Quantum correlations in a two-qubit anisotropic Heisenberg XYZ chain with uniform magnetic field*

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(Received 18 October 2013; revised manuscript received 21 January 2014; published online 29 May 2014)

Quantum correlations in an anisotropic Heisenberg XYZ chain is investigated by use of concurrence C and measurement-induced disturbance (MID). We show that the behaviors of the MID are remarkably different from the concurrence. Firstly, it is shown that there is a revival phenomenon in the concurrence but not in the MID, which is suitable for both the ground state case and the finite temperature case. Based on the analysis of the ground-state C and MID structures, we illustrate the reason why the ground-state MID does not show a revival phenomenon in detail. Then we explore different effects of the external and self parameters on entanglement and MID behaviors. It can be shown that the region of MID is evidently larger than the case of concurrence, and that the concurrence signals a quantum phase transition even at finite T while MID does not. Cases where the concurrence finally maintains one nonzero constant value regardless of the value of the variable B for a constant J_z, while MID decreases monotonously to zero with increasing B. We also show that if B can take a proper range of values, the concurrence decreases with the improvement of the anisotropic parameter γ, whereas an opposite effect for MID behaviors is presented.

Keywords: entanglement, measurement-induced disturbance, Heisenberg model

PACS: 03.67.Lx, 03.65.Ud, 75.10.Jm **DOI:** 10.1088/1674-1056/23/7/070306

1. Introduction

As a valuable resource in quantum information and quantum computation,[1–3] quantum correlation has attracted considerable attention over past decades that provides the possibility of quantum teleportation,[4,5] quantum dense coding,[6] quantum cryptographic key distribution,[7] etc. The most fascinating nonlocal correlation feature of quantum mechanics is the quantum entanglement, generally considered as an essential resource for the quantum information processing.[8–11] Intuitively, we expect that entanglement captures all of the nonclassicality in a correlation. However, recent research reveals that entanglement does not provide all aspects of quantum correlations. For mixed states, entanglement, nonclassicality and nonlocality are not identical with quantum correlations. Entanglement is a special kind of quantum correlations, but not the only kind. Recently, many authors have proposed a variety of computable measures to characterize quantum correlation in the composite states: measurement-induced disturbance (MID),[12] quantum discord (QD),[13–15] geometric discord (GD),[16–18] measurement-induced nonlocality (MIN),[19] and so on. However, so many useful measures to characterize quantum correlation leads to a lack of a uniform criterion.

Instead of resorting to two-point spin–spin correlation used in most previous methods, an operational approach that takes use of MID as a measure of quantumness has been proposed by Luo.[12] As a measure of nonclassical correlation, the MID demonstrates many different characteristics compared with the entanglement. On the other hand, due to the good integrability and scalability, the solid state systems have gained great attention.[20–22] As the natural candidates for the realization of the correlation, spin chains have demonstrated some substantial advantages compared with other physical systems.[23–26] They not only have useful applications, but also display rich correlation features.[27,28] The simplest spin chain, i.e., the Heisenberg chain, has been used to construct a quantum computer and quantum dots.[29,30] In addition, by suitable coding, the Heisenberg interaction alone can be used for quantum computation.[31–33] Therefore, it is very interesting and necessary to study the quantum correlation in spin model. In this paper, several interesting results are obtained based on the behavior of MID at zero temperature, the effects of external field parameter B, anisotropic parameter, and interaction of z-component J_z, where many of them are in contrast to the behavior of the entanglement.

In this paper, we will explore the quantum correlation behaviors of a two-qubit anisotropic Heisenberg XYZ chain under an external field parameter B with two different correlation measures. In Section 2, we first introduce the Hamiltonian of the system to obtain the thermal density matrix of the system related to H, and then briefly review the definitions of the quantum correlation measures concurrence and MID, respectively. The quantum correlation behaviors of our spin system and a detailed comparison between the entanglement and the

*Project supported by the Natural Science Foundation for Young Scientists of Shanxi Province, China (Grant No. 2012021003-3) and the National Natural Science Foundation of China (Grant No. 11247247).

