simple structure is desired from the point of application, thus observer (8) can be readily applied to real world situations.

Example 2 Let us consider the Liü chaotic system,[29] which is given as

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = -dx_1x_3 + bx_1, \\ \dot{x}_3 = hx_1^2 - cx_3, \end{cases} \tag{9}$$

where $x = (x_1, x_2, x_3)^{\mathrm{T}} \in R^{3\times 1}$ is the state vector of system (9), the constants a, b, c, d, and h are system's parameters. When $a = 10$, $b = 40$, $c = 2.5$, $d = 1$, $h = 4$, Liü system is chaotic.

By comparing system (9) with system (1), it follows that $a_1 = a$, $f_1(x_1) = -ax_1$, $b_1 = -d$, $c_1 = 0$, $g_1(x_1) = bx_1$, $h_1(x_1) = hx_1^2$, $p_1 = 0$, $d_1 = c$, and $b_1 + p_1 \neq 0$. Thus, by system (5) we derive the following observer:

$$\begin{cases} \hat{x}_2 = w_1 + \dfrac{1}{a}x_1, \\ \dot{w}_1 = -w_1 + bx_1 - dx_1\hat{x}_3 - \dfrac{1}{a}(x_1 - ax_1), \\ \hat{x}_3 = w_2, \\ \dot{w}_2 = -cw_2 + hx_1^2, \end{cases} \tag{10}$$

where $\hat{x}_2$ and $\hat{x}_3$ are the estimated values of x_2 and x_3, respectively.

Remark 3 It is not difficult to know that the observer (5) can also be used for the case of $b_1 + p_1 = 0$, however, observer (3) can not be used for the $b_1 + p_1 \neq 0$ case.

4. PC synchronization between systems (1) and (2)

In this section, we give the PC synchronization scheme between systems (1) and (2). To control system (2) and synchronize system (1), we choose the controllers $u_1(t_k)$, $u_2(t_k)$, $u_3(t_k)$, as follows:

$$\begin{cases} u_1(t_k) = -a_2y_2(t_k)) - f_2(y_1(t_k)) + a_1\hat{x}_2(t_k) \\ \qquad + f_1(x_1(t_k)) - \mu y_1(t_k) + \mu x_1(t_k), \\ u_2(t_k) = -b_2y_1(t_k)y_3(t_k) - (c_2 + \mu)y_2(t_k) \\ \qquad - g_2(y_1(t_k)) + b_1x_1(t_k)\hat{x}_3(t_k) \\ \qquad + (c_1 + \mu)\hat{x}_2(t_k) + g_1(x_1(t_k)), \\ u_3(t_k) = -p_2y_1(t_k)y_2(t_k) + (d_2 - \mu)y_3(t_k) \\ \qquad - h_2(y_1(t_k)) + p_1x_1(t_k)\hat{x}_2(t_k) \\ \qquad - (d_1 - \mu)\hat{x}_3(t_k) + h_1(x_1(t_k)), \\ t \in [t_k, t_{k+1}), \quad k = 0, 1, 2, \ldots, \end{cases} \tag{11}$$

where $\mu > 0$.

The choice of t_k encoded by the function ϕ is defined as

$$\phi(B) \triangleq ||B|| - \delta\frac{||(|e_1|, |e_2|, |e_3|)||}{2||A||}, \tag{12}$$

where $0 < \delta < 2\mu$ and A, B will be given in the sequel.

Equation (12) means that the event times are defined by $\phi(B(t_k)) = 0$, for $k = 0, 1, 2, \ldots$ At each t_k, the control is updated according to Eq. (11) and remains constants for all $t \in [t_k, t_{k+1})$.

By defining the error variables $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, and subtracting Eq. (1) from Eq. (2), we have

