Delay-dependent asymptotic stability of mobile ad-hoc networks: A descriptor system approach*

Yang Juan(杨 娟)[a)b)†], Yang Dan(杨 丹)[c)], Huang Bin(黄 彬)[d)], Zhang Xiao-Hong(张小洪)[c)], and Luo Jian-Lu(罗建禄)[e)]

[a)] College of Communication Engineering, Chongqing University, Chongqing 400030, China
[b)] The 3rd Department, People's Liberation Army, Chongqing Communication Institute, Chongqing 400035, China
[c)] School of Software Engineering, Chongqing University, Chongqing 400030, China
[d)] Foundation department, People's Liberation Army, Chongqing Communication College, Chongqing 400035, China
[e)] The Department of Electronic Technology, People's Armed Police, Police College, Chengdu 610213, China

(Received 30 October 2013; revised manuscript received 21 January 2014; published online 22 May 2014)

In order to analyze the capacity stability of the time-varying-propagation and delay-dependent of mobile ad-hoc networks (MANETs), in this paper, a novel approach is proposed to explore the capacity asymptotic stability for the delay-dependent of MANETs based on non-cooperative game theory, where the delay-dependent conditions are explicitly taken into consideration. This approach is based on the Lyapunov–Krasovskii stability theory for functional differential equations and the linear matrix inequality (LMI) technique. A corresponding Lyapunov–Krasovskii functional is introduced for the stability analysis of this system with use of the descriptor and "neutral-type" model transformation without producing any additional dynamics. The delay-dependent stability criteria are derived for this system. Conditions are given in terms of linear matrix inequalities, and for the first time referred to neutral systems with the time-varying propagation and delay-dependent stability for capacity analysis of MANETs. The proposed criteria are less conservative since they are based on an equivalent model transformation. Furthermore, we also provide an effective and efficient iterative algorithm to solve the constrained stability control model. Simulation experiments have verified the effectiveness and efficiency of our algorithm.

Keywords: stability control of capacity, Lyapunov–Krasovskii functional, time delay-dependent, descriptor

PACS: 05.45.–a, 02.30.Yy **DOI:** 10.1088/1674-1056/23/7/070502

1. Introduction

Network capacity is a key feature of mobile ad-hoc networks (MANETs).[1,2] In order to utilize the MANETs resources to the greatest extent, the data flow rate passing through the link should be balanced and get close to the link capacity instead of continuously fluctuating between insufficient use of bandwidth and complete overload, so that the capacity of MANETs can stay stable. Since MANETs are a distributed time-varying dynamic system, and while at the same time the capacity thereof is affected by many factors per se, such as power, bandwidth, mode of communication, routing policy, interference model, time-varying propagation delay, etc., the problem of capacity stability becomes a quite challenging topic.

Even though much progress has been made in the research into the stability of communication system channel capacity,[3] MANETs capacity,[4–6] network with base station,[7,8] and the distributed network without central base station,[9–11] the distributed network MANETs capacity models are generally non-linear dynamic equations.[12–14] However, there are only a few publications that have studied the stability of non-linear dynamic equations.[15,16] Consequently, even fewer studies have been carried out on MANETs capacity stability analysis. In Ref. [17], ordinary game theory was introduced to analyze the capacity of MANETs and the maximum end-to-end throughput stability. Based on the principle of flow balance, reference [18] has described the throughput stability conditions of MANETs. Furthermore, the relative entropy restriction method was used in Ref. [19] to investigate the control sphere of capacity stability for a type of MANET. However, none of the above references has analyzed the question of MANET capacity stability using the Lyapunov stability theory. Although reference [20] has introduced Lyapunov stability theory to analyze the MANET's capacity stability, the data propagation delay has not been considered. Furthermore, although the data time-varying propagation delay has considered in Ref. [21], it proposed delay-independent stability criteria.

Undoubtedly, considering that the data time-varying propagation delay MANETs capacity analysis models are non-

*Project supported by the National Natural Science Foundation of China (Grant No. 61173131), the Fundamental Research Funds for the Interdisciplinary Class Major Projects for the Central Universities of Ministry of Education of China (Grant No. CDJZR12098801), the Based and Advanced Research Projects of Chongqing of China (Grant No. cstc2013jcyjA40033), the Key Strategic Project of Chongqing of China (Grant No. CSTC2009AB2230), the Strategeic Project of Chongqing of China (Grant No. 2009AC2057), and the Special Funding for Scientific Research Project of Chongqing Postdoctoral Researchers, China (Grant No. XM20120054).

†Corresponding author. E-mail: juanyang@cqu.edu.cn

© 2014 Chinese Physical Society and IOP Publishing Ltd

linear time-varying delay systems,[21] and they are delay-dependent systems. There are two criteria concerning the stability of systems with time-delays. One is the delay-independent stability criteria, in which the size of the time-delay is not including in the formulation. The other is the delay-dependent stability criteria, in which the size of the time-delay is taken explicitly in the formulation. Delay-dependent criteria are used as the information on the length of time-delays systems, so they may be less conservative than delay-independent ones.[22] Some existing delay-dependent stability criteria for nonlinear time-varying delay systems are conservative since the model transformation will introduce additional dynamics.[23–29] A descriptor is a powerful tool to analyse the stability of nonlinear time-varying delay-dependent systems since it is based on "neutral-type" model transformation without producing any additional dynamics.[22,30]

Meanwhile, using a Lyapunov–Krasovskii functional to study the MANETs capacity stability considering the propagation delay of delay-dependent is still in its infancy, especially when the descriptor technology combined with Lyapunov–Krasovskii functional is applied in research field of MANET's capacity stability. This is still a new area and it has considerable potential. In this paper, according to the progress reported in Refs. [21]–[31], the descriptor technology combined with Lyapunov–Krasovskii functional[32,33] is used in a new perspective to analyze the delay-dependent stability criteria of the conflict-free single channel based and time-varying propagation delay considered MANETs within the normalized time, aiming at setting a unified capacity stability control frame that is less conservative for the non-competition conflict-free MAC protocol based MANETs.

The contributions of this paper can be highlighted as follows.

(i) Based on equivalent augmented model—a "descriptor form" combined with Lyapunov–Krasovskii method, a novel non-cooperative program game model delay-dependent stability control algorithm is developed to analyze the MANET's capacity stability.

(ii) The MANET's capacity delay-dependent stability control method can be generally well-adapted to the non-competition, conflict-free MAC protocol based MANETs, which can be facilely realized by regulating the physical characteristics of MANET's nodes, such as the applied power, available memory capacity, signal modulation method, encoding mode, and so on. It provides strong theoretical support for building the QoS satisfied MANETs.

(iii) The calculation for the proposed stability control algorithm in this paper is small because the MANETs capacity delay-dependent stability control criteria can be easily obtained using MATLAB toolbox.

In this paper, based on the non-cooperative program game model for MANETs capacity analysis, the MANETs source nodes propagation flow rate evolution equation set of the distributed time-varying dynamic system without considering the propagation delay has been introduced in Section 2. In Section 3, the descriptor technology combined with the Lyapunov–Krasovskii functional with delay-dependent conditions is used to systematically analyze the delay-dependent asymptotic stability conditions, the delay-dependent stability control algorithm is also given in this section. Section 4 provides three sets of simulation results. Finally, conclusions are drawn in Section 5.

2. Problem description and system model

Given a generalized MANET network topology $M = (N, L)$, where $N = \{1, 2, \ldots, n\}$ denotes the node set of containing n nodes, and $L = \{1, 2, \ldots, m, \ldots, j\}$ represents the set of shared links the containing j links. For each source node i, its routing link is formed by the set $L_i = \{l_{i_m}\}$, $l_{i_m} \in L$, and apparently, $L_i \subset L$. Each link of the network has its fixed capacity g_m and a fixed data buffer area B_i, $g_m > 0$, $B_i > 0$, so the non-cooperative program game model for the MANETs source node propagation flow rate distribution can be described as $G = \{K, \{\Omega_i\}, \{u_i(x_i)\}\}$, $i = 1, 2, \ldots, k$, where the $K = \{1, 2, \ldots, i \ldots, k\}$ is the set of source nodes, Ω_i is the strategy space of source node i, x_i stands for the flow rate from source node i, $u_i(x_i)$ is the utility function of source node i. We assume that the time has been divided into several slots with fixed length. We further assume that S refers to the situation of the links that can concurrently transmit data within the normalized time 1. That is, S describes the set of available links within the normalized time 1, namely, $S = \{s_{\{m\}}, \{m\} \subset L\}$, where each $s_{\{m\}}$ represents the available link set of a concurrent situation.

