Comparison between photon annihilation-then-creation and photon creation-then-annihilation thermal states: Non-classical and non-Gaussian properties*

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We investigate the nonclassical properties of arbitrary number photon annihilation-then-creation operation (AC) and creation-then-annihilation operation (CA) to the thermal state (TS), whose normalization factors are related to the polylogarithm function. Then we compare their quantum characters, such as photon number distribution, average photon number, Mandel Q-parameter, purity and the Wigner function. Because of the noncommutativity between the annihilation operator and the creation operator, the ACTS and the CATS have different nonclassical properties. It is found that nonclassical properties are exhibited more strongly after AC than after CA. In addition we also examine their non-Gaussianity. The result shows that the ACTS can present a slightly bigger non-Gaussianity than the CATS.

Keywords: annihilation-then-creation, creation-then-annihilation, Wigner function, nonclassicality

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1. Introduction

Nonclassical states play an important role in quantum computation and quantum information procession.[1] The nonclassicality of quantum states can be well-described by some nonclassical properties, such as sub-Poissonian photon statistics,[2] squeezing in one of the quadratures of the field,[3] and negativity of the Wigner function (WF).[4] Especially, the partial negativity of the WF implies the highly nonclassical properties of quantum states.

As the basic technique available, non-Gaussian operations, the photon subtraction a and addition $a^\dagger$, are proved to be a valuable resource for many applications in quantum information. These operations enable us to generate highly nonclassical quantum states,[5–8] including the photon-addition coherent state (thermal state),[9–11] the photon-subtraction (addition) squeezed state,[12] and so on. In addition, photon subtraction or addition can be applied to improve entanglement between Gaussian states. Furthermore, the entanglement distillation can also be achieved by combining these elementary transformations, such as $a^\dagger a$, $aa^\dagger$,[13,14] and coherent superposition of photon subtraction and addition, i.e., $ta^\dagger + ra$.[15,16] For instance, the $ta^\dagger + ra$ operator is proposed for quantum state engineering, which can transform a classical state to a nonclassical one.[16] These local coherent operations (superposition) at the level of quantum operation can provide a useful tool in quantum information processing.

On the other hand, it appears that the non-Gaussianity of a quantum state is a useful feature in quantum information processing, regardless of being non-Classical or classical according to the concepts of quantum optics.[17] To quantify this property as a resource, some non-Gaussianity measures were recently defined as distances between ρ and its associate Gaussian state τ_G. Here, τ_G is the unique Gaussian state having the same mean photon and covariance matrix as ρ. Different measures of non-Gaussianity for quantum states have been proposed.[18–20] As we all know, thermal state (TS) is a traditional classical and Gaussian state. In this work, by repeatedly operating $(a^\dagger a)$ and $(aa^\dagger)$ on the TS, we introduce two kinds of quantum states (ACTS and CATS), which present many properties that are different from the TS. Here we mainly investigate their nonclassical and non-Gaussian properties.

The rest of the paper is structured as follows. In Section 2, we introduce the ACTS and the CATS. We explicitly derive their normalization factors, which are all related to polylogarithm functions of the thermal parameter $\bar{n}$. In Section 3, we discuss their quantum characters in terms of photon number distribution (PND), average photon number, Mandel Q-parameter, and the purity. In Section 4, we give the analytical expressions of their WFs and discuss their nonclassicality. Their non-Gaussianities are considered in Section 5. The results are summarized in Section 6.

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2. ACTS and CATS

The TS is the equilibrium state for a field coupled to a reservoir at finite temperature.[21] This state is commonly discussed in statistical and thermal physics courses because it is used to derive the Planck radiation law. The density operator $\rho_{\mathrm{T}}(\bar{n})$ of the TS in Fock representation is

$$\rho_{\mathrm{T}}(\bar{n}) = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} |n\rangle\langle n|, \tag{1}$$

whose average photon number is $\bar{n}$. Alternately, the P-representation of density operator $\rho_{\mathrm{T}}(\bar{n})$ can be expressed as

$$\rho_{\mathrm{T}}(\bar{n}) = \frac{1}{\bar{n}} \int \frac{\mathrm{d}^2\alpha}{\pi} \mathrm{e}^{-|\alpha|^2/\bar{n}} |\alpha\rangle\langle\alpha|, \tag{2}$$

where $|\alpha\rangle$ is the coherent state.

