Rise of quantum correlations in non-Markovian environments in continuous-variable systems*

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(Received 30 January 2014; revised manuscript received 23 March 2014; published online 15 May 2014)

We investigate the time evolution of quantum correlations, which are measured by Gaussian quantum discord in a continuous-variable bipartite system subject to common and independent non-Markovian environments. Considering an initial two-mode Gaussian symmetric squeezed thermal state, we show that quantum correlations can be created during the non-Markovian evolution, which is different from the Markovian process. Furthermore, we find that the temperature is a key factor during the evolution in non-Markovian environments. For common reservoirs, a maximum creation of quantum correlations may occur under an appropriate temperature. For independent reservoirs, the non-Markovianity of the total system corresponds to the subsystem whose temperature is higher. In both common and independent environments, the Gaussian quantum discord is influenced by the temperature and the photon number of each mode.

Keywords: quantum correlation, non-Markovian environment, continuous-variable systems

PACS: 03.65.Yz, 03.65.Ta, 03.67.Mn **DOI:** 10.1088/1674-1056/23/7/070303

1. Introduction

A fundamental resource in quantum information processing (QIP)[1] is quantum correlations (QCs). In the old days, the research of QCs mostly focused on entanglement. However, with some new discoveries, people found that entanglement is only one kind of QCs.[2–9] Now a widely acceptable concept proposed to measure QCs is quantum discord (QD),[10] which is defined as the difference between the total correlation and the classical correlation in a bipartite state.

Over the past 10 years, studies were focused on QD. Now we know some of its formal characterization and connection with quantum phase transitions and quantum computation.[11–16] The notion of discord is extended to the domain of bipartite quantum continuous-variable (CV) systems, which is called Gaussian quantum discord (GQD).[17,18] The dynamics of open quantum systems are often considered in Markov processes,[19,20] which are described by a master equation of the reduced density matrix with a Lindblad structure[21,22] in infancy. Since non-Markovian environments retain memory, it would be of great interest to study the discord under non-Markovian environments. Therefore, many methods are devoted to define non-Markovianity in open quantum systems[23,24] and to quantify the degree of non-Markovian behavior.[25–27] Some results for the non-Markovian effect on QCs in discrete systems have been reported,[28–35] however, results on QCs in CV systems in non-Markovian processes are still very scarce. In this paper, we analyze the evolution of QCs in a CV system with common and independent non-Markovian environments and obtain some significant conclusions. The paper is structured as follows. In Section 2, we describe a definition of non-Markovianity for CV systems, which will be used in this paper. In Section 3, we review the definition of GQD and its expression to squeezed thermal states (STS). In Section 4, we show the behavior of GQD of a two-mode Gaussian symmetric STS in common and independent non-Markovian reservoirs. Finally, in Section 5, we draw our conclusions.

2. Measure of non-Markovianity in continuous-variable systems

There are several measures proposed for checking the non-Markovianity of an evolution.[23–27] In this paper, we use the method first introduced by Breuer *et al.*[15] and extended by Vasile *et al*. to CV systems.[36] In this measurement, the information flow between the open system and its environment is treated as the key feature of non-Markovianity.

A Markovian process tends to continuously reduce the distinguishability between any two states. On the contrary, non-Markovianity means the increase of the distinguishability. The reason lies in that the loss of distinguishability of states results from a flow of information from the open system to its environment and the gain of distinguishability results from the reverse process. So the measure is based on the trace distance[1] of two quantum states, which describes the probability of successfully distinguishing the states. The trace distance is defined as

$$D(\rho_1,\rho_2)=\frac{1}{2}\mathrm{Tr}\,|\rho_1-\rho_2|, \tag{1}$$

where $|A|=\sqrt{A^{+}A}$. In a Markovian dynamics, the trace dis-

*Project supported by the Fundamental Research Funds for the Central Universities, China (Grant Nos. 2013-Ia-032 and WUT:2014-Ia-026).