Definition 1[21,31] Within the normalized time, if the concurrent and non-competition maximum feasible link set $S_{\max}$ do exist in the MANETs with n nodes and j links, and $S_{\max}$ fits the feasible link set, the capacity of MANETs will be

$$C(n) = \sum_{s_{\{m\}} \in S_{\max}} \left(\sum_{\substack{i \in K \\ m \in s_{\{m\}}}} x_i^* \right),$$

where $x_i > 0$, $i = 1, 2, \ldots, k$ stands for the flow rate from source node i, the number of links is represented by $m = 1, 2, \ldots, j$ and $\{x_1^*, x_2^*, \ldots, x_k^*\}$ is the Nash equilibrium solution.

Note Given non-cooperative program game

$$G = \{K, \{\Omega_i\}, \{u_i(x_i)\}\}, \quad i = 1, 2, \ldots, k, \tag{1}$$

where the game node (source node) i is within the strategy space Ω_i of the propagation flow rate x_i with the value range of $[0, g_{l_i}]$, under the restriction of $[0, g_{l_i}]$, $g_{l_i} = \min\{g_{l_{i_m}}, l_{i_m} \in L_i\}$. Then, the maximizing utility function can be obtained as

$$u_i(x_i) = \alpha_i \sum_{l_{im}\in s_{\{m\}}} \lg\left(\frac{x^h_{i,l_{im}}}{g_{l_i}}+1\right) - \beta_i \sum_{l_{im}\in s_{\{m\}}} \frac{1}{t_{s_{l_{im}}} g_{l_i} - x_{l_{im}}}. \quad (2)$$

The obtained Nash equilibrium solution is $\{x_1^*, x_2^*, \ldots, x_k^*\}$, and x_i^* is Nash equilibrium flow rate for source node i. In Eq. (1), $x^h_{i,l_{im}}$, $h = 1,2,\ldots,k$ stands for the h-th flow rate passing through link l_{im}, and $x_{l_{im}} = \sum_{h\in K} x^h_{i,l_{im}}$ represents the sum of all the flow rates passing through link l_{im}. Obviously, under the ideal conditions, we have $x_i = x^h_{i,l_{im}}$. As a result, $\Omega_i = \{x_1, x_2, \ldots, x_i, \ldots, x_k\}$ can then be represented by

$$\left\{ x^h_{i,l_{im}} \middle| x_{l_{im}} = \sum_{h\in K} x^h_{i,l_{im}} < t_{s_{l_{im}}} g_{l_i}, \; i = 1,2,\ldots,k, m = 1,2,\ldots,j \right\}, \quad (3)$$

where $g_{l_i} = \min\{g_{l_{im}}, l_{im} \in L_i\}$ is the minimum capacity of the proposed routing link set $\{l_{im}\}$, $u_i(x_i)$ is the flow rate distribution utility function of the source node i. In addition, α_i represents the sensitivity of node i for propagation flow rate x_i, which is determined by the applied power on node i, available memory, signal modulation method, encoding mode, and the bandwidth, routing strategy of node i, and β_i represents the sensitivity of used propagation flow rate x_i on node i to the queue delay.

Note The existence of Nash equilibrium solution for this model has been proven in a previous work.[31]

Definition 2[21] The MANET's source nodes propagation flow rate evolution equation set of the distributed time-varying dynamic system without considering the propagation delay can be described as follows:

$$\dot{x}_i(t) = \omega\alpha_i \sum_{l_{im}\in s_{\{m\}}} \frac{1}{x^h_{i,l_{im}}(t) + g_{l_i}} - \omega\beta_i \sum_{l_{im}\in s_{\{m\}}} \frac{1}{\left(t_{s_{l_{im}}} g_{l_i} - \sum_{h\in K} x^h_{i,l_{im}}(t)\right)^2}, \quad (4)$$

where $i = 1,2,\ldots,k$, $m = 1,2,\ldots,j$, $h = 1,2,\ldots,k$, $\omega \in R^+$ is constant, α_i stands for the sensitivity of node i to the propagation flow rate x_i, which is determined by the power applied on node i, available memory, signal modulation method, encoding mode, etc. In addition, β_i represents the sensitivity of used propagation flow rate x_i on node i to the queue delay, and it is decided by the available memory, applied power, signal modulation method, encoding mode, etc. of the node i.[31]

Definition 3[21] MANETs source nodes propagation flow rate evolution equation set of the non-cooperative program game model for MANETs capacity analysis with time-varying propagation delay can be described as follows:

$$\dot{x}_i(t) = \alpha_i \sum_{l_{im}\in s_{\{m\}}} \frac{1}{x^h_{i,l_{im}}(t) + g_{l_i}} - \beta_i \min\{t_{sl_{im}} g_{l_i}\} \times \sum_{l_{im}\in s_{\{m\}}} \frac{1}{\left(t_{s_{l_{im}}} g_{l_i} - \sum_{h\in K} x^h_{i,l_{im}}(t - r(t))\right)^2 \min\{t_{sl_{im}} g_{l_i}\}}, \quad i = 1,2,\ldots,k, \quad m = 1,2,\ldots,j, \quad h = 1,2,\ldots,k, \quad (5)$$

where $r(t)$ indicates the resulted time-varying propagation delay when the data with propagation flow rate $x_i(t) = x^h_{i,l_{im}}(t)$ from node i pass through the link $l_{im}, m = 1,2,\ldots,j$ and $r(t)$ is a function dependent on t, and $0 \le r(t) \le r_\Gamma, \dot{r}(t) \le r_d$, r_Γ, r_d are constants.

Lemma 1[22] For any constant symmetric matrix $Q \in R^{n\times n}$, $Q = Q^{\mathrm{T}} > 0$, scalar $h_\Gamma > 0$, vector function $w : [0, h_\Gamma] \longrightarrow R^n$ such that the integrations in the following are well-defined, then

$$h_{\max} \int_{-h(t)}^{0} w^{\mathrm{T}}(\zeta) Q w(\zeta)\,\mathrm{d}\zeta \ge \left(\int_{-h(t)}^{0} w(\zeta)\,\mathrm{d}\zeta\right)^{\mathrm{T}} Q \left(\int_{-h(t)}^{0} w(\zeta)\,\mathrm{d}\zeta\right). \quad (6)$$

3. MANETs capacity stability analysis with time-varying delay-dependent

First, identical transformation is applied to the equation set (5).

Theorem 1 The source nodes propagation flow rate evolution equation set (5) of the non-cooperative program game model for MANETs capacity analysis is equivalent to the descriptor system

$$\begin{cases} \dot{x}(t) = y(t), \\ 0 = -y(t) + Cx(t) - Cx[t - r(t)] \\ \qquad - C\int_{t-r(t)}^{t} y(\xi)\,\mathrm{d}\xi + Af(x(t)) + Bg(x(t - r(t))). \end{cases} \quad (7)$$

Proof Given $f_i(x(t)) = \sum_{l_{im}\in s_{\{m\}}} \frac{1}{x^h_{i,l_{im}}(t) + g_{l_i}}$,

$$g_i(x(t - r(t))) = \sum_{l_{im}\in s_{\{m\}}} \frac{1}{\left(t_{s_{l_{im}}} g_{l_i} - \sum_{h\in K} x^h_{i,l_{im}}(t - r(t))\right)^2 \min\{t_{sl_{im}} g_{l_i}\}}.$$

Therefore, $\dot{x}_i(t) = \alpha_i f_i(x(t)) - \beta_i \min\{t_{sl_{im}} g_{l_i}\} g_i(x(t - r(t)))$, $i = 1,2\ldots,k$, $m = 1,2,\ldots,j$.