$$\begin{aligned} \dot{e}_1 &= a_2y_2 + f_2(y_1) + u_1(t_k) - a_1x_2 - f_1(x_1) \\ &= -\mu e_1 + [u_1(t_k) + a_2y_2 + f_2(y_1) \\ &\quad - a_1x_2 - f_1(x_1) + \mu y_1 - \mu x_1] \\ &= -\mu e_1 + [a_2(y_2 - y_2(t_k)) + f_2(y_1) - f_2(y_1(t_k)) \\ &\quad - a_1(x_2 - \hat{x}_2) - a_1(\hat{x}_2 - \hat{x}_2(t_k)) \\ &\quad - f_1(x_1) + f_1(x_1(t_k)) + \mu(y_1 - y_1(t_k)) \\ &\quad - \mu(x_1 - x_1(t_k))] \\ &= -\mu e_1 + [a_2e_{y_{2k}} + f_2(y_1) - f_2(y_1(t_k)) + a_1e_{x_2} - a_1\hat{e}_{x_{2k}} \\ &\quad - f_1(x_1) + f_1(x_1(t_k)) + \mu e_{y_{1k}} - \mu e_{x_{1k}}], \\ \dot{e}_2 &= b_2y_1y_3 + c_2y_2 + g_2(y_1) + u_2(t_k) \\ &\quad - (b_1x_1x_3 + c_1x_2 + g_1(x_1)) \\ &= -\mu e_2 + [u_2(t_k) + b_2y_1y_3 + c_2y_2 + g_2(y_1) \\ &\quad - (b_1x_1x_3 + c_1x_2 + g_1(x_1)) + \mu y_2 - \mu x_2] \\ &= -\mu e_2 + [b_2y_1(y_3 - y_3(t_k)) + b_2(y_1 - y_1(t_k)) \\ &\quad \times y_3(t_k) + (c_2 + \mu)y_2 - (c_2 + \mu)y_2(t_k) \\ &\quad + g_2(y_1) - g_2(y_1(t_k)) - b_1x_1(x_3 - \hat{x}_3) \\ &\quad - b_1x_1(\hat{x}_3 - \hat{x}_3(t_k)) - b_1\hat{x}_3(t_k)(x_1 - x_1(t_k)) \\ &\quad - (c_1 + \mu)x_2 + (c_1 + \mu)\hat{x}_2 - (c_1 + \mu) \\ &\quad \times (\hat{x}_2 - \hat{x}_2(t_k)) - g_1(x_1) + g_1(x_1(t_k))] \\ &= -\mu e_2 + [b_2y_1e_{y_{3k}} + b_2y_3(t_k)e_{y_{1k}} + (c_2 + \mu)e_{y_{2k}} \\ &\quad + g_2(y_1) - g_2(y_1(t_k)) + b_1x_1e_{x_3} \\ &\quad - b_1x_1\hat{e}_{x_{3k}} - b_1\hat{x}_3(t_k)e_{x_{1k}} + (c_1 + \mu)e_{x_2} \\ &\quad - (c_1 + \mu)\hat{e}_{x_{2k}} - g_1(x_1) + g_1(x_1(t_k))], \\ \dot{e}_3 &= p_2y_1y_2 - d_2y_3 + h_2(y_1) + u_3(t_k) \\ &\quad - (p_1x_1x_2 - d_1x_3 + h_1(x_1)) \\ &= -\mu e_3 + [u_3(t_k) + p_2y_1y_2 - (d_2 - \mu)y_3 + h_2(y_1) \\ &\quad - (p_1x_1x_2 - (d_1 - \mu)x_3 + h_1(x_1))] \\ &= -\mu e_3 + [p_2y_1(y_2 - y_2(t_k)) + p_2(y_1 - y_1(t_k)) \\ &\quad \times y_2(t_k) - (d_2 - \mu)y_3 + (d_2 - \mu)y_3(t_k) \\ &\quad + h_2(y_1) - h_2(y_1(t_k)) - p_1x_1(x_2 - \hat{x}_2) \\ &\quad - p_1x_1(\hat{x}_2 - \hat{x}_2(t_k)) - p_1\hat{x}_2(t_k)(x_1 - x_1(t_k)) \\ &\quad + (d_1 - \mu)x_3 - (d_1 - \mu)\hat{x}_3 + (d_1 - \mu)\hat{x}_3 \\ &\quad - (d_1 - \mu)\hat{x}_3(t_k) - h_1(x_1) + h_1(x_1(t_k))] \\ &= -\mu e_3 + [p_2y_1e_{y_{2k}} + p_2y_2(t_k)e_{y_{1k}} - (d_2 - \mu)e_{y_{3k}} \\ &\quad + h_2(y_1) - h_2(y_1(t_k)) + p_1x_1e_{x_2} \\ &\quad - p_1x_1\hat{e}_{x_{2k}} - p_1\hat{x}_2(t_k)e_{x_{1k}} - (d_1 - \mu)e_{x_3} \\ &\quad + (d_1 - \mu)\hat{e}_{x_{3k}} - h_1(x_1) + h_1(x_1(t_k))], \end{aligned}$$

where $e_{y_{ik}} = y_i(t) - y_i(t_k)$, $e_{x_{1k}} = x_1(t) - x_1(t_k)$, $\hat{e}_{x_{2k}} = \hat{x}_2 - \hat{x}_2(t_k)$, $\hat{e}_{x_{3k}} = \hat{x}_3 - \hat{x}_3(t_k)$, $i = 1, 2, 3$.

Thus, one gets the error dynamical system

$$\begin{cases} \dot{e}_1 = -\mu e_1 + [a_2 e_{y_{2k}} + f_2(y_1) - f_2(y_1(t_k)) + a_1 e_{x_2} \\ \quad -a_1 \hat{e}_{x_{2k}} - f_1(x_1) + f_1(x_1(t_k)) + \mu e_{y_{1k}} - \mu e_{x_{1k}}], \\ \dot{e}_2 = -\mu e_2 + [b_2 y_1 e_{y_{3k}} + b_2 y_3(t_k) e_{y_{1k}} + (c_2+\mu) e_{y_{2k}} \\ \quad +g_2(y_1) - g_2(y_1(t_k)) \\ \quad +b_1 x_1 e_{x_3} - b_1 x_1 \hat{e}_{x_{3k}} - b_1 \hat{x}_3(t_k) e_{x_{1k}} + (c_1+\mu) e_{x_2} \\ \quad -(c_1+\mu)\hat{e}_{x_{2k}} - g_1(x_1) + g_1(x_1(t_k))], \\ \dot{e}_3 = -\mu e_3 + [p_2 y_1 e_{y_{2k}} + p_2 y_2(t_k) e_{y_{1k}} \\ \quad +(\mu - d_2) e_{y_{3k}} + h_2(y_1) - h_2(y_1(t_k)) + p_1 x_1 e_{x_2} \\ \quad -p_1 x_1 \hat{e}_{x_{2k}} - p_1 \hat{x}_2(t_k) e_{x_{1k}} - (d_1 - \mu) e_{x_3} \\ \quad +(d_1 - \mu)\hat{e}_{x_{3k}} - h_1(x_1) + h_1(x_1(t_k))]. \end{cases} \tag{13}$$