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MID are also investigated in Section 3. Conclusions are finally drawn in Section 4.

2. Description of the model and correlation measures

In this section we first consider the Hamiltonian of the two-qubit anisotropic Heisenberg XYZ chain under an external magnetic field B. The Hamiltonian can be expressed as

$$H = J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+) + J\gamma(\sigma_1^+\sigma_2^+ + \sigma_1^-\sigma_2^-) + \frac{J_z}{2}\sigma_1^z\sigma_2^z + \frac{B}{2}(\sigma_1^z + \sigma_2^z), \quad (1)$$

where $J = (J_x + J_y)/2$, $\gamma = (J_x - Jy)/(J_x + J_y)$, and $\sigma^\pm = \frac{1}{2}(\sigma^x \pm \mathrm{i}\sigma^y)$, with $\sigma^\pm$ representing the raising and lowering operators, respectively. The γ $(0 < \gamma < 1)$ is used to measure the anisotropy in the XY plane. On the standard basis of $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the eigenvalues and eigenstates of H can be obtained as $H|\psi^\pm\rangle = (J_z/2 \pm \eta)|\psi^\pm\rangle$, $H|\phi^\pm\rangle = (-J_z/2 \pm J)|\phi^\pm\rangle$, with the eigenstates $|\psi^\pm\rangle = W^\mp[(\eta \mp B)|00\rangle \pm J\gamma|11\rangle]$, $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, the parameters $\eta = \sqrt{B^2 + J^2\gamma^2}$, and the normalization constant $W^\mp = 1/\sqrt{2\eta(\eta \mp B)}$. For a system described by H in equilibrium at temperature T, the density matrix is represented by $\rho(T) = Z^{-1}\mathrm{e}^{-\beta H}$, where $\beta = (kT)^{-1}$, Z is the partition function, and k is the Boltzmann constant. For simplicity, we write $k = 1$. Therefore, the density matrix of this system is obtained as

$$\rho = \frac{1}{Z}\begin{pmatrix} a_+ & 0 & 0 & b \\ 0 & c & d & 0 \\ 0 & d & c & 0 \\ b & 0 & 0 & a_- \end{pmatrix}. \quad (2)$$

Here the exact values of these nonzero matrix elements can be written as

$$a_\pm = \mathrm{e}^{-\beta J_z/2}[\cosh(\beta\eta) \pm B\sinh(\beta\eta)/\eta],$$
$$b = -J\gamma\mathrm{e}^{-\beta J_z/2}\sinh(\beta\eta)/\eta,$$
$$c = \mathrm{e}^{\beta J_z/2}\cosh(\beta J), \quad d = -\mathrm{e}^{\beta J_z/2}\sinh(\beta J),$$

and $Z = 2[\mathrm{e}^{-\beta J_z/2}\cosh(\beta\eta) + \mathrm{e}^{\beta J_z/2}\cosh(\beta J)]$.

Quantum correlation via entanglement For a two-qubit system the quantum thermal entanglement can be measured by the concurrence C written as $C(\rho) = \max\{0, 2\lambda_{\max} - \sum_{i=1}^{4}\lambda_i\}$,[34,35] where λ_i are the square roots of the eigenvalues of the matrix $R = \rho(\sigma_1^y \otimes \sigma_2^y)\rho^*(\sigma_1^y \otimes \sigma_2^y)$, ρ is the density matrix, and $*$ indicates the complex conjugate. With the X-structure density matrix given by Eq. (2), the expression of four λ_i are $\lambda_{1,2} = |\sqrt{a_+a_-} \pm b|$ and $\lambda_{3,4} = |c \pm d|$. No matter whether ρ is pure or mixed, the concurrence is available, and it ranges from 0 (no entanglement) to 1 (maximum entanglement). The entanglement concurrence C can be used to measure the quantum correlation behavior of this spin system.

