Mapping equivalent approach to analysis and realization of memristor-based dynamical circuit*

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A novel mapping equivalent approach is proposed in this paper, which can be used for analyzing and realizing a memristor-based dynamical circuit equivalently by a nonlinear dynamical circuit with the same topologies and circuit parameters. A memristor-based chaotic circuit and the corresponding Chua's chaotic circuit with two output differentiators are taken as examples to illustrate this approach. Equivalent dynamical analysis and realization of the memristor-based chaotic circuit are performed by using Chua's chaotic circuit. The results indicate that the outputs of memristor-based chaotic circuit and the corresponding outputs of Chua's chaotic circuit have identical dynamics. The proposed approach verified by numerical simulations and experimental observations is useful in designing and analyzing memristor-based dynamical circuits.

Keywords: dynamics, mapping equivalent approach, memristor, nonlinear circuit

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1. Introduction

As is well known, traditional electrical circuits are modeled with three fundamental elements: resistor, inductor, and capacitor, and circuit behaviors are governed by two electrical circuit laws: Kirchoff's voltage law and Kirchoff's current law. It is also known that there are four circuit attributes, i.e., electric current, voltage, charge, and magnetic flux, which describe these circuit elements, i.e., resistor with current and voltage, inductor with voltage and magnetic flux, and capacitor with current and charge. It is evident that there is not any other element described by charge and magnetic flux. With the finding, Chua proposed a fourth element, memristor,[1–3] described with charge and magnetic flux and further elaborated its existence from an elementary circuit element quadrangle. In 2008, HP Laboratory implemented the memristor.[4] The solid state implementation of the memristor triggers wide interests in theoretical investigations and circuit applications of such memristors, including different types of memristive device models proposed for circuit designs and simulations,[5–8] and dynamical modeling analyses,[8–13] and equivalent circuit realizations[13–15] of various memristor-based dynamical circuits.

Recently, specific to Chua's elementary circuit element quadrangle, Wang[16] proposed an elementary circuit element triangle, in which magnetic flux φ and charge q are thought to be two basic physical attributes that generate elementary circuit elements, and memristor as well as memcapacitor and meminductor, leading to three basic element classes, respectively. An intrinsic mathematical relationship is found to support this new classification.[16] According to this classification, the linear resistor and nonlinear memristor, belonging to the basic element class, link φ and q with a unit of Ω, the linear inductor links φ and $\mathrm{d}q$ with a unit of H, and the linear capacitor links $\mathrm{d}\varphi$ and q with a unit of F. Thus, magnetic flux φ and charge q can also be used to describe any circuit model. However, all basic circuit constraints, two electrical circuit laws and the constitutive relations of all elementary circuit elements, should be rebuilt,[17] based on which a memristor-based dynamical circuit can be described by magnetic flux φ and charge q instead of voltage v and current i, resulting in the occurrence of some interesting and meaningful conclusions.

The charge as the time integral of the current and the flux as the time integral of the voltage,[3] they represent two differential mapping relations: one is between the charge and the current, and the other is between the flux and the voltage. Based on these one-to-one relations, a novel mapping equivalent approach is established in this paper, which has not been reported in the literature so far. This novel mapping equivalent approach implies that a memristor-based dynamical circuit can be equivalently analyzed and realized by a corresponding nonlinear dynamical circuit, i.e., a memristor-based chaotic circuit and a nonlinear chaotic circuit have identical dynamics under certain conditions. In other words, if a nonlinear dynamical circuit exists, by the mapping equivalent approach, the

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corresponding memristor-based dynamical circuit exists also. This result indicates that the proposed mapping equivalent approach has theoretical guideline signification for the designs and analyses of memristor-based dynamical circuits.

In this paper we discuss an equivalent approach to analyzing and realizing a memristor-based dynamical circuit by using a nonlinear dynamical circuit. The rest of this paper is organized as follows. In Section 2, the mapping equivalent approach through two kinds of descriptions of nonlinear circuit models is proposed and illustrated. In Section 3, dynamical analysis of memristor-based chaotic circuit is conducted and the validity of equivalent analysis by Chua's chaotic circuit is shown. Circuit realization of memristor-based chaotic circuit and its equivalent realization by Chua's chaotic circuit with two output differentiators are designed in Section 4, corresponding experimental observations are tested to verify the proposed mapping equivalent approach. Finally, the conclusions are drawn from the present study in Section 5.

2. Mapping equivalent approach

A novel approach to analyzing and realizing a memristor-based dynamical circuit by using a nonlinear dynamical circuit without memristor is proposed and proved in this section.

2.1. Nonlinear circuit model described by voltage and current

A memristive dynamical circuit is a nonlinear circuit. In classical circuit theory, the nonlinear circuit model can be generally written in the form

$$f_1(\mathrm{d}i/\mathrm{d}t, \mathrm{d}v/\mathrm{d}t, i, v, t) = 0, \tag{1}$$

where $f_1(\cdot)$ captures Kirchoff's laws but also the constitutive relations of all elementary circuit elements by the voltage–current relations.