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tance always monotonically decreases, namely,

$$D(\rho_1(\tau+t),\rho_2(\tau+t)) \le D(\rho_1(t),\rho_2(t)). \tag{2}$$

When the monotonicity is not satisfied, it means that there are intervals of time in which the states become more distinguishable compared to the previous instants, which characterizes a non-Markovian evolution. For a generic CV state, there is no analytic expression for the trace distance. Alternative distinguishability signatures may be employed within the same spirit, such as the fidelity

$$F(\rho_1,\rho_2) = \mathrm{Tr}\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}, \tag{3}$$

which is a good candidate for taking the role of the trace distance in the definition of the non-Markovian measure. That is, in a CV system, the following formula holds for Markovian processes:

$$F(\rho_1(\tau+t),\rho_2(\tau+t)) \le F(\rho_1(t),\rho_2(t)). \tag{4}$$

When it fails, it defines non-Markovian processes.

3. GQD and its dynamics in a non-Markovian environment

The quantum mutual information in a bipartite quantum system, which measures the total correlation[2] of the state, is defined as

$$I(\rho_{\mathrm{AB}}) = S(\rho_{\mathrm{A}}) + S(\rho_{\mathrm{B}}) - S(\rho_{\mathrm{AB}}), \tag{5}$$

where $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy of an arbitrary state ρ, and $\rho_{\mathrm{A(B)}} = \mathrm{Tr}_{\mathrm{B(A)}}\rho_{\mathrm{AB}}$ is the reduced density operator describing the state of A (B). Another expression for the mutual information that measures the classical correlation of the state[2] defined via a measurement-based conditional density operator is

$$J_{\mathrm{A}}(\rho_{\mathrm{AB}}) = \max_{\{\Pi_i\}}\{S(\rho_{\mathrm{A}}) - \textstyle\sum_i p_i S(\rho_{A|i})\}, \tag{6}$$

where $\{\Pi_i\}$ denotes a complete set of positive operator-valued measure (POVM) performed on the subsystem B, and $\rho_{A|i} = \mathrm{Tr}_{\mathrm{B}}(\rho_{\mathrm{AB}}\Pi_i)/p_i$ is the remaining state of A after the measurement with probability $p_i = \mathrm{Tr}_{\mathrm{AB}}(\rho_{\mathrm{AB}}\Pi_i)$. The difference between the total correlation and the classical correlation is defined as the QD of A[2,10]

$$D_{\mathrm{A}}(\rho_{\mathrm{AB}}) = I(\rho_{\mathrm{AB}}) - J_{\mathrm{A}}(\rho_{\mathrm{AB}}). \tag{7}$$

Analogously, the QD of B is defined with the roles of A and B swapped.

Now a Gaussian version of QD is defined[17,18] in a bipartite CV system. We restrict ρ_{AB} to a two-mode Gaussian state whose covariance matrix can be transformed into a standard form

$$\boldsymbol{\sigma}_{\mathrm{AB}} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{C} \\ \boldsymbol{C}^{\mathrm{T}} & \boldsymbol{B} \end{pmatrix}, \tag{8}$$

where $\boldsymbol{A} = \mathrm{diag}(a,a)$, $\boldsymbol{B} = \mathrm{diag}(b,b)$, and $\boldsymbol{C} = \mathrm{diag}(c,d)$. The quantities $I_1 = \det\boldsymbol{A}$, $I_2 = \det\boldsymbol{B}$, $I_3 = \det\boldsymbol{C}$, and $I_4 = \det\boldsymbol{\sigma}$ are referred to as symplectic invariants as they are invariant under local symplectic transformations. Because $J_{\mathrm{A}}(\rho_{\mathrm{AB}})$ and $D_{\mathrm{A}}(\rho_{\mathrm{AB}})$ are invariant under local unitaries,[2,10] we can extend the QD of A to the standard form of a general two-mode Gaussian state[17,18] based on the minimization of the mismatch $I(\rho_{\mathrm{AB}}) - J_{\mathrm{A}}(\rho_{\mathrm{AB}})$ over the single-mode generalized Gaussian measurements on subsystem B as follows:

