Symmetric quantum discord for a two-qubit state*

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We discuss the symmetric quantum discord (SQD) for an arbitrary two-qubit state consisting of subsystems A and B and give the analysis formula of the symmetric quantum discord for the arbitrary two-qubit state. We also give the optimization process of the symmetric quantum discord for some states and obtain the symmetric quantum discord. We compare the quantum discord (QD) with the symmetric quantum discord, and find that the symmetric quantum discord is greater than the quantum discord. We also find that the symmetric quantum discord can be unequal to the quantum discord when the right quantum discord (measure on subsystem B) is equal to the left quantum discord (measure on subsystem A).

Keywords: quantum discord, symmetric quantum discord, optimization process

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1. Introduction

Quantum correlation has recently been identified as a general resource in quantum information protocols. Aguilar has shown that a witness of quantum correlation for a quite general class of states can be estimated through measurements corresponding to a single physical observable.[1] In addition, Silva has reported an experimental measurement of bipartite quantum correlations of an unknown two-qubit state.[2]

After Ollivier and Zurek defined the quantum discord,[3] Modi gave a unified view of quantum and classical correlations,[4] and presented the relations among entanglement, quantum discord, quantum dissonance, classical correlations, and mutual information. The quantum discords for some given states have been obtained. For example, Giorda and Paris have evaluated the quantum discord for bipartite Gaussian states,[5] Adesso and Datta have calculated the quantum discord of all two-mode Gaussian states.[6] For the no-continuous variables, Luo has calculated the quantum discord of a two-qubit Bell diagonal state,[7] Ali and Rau have computed the quantum discord of a general two-qubit X state.[8] Girolami and Adesso have discussed the quantum discord for general two-qubit states.[9] Yan and Wang have discussed the quantum discord for a qutrit–qutrit system.[10] Xu *et al.* have evaluated the quantum correlations of a three-particle W class state.[11]

For a two-qubit state, quantum discord just measures one subsystem. So quantum discord is a one-side measurement, which means that it is asymmetric and satisfies the inequality $D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB}) \neq D_{\mathrm{Q}}^{\rightarrow}(\mathrm{AB})$ in general, where $D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB})$ is the right quantum discord (measure on subsystem B) and $D_{\mathrm{Q}}^{\rightarrow}(\mathrm{AB})$ is the left quantum discord (measure on subsystem A). In the past few years, some symmetric measures of correlations were proposed.[4] As we know, a local measurement on one subsystem, say subsystem B, does not disturb the other subsystem, say subsystem A; however, the total system is certainly influenced. The symmetric quantum discord is a two-side measurement and measures the two subsystems. It is natural to be symmetric. So it may be advantageous to use the symmetric quantum discord. Recently, the symmetric quantum discord was employed in an experiment to investigate the quantumness of correlations in a spin system.[12] The symmetric quantum discord has been widely investigated for some given states. Maziero calculated the symmetric quantum discord for a two-qubit Bell diagonal state and showed that the symmetric quantum discord could be more convenient to quantify quantum correlations.[13] In this paper, we discuss the symmetric quantum discord for an arbitrary two-qubit state and give an analytical solution. By comparing the symmetric quantum discord with the quantum discord for some special states, we find that the symmetric quantum discord is larger than the quantum discord and observe that the symmetric quantum discord can be unequal to the quantum discord when the right quantum discord is equal to the left quantum discord.

This paper is organized as follows. In Section 2, we briefly introduce the definitions of quantum discord and symmetric quantum discord. In Section 3, the symmetric quantum discord for an arbitrary two-qubit state is discussed. In Section 4, we will discuss the relationship between quantum discord and symmetric quantum discord. Finally, some conclusions and discussion are presented in Section 5.

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2. Quantum discord and symmetric quantum discord