It can easily be seen that the synchronization problem between systems (1) and (2) is equivalent to the problem of stabilizing system (13).

Before presenting our main results, we introduce a Lemma which will be used in the later discussion.

Lemma 1 If $\lim_{t\to\infty} \alpha(t) = 0$, then the origin of system

$$\dot{x} = -\lambda x + \alpha(t) \tag{14}$$

is globally asymptotically stable, where $\lambda > 0$.

Proof It is not hard to know that the general solution of system (14) is

$$x = \mathrm{e}^{-\lambda t} x(0) + \mathrm{e}^{-\lambda t} \int_0^t \mathrm{e}^{\lambda \tau} \alpha(\tau) \mathrm{d}\tau, \tag{15}$$

where $x(0)$ is the initial value of x.

Since $\lim_{t\to\infty} \mathrm{e}^{-\lambda t} = 0$, we have $\lim_{t\to\infty} \mathrm{e}^{-\lambda t} x(0) = 0$. If $\lim_{t\to\infty} \int_0^t \mathrm{e}^{\lambda\tau}\alpha(\tau)\mathrm{d}\tau$ is bounded, obviously we get

$$\lim_{t\to\infty} \mathrm{e}^{-\lambda t} \int_0^t \mathrm{e}^{\lambda\tau}\alpha(\tau)\mathrm{d}\tau = 0.$$

Otherwise, if $\lim_{t\to\infty} \int_0^t \mathrm{e}^{\lambda\tau}\alpha(\tau)\mathrm{d}\tau = \infty$, then we can apply the ∞/∞ case of the L'Hôpital's rule to calculate $\lim_{t\to\infty} \mathrm{e}^{-\lambda t} \int_0^t \mathrm{e}^{\lambda\tau}\alpha(\tau)\mathrm{d}\tau$ as follows:

$$\begin{aligned} \lim_{t\to\infty} \mathrm{e}^{-\lambda t} \int_0^t \mathrm{e}^{\lambda\tau}\alpha(\tau)\mathrm{d}\tau &= \lim_{t\to\infty} \frac{\int_0^t \mathrm{e}^{\lambda\tau}\alpha(\tau)d\tau}{\mathrm{e}^{\lambda t}} \\ &= \lim_{t\to\infty} \frac{\mathrm{e}^{\lambda t}\alpha(t)}{\lambda\,\mathrm{e}^{\lambda t}} \\ &= \lim_{t\to\infty} \frac{\alpha(t)}{\lambda} = 0. \end{aligned}$$

Therefore, in view of Eqs. (15) one has $\lim_{t\to\infty} x = 0$, which means that the origin of system (14) is globally asymptotically stable.

Now, we present our main results.

Theorem 1 For the drive system (1) and response system (2), If the observer of system (1) is selected as system (3) or (5) and the controllers are chosen as Eqs. (11) and (12), then the PC synchronization between systems (1) and (2) can be achieved.

Proof Choose the following Lyapunov function:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2).$$

The derivative of the above function along the trajectories of system (13) is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= -\mu(e_1^2+e_2^2+e_3^2) + e_1[a_2 e_{y_{2k}} + f_2(y_1) \\ &\quad - f_2(y_1(t_k)) + a_1 e_{x_2} - a_1\hat{e}_{x_{2k}} \\ &\quad - f_1(x_1) + f_1(x_1(t_k)) + \mu e_{y_{1k}} - \mu e_{x_{1k}}] \\ &\quad + e_2[b_2 y_1 e_{y_{3k}} + b_2 y_3(t_k) e_{y_{1k}} \\ &\quad + (c_2+\mu) e_{y_{2k}} + g_2(y_1) - g_2(y_1(t_k)) \\ &\quad + b_1 x_1 e_{x_3} - b_1 x_1 \hat{e}_{x_{3k}} - b_1 \hat{x}_3(t_k) e_{x_{1k}} \\ &\quad + (c_1+\mu) e_{x_2} - (c_1+\mu)\hat{e}_{x_{2k}} \\ &\quad - g_1(x_1) + g_1(x_1(t_k))] + e_3[p_2 y_1 e_{y_{2k}} + p_2 y_2(t_k) e_{y_{1k}} \\ &\quad + (\mu - d_2) e_{y_{3k}} + h_2(y_1) - h_2(y_1(t_k)) \\ &\quad + p_1 x_1 e_{x_2} - p_1 x_1 \hat{e}_{x_{2k}} - p_1 \hat{x}_2(t_k) e_{x_{1k}} \\ &\quad - (d_1 - \mu) e_{x_3} + (d_1 - \mu)\hat{e}_{x_{3k}} - h_1(x_1) + h_1(x_1(t_k))]. \end{aligned}$$