For three linear circuit elements: resistor R, inductor L, and capacitor C, their constitutive relations are defined by the voltage–current relations

$$v_R(t) = Ri_R(t) \quad \text{or} \quad i_G(t) = Gv_G(t), \tag{2}$$

$$v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}, \tag{3}$$

$$i_C(t) = C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}, \tag{4}$$

respectively. For the nonlinear charge-controlled memristor and flux-controlled memristor, their constitutive relations are expressed by[3]

$$\varphi = \hat{\varphi}(q), \tag{5}$$

and

$$q = \hat{q}(\varphi), \tag{6}$$

respectively, where $\hat{\varphi}(q)$ and $\hat{q}(\varphi)$ are continuous and piecewise-differentiable functions with bounded slopes. Differentiating Eqs. (5) and (6) with respect to time t, the voltage–current relations can be obtained as

$$v_{\mathrm{M}}(t) = M(q)i_{\mathrm{M}}(t), \tag{7}$$

and

$$i_W(t) = W(\varphi)v_W(t), \tag{8}$$

respectively, where $M(q) = \mathrm{d}\hat{\varphi}/\mathrm{d}q$ is the memristance of the charge-controlled memristor, and $W(\varphi) = \mathrm{d}\hat{q}/\mathrm{d}\varphi$ is the memductance of the flux-controlled memristor.

Both $M(q)$ and $W(\varphi)$ keep track of the device history because they depend on the integral variables $q(t) = \int_{-\infty}^{t} i(\tau)\mathrm{d}\tau$ and $\varphi(t) = \int_{-\infty}^{t} v(\tau)\mathrm{d}\tau$ respectively. Equations (7) and (8) involve v, i, and q or φ. The third variable q or φ introduces memory in the device.[8]

With the definitions of four fundamental circuit elements by the voltage–current relations, all the branch voltages drops around a closed loop satisfy Kirchhoff's voltage law (KVL), and all the branch currents entering into and leaving from a node satisfy Kirchhoff's current law (KCL).

2.2. Nonlinear circuit model described by flux and charge

If magnetic flux φ and charge q are two basic physical attributes, the constitutive relations of four circuit elements can be described by magnetic flux φ and charge q.[16,17] Thus, these relations are called the flux-charge relations, which are written as

$$\varphi_R(t) = Rq_R(t) \quad \text{or} \quad q_G(t) = G\varphi_G(t), \tag{9}$$

$$\varphi_L(t) = Li_L(t) = L\frac{\mathrm{d}q_L}{\mathrm{d}t}, \tag{10}$$

$$q_C(t) = Cv_C(t) = C\frac{\mathrm{d}\varphi_C}{\mathrm{d}t}, \tag{11}$$

respectively. The flux–charge relations of the nonlinear flux-controlled memristor and charge-controlled are directly described by Eqs. (5) and (6), respectively.

Corresponding to Kirchoff's laws, two topological constraints[17] about magnetic flux φ and charge q can be derived from the law of conservation of energy and the law of conservation of charge. Thus, for all the fluxes around a closed path (or loop) in a circuit, there exists

$$\sum_{m=1}^{M} \varphi_m = 0, \tag{12}$$

where M is the number of fluxes in the loop (or the number of branches in the loop) and φ_m is the m-th flux; for all the electric charges entering into a node (or a closed boundary), there exists

$$\sum_{n=1}^{N} q_n = 0, \tag{13}$$

where N is the number of branches connected to the node, and q_n is the n-th charge entering into (or leaving from) the node.

Consequently, the nonlinear circuit models containing memristors can be rewritten as

$$f_2(\mathrm{d}q/\mathrm{d}t, \mathrm{d}\varphi/\mathrm{d}t, q, \varphi, t) = 0, \tag{14}$$

where $f_2(\cdot)$ satisfies the topological constraints (6) and (7) as well as the constitutive relations of all elementary circuit elements by the flux–charge relations.

2.3. Mapping equivalent approach

A memristive dynamical circuit consists of four fundamental circuit elements of resistor R, capacitor C, inductor L, and memristor M, its outputs are the voltage v_o and/or current i_o of any branch as shown in Fig. 1(a). A nonlinear dynamical circuit without memristor contains three fundamental circuit elements of resistor R, capacitor C, inductor L, and one nonlinear element of nonlinear resistor R_N, whose outputs are voltage v_o and/or current i_o via taking the derivative of the corresponding branch voltage v and/or current i, as shown in Fig. 1(b). Two circuits have the same topologies and the same circuit parameters of linear circuit elements. If there is the relationship of $\mathrm{d}f(i_\mathrm{N})/\mathrm{d}i_\mathrm{N} = M(i_\mathrm{N})$ between the memristor and the nonlinear resistor, their external outputs of voltage v_o and current i_o in Figs. 1(a) and 1(b) have identical dynamics.

The above conclusion can be illustrated as follows.

As shown in Figs. 1(a) and 1(b), the topological constraints of circuit connections and the constitutive relations of circuit elements are described by Eq. (1), and this nonlinear circuit model is called voltage-current model; while as shown in Fig. 2, its topological constraints of circuit connections and the constitutive relations of circuit elements are described by Eq. (14), and this nonlinear circuit model is called flux-charge model.

It is noted that the memristor M in Fig. 1(a) is assumed to be a charge-controlled memristor and its original characteristic is expressed by $\varphi_\mathrm{M} = f(q_\mathrm{M})$, and by differentiation the voltage-current relation can be obtained as shown in Fig. 1(a); while the nonlinear resistor R_N in Fig. 1(b) is regarded as a current-controlled nonlinear resistor. If the memristor M and the nonlinear resistor R_N have the same nonlinear characteristics, there exists the relationship of $\mathrm{d}f(i_\mathrm{N})/\mathrm{d}i_\mathrm{N} = M(i_\mathrm{N})$ due to the memristance of the memristor being $M(q_\mathrm{M}) = \mathrm{d}f(q_\mathrm{M})/\mathrm{d}q_\mathrm{M}$.