Quantum correlation via measurement-induced disturbance For any bipartite state ρ existing in the Hilbert space $H_A \otimes H_B$, if the reduced density matrices $\rho_A = \sum_i P_A^i \Pi_A^i$ and $\rho_B = \sum_i P_B^i \Pi_B^i$, then the measurement induced by the spectral resolution of the reduced states is

$$\Pi(\rho) = \sum_{ij} \Pi_A^i \otimes \Pi_B^j \rho \Pi_A^i \otimes \Pi_B^j, \quad (3)$$

where the state $\Pi(\rho)$ is considered as classical in the sense that there is a unique local measurement strategy, namely, Π, that leaves $\Pi(\rho)$ unchanged. This strategy produces a classical state in ρ while keeping the reduced states invariant. According to Ref. [12], measurement-induced disturbance (MID) is defined as a reasonable measure of quantumness

$$\mathrm{MID}(\rho) = I(\rho) - I[\Pi(\rho)], \quad (4)$$

where $I(\cdot)$ is mutual information and $I(\rho) = S(\rho_A) + S(\rho_B) - S(\rho)$ ($S(\cdot)$ denotes the von Neumann entropy). By taking $\Pi_i^A = |i\rangle\langle i|$ and $\Pi_j^B = |j\rangle\langle j|$ (which is unique in our case), we can obtain the density matrix of $\Pi(\rho)$, $[\Pi(\rho_A)] = \rho_A$, and $[\Pi(\rho_B)] = \rho_B$.

$$\mathrm{MID}(\rho) = S(\Pi(\rho)) - S(\rho). \quad (5)$$

3. Correlation properties of the concurrence and MID

In the above section, we give a review of the quantum correlation measures C and MID. As we know, for pure states all quantum correlation comes from entanglement. However, for mixed states, the entanglement cannot totally describe nonclassical correlation. In the following, we will concentrate on the numerical analysis about the differences between the concurrence and MID properties on different tunable system parameters at length, and explore the effects of the external and self parameters on entanglement and MID behaviors in this spin system.

With the lower temperature $T = 0.01$, figure 1 shows the novel features of the concurrence and measurement-induced disturbance (MID) as a function of B with different values of J_z. It is found that as B increases, both the concurrence and MID are initially constant and equal to its maximum value 1. When B further increases to a threshold value of B_c, the C and MID suffer a sharper dropping procession where the quantum phase transition (QPT) occurs.[36,37] The threshold value of B_c can be improved with the increase in the coupling parameter J_z for both the concurrence and MID. It is interesting that when $B > B_c$, the concurrence C undergoes a revival before decreasing to zero, whereas the MID decreases monotonously to zero with the increase in B. Therefore, the revival phenomenon is significantly different between the concurrence and MID.

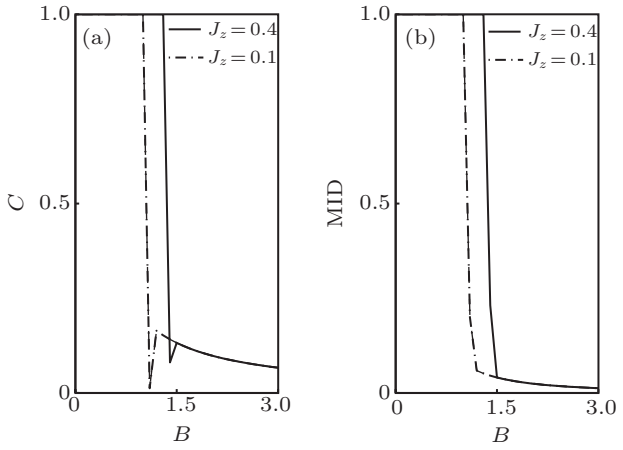


Fig. 1. With lower temperature $T = 0.01$, the concurrence (a) and MID (b) behaviors are plotted as a function of B with different values of J_z. For all plots, $J = 1.0$ and $\gamma = 0.2$.