Quantum discord (QD), which has been introduced in Ref. [3], is defined as the difference between the mutual information and the classical correlation. The quantum discord is given as

$$D(\mathrm{AB}) = I(\rho_{\mathrm{AB}}) - J(\rho_{\mathrm{AB}}), \tag{1}$$

where ρ_{AB} is the density operator of a bipartite system AB composed of subsystems A and B. The mutual correlation $I(\rho_{\mathrm{AB}})$ can be expressed as

$$I(\rho_{\mathrm{AB}}) = S(\rho_{\mathrm{A}}) + S(\rho_{\mathrm{B}}) - S(\rho_{\mathrm{AB}}), \tag{2}$$

where $\rho_{\mathrm{A}} = \mathrm{tr}_{\mathrm{B}}\rho_{\mathrm{AB}}$, $\rho_{\mathrm{B}} = \mathrm{tr}_{\mathrm{A}}\rho_{\mathrm{AB}}$, and $S(\rho) = -\mathrm{tr}(\rho\log_2\rho)$ is the von Neumann entropy of density matrix ρ. The corresponding classical correlation is defined as[3]

$$J(\rho_{\mathrm{AB}}) = S(\rho_{\mathrm{A}}) - S\left(\rho_{\mathrm{A|B}}\right), \tag{3}$$

where $S\left(\rho_{\mathrm{A|B}}\right) = \min_{\{\Pi_i^{\mathrm{B}}\}}\sum_i p_i S\left(\rho_{\mathrm{A}|i}\right)$, with the minimization being over all projection valued $\{\Pi_i^{\mathrm{B}}\}$. Here $p_i = \mathrm{tr}_{\mathrm{AB}}\left(I_{\mathrm{A}}\otimes\Pi_i^{\mathrm{B}}\rho_{\mathrm{AB}}I_{\mathrm{A}}\otimes\Pi_i^{\mathrm{B}}\right)$ is the probability of obtaining outcome i, and after the measurement, the system reduces to state $\rho_{\mathrm{A}|i} = \left(I_{\mathrm{A}}\otimes\Pi_i^{\mathrm{B}}\rho_{\mathrm{AB}}I_{\mathrm{A}}\otimes\Pi_i^{\mathrm{B}}\right)/p_i$. Then, the quantum discord (here we make measurements on subsystem B) has the following form:

$$D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB}) = S(\rho_{\mathrm{B}}) - S(\rho_{\mathrm{AB}}) + \min_{\{\Pi_i^{\mathrm{B}}\}}\sum_i p_i S\left(\rho_{\mathrm{A}|i}\right). \tag{4}$$

Braga *et al.* gave another expression for the quantum discord,[14] which is equivalent to the one given in Ref. [3]. It is given by

$$D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB}) = \min_{\{\Pi_k^{\mathrm{B}}\}}\left[I(\rho_{\mathrm{AB}}) - I\left(\Phi^{\mathrm{B}}(\rho_{\mathrm{AB}})\right)\right], \tag{5}$$

where

$$\begin{aligned}\Phi^{\mathrm{B}}(\rho_{\mathrm{AB}}) &= \sum_k\left(I_{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}\right)\rho_{\mathrm{AB}}\left(I_{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}\right),\\ k &= 0,1.\end{aligned} \tag{6}$$

The quantum discord defined in Eqs. (5) and (6) is based on a one-side measurement.

The symmetrized version of QD (SQD) is defined by[13]

$$D_{\mathrm{S}}(\mathrm{AB}) = \min_{\{\Pi_l^{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}\}}\left[I(\rho_{\mathrm{AB}}) - I\left(\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}})\right)\right], \tag{7}$$

where

$$\begin{aligned}\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}}) &= \Phi_{l,k}^{\mathrm{AB}}(\rho_{\mathrm{AB}})\\ &= \sum_{l,k}\left(\Pi_l^{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}\right)\rho_{\mathrm{AB}}\left(\Pi_l^{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}\right),\\ l &= 0,1,\ \ k = 0,1.\end{aligned} \tag{8}$$

The definition of $I\left(\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}})\right)$ is given by

$$\begin{aligned}I\left(\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}})\right) &= S(\Phi_{\mathrm{A}}(\rho_{\mathrm{AB}})) + S(\Phi_{\mathrm{B}}(\rho_{\mathrm{AB}}))\\ &\quad - S\left(\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}})\right),\end{aligned} \tag{9}$$

where $\Phi_{\mathrm{A}}(\rho_{\mathrm{AB}}) = \mathrm{tr}_{\mathrm{B}}\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}})$, $\Phi_{\mathrm{B}}(\rho_{\mathrm{AB}}) = \mathrm{tr}_{\mathrm{A}}\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}})$, and $S(\rho) = -\mathrm{tr}(\rho\log_2\rho)$ is the von Neumann entropy of density matrix ρ.