Based on the flux–charge relations of circuit elements and two topological constraints (12) and (13), the circuit equations of Fig. 2 can be built, and the flux φ and the charge q of each branch can be solved. Taking the derivative of the branch flux φ and/or charge q, the outputs of voltage v_o and/or current i_o in Fig. 2 should certainly be identical to the corresponding branch outputs in Fig. 1(a).

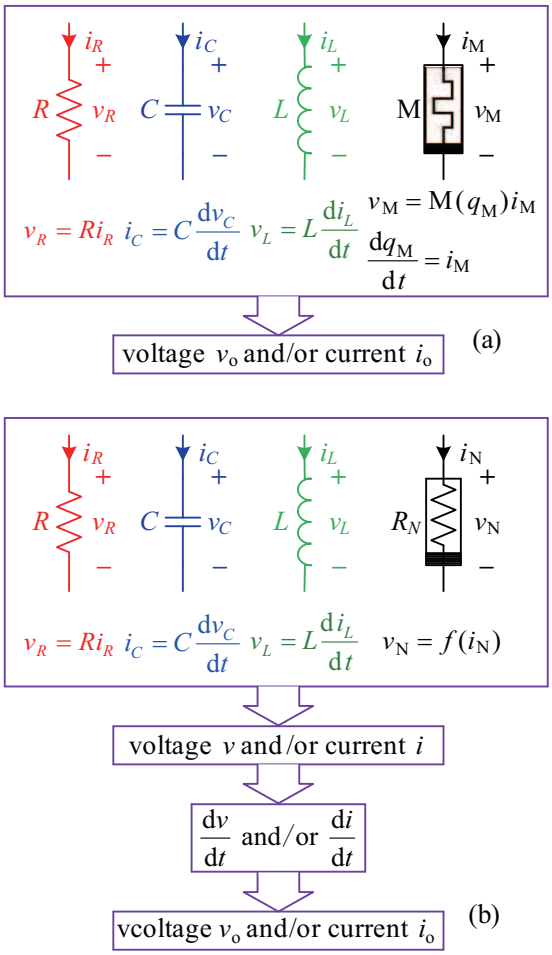


Fig. 1. (color online) Illustration of the mapping equivalent approach, showing (a) memristive dynamical circuit with the branch voltage and/or current outputs, and (b) nonlinear dynamical circuit with the differentiating outputs of the corresponding branch voltage and/or current.

Transforming by a mapping relationship of $(\varphi, q) \to (v, i)$, the fluxes and charges in all the mathematical relations of Fig. 2 are replaced by the voltages and currents. Therefore, the memristive dynamical circuit of Fig. 2 and the nonlinear dynamical circuit of Fig. 1(b) have the same solutions. Thus, the above conclusion can be established.

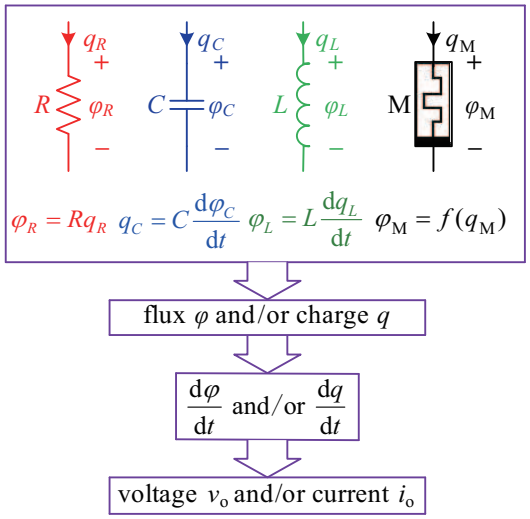


Fig. 2. (color online) Memristive dynamical circuit described by flux and charge with the differentiating outputs of the corresponding branch.

3. Equivalent dynamical analysis of memristor-based chaotic circuit

In this section, a memristor-based chaotic circuit shown in Fig. 3(a) is taken as an example. The proposed mapping equivalent approach illustrates that the dynamics of the memristor-based chaotic circuit can be equivalently analyzed by using Chua's chaotic circuit shown in Fig. 3(b).

3.1. Dynamical analysis of memristor-based chaotic circuit

The memristor-based chaotic circuit as shown in Fig. 3(a), is directly derived from Chua's chaotic circuit as shown in Fig. 3(b), by replacing Chua's diode with a flux-controlled memristor. The flux-controlled memristor is active and characterized by a smooth piecewise-quadratic nonlinearity[14] as follows:

$$q_{\mathrm{M}} = f(\varphi_{\mathrm{M}}) = -a\varphi_{\mathrm{M}} + 0.5b\,|\varphi_{\mathrm{M}}|\,\varphi_{\mathrm{M}}, \tag{15}$$

where $a, b > 0$. By differentiation the voltage-current relation is given by

$$i_{\mathrm{M}} = W(\varphi_{\mathrm{M}})v_{\mathrm{M}} = (-a + b\,|\varphi_{\mathrm{M}}|)v_{\mathrm{M}}, \tag{16}$$

where $W = \mathrm{d}f(\varphi_{\mathrm{M}})/\mathrm{d}\varphi_{\mathrm{M}} = -a + b\,|\varphi_{\mathrm{M}}|$ is the memductance of the flux-controlled memristor.