By using Assumptions 1 and 2, one obtains

$$\begin{aligned} \dot{V} &\le -2\mu V + |e_1|[|a_2||e_{y_{2k}}| + L_{f_2}|e_{y_{1k}}| + |a_1||e_{x_2}| \\ &\quad + |a_1|\hat{e}_{x_{2k}}| + L_{f_1}|e_{x_{1k}}| + \mu|e_{y_{1k}}| + \mu|e_{x_{1k}}|] \\ &\quad + |e_2|[M|b_2||e_{y_{3k}}| + M|b_2||e_{y_{1k}}| \\ &\quad + |c_2+\mu||e_{y_{2k}}| + L_{g_2}|e_{y_{1k}}| + M|b_1||e_{x_3}| \\ &\quad + M|b_1||\hat{e}_{x_{3k}}| + M|b_1||e_{x_{1k}}| + |c_1+\mu||e_{x_2}| \\ &\quad + |c_1+\mu||\hat{e}_{x_{2k}}| + L_{g_1}|e_{x_{1k}}|] + |e_3|[M|p_2||e_{y_{2k}}| \\ &\quad + M|p_2||e_{y_{1k}}| + |d_2-\mu||e_{y_{3k}}| + L_{h_2}|e_{y_{1k}}| \\ &\quad + M|p_1||e_{x_2}| + M|p_1||\hat{e}_{x_{2k}}| \\ &\quad + M|p_1||e_{x_{1k}}| + |d_1-\mu||e_{x_3}| + |d_1-\mu||\hat{e}_{x_{3k}}| + L_{h_1}|e_{x_{1k}}|] \\ &\le -2\mu V + |e_1|[(L_{f_2}+\mu)|e_{y_{1k}}| + |a_2||e_{y_{2k}}| \\ &\quad + (L_{f_1}+\mu)|e_{x_{1k}} + |a_1|\hat{e}_{x_{2k}}| + |a_1||e_{x_2}||] \\ &\quad + |e_2|[(L_{g_2}+M|b_2|)|e_{y_{1k}}| + |c_2+\mu||e_{y_{2k}}| \\ &\quad + M|b_2||e_{y_{3k}}| + (L_{g_1}+M|b_1|)|e_{x_{1k}}| \\ &\quad + |c_1+\mu||\hat{e}_{x_{2k}}| + M|b_1||\hat{e}_{x_{3k}}| \\ &\quad + |c_1+\mu||e_{x_2}| + M|b_1||e_{x_3}|] + |e_3|[(L_{h_2}+M|p_2|)|e_{y_{1k}}| \\ &\quad + M|p_2||e_{y_{2k}}| + |d_2-\mu||e_{y_{3k}}| + (L_{h_1}+M|p_1||)|e_{x_{1k}}| \\ &\quad + M|p_1||\hat{e}_{x_{2k}}| + |d_1-\mu||\hat{e}_{x_{3k}}| \\ &\quad + M|p_1||e_{x_2}| + |d_1-\mu||e_{x_3}|] \\ &\le -2\mu V + ||(|e_1|,|e_2|,|e_3|)||||A||||B|| + |e_1||a_1||e_{x_2}| \\ &\quad + |e_2|(|c_1+\mu||e_{x_2}| + M|b_1||e_{x_3}|) \\ &\quad + |e_3|(|d_1-\mu||e_{x_3}| + M|p_1||e_{x_2}|), \end{aligned}$$

where

$$A=\begin{pmatrix} L_{f_2}+\mu & |a_2| & 0 & \mu+L_{f_1} & |a_1| & 0 \\ M|b_2|+L_{g_2} & |\mu+c_2| & M|b_2| & L_{g_1}+M|b_1| & |c_1+\mu| & M|b_1| \\ M|p_2|+L_{h_2} & M|p_2| & |\mu-d_2| & L_{h_1}+M|p_1| & M|p_1| & |\mu-d_1| \end{pmatrix},$$

$$B=\begin{pmatrix} |e_{y_{1k}}| & |e_{y_{2k}}| & |e_{y_{3k}}| & |e_{x_{1k}}| & |\hat{e}_{x_{2k}}| & |\hat{e}_{x_{3k}}| \end{pmatrix}^{\mathrm{T}}.$$

In view of Eq. (12), yields

$$||B|| \le \delta \frac{||(|e_1|,|e_2|,|e_3|)||}{2||A||},$$

where $0<\delta<2\mu$.

