Adiabatic tunneling of Bose–Einstein condensates with modulated atom interaction in a double-well potential

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We study the adiabatic tunneling of Bose–Einstein condensates in a symmetric double-well potential when the interaction strength between the atoms is modulated linearly or in a cosine periodic form. It is shown that the system evolves along a nonlinear eigenstate path. In the case of linear modulation under the adiabatic approximation conditions, the tunneling probability of the condensate atoms to the other potential well is half. However, when the system is periodically scanned in the adiabatic process, we find an interesting phenomenon. A small change in the cycle period can lead to the condensate atoms returning to the right well or tunneling to the left well. The system comes from a linear eigenstate back to a nonlinear one, which is completely different from the linear eigenstate evolution. We explain the results by using the energy level and the phase diagram.

Keywords: adiabatic tunneling, Bose–Einstein condensates, double-well potential

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1. Introduction

As a basic theory of quantum mechanics, the adiabatic theorem plays an important role in understanding a quantum system and gives an opportunity to manipulate the quantum state. The control and manipulation of confined ultracold atomic gases make it possible to study the dynamics of many-body quantum states for bosons with very high level of control and precision.[1,2] Among those quantum effects, quantum tunneling is a topic of wide interest. Bose–Einstein condensates (BEC) in double-well potentials have drawn a lot of attention, both theoretically and experimentally, for the possibility to study the fundamental quantum mechanical effects at the macroscopic level.[3–6] The Josephson tunneling of a boson is found in double wells. The interaction of atoms can induce a self-trap, which was predicted in theory[7] and then confirmed by experiment.[8]

The adiabatic process of BEC in double-well potentials has also been considered. In particular, the nonlinear effect on adiabatic tunneling was considered and expected to be a possible tool to control quantum states.[9–17] In the theory of BEC, nonlinearity is caused by the interaction between the BEC atoms. To investigate this problem, some universal protocols were proposed; for example, the Landau–Zener model[9–15] and the Rosen–Zener model.[16,17] In the nonlinear Landau-Zener model, it was found that nonlinearity can lead to nonzero tunneling in an adiabatic process.[9–11,13] The period modulation of the bias has a great effect on the self-trap of BEC.[12] The study of cycle adiabatic process[15] and other studies[18–20] on such models have shown the importance of nonlinearity. In the nonlinear Rosen–Zener-type model, where the energy bias and the nonlinear parameter are set to be constants whereas the coupling strength is time dependent, nonlinearity was also found to influence the quantum tunneling.[16,17]

In this paper, considering the important role played by the nonlinearity due to the interaction strength, we study the adiabatic tunneling of BEC in a double-well potential by modulating the interaction strength. The system can be described by the two-level Landau–Zener model.[9–12] In both linear and cosine periodic modulation cases, we find that tunneling occurs with the breaking of the adiabatic process at $c = 0.0$. In particular, when the system is periodically scanned, we find an interesting phenomenon. A small change in the cycle period can lead to the condensate atoms returning to the right well or tunneling to the left one.

2. Model

The dynamics of a BEC can be described by the Gross–Pitaevskii equation (GPE) in the mean-field approximation.[21] Our study is conducted by transforming the Gross–Pitaevskii equation into a mathematically equivalent classical Hamiltonian problem where the nonlinearity is no longer a peculiar issue but rather a common character.[9,10,14]

Considering the condensation in a symmetric double-well potential and using the two-mode approximation,[9,16] we can write the order parameter of the BEC under study as $\psi(\boldsymbol{r},t) = a(t)\varphi_1(\boldsymbol{r}) + b(t)\varphi_2(\boldsymbol{r})$, where $\varphi_1(\boldsymbol{r})$ and $\varphi_2(\boldsymbol{r})$ are the local ground states of the BECs in each individual well, $a(t)$ and $b(t)$ are the probability amplitudes with the conservation of the total probability $|a(t)|^2 + |b(t)|^2 = 1$. By substituting this ansatz into the GPE, the dynamic of amplitudes $a(t)$ and $b(t)$

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is found to satisfy the discrete Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{c}{2}\left(|b|^2 - |a|^2\right) & \frac{v}{2} \\ \frac{v}{2} & -\frac{c}{2}\left(|b|^2 - |a|^2\right) \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}, \quad (1)$$

where v is the coupling between the two levels, and the nonlinear parameter c characterizes the interaction strength between the atoms, which is proportional to the atomic scattering length.