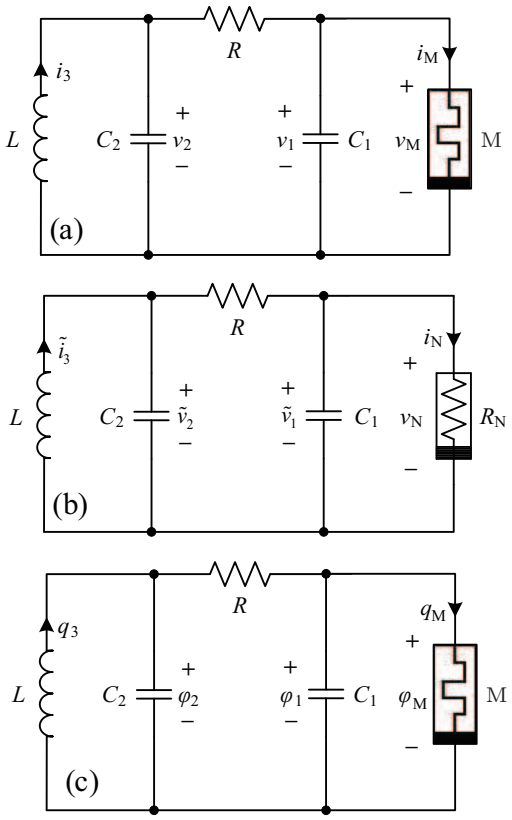


Fig. 3. (color online) Applications of the mapping equivalent approach in (a) voltage-current model of the memristor-based chaotic circuit, (b) voltage-current model of Chua's chaotic circuit, and (c) flux-charge model of the memristor-based chaotic circuit.

Consider four state variables in Fig. 3(a), i.e., capacitor C_1 (voltage v_1), capacitor C_2 (voltage v_2), inductor L (current i_3), and memristor M (flux φ_{M}). A set of four first-order differential equations for the memristor-based chaotic circuit can be established as

$$\begin{cases} \dfrac{\mathrm{d}v_1}{\mathrm{d}t} = \dfrac{1}{C_1}\left(\dfrac{v_2 - v_1}{R} - W_{\mathrm{M}}(\varphi_{\mathrm{M}})v_1\right), \\ \dfrac{\mathrm{d}v_2}{\mathrm{d}t} = \dfrac{1}{C_2}\left(\dfrac{v_1 - v_2}{R} - i_3\right), \\ \dfrac{\mathrm{d}i_3}{\mathrm{d}t} = \dfrac{1}{L}v_2, \\ \dfrac{\mathrm{d}\varphi_{\mathrm{M}}}{\mathrm{d}t} = \dfrac{1}{\tau}v_1, \end{cases} \tag{17}$$

where $\varphi_{\mathrm{M}}(t)$ is the integral of $v_{\mathrm{M}}(t)$ (or $v_1(t)$) with integral time constant τ[15] and represents the magnetic flux between two terminals of the memristor.

Taking advantage of the values of circuit parameters shown in Table 1 and choosing the initial values of four state variables as $v_1(0) = 1$ μV, $v_2(0) = 0$ V, $i_L(0) = 0$ A, and $\varphi(0) = 0$ Wb, when $R = 2.0$ kΩ and $R = 2.4$ kΩ, the phase portraits in the $v_{\mathrm{o}1}(t)$–$v_{\mathrm{o}2}(t)$ output plane are shown in Figs. 4(a)

Table 1. Circuit parameters for simulations.

Parameter	Signification	Value
C_1	capacitance	6.8 nF
C_2	capacitance	68 nF
L	inductance	17.2 mH
R	resistance	2.0 kΩ
a	coefficient	0.6667 mS
b	coefficient	0.403 mS/Wb

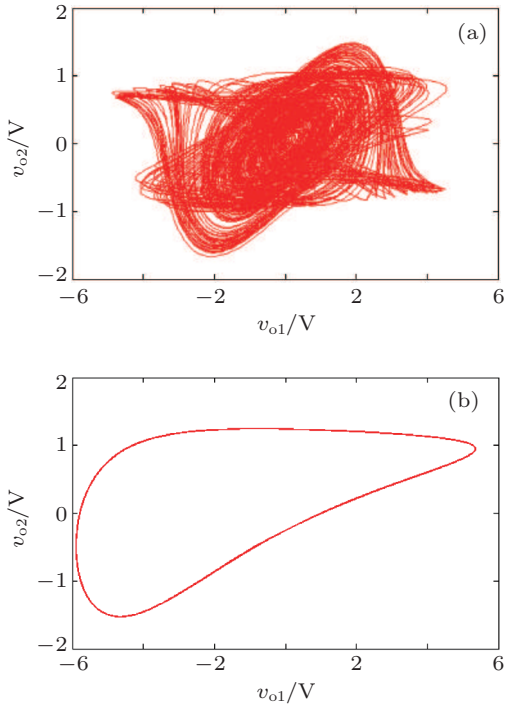


Fig. 4. (color online) Numerical phase portraits of memristor-based chaotic circuit with (a) $R = 2.0$ kΩ and (b) $R = 2.4$ kΩ.

and 4(b), respectively, where $v_{o1}(t) = v_1(t)$ and $v_{o2}(t) = v_2(t)$ are two output voltages of the memristor-based chaotic circuit. The numerical simulations demonstrate that the circuit can generate chaotic and periodic signals.

3.2. Equivalent analysis by using Chua's chaotic circuit

Correspondingly, for Chua's chaotic circuit[18] in Fig. 3(b), the Chua's diode with the same nonlinearity as that of Eq. (15) is characterized by

$$i_N = f(v_N) = -av_N + 0.5b|v_N|v_N, \tag{18}$$

where a, $b > 0$, from which it can be found that the Chua's diode is an active current-controlled nonlinear resistor.