Because the mutual correlation $I(\rho_{\mathrm{AB}})$ can always be evaluated in principle, we just focus on $I\left(\Phi^{\mathrm{B}}(\rho_{\mathrm{AB}})\right)$ and $I\left(\Phi^{\mathrm{AB}}(\rho_{\mathrm{AB}})\right)$ in this paper.

3. The symmetric quantum discord for an arbitrary two-qubit state

As mentioned above, the symmetric quantum discord is defined by Eq. (7), and $I(\rho_{\mathrm{AB}})$ can always be evaluated in principle. So we just focus on $I\left(\Phi^{\mathrm{AB}}\right)$. We know that an arbitrary two-qubit state can be brought through local unitary to the following form:

$$\begin{aligned}\rho_{\mathrm{AB}} &= \frac{1}{4}\Bigg(\boldsymbol{I}_{4\times4} + \boldsymbol{a}\cdot\boldsymbol{\sigma}\otimes\boldsymbol{I}_{2\times2} + \boldsymbol{I}_{2\times2}\otimes\boldsymbol{b}\cdot\boldsymbol{\sigma}\\ &\quad + \sum_{j=1}^{3}c_j\sigma_j\otimes\sigma_j\Bigg),\end{aligned} \tag{10}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the Bloch vectors, which means $|\boldsymbol{a}|\le1$ and $|\boldsymbol{b}|\le1$, $\boldsymbol{\sigma}_j$ are the Pauli operators, $c_j\in\mathbf{R}$, and $\boldsymbol{I}$ is the identity matrix.

Any von Neumann measurement for parities A and B can be written as

$$\Pi_{l,k}^{\mathrm{AB}} = \Pi_l^{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}, \tag{11}$$

where $\Pi_l^{\mathrm{A}} = V_{\mathrm{A}}|l\rangle_{\mathrm{A}\,\mathrm{A}}\langle l|V_{\mathrm{A}}^{+}$ $(l=0,1)$ and $\Pi_k^{\mathrm{B}} = V_{\mathrm{B}}|k\rangle_{\mathrm{B}\,\mathrm{B}}\langle k|V_{\mathrm{B}}^{+}$ $(k=0,1)$. Here $|l\rangle_{\mathrm{A}\,\mathrm{A}}\langle l|$ and $|k\rangle_{\mathrm{B}\,\mathrm{B}}\langle k|$ are the projection operators on subsystems A and B, respectively, V_{A} and V_{B} are the unitary matrices acting on the two subsystems, respectively, and V_{A}, $V_{\mathrm{B}}\in\mathrm{SU}(2)$. So they can be expressed as $V_{\mathrm{A(B)}} = t^{\mathrm{A(B)}}\boldsymbol{I} + \mathrm{i}\boldsymbol{y}^{\mathrm{A(B)}}\boldsymbol{\sigma}$, where $t^{\mathrm{A(B)}}\in\mathbf{R}$ is a real number, $\boldsymbol{y}^{\mathrm{A(B)}} = \left(y_1^{\mathrm{A(B)}}, y_2^{\mathrm{A(B)}}, y_3^{\mathrm{A(B)}}\right)$ is a vector. For unitary $V_{\mathrm{A(B)}}\in\mathrm{SU}(2)$, we have the following equation:

$$\left(t^{\mathrm{A(B)}}\right)^2 + \left(y_1^{\mathrm{A(B)}}\right)^2 + \left(y_2^{\mathrm{A(B)}}\right)^2 + \left(y_3^{\mathrm{A(B)}}\right)^2 = 1. \tag{12}$$

By using the computation method given by Luo,[7] we can obtain

$$\begin{aligned}\Phi_{l,k}^{\mathrm{AB}}(\rho_{\mathrm{AB}}) &= \Pi_l^{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}\rho_{\mathrm{AB}}\Pi_l^{\mathrm{A}}\otimes\Pi_k^{\mathrm{B}}\\ &= V_{\mathrm{A}}|l\rangle_{\mathrm{A}\,\mathrm{A}}\langle l|V_{\mathrm{A}}^{+}\otimes V_{\mathrm{B}}|k\rangle_{\mathrm{B}\,\mathrm{B}}\langle k|V_{\mathrm{B}}^{+}\rho V_{\mathrm{A}}|l\rangle_{\mathrm{A}}\\ &\quad\times{}_{\mathrm{A}}\langle l|V_{\mathrm{A}}^{+}\otimes V_{\mathrm{B}}|k\rangle_{\mathrm{B}\,\mathrm{B}}\langle k|V_{\mathrm{B}}^{+}\\ &= \frac{1}{4}V_{\mathrm{A}}|l\rangle_{\mathrm{A}\,\mathrm{A}}\langle l|V_{\mathrm{A}}^{+}\otimes V_{\mathrm{B}}|k\rangle_{\mathrm{B}\,\mathrm{B}}\langle k|V_{\mathrm{B}}^{+}\end{aligned}$$