$$\begin{aligned} C_{\mathrm{A}}(\rho_{\mathrm{AB}}) &= h(\sqrt{I_2}) - h(\sqrt{d_-}) - h(\sqrt{d_+}) \\ &\quad + \inf_{\boldsymbol{\sigma}_{\mathrm{M}}} h(\sqrt{\det\boldsymbol{\sigma}_{\mathrm{A}}}), \end{aligned} \tag{9}$$

where $h(x) = (x+1/2)\ln(x+1/2) - (x-1/2)\ln(x-1/2)$, $d_\pm$ are the symplectic eigenvalues of ρ_{AB}, $d_\pm^2 = (\Delta \pm \sqrt{\Delta^2 - 4I_4})/2$, with $\Delta = I_1 + I_2 + 2I_3$, σ_{M} is the covariance matrix of single-mode Gaussian states for the Gaussian measurement on subsystem B, and $\boldsymbol{\sigma}_{\mathrm{A}} = \boldsymbol{A} - \boldsymbol{C}(\boldsymbol{B} + \sigma_{\mathrm{M}})^{-1}\boldsymbol{C}^{\mathrm{T}}$ is the covariance matrix of subsystem A after the measurement.[37] In this paper, we will focus on the relevant subclass of two-mode STS with a generic state

$$\rho = V(r)\rho_1 \otimes \rho_2 V^+(r), \tag{10}$$

where $V(r) = \mathrm{e}^{r(a^+b^+ - ab)}$ is the two-mode squeezing operator, and ρ_i $(i=1,2)$ is given by $\rho_i = \sum_n N_i^n(1+N_i)^{-n-1}|n\rangle_i\langle n|$ with N_i being the average number of thermal photons. For the case of STS, we can obtain the diagonal matrix elements defining A, B, and C

$$a = \left(N_r + \frac{1}{2}\right) + N_1(1+N_r) + N_2 N_r, \tag{11}$$

$$b = \left(N_r + \frac{1}{2}\right) + N_2(1+N_r) + N_1 N_r, \tag{12}$$

$$c = -d = (1+N_1+N_2)\sqrt{N_r(1+N_r)}, \tag{13}$$

where $N_r = \sinh^2 r$. The Gaussian QD of A for the generic bipartite STS can then be written as

$$\begin{aligned} C_{\mathrm{A}}(\rho_{\mathrm{AB}}) &= h(\sqrt{I_2}) - h(\sqrt{d_-}) - h(\sqrt{d_+}) \\ &\quad + h\left(\frac{\sqrt{I_1} + 2\sqrt{I_1 I_2} + 2I_3}{1+2\sqrt{I_2}}\right). \end{aligned} \tag{14}$$

By exchanging $I_1 \leftrightarrow I_2$, the Gaussian QD of B is obtained with Gaussian measurements on subsystem A. In the following part, we will use the Gaussian QD of A as the GQD.

In this paper, we consider the evolution of the GQD in both common and independent non-Markovian environments.

For the common environments, the parameters of the covariance matrix evolve as[38]

$$\begin{cases} a(t) = a\mathrm{e}^{-x(t)} + (1-\mathrm{e}^{-x(t)})(M+1/2), \\ b(t) = b\mathrm{e}^{-x(t)} + (1-\mathrm{e}^{-x(t)})(M+1/2), \\ c(t) = c\mathrm{e}^{-x(t)}, \\ d(t) = d\mathrm{e}^{-x(t)}, \end{cases} \tag{15}$$

where $x(t) = \alpha \int_0^t 2\gamma(s)\,\mathrm{d}s$, the damping rate $\gamma(t)$ is determined by the spectral density $J(w)$ of the reservoir, α is a coupling constant, and M is the average number of thermal photons in the reservoir, which features the environment's temperature.