Here we introduce two kinds of quantum states. Theoretically, by repeatedly operating $a^\dagger a$ on a TS, the ACTS can be obtained

$$\rho_m^{\mathrm{AC}}(\bar{n}) = N_{\mathrm{AC}}^{-1} \left(a^\dagger a\right)^m \rho_{\mathrm{T}}(\bar{n}) \left(a^\dagger a\right)^m. \tag{3}$$

Similarly, by repeatedly operating $aa^\dagger$ on a TS, one can generate another non-Gaussian state, i.e., the CATS

$$\rho_m^{\mathrm{CA}}(\bar{n}) = N_{\mathrm{CA}}^{-1} \left(aa^\dagger\right)^m \rho_{\mathrm{T}}(\bar{n}) \left(aa^\dagger\right)^m, \tag{4}$$

where m is a positive integer, and N_{AC} (N_{CA}) is the normalization constant determined by $\mathrm{Tr}\left[\rho_m^{\mathrm{AC}}(\bar{n})\right] = 1$ $\left(\mathrm{Tr}\left[\rho_m^{\mathrm{CA}}(\bar{n})\right] = 1\right)$.

To fully describe a quantum state, its normalization is usually necessary. For $m \neq 0$, we have

$$N_{\mathrm{AC}} = \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} n^{2m} = \frac{1}{\bar{n}+1} \mathrm{Li}_{-2m}(\Lambda), \tag{5}$$

$$N_{\mathrm{CA}} = \frac{1}{\bar{n}} \mathrm{Li}_{-2m}(\Lambda), \tag{6}$$

where $\mathrm{Li}_n(x) = \sum_{k=1}^{\infty}(x^k/k^n)\,(n \neq 0)$ is the polylogarithm function[22] and $\Lambda = \bar{n}/(\bar{n}+1)$. The normalization factors of the ACTS and the CATS are all polylogarithm functions. In particular, when $m = 0$ corresponding to the TS, $N_{\mathrm{AC}} = N_{\mathrm{CA}} = 1$ as expected.

3. Some quantum characters of the ACTS and the CATS

In this section, we investigate and compare some quantum characters of the ACTS and the CATS, such as PND, average photon number, Mandel Q-parameter, and purity.

3.1. Photon number distribution

The PND is a key characteristic of every optical field. All interesting states of the fields are constructed as a combination of Fock states, and different combinations have different quantum properties. The PNDs, i.e., the probability of finding n photons in ρ, is defined as $p_n = \langle n|\rho|n\rangle$. Obviously the PNDs of the ACTS and the CATS are respectively

$$p_n^{\mathrm{AC}} = \frac{N_{\mathrm{AC}}^{-1} \bar{n}^n n^{2m}}{(\bar{n}+1)^{n+1}}, \tag{7}$$

$$p_n^{\mathrm{CA}} = \frac{N_{\mathrm{CA}}^{-1} \bar{n}^n (n+1)^{2m}}{(\bar{n}+1)^{n+1}}. \tag{8}$$

Using Eqs. (7) and (8), we plot the PNDs of the ACTS and the CATS in the same bar graph in Fig. 1 for different m and $\bar{n}$. Obviously, the PNDs of the ACTS and the CATS lose the distribution character of the TS $p_n^{\mathrm{T}} = \bar{n}^n/(\bar{n}+1)^{n+1}$.