Since the entanglement and MID can behave very differently with revival phenomenon, in what follows, we will illustrate why it occurs. For simplicity, we will analyze the limit case of $T \to 0$, which can not only reveal the fundamental reason but also can make us understand the ground state C and MID structures. In the limit case of $T \to 0$, according to the definitions about correlation measures (the concurrence and MID) and the spectrum of H, after some calculations the analytical solutions of the ground-state concurrence C and MID can be written as the following piecewise functions:

$$\eta < J + J_z : \begin{cases} C = 1, \\ \text{MID} = 1, \end{cases}$$

$$\eta = J + J_z : \begin{cases} C = \dfrac{\eta - J\gamma}{2\eta}, \\ \text{MID} = \dfrac{1}{2}\Delta + 1, \end{cases}$$

$$\eta > J + J_z : \begin{cases} C = \dfrac{J\gamma}{\eta}, \\ \text{MID} = \Delta + 1, \end{cases}$$

where

$$\Delta = \frac{B}{\eta} \log_2 \frac{\eta - B}{J\gamma} + \log_2 \frac{\eta}{J\gamma}.$$

These piecewise functions can be observed directly by calculating the correlation measures entanglement and MID about the ground-state spectrum of H. When $E_4 < E_2$ (or $\eta < J + J_z$), the ground state of H is $|\phi^-\rangle$ and then $C = \text{MID} = 1$. When $E_2 < E_4$ (or $\eta > J + J_z$), the ground state of H is $|\psi^-\rangle$, then $C = \frac{J\gamma}{\eta}$ and $\text{MID} = \Delta + 1$. On the other hand, at the critical point ($E_2 = E_4$ or $\eta = J + J_z$), the ground state of the system is a mixture state about $|\phi^-\rangle$ and $|\psi^-\rangle$; therefore, $C = \frac{\eta - J\gamma}{2\eta}$ and $\text{MID} = \frac{1}{2}\Delta + 1$. It follows that the threshold value of B_c at which C and MID become a nonanalytic function of B is given by $\eta = J + J_z$. Therefore, a quantum phase transition occurs in the limit case of $T = 0$ for $B = B_c$, without the correlation vanishing. Moreover, a revival entanglement for magnetic fields $B \geq B_c$ occurs, as $C(B)$ becomes larger than its value at $B = B_c$. However, it can be seen that $\Delta \leq 0$ due to $\text{MID} \leq 1$. In other words, the value of MID for the case of $B \geq B_c$ is lower than the case of $B = B_c$ ($\Delta + 1 \leq \frac{1}{2}\Delta + 1$). Therefore, it is obvious that the MID only undergoes a decreasing procession, and the revival phenomenon is not included in the MID behaviors.

To visualize different effects of the coupling parameters on C and MID behaviors, in Fig. 2, we plot the quantum correlation measures C and MID as a function of T, where J_z varies and B and γ are determined values. It can be seen that there is a threshold value of T_c at which the entanglement vanishes, and T_c is improved monotonously with the increase in J_z. For MID, the value of T_c is fully independent of J_z (three curve lines of MID are superposed finally), and it is larger than the value of concurrence C. That is to say, the life of MID is longer than C with the fixed T and J_z. Namely, the MID is expected to be more robust than entanglement from this aspect. We also note that the entanglement and MID can be enhanced evidently by improving the coupling intensity J_z. When $J_z = 0$ the curve lives of MID and C indicate that, the concurrence exhibits a regrowth phenomenon, while MID first increases and then decreases monotonously to zero. Therefore, there is no revival phenomenon in MID. In addition, in the case of $J_z = 0$, the maximal values of the concurrence in both areas are all smaller than the maximal value of MID.

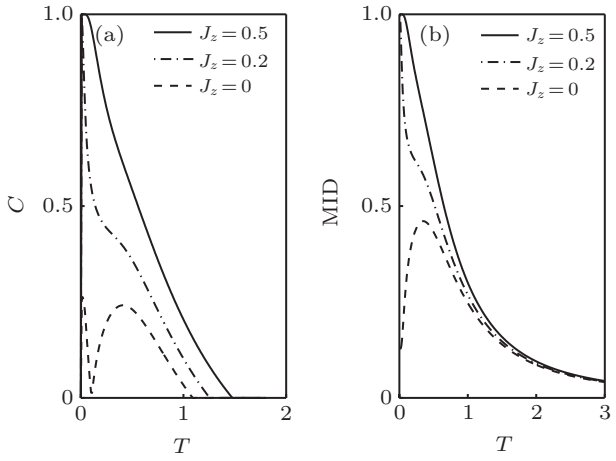


Fig. 2. Quantum correlation measures C (a) and MID (b) as a function of the temperature T. For all plots, $J = 1.0$, $\gamma = 0.3$, and $B = 1.1$. From top to bottom: J_z equals 0.5, 0.2, and 0, respectively.