For the independent environments, the parameters of the covariance matrix evolve as[17]

$$\begin{cases} a(t) = a\mathrm{e}^{-x_1(t)} + (1-\mathrm{e}^{-x_1(t)})(M_1+1/2), \\ b(t) = b\mathrm{e}^{-x_2(t)} + (1-\mathrm{e}^{-x_2(t)})(M_2+1/2), \\ c(t) = c\mathrm{e}^{-\frac{1}{2}[x_1(t)+x_2(t)]}, \\ d(t) = d\mathrm{e}^{-\frac{1}{2}[x_1(t)+x_2(t)]}, \end{cases} \tag{16}$$

where $x_1(t)$, $x_2(t)$, M_1, M_2 have the same definitions as those above but belong to the two reservoirs respectively.

It has been proved[25,36] that whenever the damping rate $\gamma(t) < 0$, equation (2) or (4) is not satisfied. Then we can infer that the evolution is non-Markovian. As we mentioned above, $\gamma(t)$ is determined by the spectral density $J(w)$ of the environment. We consider the case of a Lorentzian spectral density with the coupling constant α being much smaller than 1, $\gamma(t)$ then describes an exponentially damped oscillation and takes on negative values within certain intervals of time in the system. If there exists some negativity intervals of $\gamma(t)$, it means that equation (4) fails for a certain time in the physical process, then it is a non-Markovian process. In addition, in an experiment, one can determine whether there is any increase in the fidelity for two states of the open system under study at a different time. If the increase appears, one can conclude that in this time interval, $\gamma(t)$ is negative and the non-Markovian effects are crucial. In this way, one can also gain some knowledge of the nature of the reservoir and the interactions, which may be helpful for controlling QCs in experiments.

4. Evolution of quantum correlations with non-Markovianity

In this section, we study the important features of the GQD in common and independent reservoirs during the non-Markovian evolution which exists in certain time intervals with $\gamma(t) < 0$. We analyze the case of a two-mode symmetric STS in which $N_1 = N_2$ and the spectral density $J(w)$ of the reservoir is of the Lorentzian spectral form. In a common environment with zero temperature ($M = 0$), by setting $N_1 = N_2 = 10$, $r = 2$, and $\alpha = 0.1$, we compare in Fig. 1(a) two different non-Markovian cases whose damping rates $\gamma(t)$ are given by Eqs. (17) and (18), respectively, with the Markovian case whose damping rate $\gamma(t)$ is given by Eq. (19)

$$\gamma(t) = \frac{1}{2} \times \begin{cases} \mathrm{e}^{-t/5}\sin t, & \text{if } t < 5\pi/2, \\ \mathrm{e}^{-\pi/2}, & \text{if } t \geq 5\pi/2, \end{cases} \tag{17}$$

$$\gamma(t) = \frac{1}{2} \times \begin{cases} \mathrm{e}^{-t/5}\sin t, & \text{if } t < 9\pi/2, \\ \mathrm{e}^{-\pi/2}, & \text{if } t \geq 9\pi/2, \end{cases} \tag{18}$$

$$\gamma(t) = \frac{1}{2} \times \begin{cases} \mathrm{e}^{-t/5}\sin t, & \text{if } t < \pi, \\ \mathrm{e}^{-\pi/2}, & \text{if } t \geq \pi. \end{cases} \tag{19}$$