Given $\dot{x}(t) = [\dot{x}_1(t) \quad \dot{x}_2(t) \quad \cdots \quad \dot{x}_k(t)]^{\mathrm{T}}$, $y(t) = \dot{x}(t)$, $f(x(t)) = [f_1(x(t)) \quad f_2(x(t)) \quad \ldots \quad f_k(x(t))]^{\mathrm{T}}$, $g(x(t - r(t))) = [g_1(x(t - r(t))) \; g_2(x(t - r(t))) \; \ldots \; g_k(x(t - r(t)))]^{\mathrm{T}}$,

$$A=\begin{bmatrix}\alpha_1 & 0 & \dots & 0\\ 0 & \alpha_2 & \dots & 0\\ \dots & \dots & & \\ 0 & 0 & \dots & \alpha_k\end{bmatrix},\quad B=\begin{bmatrix}-\beta_1\min\{t_{sl_{1_m}}g_{l_1}\} & 0 & \dots & 0\\ 0 & -\beta_2\min\{t_{sl_{2_m}}g_{l_2}\} & \dots & 0\\ \dots & \dots & \dots & \\ 0 & 0 & \dots & -\beta_k\min\{t_{sl_{k_m}}g_{l_k}\}\end{bmatrix}.$$

Therefore, equation set (5) can be equivalent to

$$\begin{cases}\dot{x}(t)=y(t),\\ y(t)=Af(x(t))+Bg(x(t-r(t))).\end{cases}\tag{8}$$

Then, system (8) can be represented in the form of descriptor system with discrete and distributed delays in the "fast variable" y[30,32]

$$\begin{cases}\dot{x}(t)=y(t),\\ 0=-y(t)+Cx(t)-Cx(t-r(t))\\ \quad -C\int_{t-r(t)}^{t}y(\xi)\,\mathrm{d}\xi+Af(x(t))+Bg(x(t-r(t))).\end{cases}$$

Clearly, as the non-linear term $f(x(t))$, $g(x(t-r(t)))$ of the descriptor system (7), it satisfies the condition (9) restricted by Eq. (3)

$$\begin{aligned}&\|f(x(t))\|\le\varphi^2\|x(t)\|,\\ &\|g(x(t-r(t)))\|\le\theta^2\|x(t-r(t))\|,\end{aligned}\tag{9}$$

where $\varphi\ge 0$, $\theta\ge 0$ are the given constants. The constraint condition (15) can also be equivalent to

$$\begin{aligned}&f^{\mathrm{T}}(x(t))f(x(t))\le\varphi^2x^{\mathrm{T}}(t)x(t),\\ &g^{\mathrm{T}}(x(t-r(t)))g(x(t-r(t)))\\ &\le\theta^2x^{\mathrm{T}}(t-r(t))x(t-r(t)).\end{aligned}\tag{10}$$

Theorem 2 The system described by Eq. (7) or (8), with uncertainty described by Eq. (10) is asymptotically stable if there exist real matrices P_2, P_3 and symmetric positive definite matrices P_1, Q, R and scalar $\varepsilon_1>0$, $\varepsilon_2>0$, such that the following LMI holds:

$$\Xi=\begin{bmatrix}\Omega_{11} & \Omega_{12} & -P_2^{\mathrm{T}}C & -r_\Gamma P_2^{\mathrm{T}}C & P_2^{\mathrm{T}}A & P_2^{\mathrm{T}}B\\ \Omega_{12}^{\mathrm{T}} & \Omega_{22} & -P_3^{\mathrm{T}}C & -r_\Gamma P_3^{\mathrm{T}}C & P_3^{\mathrm{T}}A & P_3^{\mathrm{T}}B\\ -C^{\mathrm{T}}P_2 & -C^{\mathrm{T}}P_3 & -(1-r_{\mathrm{d}})Q+\varepsilon_2\theta^2I & 0 & 0 & 0\\ -r(t)_{\max}C^{\mathrm{T}}P_2 & -r_\Gamma C^{\mathrm{T}}P_3 & 0 & -(1-r_{\mathrm{d}})r_\Gamma R & 0 & 0\\ A^{\mathrm{T}}P_2 & A^{\mathrm{T}}P_3 & 0 & 0 & -\varepsilon_1I & 0\\ B^{\mathrm{T}}P_2 & B^{\mathrm{T}}P_3 & 0 & 0 & 0 & -\varepsilon_2I\end{bmatrix}<0,\tag{11}$$

where

$$\begin{aligned}\Omega_{11}&=P_2^{\mathrm{T}}C+C^{\mathrm{T}}P_2+Q+\varepsilon_1\varphi^2I,\\ \Omega_{12}&=P_1-P_2^{\mathrm{T}}+C^{\mathrm{T}}P_3,\\ \Omega_{22}&=-P_3-P_3^{\mathrm{T}}+r_\Gamma R.\end{aligned}$$

Proof Choose the Lyapunov–Krasovskii functional candidate for system (7) as

$$V(t)=V_1(t)+V_2(t)+V_3(t),\tag{12}$$

$$\begin{aligned}V_1(t)&=(x^{\mathrm{T}}(t)y^{\mathrm{T}}(t))\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}P_1 & 0\\ P_2 & P_3\end{bmatrix}\begin{pmatrix}x(t)\\ y(t)\end{pmatrix},\\ V_2(t)&=\int_{t-r(t)}^{t}x^{\mathrm{T}}(\xi)Qx(\xi)\,\mathrm{d}\xi,\\ V_3(t)&=\int_{t-r(t)}^{t}(r(t)-t+\xi)y^{\mathrm{T}}(\xi)Ry(\xi)\,\mathrm{d}\xi,\end{aligned}\tag{13}$$

where P_2, P_3 and symmetric positive definite matrices P_1, Q, R are the solutions of Eq. (11). This functional is degenerate as it is usual for descriptor systems.[32,33] The first term of Eq. (12) corresponds to the descriptor system (see Refs. [32] and [33]), the second—to the delay-independent conditions with respect to the discrete delays $r(t)$, while the third—to the delay-dependent conditions with respect to the delays $r(t)$.

The derivative of $V(t)$ along the trajectory of system (7) is given by $\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)$. Since

$$\begin{aligned}\dot{V}_1(t)&=2(x^{\mathrm{T}}(t)y^{\mathrm{T}}(t))\begin{bmatrix}P_1 & P_2^{\mathrm{T}}\\ 0 & P_3^{\mathrm{T}}\end{bmatrix}\begin{pmatrix}y(t)\\ 0\end{pmatrix}\\ &=2(x^{\mathrm{T}}(t)y^{\mathrm{T}}(t))\begin{bmatrix}P_1 & P_2^{\mathrm{T}}\\ 0 & P_3^{\mathrm{T}}\end{bmatrix}\\ &\quad\times\begin{pmatrix}y(t)\\ -y(t)+Cx(t)-Cx(t-r(t))-C\int_{t-r(t)}^{t}y(\xi)\,\mathrm{d}\xi\end{pmatrix}\\ &\quad+2(x^{\mathrm{T}}(t)y^{\mathrm{T}}(t))\begin{bmatrix}P_1 & P_2^{\mathrm{T}}\\ 0 & P_3^{\mathrm{T}}\end{bmatrix}\\ &\quad\times\begin{pmatrix}0\\ Af(x(t))+Bg(x(t-r(t)))\end{pmatrix}\\ &=2x^{\mathrm{T}}(t)(P_1-P_2^{\mathrm{T}}+C^{\mathrm{T}}P_3)y(t)+x^{\mathrm{T}}(t)(P_2^{\mathrm{T}}C+C^{\mathrm{T}}P_2)\\ &\quad\times x(t)-2x^{\mathrm{T}}(t)P_2^{\mathrm{T}}Cx(t-r(t))\\ &\quad+2x^{\mathrm{T}}(t)(-r_mP_2^{\mathrm{T}}C)\left(\frac{1}{r_m}\int_{t-r(t)}^{t}y(\xi)\,\mathrm{d}\xi\right)\end{aligned}$$