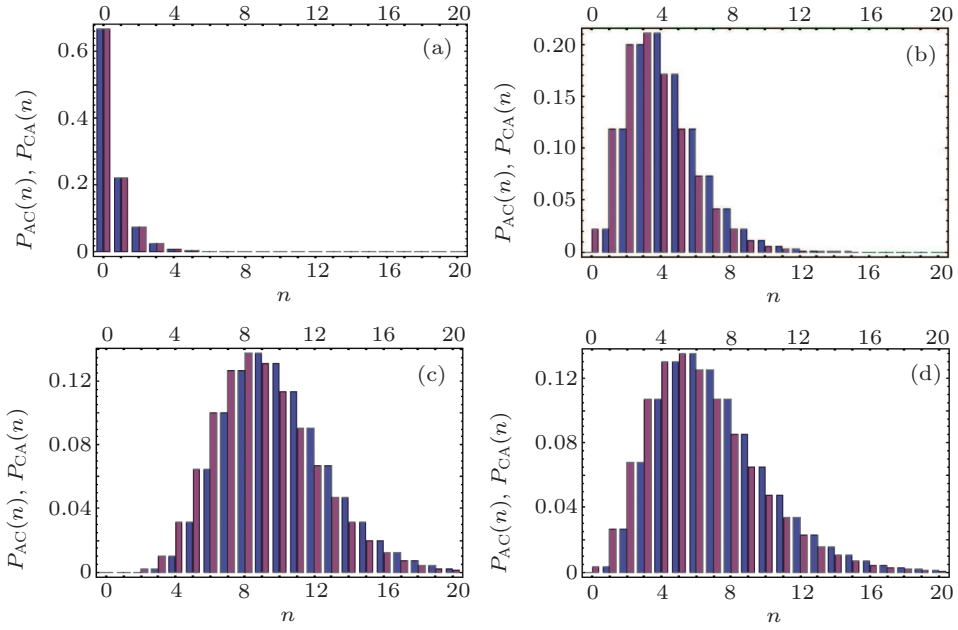


Fig. 1. (color online) Photon number distribution of the ACTS (left bar) and the CATS (right bar) in the same graph for different cases. (a) $m = 0$, $\bar{n} = 0.5$; (b) $m = 2$, $\bar{n} = 0.5$; (c) $m = 5$, $\bar{n} = 0.5$; (d) $m = 2$, $\bar{n} = 1$.

3.2. Average photon number

The average photon number for the quantum state ρ is defined as $\langle n\rangle = \mathrm{Tr}\left[\left(a^{\dagger}a\right)\rho\right]$, then we know

$$\langle n\rangle_{\mathrm{AC}} = \frac{\mathrm{Li}_{-2m-1}(\Lambda)}{\mathrm{Li}_{-2m}(\Lambda)}. \tag{9}$$

Similarly we know

$$\langle n\rangle_{\mathrm{CA}} = \frac{\mathrm{Li}_{-2m-1}(\Lambda)}{\mathrm{Li}_{-2m}(\Lambda)} - 1. \tag{10}$$

From above, we readily know that $\langle n\rangle_{\mathrm{AC}} = \langle n\rangle_{\mathrm{CA}} + 1$. That is to say, the mean photon of ACTS is always larger by exactly one than that of CATS regardless of m.

3.3. Mandel Q-parameter

In the early days, Mandel introduced the so-called Mandel Q-factor[23] to describe the violation from the PND of the state of Poissonian statistics. To see clearly the photon statistical properties of the quantum optical field, we examine Mandel's Q-parameter defined by

$$Q = \frac{\langle a^{\dagger 2}a^2\rangle}{\langle a^{\dagger}a\rangle} - \langle a^{\dagger}a\rangle, \tag{11}$$

which measures the deviation of the photon number statistics from the Poissonian distribution, characteristic of coherent states. If $Q = 0$, the radiation field has Poissonian photon statistics, while if $Q > 0$ ($Q < 0$) we say that the radiation field has super- (sub-) Poissonian photon statistics.

Noticing that $a^{\dagger 2}a^2 = \left(a^{\dagger}a\right)^2 - a^{\dagger}a = \left(aa^{\dagger}\right)^2 - 3aa^{\dagger} + 2$, we readily know $Q_{\mathrm{T}} = \bar{n}$ for the TS and

$$Q_{\mathrm{AC}} = \frac{\mathrm{Li}_{-2m-2}(\Lambda)}{\mathrm{Li}_{-2m-1}(\Lambda)} - \frac{\mathrm{Li}_{-2m-1}(\Lambda)}{\mathrm{Li}_{-2m}(\Lambda)} - 1, \tag{12}$$

as well as

$$Q_{\mathrm{CA}} = \frac{\mathrm{Li}_{-2m-2}(\Lambda) - \mathrm{Li}_{-2m-1}(\Lambda)}{\mathrm{Li}_{-2m-1}(\Lambda) - \mathrm{Li}_{-2m}(\Lambda)} - \frac{\mathrm{Li}_{-2m-1}(\Lambda)}{\mathrm{Li}_{-2m}(\Lambda)} - 1, \tag{13}$$

for the ACTS and the CATS respectively.