Then we have

$$\begin{aligned}\dot{V} \le\ & (-2\mu+\delta)V+|e_1||a_1||e_{x_2}|+|e_2|(|c_1+\mu||e_{x_2}| \\ & +M|b_1||e_{x_3}|)+|e_3|(|d_1-\mu||e_{x_3}|+M|p_1||e_{x_2}|).\end{aligned}$$

Note that

$$\lim_{t\to\infty} e_{x_2}=\lim_{t\to\infty} e_{x_3}=0,$$

and

$$-2\mu+\delta<0.$$

By using Lemma 1, we obtain

$$\lim_{t\to\infty} V=0.$$

Thus, one has

$$\lim_{t\to\infty} e_1=\lim_{t\to\infty} e_2=\lim_{t\to\infty} e_3=0,$$

and the proof is complete.

In order to give the lower bound of inner event-triggered time $T_k=t_{k+1}-t_k$, we make the following Assumption 4.

Assumption 4 There exists positive constants ϑ, ϖ, such that $\vartheta=\min\{||B||\}$ and $||\omega||\le\mu\varpi$, where

$$\begin{aligned}\omega =\ & (a_2e_{y_{2k}}+f_2(y_1)-f_2(y_1(t_k))+a_1e_{x_2} \\ & -a_1\hat{e}_{x_{2k}}-f_1(x_1)+f_1(x_1(t_k))+\mu e_{y_{1k}}-\mu e_{x_{1k}}, \\ & b_2y_1e_{y_{3k}}+b_2y_3(t_k)e_{y_{1k}}+(c_2+\mu)e_{y_{2k}}+g_2(y_1) \\ & -g_2(y_1(t_k))+b_1x_1e_{x_3}-b_1x_1\hat{e}_{x_{3k}}-b_1\hat{x}_3(t_k)e_{x_{1k}} \\ & +(c_1+\mu)e_{x_2}-(c_1+\mu)\hat{e}_{x_{2k}}-g_1(x_1)+g_1(x_1(t_k)), \\ & p_2y_1e_{y_{2k}}+p_2y_2(t_k)e_{y_{1k}}+(\mu-d_2)e_{y_{3k}} \\ & +h_2(y_1)-h_2(y_1(t_k)) \\ & +p_1x_1e_{x_2}-p_1x_1\hat{e}_{x_{2k}}-p_1\hat{x}_2(t_k)e_{x_{1k}}-(d_1-\mu)e_{x_3} \\ & +(d_1-\mu)\hat{e}_{x_{3k}}-h_1(x_1)+h_1(x_1(t_k)))^{\mathrm{T}}.\end{aligned}$$

Theorem 2 Consider the drive system (1) and response system (2). Suppose that Assumption 4 is satisfied, then the inner event-triggered time T_k is lower bounded by a positive time T, which is given by

$$T=\frac{1}{\mu}\log\left(1+\frac{2||A||\vartheta}{\delta\varpi}\right)>0.$$

Proof Let $e=(e_1,e_2,e_3)^{\mathrm{T}}$, by Eq. (13) one obtains

$$\dot{e}=-\mu e+\omega.$$

By using

$$\frac{\mathrm{d}||e||}{\mathrm{d}t}\le||\dot{e}||,$$

we have

$$\frac{\mathrm{d}||e||}{\mathrm{d}t}\le\mu(||e||+\varpi).$$

According to the Comparison Lemma,[32,33] one has

$$||e||\le\varpi(\mathrm{e}^{\mu(t-t_k)}-1),\quad \text{for}\quad t\ge t_k.$$

i.e.,

$$t-t_k\ge\frac{1}{\mu}\log\left(1+\frac{||e||}{\varpi}\right),\quad \text{for}\quad t\ge t_k.$$

Note that $||e(t)||=||(|e_1|,|e_2|,|e_3|)||$, and

$$||B||\le\delta\frac{||(|e_1|,|e_2|,|e_3|)||}{2||A||},$$

we have

$$t-t_k\ge\frac{1}{\mu}\log\left(1+\frac{2||A||||B||}{\delta\varpi}\right).$$

In view of Assumption 4, the inner event-triggered time is uniformly lower bounded by T, which satisfies

$$T\ge\frac{1}{\mu}\log\left(1+\frac{2||A||\vartheta}{\delta\varpi}\right)>0.$$

5. Numerical simulations

In this section, we take the Lü system and Chen system as examples to illustrate the efficiency of the obtained results.

The Lü system and Chen system are famous chaotic systems. The dynamical equation of the Lü system[25] can be described by

$$\begin{cases}\dot{x}_1=\alpha_1(x_2-x_1),\\ \dot{x}_2=-x_1x_3+\beta_1x_2,\\ \dot{x}_3=x_1x_2-\gamma_1x_3,\end{cases}\tag{16}$$

where $x=(x_1,x_2,x_3)^{\mathrm{T}}\in R^{3\times1}$ is the state vector of system (16), the positive constants $\alpha_1,\gamma_1,\beta_1$ are system's parameters, which are known in advance. When $\alpha_1=36$, $\gamma_1=3$, and $\beta_1=20$, Lü system is chaotic.