$$
\begin{aligned}
&+2x^{\mathrm{T}}(t)P_2^{\mathrm{T}}Af(x(t))2x^{\mathrm{T}}(t)P_2^{\mathrm{T}}Bg(x(t-r(t)))\\
&-y^{\mathrm{T}}(t)(P_3+P_3^{\mathrm{T}})y(t)+2y^{\mathrm{T}}(t)(-P_3^{\mathrm{T}}C)x(t-r(t))\\
&+2y^{\mathrm{T}}(t)(-r_mP_3^{\mathrm{T}}C)\left(\frac{1}{r_m}\int_{t-r(t)}^{t}y(\xi)\,\mathrm{d}\xi\right)\\
&+2y^{\mathrm{T}}(t)P_3^{\mathrm{T}}Af(x(t))+2y^{\mathrm{T}}(t)P_3^{\mathrm{T}}Bg(x(t-r(t))),\\
\dot{V}_2(t) &= x^{\mathrm{T}}(t)Qx(t)-(1-\dot{r}(t))x^{\mathrm{T}}(t-r(t))Qx(t-r(t))\\
&\le x^{\mathrm{T}}(t)Qx(t)-(1-r_{\mathrm{d}})x^{\mathrm{T}}(t-r(t))Qx(t-r(t)),\\
\dot{V}_3(t) &= \dot{r}(t)\int_{t-r(t)}^{t}y^{\mathrm{T}}(\xi)Ry(\xi)\,\mathrm{d}\xi+r(t)y^{\mathrm{T}}(t)Ry(t)\\
&\quad-\int_{t-r(t)}^{t}y^{\mathrm{T}}(\xi)Ry(\xi)\,\mathrm{d}\xi.
\end{aligned}
$$

Use Lemma 1 to obtain

$$
\begin{aligned}
\dot{V}_3(t) &= \dot{r}(t)\int_{t-r(t)}^{t}y^{\mathrm{T}}(\xi)Ry(\xi)\,\mathrm{d}\xi+r(t)y^{\mathrm{T}}(t)Ry(t)\\
&\quad-\int_{t-r(t)}^{t}y^{\mathrm{T}}(\xi)Ry(\xi)\,\mathrm{d}\xi\\
&\le y^{\mathrm{T}}(t)r_\Gamma Ry(t)-\left(\frac{1}{r_\Gamma}\int_{t-r(t)}^{t}y(\xi)\,\mathrm{d}\xi\right)^{\mathrm{T}}\\
&\quad\times[(1-r_{\mathrm{d}})r_\Gamma R]\left(\frac{1}{r_\Gamma}\int_{t-r(t)}^{t}y(\xi)\,\mathrm{d}\xi\right)).
\end{aligned}
$$

Therefore, we have

$$
\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)\le\lambda^{\mathrm{T}}(t)\Xi_0\lambda(t), \tag{14}
$$

where

$$
\lambda^{\mathrm{T}}(t)=(x^{\mathrm{T}}(t)\quad y^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-r(t))\left(\frac{1}{r_\Gamma}\int_{t-r(t)}^{t}y(\xi)\,\mathrm{d}\xi\right)^{\mathrm{T}}\quad f^{\mathrm{T}}(x(t))\quad g^{\mathrm{T}}(x(t-r(t)))),
$$

and

$$
\Xi_0=\begin{bmatrix}
\Omega_{11} & \Omega_{012} & -P_2^{\mathrm{T}}C & -r_\Gamma P_2^{\mathrm{T}}C & P_2^{\mathrm{T}}A & P_2^{\mathrm{T}}B\\
\Omega_{012}{}^{\mathrm{T}} & \Omega_{022} & -P_3^{\mathrm{T}}C & -r_\Gamma P_3^{\mathrm{T}}C & P_3^{\mathrm{T}}A & P_3^{\mathrm{T}}B\\
-C^{\mathrm{T}}P_2 & -C^{\mathrm{T}}P_3 & -(1-r_{\mathrm{d}})Q & 0 & 0 & 0\\
-r(t)_{\max}C^{\mathrm{T}}P_2 & -r_\Gamma C^{\mathrm{T}}P_3 & 0 & -(1-r_{\mathrm{d}})r_\Gamma R & 0 & 0\\
A^{\mathrm{T}}P_2 & A^{\mathrm{T}}P_3 & 0 & 0 & 0 & 0\\
B^{\mathrm{T}}P_2 & B^{\mathrm{T}}P_3 & 0 & 0 & 0 & 0
\end{bmatrix}<0, \tag{15}
$$

with

$$
\begin{aligned}
\Omega_{011}&=P_2^{\mathrm{T}}C+C^{\mathrm{T}}P_2+Q,\\
\Omega_{012}&=\Omega_{12}=P_1-P_2^{\mathrm{T}}+C^{\mathrm{T}}P_3,\\
\Omega_{022}&=\Omega_{22}=-P_3-P_3^{\mathrm{T}}+r_\Gamma R.
\end{aligned} \tag{16}
$$

A sufficient condition for asymptotic stability of system (7) is that there exist real matrices P_2, P_3 and symmetric positive definite matrices P_1, Q, and R such that $\dot{V}(t)<0$. However, we can see

$$
\dot{V}(t)=\lambda^{\mathrm{T}}(t)\Xi_0\lambda(t)\le 0, \tag{17}
$$

for all $\lambda(t)\neq 0$ (see Ref. [21]) satisfying Eq. (8), where equation (17) means that it is negatively semi-definite whenever neither $x(t)$ nor $x(t-r(t))$ is zero. Using the S-procedure,[22] we see that this condition is implied by the existence of scalar $\varepsilon_1>0$, $\varepsilon_2>0$ such that

$$
\begin{aligned}
&\lambda^{\mathrm{T}}(t)\Xi_0\lambda(t)+\varepsilon_1(\varphi^2x^{\mathrm{T}}(t)x(t)-f^{\mathrm{T}}(x(t))f(x(t)))\\
&+\varepsilon_2(\theta^2x^{\mathrm{T}}(t-r(t))x(t-r(t))\\
&-g^{\mathrm{T}}(x(t-r(t)))g(x(t-r(t))))<0,
\end{aligned}
$$

for all $\lambda(t)\neq 0$. Therefore, if there exist real matrices P_2, P_3 and symmetric positive definite matrices P_1, Q, and R, and scalar $\varepsilon_1>0$ and $\varepsilon_2>0$ such that LMI (Eq. (11)) is satisfied, then system (7) or (8), with system (10) is asymptotically stable.[22,34]

Remark 1 Theorem 2 provides a delay-dependent stability criterion for nonlinear time-varying delay systems with nonlinear parts in terms of the solvability of an LMI.

Since α_i, β_i are undetermined coefficients, it can be seen from Theorem 2 that by using the LMI tool box in MATLAB, the capacity of time-varying propagation delay-dependent considered non-cooperative program game MANETs can stay stable as long as certain α_i, β_i can be obtained to guarantee the existence of P_1, P_2, P_3, Q, R, ε_1, ε_2 and minimum r_Γ, and the α_i, β_i satisfying the requirements are not unique within their effective data range, so it is possible to get suitable α_i, β_i to guarantee both the existence of P_1, P_2, P_3, Q, R, ε_1, ε_2 and $r_\Gamma=r_{\Gamma_{\min}}$.

Therefore, we have designed the non-cooperative program game model delay-dependent stability control algorithm for the time-varying propagation delay considered, non-compete channel shared MANET capacity analysis.