3. Adiabatic tunneling

The interaction between BEC atoms can be tuned by the Feshbach resonance,[22,23] which has become an important tool in investigations with cold atoms. Our concern here is how the adiabatic tunneling is influenced by the interaction between the atoms.

A linear modulation and a cosine periodic modulation of the interaction strength between the atoms are applied in our study on the BEC in a symmetric double-well potential. For the two cases, we assume that the quantum state is initially prepared on one well, that is $a(0) = 0.0$ and $b(0) = 1.0$.

First, we study the evolution of the system when the interaction strength c is modulated with time as $c = \alpha t$ for $-\infty < t < +\infty$, where α is the sweeping rate. We focus on the adiabatic limit; that is, the sweeping rate α is small enough, tending to zero ($\alpha \to 0$).

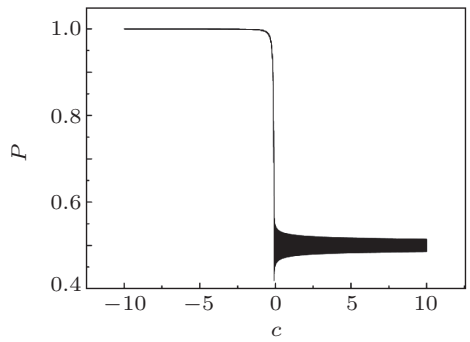


Fig. 1. The tunneling probability as a function of the nonlinear c with $v = 0.1$ and $\alpha = 0.0001$.

In Fig. 1, the final tunneling populations of the second well, $p = |b(t)|^2$, is plotted as a function of the nonlinear parameter c with $v = 0.1$ and $\alpha = 0.0001$. In this case, under the adiabatic approximation conditions, we note that when the interaction strength varies from the initial values to zero, the population of atoms in the right well keeps its initial value, and there is no tunneling. However, when c is close to zero, the population of atoms in the right well starts to decrease, and a sudden change of the population occurs close to $c = 0$; that is, half of the atoms tunnel from the right well to the left one. After this crossover, the population will not be a constant as described by the adiabatic theory but does a small oscillation around 0.5 when c increases from zero to a high value. That is to say, the quantum adiabatic theorem is not obeyed when c crosses over zero.

To obtain further insight into the behavior of the tunneling probability, we apply a cosine periodic modulation on the interaction strength as $c = -c_0\cos(2\pi t/T)$, where T is the scanning period. The population of the BEC in the right well is shown as a function of time in Fig. 2.

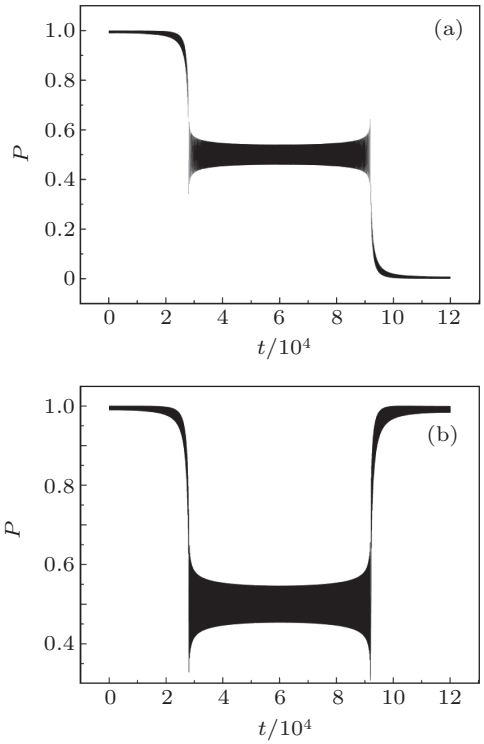


Fig. 2. Numerical results for the tunneling probability as a function of time for $c = -c_0\cos(2\pi t/T)$ with $c_0 = 1.0$ and $v = 0.1$. Entirely different tunneling probabilities are observed for a slight variation of the scanning period: (a) $T = 120000$, (b) $T = 120070$.

When c varies from $-c_0$ to 0 in a quarter period, we observe that a same dynamic like that in the linear modulation occurs; that is, a sudden jump of the population at $c = 0$ breaking the adiabatic dynamic. In the period of c changing from 0 to c_0 and then to 0 again, a small oscillation of the population between the two wells occurs. In the last quarter period, with the change of c from 0 to $-c_0$, including across 0, we note a second jump. However, the state of the BEC atoms is not unique, and is dependent on the period of the modulation. On the one hand, as shown in Fig. 2(a), when $T = 120000$, the atoms in the left well all tunnel to the right well and all the BEC stays in one well. That is to say, after a period, all of the atoms tunnel from the one well to the other. On the other hand, in Fig. 2(b) where $T = 120070$, we note that after crossing $c = 0$ again, half of the total population of BEC transferred to the left well in the first crossover comes back to the right well. The condensate atoms return to the initial state after a period.