With $\gamma = 0.3$ and $J_z = 0.9$, we show the influence of the two quantities B and T for the concurrence and MID in Fig. 3. It is shown that in the lower region of B (about 2), the concurrence C only undergoes a decreasing procession with improving T, but in the larger region of B, the C behaves complicated. It is first improved and then decreased till the threshold value point T_c. When T further increases, the revival phenomenon occurs with C, and finally maintains the value of zero for larger T (it is about 1.7). The threshold value of T_c is increased with the increase of B. However, in the case of MID behaviors,

in the lower region of B, MID decreases monotonously from the maximal value 1 to 0, whereas in the larger region of B, it is first enhanced and then decreased slowly to zero with increasing T, and the threshold value of T_c vanishes (appeared in C). That is to say, the concurrence demonstrates a revival phenomenon but MID does not. Comparing the two plots, we can also observe that the region of MID is evidently larger than the concurrence, which means that we can obtain the value of MID at higher temperature T and larger magnitude of B, and the life of MID is longer than C.

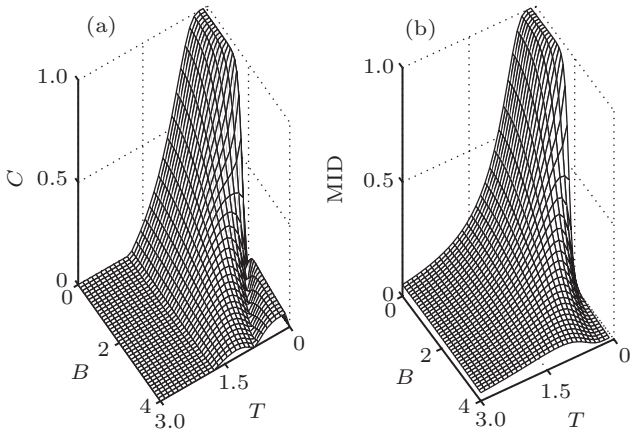


Fig. 3. Correlation measures C (a) and MID (b) are plotted versus temperature T and external field parameter B. For all plots, $J = 1.0$, $\gamma = 0.3$, and $J_z = 0.9$.

In Fig. 4, we show concurrence and MID as a function of B and γ with the determined values $T = 0.2$ and $J_z = 0.9$. It is shown that both the concurrence C and MID are initially constant and equal to its maximum value 1 with smaller value of region B, and the region of (B, γ) parameter space, where the MID maintains the maximal value 1, and is obviously larger than the case of C. It is possible to obtain the maximal value of MID through increasing γ with a lower value of B. In this sense, it is argued that the quantum correlation based only on the MID is expected to be more robust than it based on the concurrence. Obviously, the concurrence exists a threshold value of magnetic field B_c, at which C suddenly changes, quantum phase transition occurs, and the value of B_c increases with the decrease in γ. With the determined value of anisotropic parameter γ, near the region of B_c, the concurrence initially decreases and then undergoes a revival procession and finally decreases to zero with the improvement of B. As for the MID behaviors, instead of the revival phenomenon, the value of MID monotonously decreases with the increase in B for the determined value γ. In short, there is a revival phenomenon in the concurrence but not in the MID. We also show that with a proper range of B (for example, it is about $B = 2.4$), the concurrence decreases with the improvement of γ, whereas we present an effect that is opposite to the MID behaviors, and the value of MID is enhanced with the increase in γ. In the larger value of region B, both the values of MID and C increase with the increase in γ.