According to the real physical characteristics of MANETs, the value ranges of α_i, β_i are $\alpha_i\in[a_{\alpha i}\quad b_{\alpha i}]$, $\beta_i\in[a_{\beta i}\quad b_{\beta i}]$, which are determined to search the step lengths $h_{\alpha_i}=(b_{\alpha i}-a_{\alpha i})/\tau$, $h_{\beta_i}=(b_{\beta i}-a_{\beta i})/\tau$, τ is a natural number.

Input $a_{\alpha i}, b_{\alpha i}, a_{\beta i}, b_{\beta i}, r_{\Gamma_{\min}} = 1, h_{\alpha_i}, h_{\beta_i}, i = 1,2,\ldots,k$;

step 1 Given $i = 1$;

step 2 Given $\alpha_i = a_{\alpha i}, i = 1,2,\ldots,k$;

step 3 Given $\beta_i = a_{\beta i}, i = 1,2,\ldots,k$;

step 4 Calculate P_{1i}, P_{2i}, P_{3i}, Q_i, R_i, ε_{1i}, ε_{2i}, $r_{\Gamma i}$ by Eq. (11) of Theorem 2; go to step 5 if P_{1i}, P_{2i}, P_{3i}, Q_i, R_i, ε_{1i}, ε_{2i}, $r_{\Gamma i}$ exists and $r_{\Gamma i} < r_{\Gamma_{\min}}$. Otherwise go to step 6;

step 5 Given $r_{\Gamma_{\min}} = r_{\Gamma i}$, $\tilde{\alpha}_i = \alpha_i$, $\tilde{\beta}_i = \beta_i$, $\tilde{P}_{1_i} = P_{1_i}$, $\tilde{P}_{2i} = P_{2i}$, $\tilde{P}_{3i} = P_{3i}$, $\tilde{Q}_i = Q_i$, $\tilde{\varepsilon}_{1i} = \varepsilon_{1i}$, $\tilde{\varepsilon}_{2i} = \varepsilon_{2i}$, $\tilde{R}_i = R_i$, go to step 6;

step 6 Given $\beta_i = a_{\beta i} + h_{\beta_i}$, go to step 4 if $\beta_i \le b_{\beta i}$, Otherwise given $\beta_i = a_{\beta i}$, go to step 7;

step 7 Given $\alpha_i = a_{\alpha i} + h_{\alpha i}$, go to step 3 if $\alpha_i \le b_{\alpha i}$ Otherwise $i = i + 1$; go to step 2;

step 8 Output: $\tilde{P}_{1i}, \tilde{P}_{2i}, \tilde{P}_{3i}, \tilde{Q}_i, \tilde{R}_i, \tilde{\varepsilon}_{1i}, \tilde{\varepsilon}_{2i}, \tilde{\alpha}_i, \tilde{\beta}_i, r_{\Gamma_{\min}}$.

Remark 2 The differences between the delay-dependent stability control algorithm and the delay-independent stability control algorithm are as follows:

(i) The 6×6 matrix is used in the delay-dependent stability control algorithm, while the 5×5 matrix is used in the delay-independent stability control algorithm (see Ref. [21]).

(ii) The differences of stability criteria are that the delay-dependent stability control algorithm is delay $r_\Gamma = r_{\Gamma_{\min}}$, while the delay-independent stability control algorithm is derivative of the delay $r_{\rm d} = r_{\rm d_{min}}$ (see Ref. [21]. The former is more than the latter a control matrix R.

The above differences are due to the fact that, when compared with the delay-independent case, the consideration of delay-dependence increases the difficulty for the control of the system. However, in engineering practice, the study of delay-dependent stability of MANET's capacity is of more practical value. Therefore, the delay-dependent stability control algorithm has more significant applied values in the analysis of a MANET's capacity stability.

4. Simulation experiments

To validate the performance of this algorithm, we use the experimental environment of Ref. [21]. MATLAB 7.0 and OPNET 14.5 are used to perform four joint simulation experiments.

4.1. Stimulation experiment 1

Assuming that there are three nodes: node 1, node 2, and node 3 and two links l_{1_1}, $l_{1_2} = l_{2_1}$ with the capacity of $g_{l_{1_1}} = 10$ kb/s, $g_{l_{1_2}} = g_{l_{2_1}} = 7$ kb/s, respectively, in wireless MANETs. The scheduling time of links within the normalized time are $t_{l_{1_1}} = 2/3$, $t_{l1_2} = t_{l_{2_1}} = 1/3$, the flow rates from node 1 to node 3 and from node 2 to node 3 are x_1 and x_2, respectively. Given the ranges of α_i and β_i are $\alpha_1 \in [18]$, $\alpha_2 \in [18]$, $\beta_1 \in [112]$, and $\beta_2 \in [1\,12]$. The network topology is shown in Fig. 1.

If the algorithm is not used to calculate α_i and β_i for the capacity stability control, when the values of α_i and β_i are randomly chosen from the values in Table 1, the propagation flow rates of node 1 and node 2 are shown in Fig. 2. Apparently, the network capacity cannot reach a stable condition.

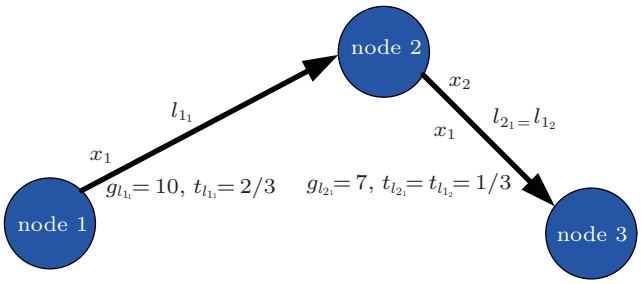


Fig. 1. (color online) The MANET capacity stability analysis of experiment 1.

Table 1. Values of α_i and β_i chosen randomly in simulation experiment 1.

Source nodes	α	β
Node 1	4	10
Node 2	10.5	1.5

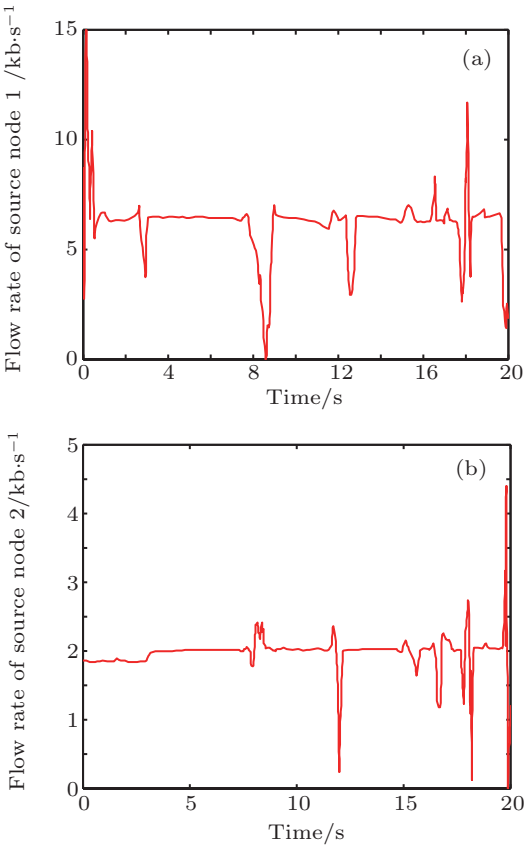


Fig. 2. (color online) Flow rates of node 1 (a) and node 2 (b) without using algorithm control.

Then, we use the stability control algorithm of non-cooperative program game model with time-varying propagation delay for MANET's capacity analysis to calculate

$$P_1 = \begin{bmatrix} 52.9319 & 0 \\ 0 & 264.4571 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} -39.6940 & 0 \\ 0 & -219.3413 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} -39.7136 & 0 \\ 0 & -212.7568 \end{bmatrix},$$

$$Q = \begin{bmatrix} 58.7367 & 0 \\ 0 & 57.6140 \end{bmatrix}, \quad R = \begin{bmatrix} 130 & 0 \\ 0 & 500800 \end{bmatrix},$$

$$\varepsilon_1 = 1.7961 \times 10^4, \quad \varepsilon_2 = 1.9082 \times 10^6,$$

and $r_{\Gamma_{\min}} = 5.0 \times 10^{-6}$ s, α_i and β_i can be obtained in this situation (Table 2), which are then substituted into the the MANET's source node propagation flow rate evolution equation set (7) to perform simulation for the MANET shown in Fig. 1. The simulation time is recorded in seconds (s) and the propagation flow rate of source nodes is updated every second. Figure 3 shows the propagation flow rates and network capacity for node 1 and node 2.