$$\times\left(I+\boldsymbol{a}\cdot\boldsymbol{\sigma}\otimes\boldsymbol{I}+\boldsymbol{I}\otimes\boldsymbol{b}\cdot\boldsymbol{\sigma}+\sum_{j=1}^{3}c_j\sigma_j\otimes\sigma_j\right)$$
$$V_{\rm A}\left|l\right\rangle_{\rm A\,A}\!\left\langle l\right|V_{\rm A}^{+}\otimes V_{\rm B}\left|k\right\rangle_{\rm B\,B}\!\left\langle k\right|V_{\rm B}^{+}$$
$$=\frac{1}{4}\Bigg(I+\sum_{j=1}^{3}\left((-1)^{l}a_jz_j^{\rm A}+(-1)^{k}b_jz_j^{\rm B}\right.$$
$$\left.+c_j(-1)^{l}(-1)^{k}z_j^{\rm A}z_j^{\rm B}\right)\Bigg)$$
$$V_{\rm A}\left|l\right\rangle_{\rm A\,A}\!\left\langle l\right|V_{\rm A}^{+}\otimes V_{\rm B}\left|k\right\rangle_{\rm B\,B}\!\left\langle k\right|V_{\rm B}^{+}$$
$$=\frac{1}{4}\left(I+(-1)^{l}H_1+(-1)^{k}H_2+(-1)^{l}(-1)^{k}H_3\right)$$
$$V_{\rm A}\left|l\right\rangle_{\rm A\,A}\!\left\langle l\right|V_{\rm A}^{+}\otimes V_{\rm B}\left|k\right\rangle_{\rm B\,B}\!\left\langle k\right|V_{\rm B}^{+},\qquad(13)$$

where

$$z_1^{\rm A(B)}=2\left(-t^{\rm A(B)}y_2^{\rm A(B)}+y_1^{\rm A(B)}y_3^{\rm A(B)}\right),$$
$$z_2^{\rm A(B)}=2\left(t^{\rm A(B)}y_1^{\rm A(B)}+y_2^{\rm A(B)}y_3^{\rm A(B)}\right),$$
$$z_3^{\rm A(B)}=\left(t^{\rm A(B)}\right)^2+\left(y_3^{\rm A(B)}\right)^2-\left(y_1^{\rm A(B)}\right)^2-\left(y_2^{\rm A(B)}\right)^2,$$
$$\left|\boldsymbol{z}^{\rm A(B)}\right|=1,\qquad(14)$$

$$H_1=\boldsymbol{a}\cdot\boldsymbol{z}^{\rm A},\ \ H_2=\boldsymbol{b}\cdot\boldsymbol{z}^{\rm B},\ \ H_3=\sum_{j=1}^{3}c_jz_j^{\rm A}z_j^{\rm B}.\qquad(15)$$

Then, we can obtain

$$\Phi^{\rm AB}=\sum_{l,k}\Phi_{l,k}^{\rm AB}(\rho)$$
$$=\frac{1}{4}\left(I+H_1M_{\rm A}\otimes I+I\otimes H_2M_{\rm B}+H_3M_{\rm A}\otimes M_{\rm B}\right),\qquad(16)$$

where

$$M_{\rm A}=V_{\rm A}\left|0\right\rangle_{\rm A\,A}\!\left\langle 0\right|V_{\rm A}^{+}-V_{\rm A}\left|1\right\rangle_{\rm A\,A}\!\left\langle 1\right|V_{\rm A}^{+},$$
$$M_{\rm B}=V_{\rm B}\left|1\right\rangle_{\rm B\,B}\!\left\langle 1\right|V_{\rm B}^{+}-V_{\rm B}\left|1\right\rangle_{\rm B\,B}\!\left\langle 1\right|V_{\rm B}^{+}.\qquad(17)$$