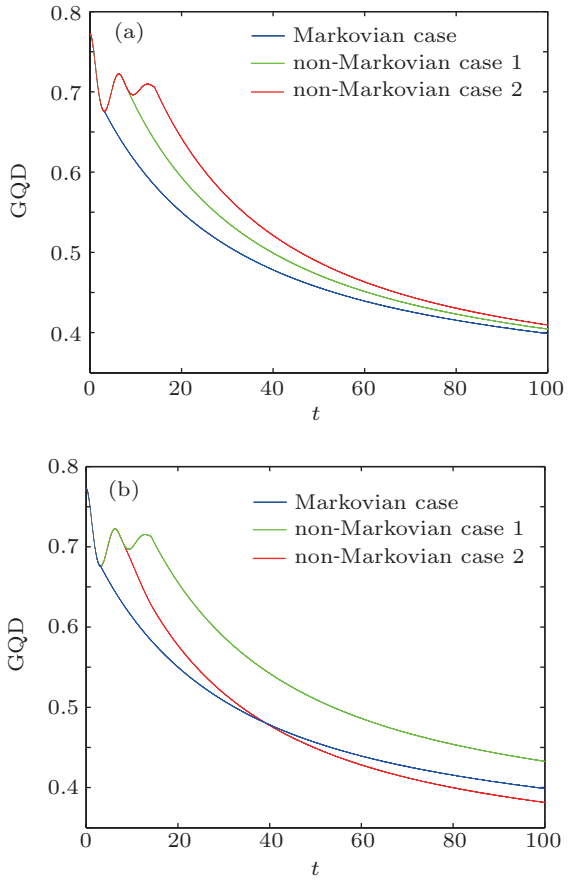


Fig. 1. (color online) Evolution of GQD with $N_1 = N_2 = 10$, $r = 2$, $\alpha = 0.1$ in Markovian and non-Markovian cases: (a) in common environments, and (b) in independent environments.

We can see that the GQD monotonically decreases in the Markovian case as expected, however the situation is very different in non-Markovian cases. Although it decreases as a whole, the GQD has an obvious rise whenever $\gamma(t) < 0$. In other words, when the information flows from the environment to the system, which features non-Markovianity, the QCs are created. In independent environments with zero temperature ($M = 0$), by setting $N_1 = N_2 = 10$, $r = 2$, and $\alpha = 0.1$, we consider the damping rate $\gamma(t)$ of the form (18) for subsystem A and of the form (17) for subsystem B singly in non-Markovian case 1, and swap the damping rates $\gamma(t)$ in non-Markovian case 2. In contrast to the Markovian case in Fig. 1(b), we find a special dynamics feature different from that in the common environment. Between the two different damping rates

belonging to subsystems A and B in the non-Markovian process, the leading role is played by that of system A (B) which is not measured, it means that the time QCs are created corresponds to the non-Markovianity of subsystem A (B). That is to say at zero temperature, the characters of the whole system caused by the non-Markovianity may depend on the environmental property of the subsystem, which is not measured. In this situation, it offers us the ability to perform some operation on one subsystem without considering the influence of the environment. Then we study the influence of the average number of thermal photons N_i $(i = 1,2)$, the result is shown in Fig. 2. We find both in common and independent environments, the GQD has smaller values with larger N_i, including the section created by the information flowing back.

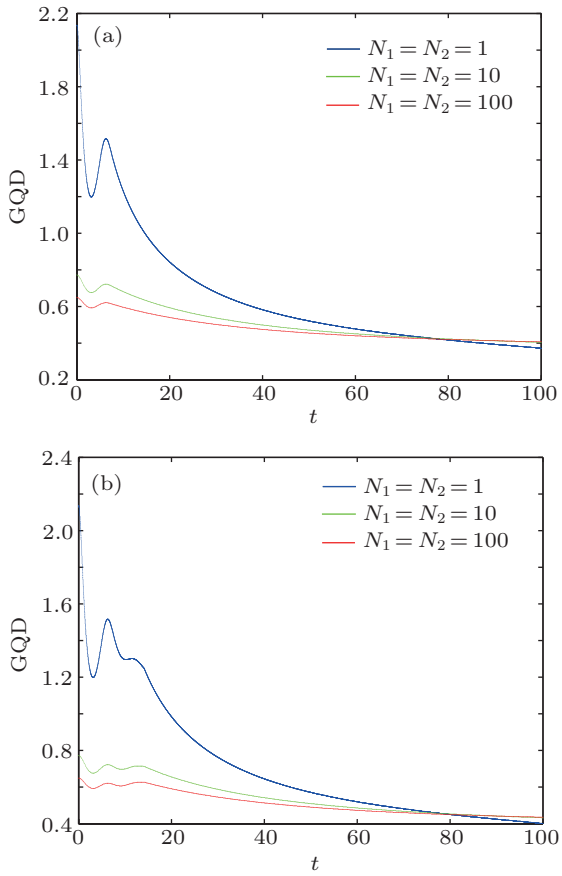


Fig. 2. (color online) Evolution of GQD as a function of N_i: (a) in non-Markovian case 1 mentioned before in common environments, and (b) in non-Markovian case 1 mentioned before in independent environments.