Using Eqs. (12) and (13), the variation of the Mandel Q-parameter for ACTS for CATS with $\bar{n}$ for different number m are shown in Fig. 2. Figure 2(a) shows that for ACTS with different m, the value of Q_{AC} monotonically increases with $\bar{n}$ and becomes positive near the threshold value of $\bar{n} \approx 0.57$. For the CATS, the variation of the Mandel Q-parameter with $\bar{n}$ is shown in Fig. 2(b), which is different from the case of the ACTS. For small $\bar{n}$, the Mandel Q-parameter has a negative value, which implies sub-Poissonian statistics. On the other hand, when $\bar{n}$ is large enough, Q is always positive for any m.

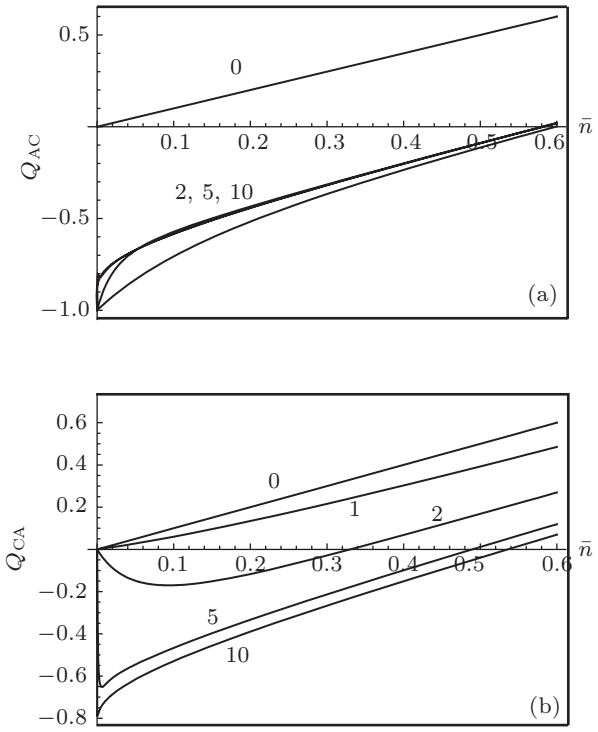


Fig. 2. Mandel Q-parameter of the ACTS (a) and the CATS (b) with $m = 0, 1, 2, 5, 10$.

3.4. Purity

The purity is fundamental in distinguishing a pure state from a mixed one and determining the degree of mixing of the quantum state ρ. Thus we define $\mu(\rho) \equiv \mathrm{Tr}[\rho^2]$ as purity. For continuous variable systems, one has $0 < \mu \leqslant 1$. Obviously, the purity of the TS is $\mu_{\mathrm{T}} = (2\bar{n}+1)^{-1}$. According to the definition, we also have

$$\mu_{\mathrm{AC}} = \mu_{\mathrm{CA}} = \frac{\mathrm{Li}_{-4m}\left(\Lambda^2\right)}{\left[\mathrm{Li}_{-2m}(\Lambda)\right]^2}. \tag{14}$$

Equation (14) shows that the ACTS and the CATS with the same number m have the same purity. As shown in Fig. 3, the purity monotonically decreases with $\bar{n}$. In addition, the bigger the m, the smaller the purity.

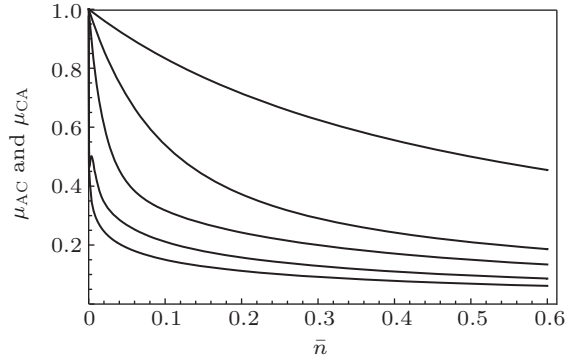


Fig. 3. Purity of the ACTS and the CATS with different $m = 0, 1, 2, 5, 10$.