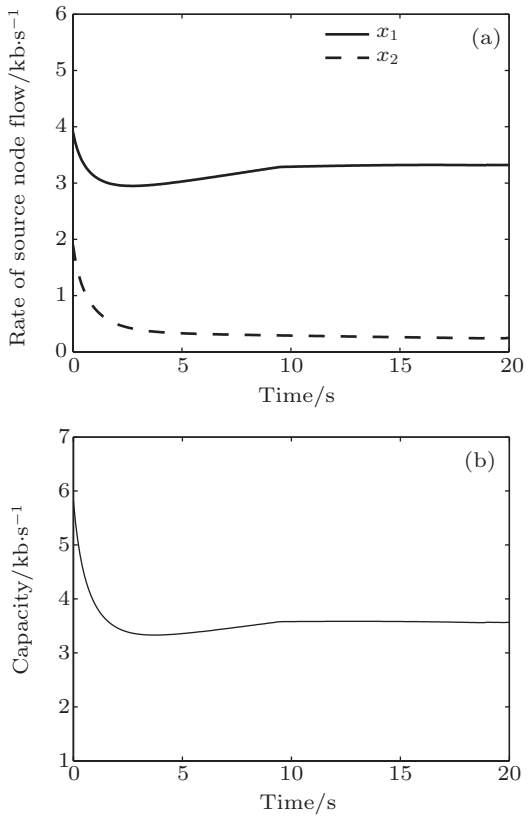


Fig. 3. The algorithm applied flow rates (a) and capacity stability (b) of node 1 and node 2.

From Fig. 3 we can observe that the propagation flow rates from both node 1 and node 2 converge to the equilibrium point within 10.2 s, and the MANET's capacity can also reach a stable capacity within less than 10.2 s. This experiment certifies the good stability control effectiveness of the stability control algorithm of non-cooperative program game model with time-varying propagation delay for MANET capacity analysis on the topology shown in Fig. 1. Therefore, we can, within the range of the MANET's physical characteristics (i.e., the valid data ranges of α_i and β_i, $i = 1,2$), carry out the capacity stability control by regulating the values of α_i and β_i, $i = 1,2$, and optimize the propagation flow rates allocation for each source node to make good use of the network resources and maximize the network's capacity.

Table 2. Values of α_i and β_i obtained by using algorithm in simulation experiment 1.

Source nodes	α	β
Node 1	1	4
Node 2	1.5	10.5

4.2. Simulation experiment 2

Assuming that there are four nodes: node 1, node 2, node 3, node 4 and three links: l_{1_1}, l_{2_1}, $l_{3_1} = l_{1_2} = l_{2_2}$ with capacity of $g_{l_{1_1}} = 6$ kb/s, $g_{l_{2_1}} = 4$ kb/s, $g_{l_{3_1}} = g_{l_{1_2}} = g_{l_{2_2}} = 10$ kb/s. The data flow rate from node 1 to node 3 is x_1, from node 2 to node 3 is x_2, and from node 3 to node 4 is x_3. In this situation, x_1 and x_2 transmit data at the same time, available links set is $S = \{s\{l_{1_1}, l_{2_1}\}, s\{l_{3_1}\}\}$ within the normalized time 1, and the scheduling time of each link are $t_{s_{\{l_{1_1}, l_{2_1}\}}} = 1/3$, $t_{s_{\{l_{3_1}\}}} = 2/3$. Given the data ranges of α_i and β_i as: $\alpha_1 \in [1\,12]$, $\alpha_2 \in [1\,14]$, $\alpha_3 \in [1\,15]$, $\beta_1 \in [1\,12]$, $\beta_2 \in [1\,14]$, $\beta_3 \in [1\,14]$, the network topology is shown in Fig. 4.

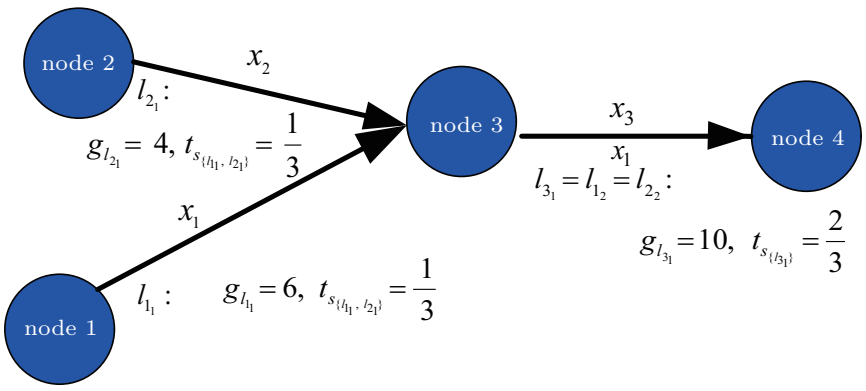


Fig. 4. (color online) The MANET capacity stability analysis of experiment 2.

If the stability control algorithm of non-cooperative program game model with time-varying propagation delay for MANETs capacity analysis is not used to calculate α_i and β_i for capacity stability control and the values are randomly picked within the data ranges of α_i and β_i (Table 3), then the information of propagation data flow rates from node 1, node 2, and node 3 shown in Fig. 5 can be obtained. Obviously, the MANETs capacity cannot reach a stable status.

Table 3. Values of α_i and β_i chosen randomly in simulation experiment 2.

Source nodes	α	β
Node 1	2	1.5
Node 2	7	5
Node 3	5	8

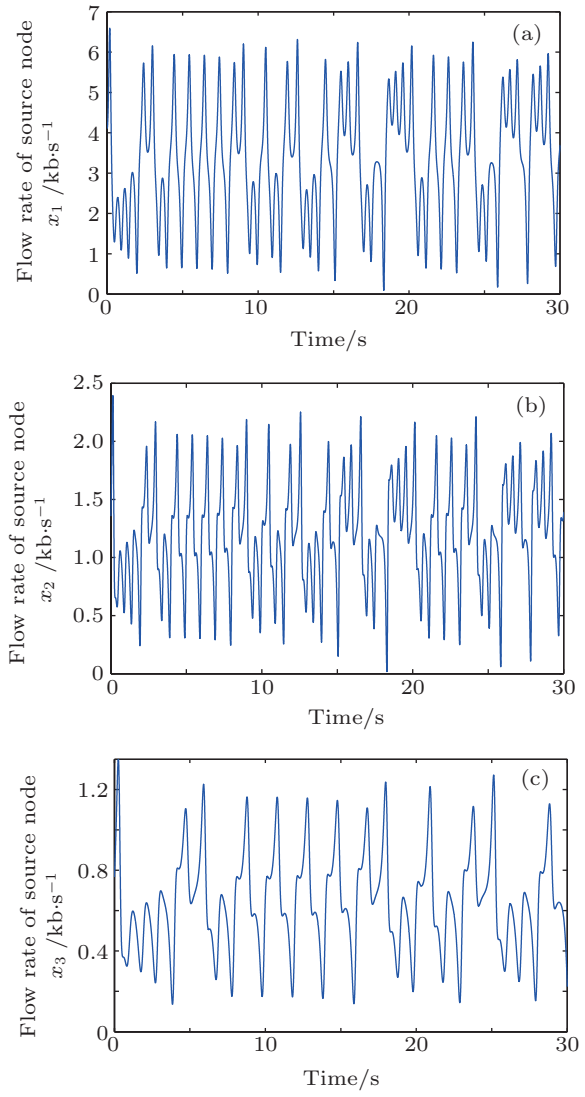


Fig. 5. (color online) Flow rates of node 1 (a), node 2 (b), and node 3 (c) without algorithm control.