Now we study how the evolution especially the increasing part in non-Markovian processes is affected by the temperature of the reservoir. In Fig. 3(a), we display the dynamic of the GQD in a common environment for growing values of the reservoir average photon number M. Obviously, with the raising temperature, the GQD becomes smaller. Especially, we notice that there exists an optimal temperature which can cause a maximum increment of the GQD. The increased value of the GQD is larger with increasing temperature until a certain temperature, then it begins to be smaller with the temperature rising further. It means that we may create more QCs by controlling the temperature in the common environment. In our situation in Fig. 3(a), the red curve stands for the optimal temperature with $M = 3$. The evolution as a function of temperature in three-dimension is shown in Fig. 3(b). We can see that no matter what the temperature is, the GQD will drop to zero eventually.

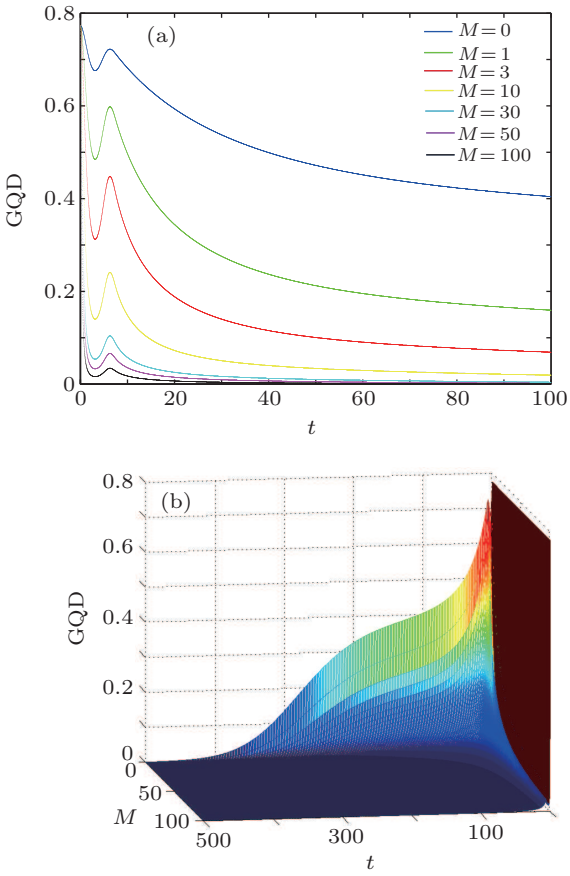


Fig. 3. (color online) The evolution of GQD in non-Markovian case 1 mentioned before in a common environment: (a) in different temperatures, (b) as a function of temperature in three-dimension.

In the independent environments, the GQD shows a fantastic feature caused by the temperature, which can be seen in Fig. 4. In Fig. 4(a), we consider non-Markovian case 2 in the independent environments mentioned above, with the temperature increasing in reservoir A and remaining unchanged in reservoir B. In Fig. 4(b), the situation is the same except the temperature is increasing in reservoir B and remaining unchanged in reservoir A. We find that the temperature is the determining factor in choosing the dominant reservoir of the subsystems. The non-Markovian characteristic of the total system corresponds to the subsystem whose temperature is higher. When the temperatures of the two subsystems are the same, the leading role is played by the subsystem which is not measured. It provides us with an effective method to adjust

the non-Markovianity through the measured subsystem in independent environments in experiments, so we can study the QCs in non-Markovian processes further. The result is confirmed clearly in Figs. 4(c) and 4(d), where we use the black split line to represent the threshold when the two reservoirs have the same temperature.