4. Wigner function of the ACTS and the CATS

The WF, first introduced by Wigner in 1932,[24] has now become a very popular tool to study the nonclassical properties of quantum states. The presence of the negativity of the WF is a signature of its nonclassicality.[25] The WF of ρ given in the coherent representation $|\beta\rangle$ is expressed as[26,27]

$$W(z)=\frac{2\mathrm{e}^{2|z|^2}}{\pi}\int\frac{\mathrm{d}^2\beta}{\pi}\langle-\beta|\rho|\beta\rangle\,\mathrm{e}^{-2(\beta z^*-\beta^* z)}, \tag{15}$$

where $z=(q+\mathrm{i}p)/\sqrt{2}$. It is easy to obtain the WF for $\rho_{\mathrm{T}}(\bar{n})$ as follows:

$$W_{\mathrm{T}}(z)=\frac{2}{\pi(2\bar{n}+1)}\exp\left(-\frac{2|z|^2}{2\bar{n}+1}\right). \tag{16}$$

The WF in Eq. (16) is Gaussian and positive in the phase space, which indicates that $\rho_{\mathrm{T}}(\bar{n})$ is a Gaussian state.

Employing operations $a^\dagger a$ and $a^\dagger a$ repeatedly on the TS, we may obtain the ACTS and the CATS, two kinds of non-Gaussian states, which can also be characterized by the WF in phase space. Next we derive the WFs of the ACTS and the CATS in Fock representation and P-representation of the TS.

4.1. Obtained by Fock representation

Using the definitions of the ACTS and the CATS in Fock representation and Eq. (15), the WFs for the ACTS and the CATS can be expressed as

$$W^{\mathrm{AC}}(z)=\sum_{n=0}^{\infty}p_n^{\mathrm{AC}}W_{|n\rangle\langle n|}(z), \tag{17}$$

$$W^{\mathrm{CA}}(z)=\sum_{n=0}^{\infty}p_n^{\mathrm{CA}}W_{|n\rangle\langle n|}(z), \tag{18}$$

respectively. Here $W_{|n\rangle\langle n|}(z)$ is the WF of the Fock state $|n\rangle$

$$W_{|n\rangle\langle n|}(z)=\frac{2(-1)^n}{\pi}\mathrm{e}^{-2|z|^2}L_n(4|z|^2), \tag{19}$$

where $L_n(x)$ is the n-order of the Laguerre function. The expressions in Eqs. (17) and (18) include the forms of the infinity summation.

4.2. Obtained by P-representation

Using the P-representation of the TS and noticing that

$$\begin{aligned}\left(a^\dagger a\right)^m|\alpha\rangle&=\mathrm{e}^{-|\alpha|^2/2}\partial_s^m\,\mathrm{e}^{sa^\dagger a}\,\mathrm{e}^{\alpha a^\dagger}|0\rangle|_{s=0}\\&=\mathrm{e}^{-|\alpha|^2/2}\partial_s^m\,\mathrm{e}^{\alpha\mathrm{e}^s a^\dagger}|0\rangle|_{s=0},\end{aligned} \tag{20}$$

$$\langle\alpha|\left(a^\dagger a\right)^m=\mathrm{e}^{-|\alpha|^2/2}\partial_\tau^m\langle 0|\,\mathrm{e}^{\alpha^*\mathrm{e}^\tau a}|_{\tau=0}, \tag{21}$$

where we have used $a|0\rangle=0$ and Baker–Hausdoff theorem,[28] then we have

$$\begin{aligned}\rho_m^{\mathrm{AC}}(\bar{n})&=\frac{N_{\mathrm{AC}}^{-1}}{\bar{n}}\partial_s^m\partial_\tau^m\int\frac{\mathrm{d}^2\alpha}{\pi}\,\mathrm{e}^{-\Lambda^{-1}|\alpha|^2}\\&\quad\times\mathrm{e}^{\alpha\mathrm{e}^s a^\dagger}|0\rangle\langle 0|\,\mathrm{e}^{\alpha^*\mathrm{e}^\tau a}|_{s=\tau=0},\end{aligned} \tag{22}$$

$$\begin{aligned}\rho_m^{\mathrm{CA}}(\bar{n})&=\frac{N_{\mathrm{CA}}^{-1}}{\bar{n}}\partial_s^m\partial_\tau^m\int\frac{\mathrm{d}^2\alpha}{\pi}\,\mathrm{e}^{-\Lambda^{-1}|\alpha|^2+s+\tau}\\&\quad\times\mathrm{e}^{\alpha\mathrm{e}^s a^\dagger}|0\rangle\langle 0|\,\mathrm{e}^{\alpha^*\mathrm{e}^\tau a}|_{s=\tau=0}.\end{aligned} \tag{23}$$