Three state variables in Fig. 3(b) are capacitor C_1 (voltage $\tilde{v}_1$), capacitor C_2 (voltage $\tilde{v}_2$), and inductor L (current $\tilde{i}_3$), and the dynamical equations of Chua's chaotic circuit can then be constructed as

$$\begin{cases} \dfrac{d\tilde{v}_1}{dt} = \dfrac{1}{C_1}\left(\dfrac{\tilde{v}_2 - \tilde{v}_1}{R} - i_N\right), \\ \dfrac{d\tilde{v}_2}{dt} = \dfrac{1}{C_2}\left(\dfrac{\tilde{v}_1 - \tilde{v}_2}{R} - \tilde{i}_3\right), \\ \dfrac{d\tilde{i}_3}{dt} = \dfrac{1}{L}\tilde{v}_2, \end{cases} \tag{19}$$

where $v_N = \tilde{v}_1$.

When R = 2.0 kΩ and R = 2.4 kΩ, for the same circuit parameters listed in Table 1 and the initial values of three state variables $\tilde{v}_1(0) = 1$ μV, $\tilde{v}_2(0) = 0$ V, and $\tilde{i}_L(0) = 0$ A, the phase portraits in the $\tilde{v}_1(t)$–$\tilde{v}_2(t)$ plane are shown in Figs. 5(a) and 5(b), respectively.

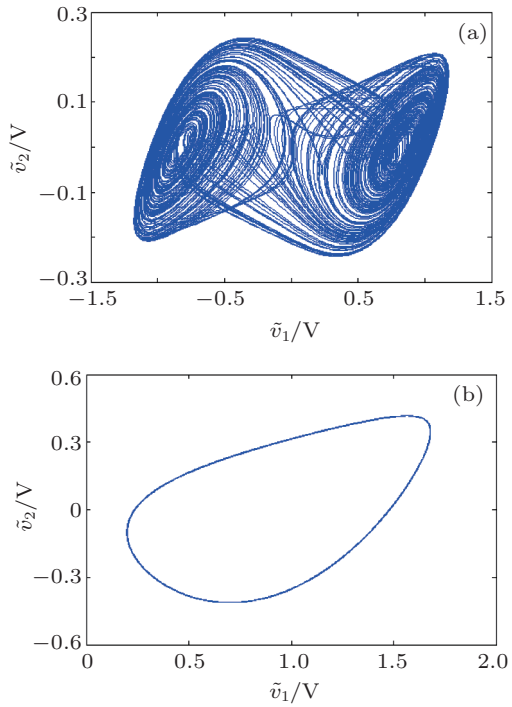


Fig. 5. (color online) Numerical phase portraits of Chua's chaotic circuit with (a) R = 2.0 kΩ and (b) R = 2.4 kΩ.

Take $v_{o1} = \tau d\tilde{v}_1/dt$ and $v_{o2} = \tau d\tilde{v}_2/dt$ as two output voltages of Chua's chaotic circuit, where τ is a rescaling factor with the same value as the above integral time constant. The corresponding phase portraits in the $v_{o1}(t)$–$v_{o2}(t)$ output plane are shown in Figs. 6(a) and 6(b), respectively. The numerical simulations illustrate that the circuit can oscillate in chaotic and periodic states.

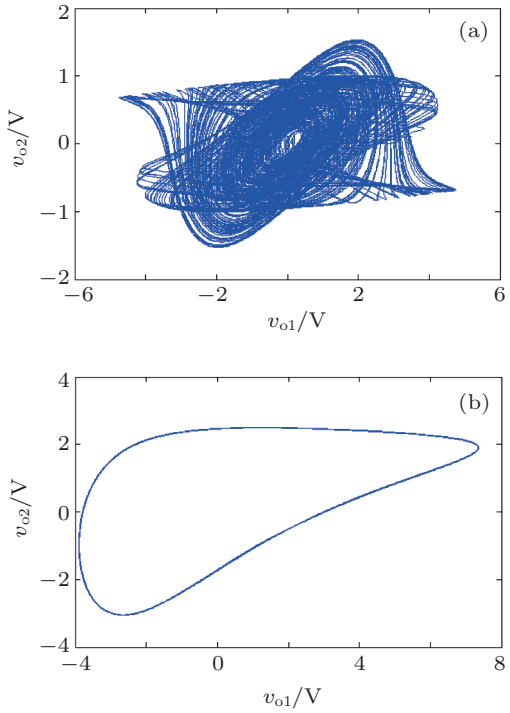


Fig. 6. (color online) Equivalent phase portraits by Chua's chaotic circuit with two output differentiators with (a) R = 2.0 kΩ and (b) R = 2.4 kΩ.