How do we explain this phenomenon? The study of the Landau–Zener transition in the case is facilitated by solving the eigenenergies and eigenstates of the Hamiltonian.[11] According to the adiabatic theory of nonlinear quantum mechanics, the characters of quantum transition in the adiabatic limit should be entirely determined by the structure of the energy levels and the properties of the corresponding eigenstates.[9,11,12]

The nonlinear eigenstates are defined as the solution of the time-independent version of Eq. (1). We can obtain

$$\mu \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{c}{2}\left(|b|^2 - |a|^2\right) & \frac{v}{2} \\ \frac{v}{2} & -\frac{c}{2}\left(|b|^2 - |a|^2\right) \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \quad (2)$$

The chemical potential μ for different nonlinear parameter c with $v = 0.1$ is shown in Fig. 3. We can see that the chemical potential is influenced dramatically by the interaction between the atoms. We can readily obtain the eigenstates of the model. When $|c| \gg v$, there are four eigenstates in this model, which are $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$, and $\begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$. When $|c| < v$, the model has two eigenstates $\begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$ and $\begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$.

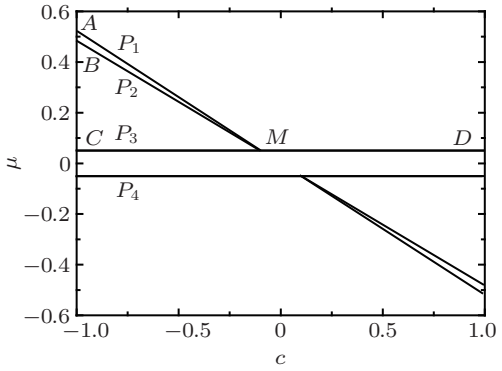


Fig. 3. Chemical potential for different nonlinear parameter c with $v = 0.1$.