Then, we use stability control algorithm of non-cooperative program game model with time-varying propagation delay for the MANET's capacity analysis to calculate

$$P_1 = \begin{bmatrix} 316.375 & 0 & 0 \\ 0 & 226.574 & 0 \\ 0 & 0 & 532.028 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} -168.173 & 0 & 0 \\ 0 & 235.024 & 0 \\ 0 & 0 & -531.024 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 791.560 & 0 & 0 \\ 0 & 258.353 & 0 \\ 0 & 0 & 194.251 \end{bmatrix},$$

$$Q = \begin{bmatrix} 536.041 & 0 & 0 \\ 0 & 336.104 & 0 \\ 0 & 0 & 536.270 \end{bmatrix},$$

$$R = \begin{bmatrix} 147.1241 & 0 & 0 \\ 0 & 439.210 & 0 \\ 0 & 0 & 267.047 \end{bmatrix},$$

$$\varepsilon_1 = 3.1327 \times 10^4, \quad \varepsilon_2 = 7.1973 \times 10^4,$$

and $r_{\Gamma_{\min}} = 1.5 \times 10^{-6}$ s, α_i and β_i can be obtained for this situation (Table 4), which are then substituted into the MANET's source node propagation flow rate evolution equation set (7) to

Table 4. The results obtained by applying the algorithm in simulation experiment 2.

Source nodes	α	β
Node 1	9	5
Node 2	13	10
Node 3	11	3

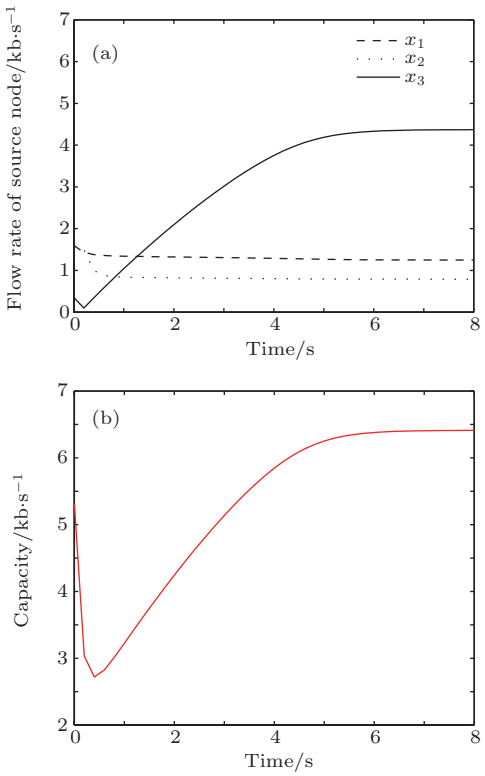


Fig. 6. (color online) The algorithm applied flow rates (a) and capacity stability (b) of node 1, node 2, and node 3.

perform simulation of the MANET shown in Fig. 7. The unit of simulation time is second (s) and the propagation flow rates of source nodes are updated every second. Figure 6 shows the propagation flow rates and network capacity for node 1, node 2, and node 3. It can be seen that the propagation flow rates from node 1, node 2, and node 3 all converge to balance within less than 6.3 s, and the MANET's capacity can also reach the stable capacity within 6.3 s. This experiment indicates the good stability control effect of the stability control algorithm for the MANET's capacity analysis, time-varying, propagation delay-considered non-cooperative program game model. Therefore, we can, within the ranges of the MANET's physical characteristics (or the effective data ranges of α_i and β_i, $i = 1,2,3$), carry out capacity stability control by regulating the values of α_i and β_i, $i = 1,2,3$ and optimizing the propagation flow rates allocation for each source node to make good use of the network resources and maximize the network capacity.

4.3. Simulation experiment 3

To further test the performance of this algorithm, we built the simulation network on the OPNET 14.5 platform after the algorithm based stability control in simulation experiment 2, as shown in Fig. 7. Assuming that: the mobile node moves according to the wlan-roaming track within the range of 10^2 km; the TDMA protocol is used in MAC layer; and the AODV and DSR protocols are used in the routing layer. The other parameters are the same as in the simulation in experiment 2. The final results are shown in Figs. 8 and 9, indicating that although the MANETs capacity requires a longer time to converge to the stable situation compared with simulation experiment 2, its capacities under different routing protocols can all eventually converge to the stable status. In this experiment, it takes a relatively long time to converge under DSR protocol (7.95 s), which is largely due to the larger end-to-end delay under DSR protocol than the end-to-end delay under AODV with the same network conditions.

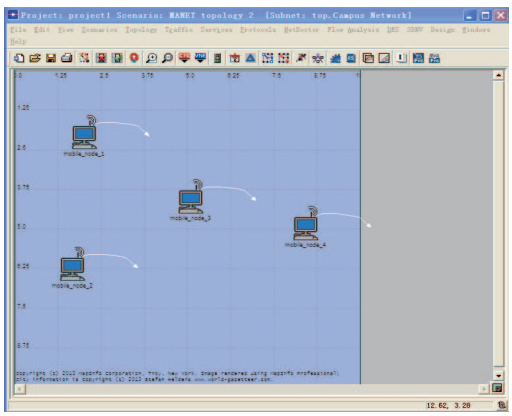


Fig. 7. (color online) OPNET 14.5 simulation network.

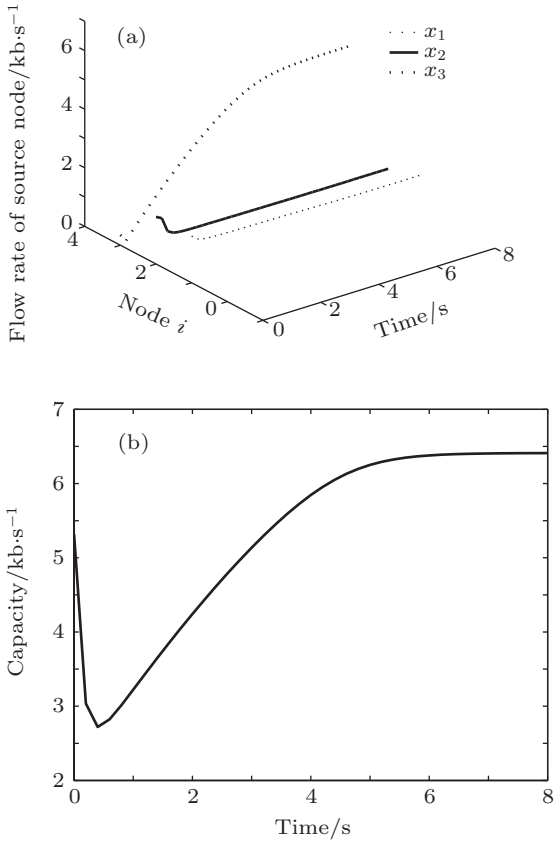


Fig. 8. (color online) The propagation flow rates (a) and capacity stability (b) of node 1, node 2, and node 3 under TDMA, DSR protocols.