3.3. Remarks for dynamical behaviors of two chaotic circuits

Different from the voltage–current model in Fig. 3(a), the flux–charge model of the memristor-based chaotic circuit is shown in Fig. 3(c). Based on the flux–charge relations of Eqs. (9)–(11) and (15), three state variables of capacitor C_1 (flux φ_1), capacitor C_2 (flux φ_2), and inductor L (charge q_3) in Fig. 3(c) can be selected, and then a set of three first-order differential equations can be obtained as

$$\begin{cases} \dfrac{d\varphi_1}{dt} = \dfrac{1}{C_1}\left(\dfrac{\varphi_2 - \varphi_1}{R} - q_M\right), \\ \dfrac{d\varphi_2}{dt} = \dfrac{1}{C_2}\left(\dfrac{\varphi_1 - \varphi_2}{R} - q_3\right), \\ \dfrac{dq_3}{dt} = \dfrac{1}{L}\varphi_2, \end{cases} \tag{20}$$

where q_M is expressed by Eq. (15). Since $q_M = f(\varphi_M)$ and $i_N = f(v_N)$ have the same nonlinearities, the dynamical equations of Eq. (20) are the same as those of Eq. (19), implying that they have the identical solutions for the same initial states. Consequently, the mapping equivalent approach satisfies in this case that the dynamical behaviors of the memristive

dynamical circuit in Fig. 3(a) can be equivalently analyzed by using the nonlinear dynamical circuit in Fig. 3(b).

It is remarkable that the memristor-based chaotic circuit in Fig. 3(a) and Chua's chaotic circuit in Fig. 3(b) have the same topologies, and there exists the following relationship:

$$\frac{\mathrm{d}f(v_{\mathrm{N}})}{\mathrm{d}v_{\mathrm{N}}} = W_{\mathrm{M}}(v_{\mathrm{N}}) = -a + b|v_{\mathrm{N}}|. \tag{21}$$

Furthermore, comparing Fig. 6 with Fig. 4, it can be found that the output voltages of Chua's chaotic circuit via two output differentiators and the memristor-based chaotic circuit have the similar dynamical characteristics, which verify that the mapping equivalent approach is effective for the equivalent analysis of the memristor-based chaotic circuit by utilizing Chua's chaotic circuit.

For the circuit parameters listed Table 1, the resistor R is a varying parameter. For the case that R increases gradually, the bifurcation diagram of state variable $v_1(t)$ of the memristor-based chaotic circuit and the bifurcation diagram of state variable $\tilde{v}_1(t)$ of Chua's chaotic circuit are depicted in Figs. 7(a1) and 7(b1), respectively. Note that the Poincaré sections $v_2(t) = 0$ V and $\tilde{v}_2(t) = 0$ V are selected to plot the corresponding bifurcation diagrams. Making use of Wolf's method to calculate Lyapunov exponents,[19] the results corresponding to Figs. 7(a1) and 7(b1) are plotted in Figs. 7(a2) and 7(b2), respectively.

It can be observed that the dynamical characteristic of the memristor-based chaotic circuit is similar to that of Chua's chaotic circuit. However, when R is located in some parameter regions, there appear some differences in dynamical behavior between Fig. 7(a) and Fig. 7(b), including transient chaotic and periodic orbits in Fig. 7(a)[10,20] and periodic windows in Fig. 7(b). Additionally, the dynamical behaviors of the memristor-based chaotic circuit are greatly dependent on the initial state of the memristor.[21] Despite all this, the mapping equivalent approach is useful to equivalently analyze the dynamics of the memristor-based chaotic circuit by using the Chua's chaotic circuit.

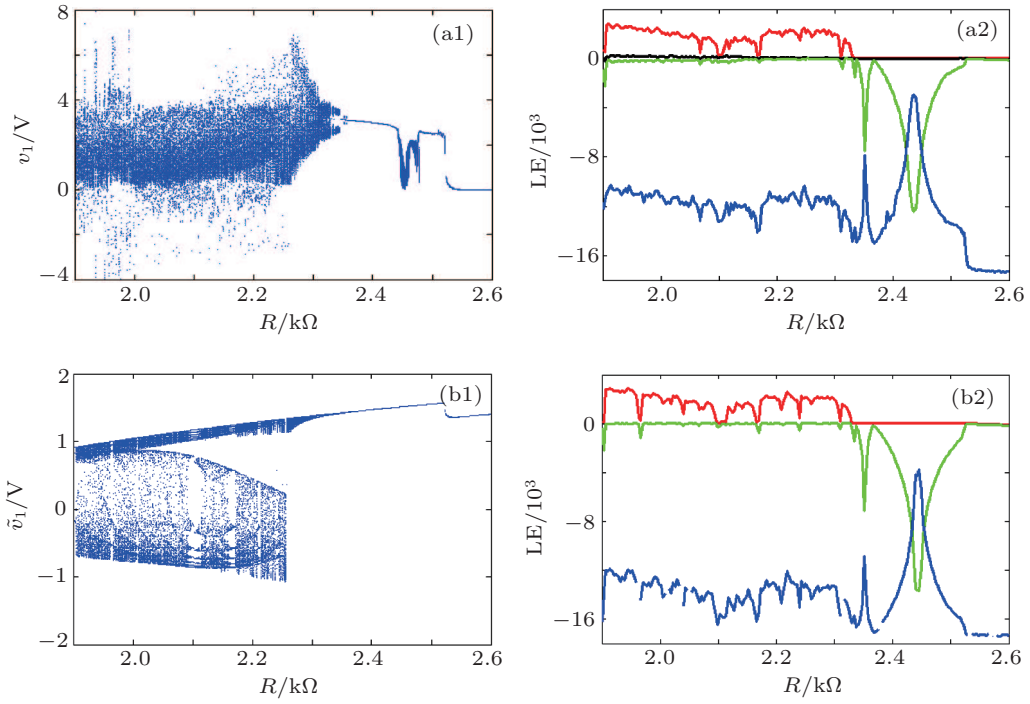


Fig. 7. (color online) Dynamic behaviors varying with resistor R in (a) memristor-based chaotic circuit, (b) Chua's chaotic circuit. Panels (a1) and (b1) are bifurcation diagrams, and panels (a2) and (b2) show the corresponding Lyapunov exponent spectra.