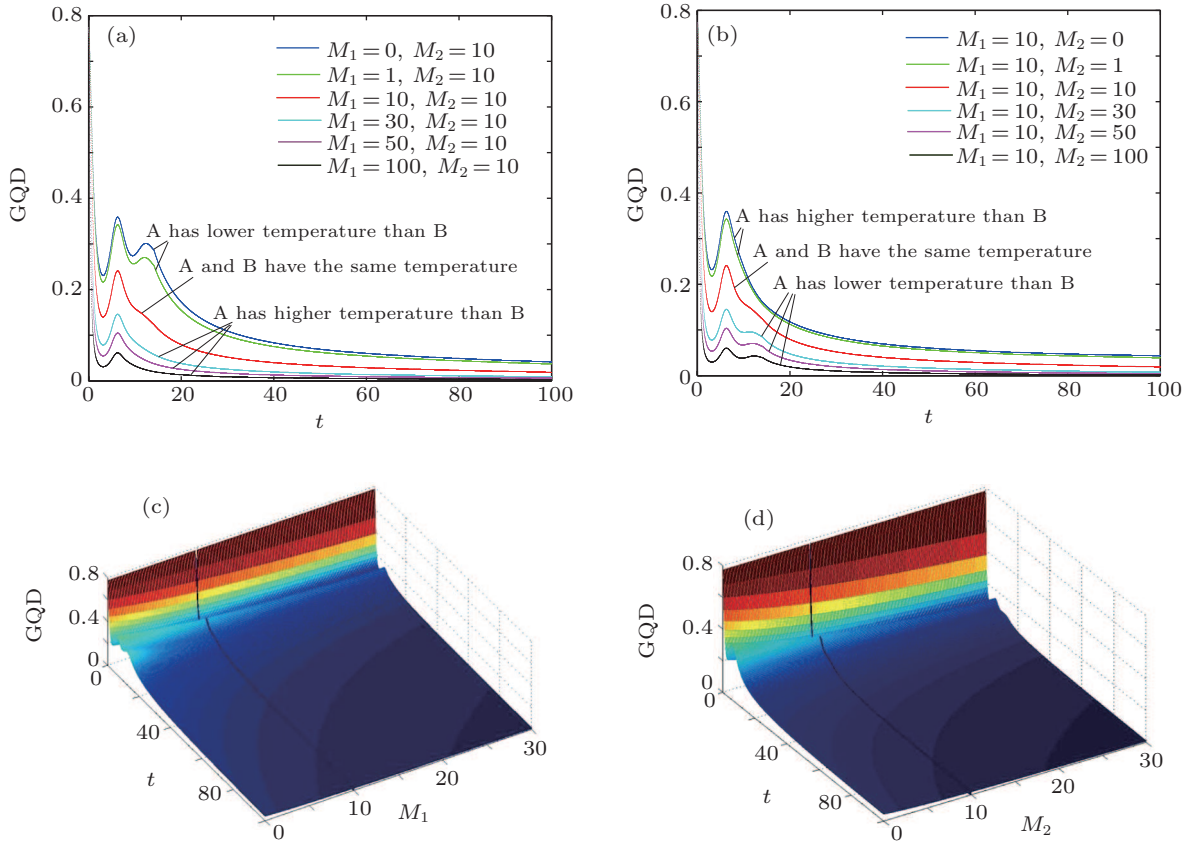


Fig. 4. (color online) The evolution of GQD in non-Markovian case 2 in independent environments: (a) the temperature increasing in reservoir A, (b) the temperature increasing in reservoir B. The GQD as a function of (c) the temperature of reservoir A in three-dimension, and (d) the temperature of reservoir B in three-dimension.

5. Conclusions

In this paper, we study the evolution of the QCs in CV bipartite systems subject to common and independent non-Markovian environments. By comparing the different dynamics of GQD in Markovian and non-Markovian reservoirs, we find that while the behavior of the QCs is monotonically decreasing in the Markovian case, it can give rise to a nonmonotonic behavior in the non-Markovian process. It means that QCs can be created in a non-Markovian evolution. We observe that the GQD is influenced by the photon number of each mode and the temperatures of the environments. Furthermore, we discover that in the common environments, there is an optimal temperature which can cause a maximum creation of QCs, and in the independent environments, the temperature is the key factor deciding which subsystem's reservoir is more important. It offers a new concept in controlling the non-Markovian features through the measured subsystem for practical purposes.

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