The eigenvalues of Eq. (16) are $\lambda_i^{\rm AB}/4$, where $\lambda_i^{\rm AB}$ is the solution of the following equation:

$$H_1^4+H_2^4+H_3^4-2\left(H_2^2H_1^2+H_2^2H_3^2+H_1^2H_3^2\right)$$
$$-2\left(H_2^2+H_1^2+H_3^2\right)(\lambda-1)^2-8H_2H_1H_3(\lambda-1)+\lambda^4$$
$$-4\lambda^3+6\lambda^2-4\lambda+1=0.\qquad(18)$$

So we can obtain

$$\lambda_1=H_3-H_2-H_1+1,$$
$$\lambda_2=H_2-H_1-H_3+1,$$
$$\lambda_3=H_1-H_2-H_3+1,$$
$$\lambda_4=H_1+H_2+H_3+1.\qquad(19)$$

With the eigenvalues of Eq. (16), we can give the analysis formula of SQD for an arbitrary two-qubit state. By using the result of Eq. (19), we can obtain the von Neumann entropy of $\Phi^{\rm AB}$, which can be expressed as

$$S\left(\Phi^{\rm AB}\right)=-\sum_{i=1}^{4}\frac{1}{4}\lambda_i\log_2\left(\frac{1}{4}\lambda_i\right).\qquad(20)$$

The reduced density operators can be obtained by tracing over one subsystem

$$\Phi_{\rm A}={\rm tr}_{\rm B}\left(\Phi^{\rm AB}\right)=\frac{1}{2}\left(1+H_1M_{\rm A}\right),$$
$$\Phi_{\rm B}={\rm tr}_{\rm A}\left(\Phi^{\rm AB}\right)=\frac{1}{2}\left(1+H_2M_{\rm B}\right).\qquad(21)$$

As a result, the von Neumann entropies of $\Phi^{\rm A}$ and $\Phi^{\rm B}$ are given by

$$S(\Phi_{\rm A})=-\frac{(1+H_1)}{2}\log_2\frac{(1+H_1)}{2}$$
$$-\frac{(1-H_1)}{2}\log_2\frac{(1-H_1)}{2},$$
$$S(\Phi_{\rm B})=-\frac{(1+H_2)}{2}\log_2\frac{(1+H_2)}{2}$$
$$-\frac{(1-H_2)}{2}\log_2\frac{(1-H_2)}{2}.\qquad(22)$$

Finally, we can obtain the classical correlations of SQD as

$$I\left(\Phi^{\rm AB}\right)=S(\Phi_{\rm A})+S(\Phi_{\rm B})-S\left(\Phi^{\rm AB}\right).\qquad(23)$$

Then we obtain the analysis formula of SQD for an arbitrary two-qubit state since $I(\rho_{\rm AB})$ can always be evaluated in principle. However, it is difficult to perform the optimization process of SQD for an arbitrary two-qubit state, so we just consider four special states: (i) $a_j=0$, $b_j\neq0$, $c_j\neq0$, (ii) $b_j=0$, $a_j\neq0$, $c_j\neq0$, (iii) $a_j=0$, $b_j=0$, $c_j\neq0$ (Maziero has considered this case),[13] and (iv) $a_j=b_j=0$, $j=1,2$ and $c_k=0$, $k=2,3$.

(i) For $a_j=0$, $b_j\neq0$, $c_j\neq0$, we obtain the analytical solution of Eq. (18) as

$$\lambda_{1,2}^{\rm AB}=1\pm(H_2+H_3),$$
$$\lambda_{3,4}^{\rm AB}=1\pm(H_2-H_3).\qquad(24)$$

We want to find the maximal values of $I\left(\Phi^{\rm AB}\right)$. In this case, $I\left(\Phi^{\rm AB}\right)$ has the properties $\partial\left[I\left(\Phi^{\rm AB}\right)\right]/\partial\left|H_2\right|\geq0$ and $\partial\left[I\left(\Phi^{\rm AB}\right)\right]/\partial\left|H_3\right|\geq0$. However, $|H_2|$ and $|H_3|$ are not independent of each other. For convenience, we further assume $c_1=c_2=c_3=c$. In this situation, $H_2=\boldsymbol{b}\cdot\boldsymbol{z}^{\rm B}$ and $H_3=c\boldsymbol{z}^{\rm A}\cdot\boldsymbol{z}^{\rm B}$ can approach the maximal values simultaneously, that is, $\max|H_2|=|\boldsymbol{b}|$, $\max|H_3|=|c|$ when $\boldsymbol{b}\cdot\boldsymbol{z}^{\rm B}=\pm|\boldsymbol{b}|$ and $\boldsymbol{z}^{\rm A}\cdot\boldsymbol{z}^{\rm B}=\pm1$. So the classical correlation is given by