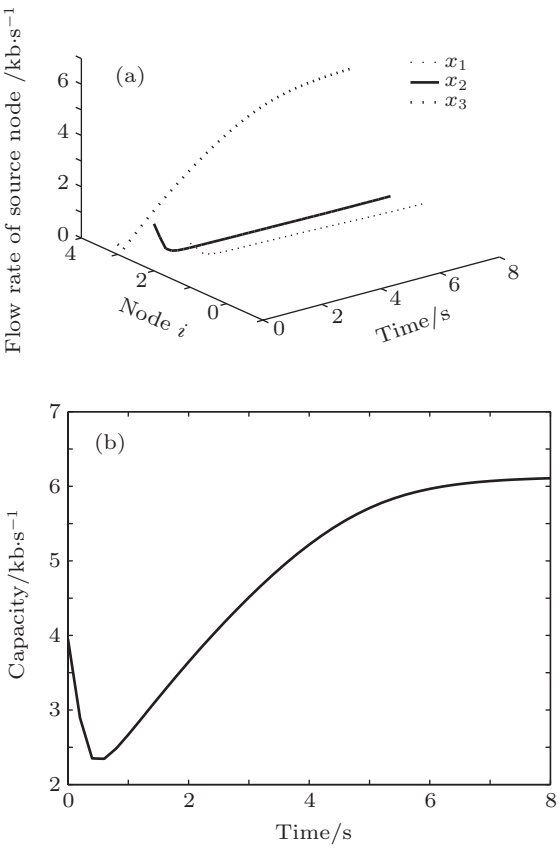


Fig. 9. The propagation flow rates (a) and capacity stability (b) of node 1, node 2, and node 3 under TDMA, AODV protocols.

4.4. Simulation experiment 4

In order to compare the difference between the delay-dependent stability and the delay-independent stability, we operate the delay-dependent stability control algorithm and the delay-independent stability control algorithm in simulation experiment 2. The simulation results are shown as follows.

Table 5. The obtained and shared α and β by applying the delay-independent stability control algorithm and the delay-dependent stability control algorithm in simulation experiment 2.

Source nodes	α	β
Node 1	8.5	5
Node 2	12.5	10
Node 3	11.5	2.5

As Fig. 10 shows, the time required to achieve system stability is obviously longer when delay-dependence is taken into account than when delay-independence is under the same conditions. The reason for this, as can be seen, lies in that, when compared with delay-independent stability, delay-dependent stability demands more control criteria, which is more difficult.

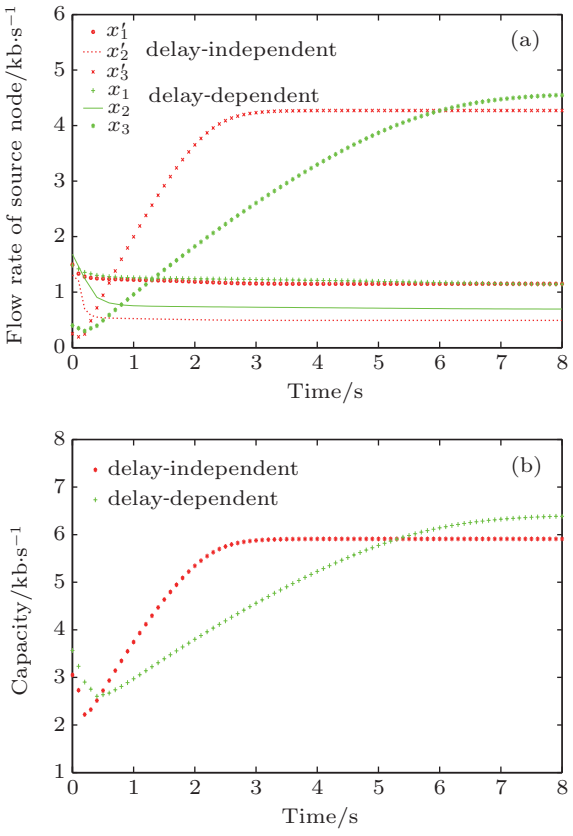


Fig. 10. (color online) Comparison between the delay-dependent stability (a) and the delay-independent stability (b).

5. Conclusion

In this paper, we have developed a novel delay-dependent stability control method for delay-dependent non-cooperative program game MANETs capacity analysis model. First, we have obtained the propagation flow rate evolution equation set from source nodes of the time-varying propagation delay considered non-cooperative program game MANETs capacity analysis model; that is, the MANET's source nodes propagation flow rate evolution equation set of the distributed time-varying dynamic system with the time-varying propagation delay term. Then, based on the descriptor technology combined with the Lyapounov–Krasovskii method, a Lyapounov–Krasovskii functional for the delay-dependent conditions with respect to the discrete delays has been built to prove the asymptotic stability conditions of the delay-dependent non-cooperative program game model for the MANET's capacity analysis. In addition, we have also developed the MANET's capacity delay-dependent stability control algorithm. The simulation experiments have also confirmed the good delay-dependent stability control property of this algorithm.

References

[1] Gupta P and Kumar P 2000 *IEEE Trans. Inf. Theory* **46** 388
[2] Grossglauser M and Tse D N C 2002 *IEEE ACM Trans. Netw.* **10** 477
[3] Zhao S M and Liu J 2010 *Acta Phys. Sin.* **59** 771 (in Chinese)
[4] Shirong D, John N D and Bahram A 2011 *Ad Hoc Netw.* **9** 120
[5] Zhao H T, Emiliano G P, Jibo W and Yong X 2011 *Phys. Commun.* **4** 98
[6] Zhong L, Wang C, Jiang C J and Li X Y 2013 *Ad Hoc Netw.* **11** 29
[7] Wang L J, Cai L and Liu X Z 2009 *Comput. Netw.* **53** 338
[8] Georgiadis L, Szpankowski W and Tassiulas L 1997 *IEEE Trans. Inform. Theory* **43** 923
[9] Li W X, Su H and Wang K 2011 *Automatica* **47** 215
[10] Feijer D and Paganini F. 2010 *Automatica* **46** 1974
[11] Dargie W and Schill A 2011 *J. Comput. Syst. Sci.* **77** 852
[12] Chen C H, Yin C C and Yue G X 2011 *The Journal of China Universities of Posts and Telecommunications* **18** 22
[13] Jae Y S and Seong L K 2012 *Comput. Commun.* **35** 1345
[14] Mansoori M and Mahdavi M 2013 *Performance Evaluation* **70** 45
[15] Liu S, Liu B, Zhang Y K and Wen Y 2010 *Acta Phys. Sin.* **59** 38 (in Chinese)
[16] Shi P M, Li J Z, Liu B and Han D Y 2011 *Acta Phys. Sin.* **60** 094501 (in Chinese)
[17] Yamamoto K 2005 *IEICE Trans. Commun.* **E88-B** 1009
[18] Kherani A, El-Khoury R and El-Azouzi R 2008 *Comput. Netw.* **52** 1365
[19] Farhadi A and Charalambous C D 2010 *Automatica* **46** 889
[20] Yang J, Li Y, Zhang Z J and Li J Q 2012 *J. Elect. Inf. Technol.* **34** 75
[21] Yang J, Yang D, Huang B, Zhang X H and Yang C 2014 *Acta Phys. Sin.* (in press) (in Chinese)
[22] Liao X F, Liu Y B, Guo S T and Mai H H 2009 *Commun. Nonlinear Sci. Numer. Simulat.* **14** 3120
[23] Li C D and Liao X F 2006 *Int. J. Bifurc. Chaos* **16** 3323
[24] Arik S 2002 *IEEE Trans. Neural Netw.* **13** 1239
[25] Cao J D 2001 *IEEE Trans. Circ. Syst.* **48** 1330
[26] Hale J and Verduyn L S M 1993 *Introduction to Functional Differential Equations* (New York: Springer) p. 332
[27] Kolmanovskii V and Nosov V 1986 *Stability of Functional Differential Eequations* (New York: Academic Press) p. 214
[28] Kolamnovskii V and Myshkis A 1999 *Applied Theory of Functional Differential Equations* (Boston (MA): Kluwer) p. 59
[29] Gopalsamy K 1992 *Ph.D. Dissertation* (Netherlands: Kluwer Academic)
[30] Fridman E 2001 *Syst. Control Lett.* **43** 309
[31] Yang J 2011 Ph.D. Dissertation (Chongqing: Chongqing University) (in Chinese)
[32] Fridman E 2002 *J. Math. Anal. Appl.* **273** 24
[33] Fridman E and Shaked U 2002 *Linear Algebra Appl.* **351** 271
[34] Wu M and He Y 2008 *Delay System Robust Control-Liberty Matrix Method* (Beijing: Science Press) p. 1 (in Chinese)