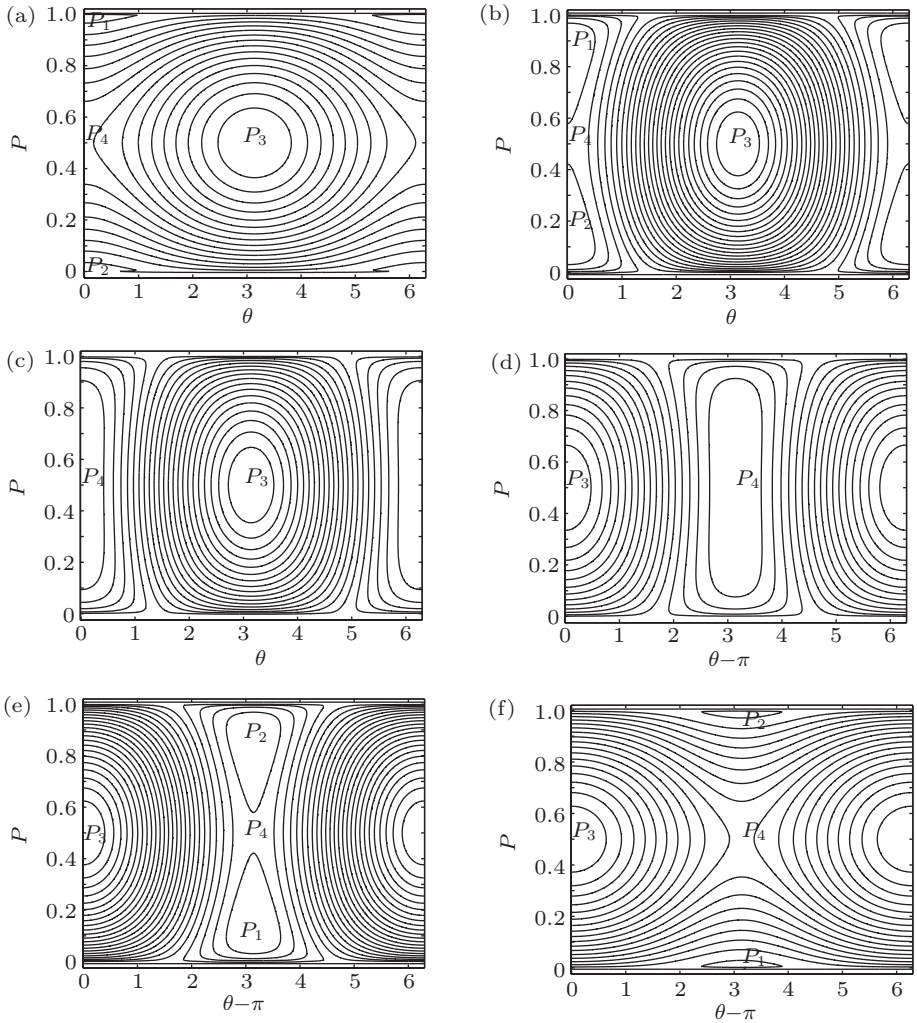


Fig. 4. Evolution in the phase space as the nonlinear parameter c changes adiabatically at $v = 0.1$: (a) $c = -0.5$, (b) $c = -0.15$, (c) $c = -0.1$, (d) $c = 0.1$, (e) $c = 0.15$, (f) $c = 0.5$.

The initial state we previously used is the nonlinear eigenstate $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, which is denoted by A. The system follows a slow scanning of c starting from point A. When the system evolves to point M, namely, at $c = -\nu$, the system jumps to another eigenstate $\begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$, This means that half of the BEC tunnels to the other well, then the system evolves along the path MD. If scanning in a cycle, the system will evolve from D to M again, then a question is raised, which path will the system evolve along. In the adiabatic theorem, it should keep the state. However an interesting phenomenon appears, the system will jump to nonlinear eigenstate $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and evolve along path MA or path MB. A slight variation of the period can lead to dramatic different results, the condensate atoms come back to the initial state (return to the right well) (Fig. 2(b)), or the condensate atoms do not return to the initial state and all the atoms tunnel to the left well (Fig. 2(a)).

In the following, we study the model with an equivalent classical Hamiltonian using the method used in Refs. [9]–[12] and [16]. By analyzing the phase space, we obtain an insight into the adiabatic evolution of the above nonlinear quantum system. The eigenstates become the fixed points in the classical problem, whose adiabatic evolution can be understood from a stability analysis of such points. The results are shown in Fig. 4.

In Fig. 4(a), when $c = -0.5$, the system presents the nonlinear eigenstates corresponding to fixed points P_1 and P_2, and the linear eigenstates corresponding to points P_3 and P_4, as shown show in Fig. 3. The adiabatic evolution of the system can, therefore, be evaluated by tracing the fixed point P_1 in the phase space when c varies, such as the adiabatic evolution of the system starting from point A in Fig. 3. When we increase c to $c = -0.15$, which is close to point M in Fig. 3, points P_1 and P_2 will be close to a new fixed point P_3 in Fig. 4(b). When $c = -\nu$, at point M in Fig. 3, points P_1 and P_2 will merge into a new stable fixed point P_3 in Fig. 4(c). As a result, the system evolves into eigenstate $\begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$, the tunneling probability of the atoms in the right well to the left well is 0.5.

When we scan the system back along the path DM, at point M, points P_1, P_2, and P_3 come into one fixed point again. After that, when c departs M again, three fixed points will separate again, since one of them, P_3, is not stable, the corresponding eigenstate $\begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}$ will not be approached. The other two fixed points P_1 and P_2 are stable, so the system can stay there. It is shown that the tunneling characteristics change accordingly with the slight variation of the cycle, resulting in the condensate atoms returning to the right well or tunneling to the left well.

4. Conclusion

By applying a linear or a cosine periodic modulation on the interaction strength between atoms, in this paper we present an investigation of the adiabatic tunneling of BEC in a symmetric double-well potential. Assuming that all of the atoms are in one well initially, we find that in the case of linear modulation interaction, the adiabatic process is broken at $c = 0$ with half atoms tunneling to the other potential. When the interaction strength is periodically modulated, we find an interesting phenomenon. A small change in the period of modulation leads to all condensate atoms returning to the initial well or tunneling to the other well. These results show that the adiabatic tunneling is dramatically influenced by the interaction between atoms in BECs.

In practice, the nonlinear parameter c may be modulated arbitrarily by using the Feshbach resonance technology, and such a symmetric double-well potential may be also achieved by a laser sheet dividing a trap. Then, one can directly verify our results in the system without intrinsic difficulty. An experiment can be designed to observe the adiabatic tunneling. The novel phenomenon described by this model can be confirmed in a quantum system. Our discussion may stimulate experimental works in this direction.

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