$$I_{\rm m}\left(\Phi^{\rm AB}\right)=1-\frac{(1+|\boldsymbol{b}|)}{2}\log_2\frac{(1+|\boldsymbol{b}|)}{2}$$
$$-\frac{(1-|\boldsymbol{b}|)}{2}\log_2\frac{(1-|\boldsymbol{b}|)}{2}$$
$$+\frac{1}{4}(1+|\boldsymbol{b}|+c)\log_2\frac{(1+|\boldsymbol{b}|+c)}{4}$$
$$+\frac{1}{4}(1+|\boldsymbol{b}|-c)\log_2\frac{(1+|\boldsymbol{b}|-c)}{4}$$
$$+\frac{1}{4}(1-|\boldsymbol{b}|+c)\log_2\frac{(1-|\boldsymbol{b}|+c)}{4}$$

$$+\frac{1}{4}(1-|\boldsymbol{b}|-c)\log_2\frac{(1-|\boldsymbol{b}|-c)}{4}. \quad (25)$$

This result is the same as the classical correlation of quantum discord.

(ii) For $b_j=0$, $a_j\neq 0$, $c_j\neq 0$, we obtain

$$\begin{aligned}\lambda_{1,2}^{\mathrm{AB}} &= 1\pm(H_1+H_3),\\ \lambda_{3,4}^{\mathrm{AB}} &= 1\pm(H_1-H_3).\end{aligned} \quad (26)$$

The result is similar to that in case (i). We assume $c_1=c_2=c_3=c$, the maximal values of $|H_1|$ and $|H_3|$ are $|H_1|=|\boldsymbol{a}|$ and $|H_3|=|c|$. Then, we can obtain $I_{\mathrm{m}}\left(\Phi^{\mathrm{AB}}\right)$. This result is also the same as the classical correlation of quantum discord.

(iii) For $a_j=0$, $b_j=0$, $c_j\neq 0$, we obtain

$$\begin{aligned}\lambda_{1,2}^{\mathrm{AB}} &= 1+H_3,\\ \lambda_{3,4}^{\mathrm{AB}} &= 1-H_3.\end{aligned} \quad (27)$$

An optimization process can give that $|H_3|=c=\max(|c_1|,|c_2|,|c_3|)$. In fact, in this case, the SQD and the QD have been studied by Maziero[11] and Luo,[7] respectively. Their results agree with each other.

(iv) For $a_j=b_j=0$, $j=1,2$, and $c_k=0$, $k=2,3$, the states are the X-real states, the quantum discords of these states have been given in Ref. [8]. Here we investigate the SQD. By numerical analysis, for $0.4\geq a_3\geq 0$, $0.4\geq b_3\geq 0$, $1\geq c_1\geq 0.2$, we obtain the maximal values of $I\left(\Phi^{\mathrm{AB}}\right)$ when $z_3^{\mathrm{A}}=z_2^{\mathrm{A}}=z_3^{\mathrm{B}}=z_2^{\mathrm{B}}=0$ and $z_1^{\mathrm{A}}=z_1^{\mathrm{B}}=1$. So the classical correlation of SQD can be obtained, which is not the same as that of QD. We will discuss this case in detail in Section 4.

We find that the SQD is equal to the QD in cases (i)–(iii). In order to give the difference between the symmetric discord and the quantum discord, we consider case (iv) and give its optimization process.

4. The relationship between SQD and QD

Quantum discord and symmetric quantum discord have the same mutual correlations, so we just pay attention to the classical correlations. In this section, we will discuss the difference between SQD and QD. When $a_j,b_j,c_j\neq 0$, it is difficult to give the optimization process of SQD. So we consider the four cases above. Because we obtain $D_{\mathrm{S}}=D_{\mathrm{Q}}$ for cases (i)–(iii), we will discuss case (iv) to show the difference. To compare the symmetric discord with the quantum discord, we give the difference as

$$\begin{aligned}D_\Delta &= D_{\mathrm{S}}(\mathrm{AB})-D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB})\\ &= \left(I(\rho)-I_{\mathrm{m}}\left(\Phi^{\mathrm{AB}}\right)\right)-\left(I(\rho)-I_{\mathrm{m}}\left(\Phi^{\mathrm{B}}\right)\right)\\ &= I_{\mathrm{m}}\left(\Phi^{\mathrm{B}}\right)-I_{\mathrm{m}}\left(\Phi^{\mathrm{AB}}\right).\end{aligned} \quad (28)$$