Suppose system (16) is the drive system, the response system is Chen system [26] which is given by

$$\begin{cases}\dot{y}_1=\alpha_2(y_2-y_1)+u_1(t_k),\\ \dot{y}_2=-y_1y_3+(\gamma_2-\alpha_2)y_1+\gamma_2y_2+u_2(t_k),\\ \dot{y}_3=y_1y_2-\beta_2y_3+u_3(t_k),\ \ k=0,1,2,\ldots,\end{cases}\tag{17}$$

where $y = (y_1, y_2, y_3)^{\mathrm{T}} \in R^{3\times 1}$ is the state vector of system (17), u_1, u_2, u_3 are the controllers to be designed later. Chen system exhibits a chaotic attractor at the parameters $\alpha_2 = 35, \beta_2 = 3, \gamma_2 = 28$ and $u_1 = u_2 = u_3 = 0$.

Considering the Lü chaotic system described by Eq. (16), it is obvious that $a_1 = \alpha_1$, $f_1(x_1) = -\alpha_1 x_1$, $b_1 = -1$, $c_1 = \beta_1$, $g_1(x_1) = h_1(x_1) = 0$, $p_1 = 1$, $d_1 = \gamma_1$, and $b_1 + p_1 = 0$. Thus, by system (3) we derive the following observer:

$$\begin{cases} \hat{x}_2 = w + \dfrac{\beta_1 + 1}{\alpha_1} x_1, \\ \dot{w} = -w + \left(\beta_1 + 1 - \dfrac{\beta_1 + 1}{\alpha_1}\right) x_1 - x_1 \hat{x}_3, \\ \dot{\hat{x}}_3 = x_1 \hat{x}_2 - \gamma_1 \hat{x}_3, \end{cases} \tag{18}$$

where $\hat{x}_2$ and $\hat{x}_3$ are the estimated values of x_2 and x_3, respectively.

In view of system (17), we get $f_2(y_1) = -\alpha_2 y_1$, $g_2(y_1) = (\gamma_2 - \alpha_2) y_1$, $h_2(y_1) = 0$. Thus, by using (11) we have the following controllers:

$$\begin{cases} u_1(t_k) = -\alpha_2 y_2(t_k)) + \alpha_2 y_1(t_k)) + \alpha_1 \hat{x}_2(t_k) \\ \qquad -\alpha_1 x_1(t_k) - \mu y_1(t_k) + \mu x_1(t_k), \\ u_2(t_k) = -y_1(t_k) y_3(t_k) - (\gamma_2 + \mu) y_2(t_k) \\ \qquad -(\gamma_2 - \alpha_2) y_1(t_k) - x_1(t_k) \hat{x}_3(t_k) \\ \qquad +(\beta_1 + \mu) \hat{x}_2(t_k), \\ u_3(t_k) = -y_1(t_k) y_2(t_k) + (\beta_2 - \mu) y_3(t_k) + x_1(t_k) \hat{x}_2(t_k) \\ \qquad -(\gamma_1 - \mu) \hat{x}_3(t_k), \\ t \in [t_k, t_{k+1}), \quad k = 0, 1, 2, \ldots \end{cases} \tag{19}$$

It is easy to know that $L_{f_1} = |\alpha_1|$, $L_{g_1} = L_{h_1} = L_{h_2} = 0$, $L_{f_2} = |\alpha_2|$, $L_{g_2} = |\gamma_2 - \alpha_2|$, and

$$A = \begin{pmatrix} |\alpha_2| + \mu & |\alpha_2| & 0 & \mu + |\alpha_1| & |\alpha_1| & 0 \\ M + |\gamma_2 - \alpha_2| & |\mu + \gamma_2| & M & M & |\beta_1 + \mu| & M \\ M & M & |\mu - \beta_2| & M & M & |\mu - \gamma_1| \end{pmatrix}.$$

The systems parameters are chosen as $\alpha_1 = 36$, $\gamma_1 = 3$, $\beta_1 = 20$, $\alpha_2 = 35$, $\beta_2 = 3$, $\gamma_2 = 28$, so that the drive system and the response system operate in a chaotic regime. If we set $M = 40$ and $\mu = 5$, then $||A|| = 137.1165$. Suppose the initial states for the drive system and response system are taken as $(x_1(0), x_2(0), x_3(0)) = (8, 6, 10)$, $(y_1(0), y_2(0), y_3(0)) = (7, 3, 12)$ and $(\hat{x}_2(0), \hat{x}_3(0)) = (6.3, 9.7)$ then $(e_1(0), e_2(0), e_3(0)) = (-1, -3, 2)$ and $(\hat{e}_{x_2}(0), \hat{e}_{x_3}(0)) = (0.3, -0.3)$. The corresponding numerical results are shown in Figs. 1–6.