4. Equivalent realization of memristor-based chaotic circuit

The mapping equivalent approach is applied to the equivalent realization of memristor-based chaotic circuit by using Chua's chaotic circuit in this section. Correspondingly, the circuit experimental results are obtained.

4.1. Design of unified equivalent circuit

The unified circuit schematic to realize the flux controlled memristor characterized by Eq. (15) and the Chua's diode characterized by Eq. (18) is shown in Fig. 8(a), where $H(\cdot)$ circuit is an absolute function circuit in Fig. 8(b) and $\int v$ circuit is an integrator in Fig. 8(c).[14,15] When the switch S is located at position 1, the circuit is used for realizing the flux controlled

memristor in Fig. 3(a); while when S is located at position 2, the circuit is used for implementing the Chua's diode in Fig. 3(b).

The circuit in Fig. 8(b) consisting of two operational amplifiers U_2 and U_3, two resistors $R_{\rm sat}$ and R_4, and one multiplier M_2 is an absolute function circuit, which realizes the relationship as

$$v_{\rm b}(t) = \frac{g_2 R_4 E_{\rm sat}}{R_{\rm sat}} |v_{\rm a}(t)|. \tag{22}$$

The output of the multiplier M_1 in Fig. 8(a) is

$$v_{\rm c}(t) = g_1 v_{\rm b}(t) v(t). \tag{23}$$

Thus, when $R_2 = R_3$, the equivalent constitutive relation of the circuit in Fig. 8(a) can be given by[14]

$$i(t) = -\frac{v(t)}{R_1} + \frac{v_{\rm c}(t)}{R_1} = \left(-\frac{1}{R_1} + \frac{g_1 g_2 R_4 E_{\rm sat}}{R_1 R_{\rm sat}} |v_{\rm a}|\right) v(t), \tag{24}$$

where g_1 and g_2 are variable scale factors of the multipliers M_1 and M_2 respectively, $E_{\rm sat}$ is the saturated output voltage of operational amplifier and equals 13.5 V measured in the experimental setup.

If the switch S is located at position 1, there exists $v_a(t) = \varphi(t)$; while if S is located at position 2, there is $v_a(t) = v(t)$. Comparing Eq. (24) with Eq. (16) or Eq. (18), the following relationships for a and b in terms of the component values in Fig. 8 are obtained

$$a = \frac{1}{R_1}, \quad b = \frac{g_1 g_2 R_4 E_{\rm sat}}{R_1 R_{\rm sat}}. \tag{25}$$

When the parameters of the nonlinearity of Eq. (15) or (18) are $a = 0.6667$ mS and $b = 0.403$ mS/Wb, the designed circuit parameters in Figs. 8(a) and 8(b) are selected as listed in Table 2.

Table 2. Circuit parameters for equivalent realization.

Parameter	Signification	Value
R_1	resistance	1.5 kΩ
R_2, R_3	resistance	2 kΩ, 2 kΩ
R_4	resistance	6.05 kΩ
$R_{\rm sat}$	resistance	13.5 kΩ
g_1	scale factor of multiplier M_1	0.1
g_2	scale factor of multiplier M_2	1
C_0	capacitance	68 nF
R_0	resistance	4 kΩ
R_5, R_6	resistance	2 kΩ, 2 kΩ

The circuit in Fig. 8(c) is an integrator with time constant $\tau = R_0 C_0$ that implements (when $R_5 = R_6$)

$$\frac{{\rm d}v_{\rm a}(t)}{{\rm d}t} = \frac{R_6}{R_5}\frac{v(t)}{R_0 C_0} = \frac{v(t)}{\tau}. \tag{26}$$

The circuit in Fig. 8(d) is a differentiator with time constant $\tau = R_0 C_0$, which serves as the output circuit of Chua's chaotic circuit in Fig. 3(b). When $R_5 = R_6$, the differentiator yields

$$v_{\rm o}(t) = \frac{R_6}{R_5} R_0 C_0 \frac{{\rm d}v(t)}{{\rm d}t} = \tau \frac{{\rm d}v(t)}{{\rm d}t}. \tag{27}$$

For the circuits in Figs. 8(c) and 8(d), the values of the circuit parameters are chosen as listed in Table 2.

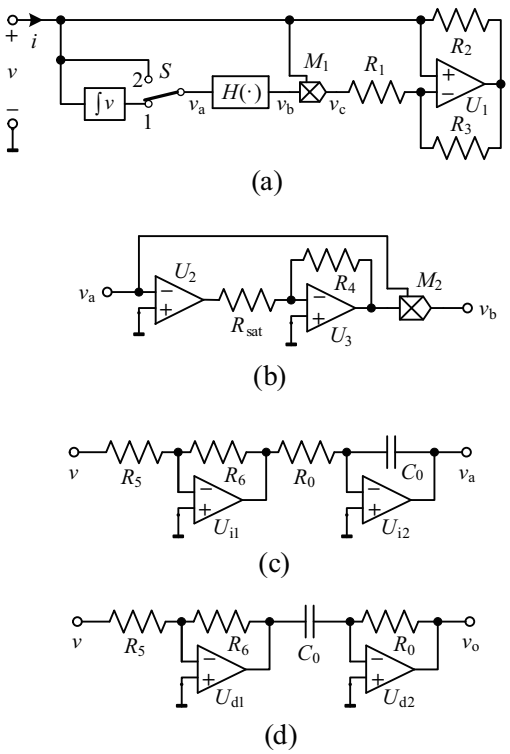


Fig. 8. Unified circuit realization of the flux controlled memristor and the Chua's diode: (a) main circuit; (b) absolute function circuit; (c) integrating circuit; and (d) differentiating circuit.