Here we consider the case with $a_j=b_j=0$, $j=1,2$; $c_k=0$, $k=2,3$; $0.4\geq a_3=a\geq 0$; $0.4\geq b_3=b\geq 0$; $1\geq c_1=c\geq 0.2$. The quantum discord of these states has been given in Ref. [8]. We obtain the classical correlation of SQD in the following equation:

$$I_{\mathrm{m}}\left(\Phi^{\mathrm{AB}}\right)=2+\frac{1+c}{2}\log_2\frac{1+c}{4}+\frac{1-c}{2}\log_2\frac{1-c}{4}. \quad (29)$$

Then, D_Δ is given by

$$\begin{aligned}D_\Delta &= -\frac{(1+a)}{2}\log_2\frac{(1+a)}{2}-\frac{(1-a)}{2}\log_2\frac{(1-a)}{2}\\ &\quad+\frac{1+\sqrt{a^2+c^2}}{2}\log_2\frac{1+\sqrt{a^2+c^2}}{4}\\ &\quad+\frac{1-\sqrt{a^2+c^2}}{2}\log_2\frac{1-\sqrt{a^2+c^2}}{4}\\ &\quad-\frac{1+c}{2}\log_2\frac{1+c}{4}-\frac{1-c}{2}\log_2\frac{1-c}{4}-1,\end{aligned} \quad (30)$$

and plotted in Fig. 1 for $c=0.5$ and $c=0.9$.

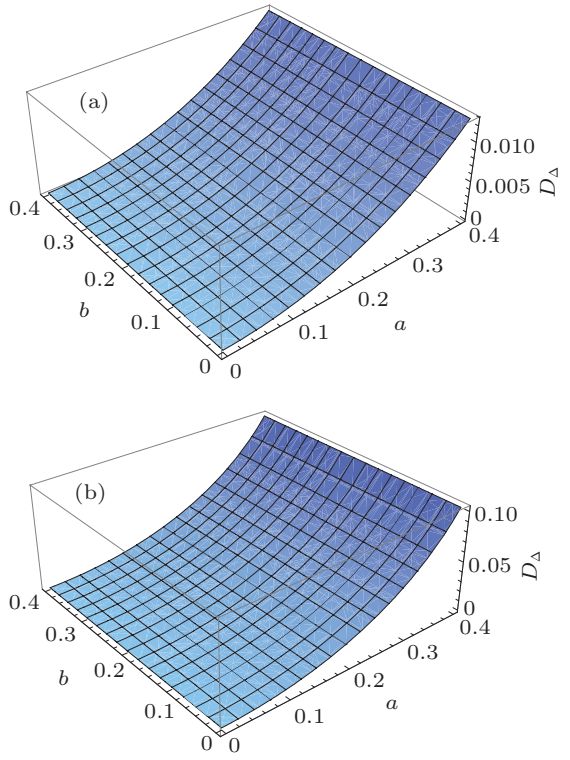


Fig. 1. (color online) The difference between SQD and QD ($D_\Delta(c)=D_{\mathrm{S}}-D_{\mathrm{Q}}$), where $a_1=a_2=b_1=b_2=c_2=c_3=0$, $a_3=a$, $b_3=b$, $c_1=c$; and (a) $c=0.5$, (b) $c=0.9$.

We obtain $D_\Delta\geq 0$, that is, $D_{\mathrm{S}}(\mathrm{AB})\geq D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB})$. We also obtain $\partial D_\Delta(c)/\partial c>0$, which means $D_\Delta(c)$ is a monotone increasing function by considering c as an independent variable and b does not influence D_Δ. When $a=0$, we obtain $D_\Delta(c)=0$, then we have $D_{\mathrm{S}}(AB)=D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB})$. We find that the classical correlation obtained from the one-side measurement is greater than that obtained from the two-side measurement. Because symmetric quantum discord and quantum discord have the same mutual correlations, we can obtain $D_{\mathrm{S}}(\mathrm{AB})\geq D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB})$. Non-selective von Neumann measurements can destroy quantum correlations, so we have $D_{\mathrm{S}}(\mathrm{AB})\geq D_{\mathrm{Q}}^{\leftarrow}(\mathrm{AB})$. Our calculations support this result.