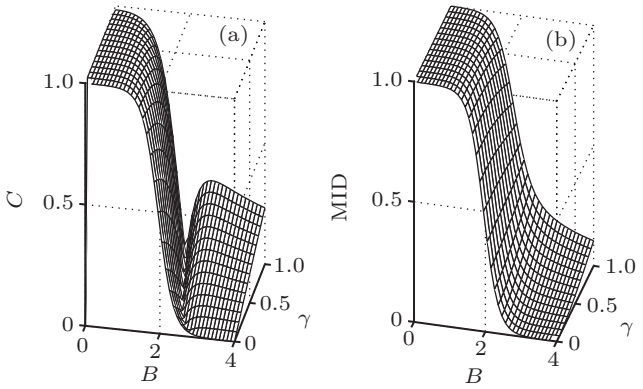


Fig. 4. Concurrence (a) and MID (b) versus B and anisotropy parameter γ. For all plots, $J = 1.0$, $T = 0.2$, and $J_z = 0.9$.

In Fig. 5, we plot the concurrence C and MID with two dependent variables B and J_z for $\gamma = 0.3$ and $T = 0.6$. From this figure, one can see clearly that the revival phenomenon is still significantly different between the concurrence and MID. Specifically, the entanglement can be revived, while the MID is dropping monotonously with the enhancing magnetic field B, and the threshold value of B_c in C behaviors increases with the increasing coupling parameter J_z. In addition, in the larger region of B, the revival entanglement keeps at one nonzero constant value, which is independent of the coupling parameter J_z and field B, and it is invariable regardless of the value of J_z and B. However, the values of MID decreases monotonously from the maximal value to zero with the increase in B. In other words, figure 5 also reveals that the influences about parameter J_z on MID are obviously weaker than the effects on entanglement.

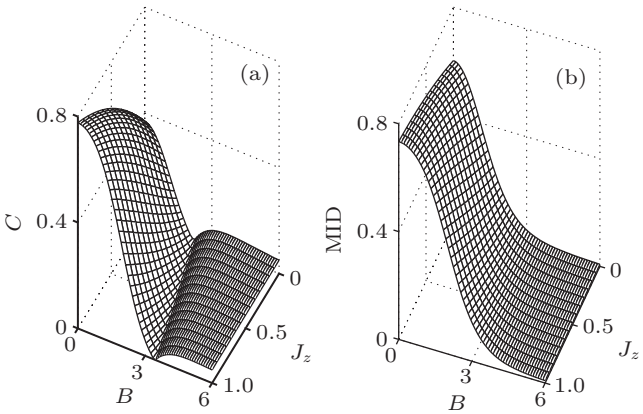


Fig. 5. Concurrence (a) and MID (b) as a function of B and J_z. For all plots, $J = 1.0$, $\gamma = 0.3$, and $T = 0.6$.

4. Conclusion

In conclusion, we illustrate the difference between the quantum correlation measures (concurrence and MID) about

an anisotropic two-qubit Heisenberg XYZ chain. These studies could inspire some theoretical research on constructing one uniform quantum correlation measure. The results show that the entanglement and MID can behave very differently, with the revival phenomenon of the former not included in the latter. This novel difference appears in the ground state, and is also suitable for the finite temperature case. Through analysis of the ground state case, we offer a detailed reason why the concurrence undergoes a revival procession, but MID decreases monotonously from the maximal value 1 to 0. This conclusion can be drawn from the analytic solutions about the ground state concurrence and MID. We also explore the different effects about the external and self coupling parameters on entanglement and MID behaviors. Several remarkable differences between them are observed. The value of MID is larger than the concurrence with a determined temperature and J_z, and the region of MID is always larger than the concurrence. That is to say, the quantum correlation based only on MID is expected to be more robust than based on the concurrence. If B can take a proper range of values, the concurrence decreases with the increase in γ, whereas we present an effect that is opposite to the MID behaviors, the value of MID is enhanced by increasing γ. When it is plotted as a function of B and J_z, the concurrence finally maintains one nonzero constant value, while the values of MID decrease monotonously from the maximal value to zero with the increases in B, and the influence of parameter J_z on the MID is obviously weaker than the effects on entanglement.

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