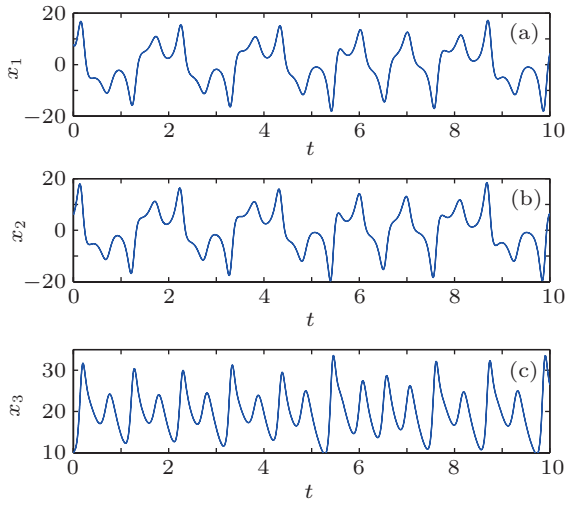


Fig. 1. (color online) The trajectories of drive system (16). (a) x_1, (b) x_2, and (c) x_3.

Figures 1 and 2 show the trajectories of the drive system (16) and response system (17) with input control (11), respectively. Figure 3 depicts the time evolution of errors e_{x_2}, e_{x_3} between drive system (16) and observer system (18).

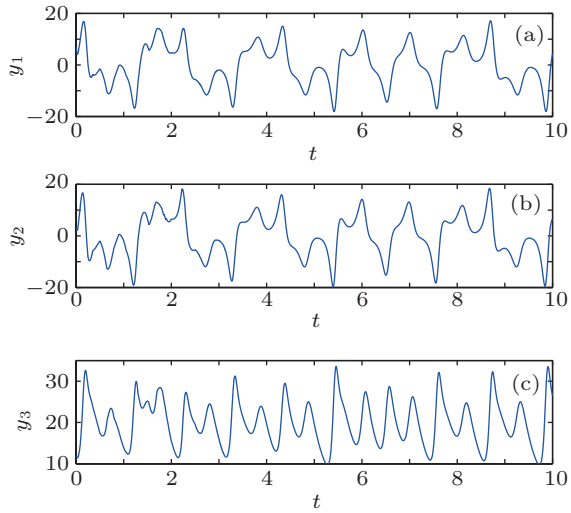


Fig. 2. (color online) The trajectories of response system (17) with input control (11). (a) y_1, (b) y_2, and (c) y_3.

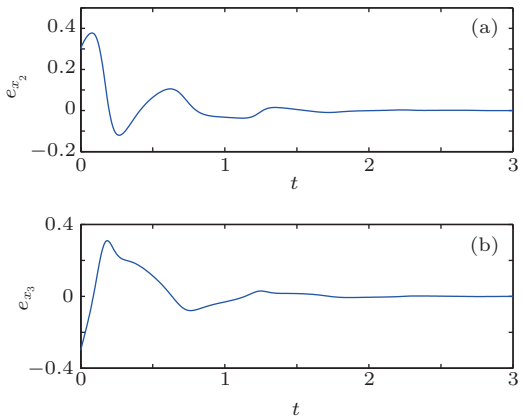


Fig. 3. (color online) The time evolution of errors e_{x_2}, e_{x_3} between drive system (16) and observer system (18). (a) e_{x_2} and (b) e_{x_3}.

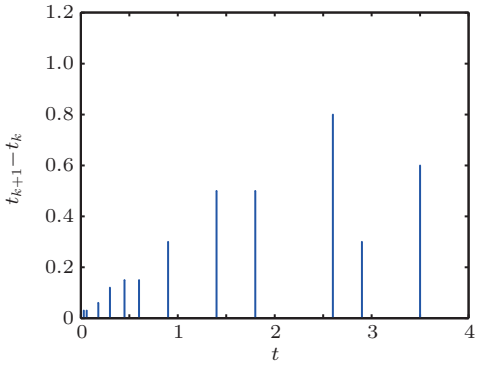


Fig. 4. (color online) The event-triggered time $t_{k+1}-t_k$ between drive system (16) and response system (17).

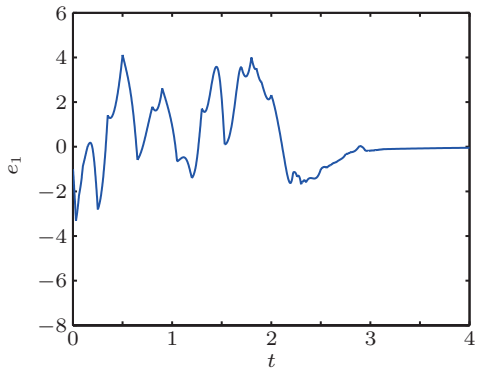


Fig. 5. (color online) The time evolution of error e_1 between drive system (16) and response system (17).

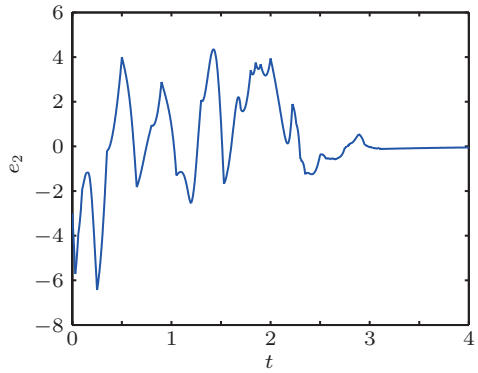


Fig. 6. (color online) The time evolution of error e_2 between drive system (16) and response system (17).