We also give the difference between the symmetric quantum discord and the average of the right quantum discord and the left quantum discord as

$$\begin{aligned} D_{1\Delta} &= D_{\rm S}({\rm AB}) - \frac{D_{\rm Q}^{\leftarrow}(AB) + D_{\rm Q}^{\rightarrow}({\rm AB})}{2} \\ &= \left(I(\rho) - I_{\rm m}\left(\Phi^{\rm AB}\right)\right) \\ &\quad - \frac{\left(I(\rho) - I_{\rm m}\left(\Phi^{\rm B}\right)\right) + \left(I(\rho) - I_{\rm m}\left(\Phi^{\rm A}\right)\right)}{2} \\ &= \frac{I_{\rm m}\left(\Phi^{\rm B}\right) + I_{\rm m}\left(\Phi^{\rm A}\right)}{2} - I_{\rm m}\left(\Phi^{\rm AB}\right), \end{aligned} \tag{31}$$

where $D_{\rm Q}^{\leftarrow}({\rm AB})$ is the right quantum discord and $D_{\rm Q}^{\rightarrow}({\rm AB})$ is the left quantum discord. The $D_{1\Delta}$ is plotted in Fig. 2. As we know, when $a = b$, we can obtain $D_{\rm Q}^{\leftarrow}({\rm AB}) = D_{\rm Q}^{\rightarrow}(AB)$. We have $D_{\rm S}({\rm AB}) > D_{\rm Q}^{\leftarrow}(AB) = D_{\rm Q}^{\rightarrow}({\rm AB})$ when $a = b \neq 0$. This means that even the right quantum discord is equal to the left one, the symmetric quantum discord can be unequal to the quantum discord.

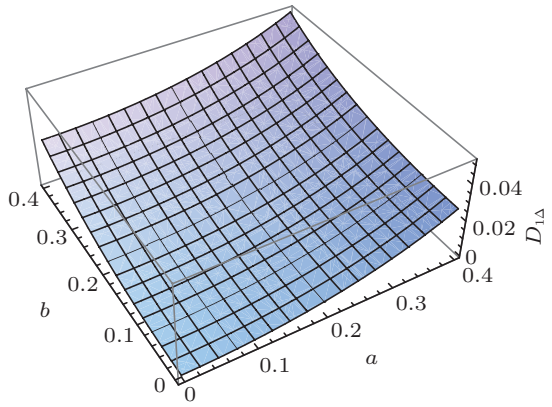


Fig. 2. (color online) The difference between the symmetric quantum discord and the average of the right quantum discord and the left quantum discord ($D_{1\Delta} = D_{\rm S}({\rm AB}) - (D_{\rm Q}^{\leftarrow}({\rm AB}) + D_{\rm Q}^{\rightarrow}({\rm AB}))/2$), where $a_1 = a_2 = b_1 = b_2 = c_2 = c_3 = 0$, $a_3 = a$, $b_3 = b$, $c_1 = c = 0.8$.

5. Discussion and conclusion

In this paper, we discussed the symmetric quantum discord for an arbitrary two-qubit state and obtained the analysis formula of SQD for an arbitrary two-qubit state. However, it is difficult to give the optimization process of SQD for an arbitrary two-qubit state, so we just considered some special states: (i) $a_j = 0, b_j \neq 0, c_j \neq 0$, (ii) $b_j = 0, a_j \neq 0, c_j \neq 0$, (iii) $a_j = 0, b_j = 0, c_j \neq 0$ (Maziero has considered the case),[13] and (iv) $a_j = b_j = 0$, $j = 1,2$ and $c_k = 0$, $k = 2,3$. Maziero has investigated case (iii). To our knowledge, the other three cases have not been considered. For the four special states, we found the optimization process and gave the classical correlation $I_{\rm m}\left(\Phi^{\rm AB}\right)$ for the symmetric quantum discord. As a result, we obtained $D_{\rm S}({\rm AB}) = D_{\rm Q}^{\leftarrow}({\rm AB})$ for some states in cases (i)–(iii). We found that the symmetric quantum discord is greater than the quantum discord in case (iv). We also observed that the symmetric quantum discord can be unequal to the quantum discord when the right quantum discord is equal to the left quantum discord.

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