It can be found that the circuit parameter values of the flux controlled memristor are identical to those of the Chua's diode, which ensures that the relation in Eq. (21) holds true.

4.2. Circuit experimental observations

Based on the circuit realizations of the flux controlled memristor and the Chua's diode in Fig. 8, the experimental circuits of Figs. 3(a) and 3(b) can be built, where the parameters of experimental components are given in Tables 1 and 2 respectively, and the others are AD633JN as analog multiplier, AD711kN as operational amplifier, and operation voltages of ±15 V.

First, the experimental observations for the memristor-based chaotic circuit are performed. When $R = 2.0$ kΩ and $R = 2.4$ kΩ, chaotic and periodic orbits are captured and exhibited in Figs. 9(a) and 9(b), respectively. The experimental results reveal that the memristor-based chaotic circuit can exhibit chaotic and periodic behaviors.

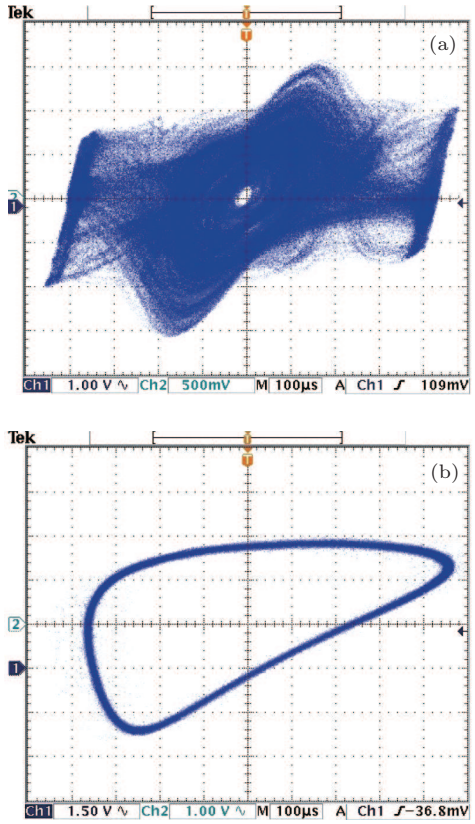


Fig. 9. (color online) Measured phase portraits of memristor-based chaotic circuit with (a) $R = 2.0$ kΩ and (b) $R = 2.4$ kΩ. The horizontal axis is $v_1(t)$ and the vertical axis is $v_2(t)$.

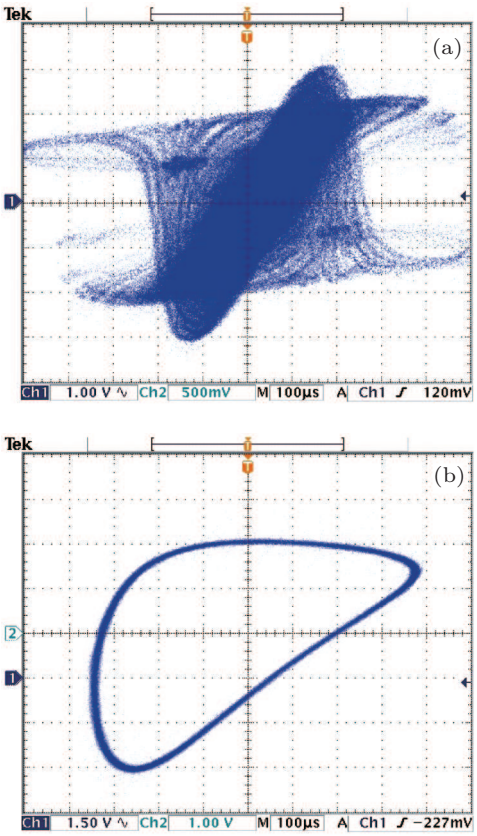


Fig. 10. (color online) Measured phase portraits of Chua's chaotic circuit via two output differentiators with (a) $R = 2.0$ kΩ and (b) $R = 2.4$ kΩ. The horizontal axis is $v_{o1}(t)$ and the vertical axis is $v_{o2}(t)$.

Next, the effectiveness of equivalent realization by using Chua's chaotic circuit is tested to verify the proposed mapping equivalent approach. When $R = 2.0$ kΩ and $R = 2.4$ kΩ, the corresponding oscillation states can be measured from two output differentiators connecting Chua's chaotic circuit and displayed in Figs. 10(a) and 10(b), respectively. The experimental results are in good agreement with numerical simulations.

5. Conclusions

In this paper, by Chua's elementary circuit element quadrangle and Wang's elementary circuit element triangle, the mapping relations of two basic electrical circuit laws and the constitutive relations of four fundamental circuit elements are established. Correspondingly, a novel mapping equivalent approach is presented, based on which the equivalent dynamical analysis and realization of the memristor-based chaotic circuit are performed by using Chua's chaotic circuit. Numerical and experimental results illustrate that the proposed approach is effective and imply that a memristor-based chaotic circuit can be directly derived from a nonlinear chaotic circuit. However, the memristive dynamical circuit in fact has different dynamical behaviors from the nonlinear dynamical circuit for some circuit parameters, such as transient chaotic orbits in the memristive dynamical circuit, periodic windows in the nonlinear dynamical circuit and so on. More details are not discussed in this paper and will be reported later.

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