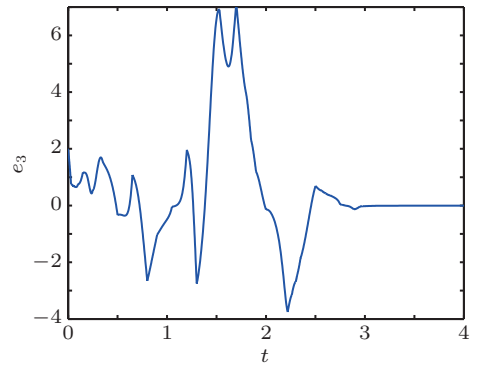


Fig. 7. (color online) The time evolution of error e_3 between drive system (16) and response system (17).

From Fig. 3, it can be seen that the observer error converge to zero, which means that the two observers (3) and (5) are effective and feasible. Figure 4 shows the event-triggered time $t_{k+1}-t_k$ between drive system (16) and response system (17). Figures 5–7 display the time response of the synchronization error $e=(e_1,e_2,e_3)^{\mathrm{T}}$. As expected, one can observe that the error vector e converges to zero. From Figs. 5–7, it is easy to see that the PC synchronization between drive system (16) and response system (17) is reached.

6. Conclusion

This work investigates the PC synchronization of a class of chaotic systems with only one state variable is available in the drive system. Some observers are proposed for this chaotic system. Based on the observers designed for the drive system some new sufficient conditions are presented for achieving PC synchronization via event-triggered control. In our control scheme we do not need to use the unavailable states of the drive system, thus our approach is more realistic and practical than most of the existing ones because they assume that all of the states are available. Finally, numerical simulations are carried out to verify the efficiency of the proposed controller.

References

[1] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[2] Pourdehi S and Karimaghaee P 2012 *Chaos* **22** 023145
[3] Kitajima H and Yoshihara T 2012 *Physica D* **241** 1804
[4] Ahmadlou M and Adeli H 2012 *Physica D* **241** 326
[5] Guo X Y and Li J M 2011 *Chin. Phys. Lett.* **28** 120503
[6] Li G H 2007 *Chaos Soliton. Fract.* **32** 1786
[7] Luo R Z, Wang Y L and Deng S C 2011 *Chaos* **21** 043114
[8] Odibat Z M, Corson N, Aziz-Alaoui M A and Bertelle C 2010 *Int. J. Bifurc. Chaos* **20** 81
[9] Peng C and Chen C 2008 *Chaos Soliton. Fract.* **37** 598
[10] Zhao Y and Wang W 2009 *Chaos Soliton. Fract.* **41** 60
[11] Chen C S 2011 *Appl. Math. Comput.* **217** 10377
[12] Yu H T, Wang J, Deng B, Wei X L and Chen Y Y 2013 *Chin. Phys. B* **22** 058701
[13] Liu J G 2013 *Chin. Phys. B* **22** 060510
[14] Garcia E and Antsaklis P J 2013 *IEEE Trans. Auto. Control* **58** 422
[15] Peng C and Yang T C 2013 *Automatica* **49** 1326
[16] Yu H and Antsaklis P J 2013 *Automatica* **49** 30
[17] Zhu H L and Zhang X B 2009 *J. Inf. Comput. Sci.* **4** 033
[18] Farivar F, Shoorehdeli M A, Nekoui M A and Teshnehlab M 2012 *Nonlinear Dyn.* **67** 1913
[19] Bai J, Yu Y G, Wang S and Song Y 2012 *Commun. Nonlinear Sci. Numer. Simulat.* **17** 1921
[20] Koofigar H R, Sheikholeslam F and Hosseinnia S 2011 *Chaos* **21** 043134
[21] Chen D, Zhang R, Sprott J C, Chen H and Ma X 2012 *Chaos* **22** 023130
[22] Banerjee S, Jeeva S T S and Kurths J 2013 *Chaos* **23** 013118
[23] Zhang K, Wang H and Fang H 2012 *Commun. Nonlinear Sci. Numer. Simulat.* **17** 317
[24] Asheghan M and Míguez J 2013 *Chaos* **23** 023108
[25] Lü J and Chen G 2002 *Int. J. Bifurc. Chaos* **12** 659
[26] Chen G and Ueta T 1999 *Int. J. Bifurc. Chaos* **9** 1465
[27] Lorenz E N 1963 *J. Atmos. Sci.* **20** 130
[28] Lü J H, Chen G R, Cheng D Z and Celikovsky S 2002 *Int. J. Bifurc. Chaos* **12** 2917
[29] Liu C, Liu T, Liu L and Liu K 2004 *Chaos Soliton. Fract.* **22** 1031
[30] Sun Y J 2010 *Phys. Lett. A* **374** 933
[31] Zhang Z Q, Shao H Y, Wang Z and Shen H 2012 *Appl. Math. Comput.* **218** 7614
[32] Khalil H 2002 *Nonlinear Sys.* (Prentice Hall)
[33] Tallapragada P and Chopra N 2011 50*th IEEE Conference on Decision and Control and European Control Conference (CDC–ECC)* 5377