Substituting Eqs. (22) and (23) into Eq. (15), we have the WF of the ACTS

$$\begin{aligned}W_{\mathrm{AC}}(z)&=\frac{2N_{\mathrm{AC}}^{-1}}{\bar{n}\pi}\partial_s^m\partial_\tau^m\left[\frac{1}{\Lambda^{-1}+\mathrm{e}^{s+\tau}}\right.\\&\quad\left.\times\exp\left(2\frac{\Lambda\,\mathrm{e}^{s+\tau}-1}{\Lambda\,\mathrm{e}^{s+\tau}+1}|z|^2\right)\right]|_{s=\tau=0},\end{aligned} \tag{24}$$

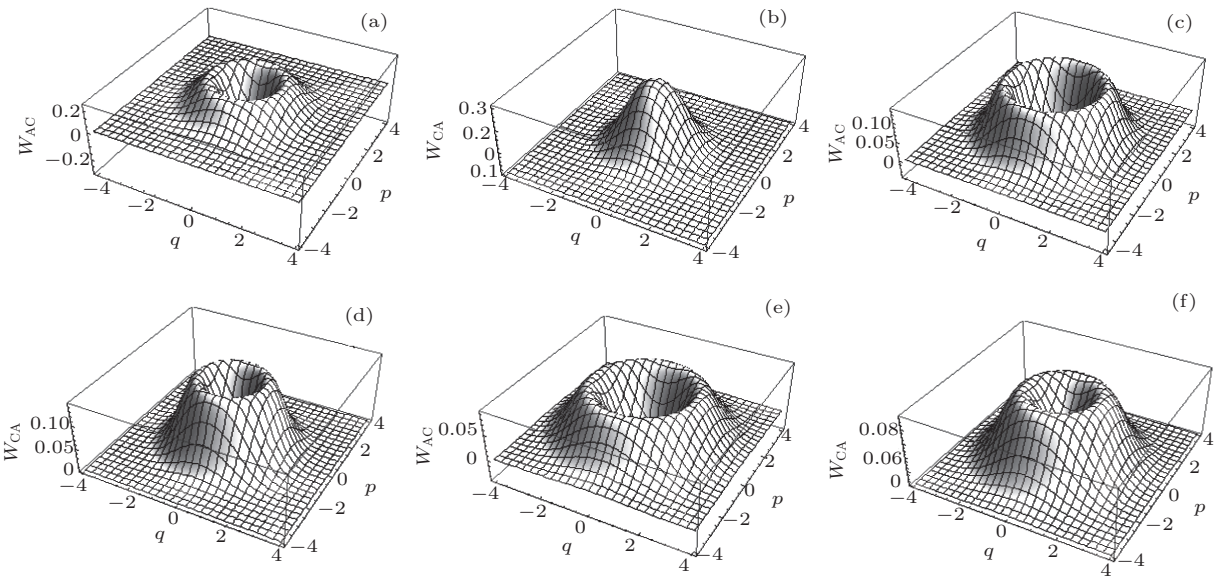


Fig. 4. Wigner functions of the ACTS and the CATS with cases (a) $m=1$, $\bar{n}=0.1$, ACTS; (b) $m=1$, $\bar{n}=0.1$, CATS; (c) $m=2$, $\bar{n}=0.1$, ACTS ; (d) $m=2$, $\bar{n}=0.1$, CATS; (e) $m=1$, $\bar{n}=0.5$, ACTS; (f) $m=1$, $\bar{n}=0.5$, CATS.

and the WF of the CATS

$$W_{\mathrm{CA}}(z) = \frac{2N_{\mathrm{CA}}^{-1}}{\bar{n}\pi}\partial_s^m\partial_\tau^m\left[\frac{\mathrm{e}^{s+\tau}}{\Lambda^{-1}+\mathrm{e}^{s+\tau}} \times \exp\left(2\frac{\Lambda\,\mathrm{e}^{s+\tau}-1}{\Lambda\,\mathrm{e}^{s+\tau}+1}|z|^2\right)\right]|_{s=\tau=0}, \quad (25)$$

respectively. In particular, when $m=0$ corresponding to the TS, equations (24) and (25) are reduced to $W_{\mathrm{T}}(z)$ as expected.

As is well known, when $m=0$, the TS is Gaussian, which can be judged from the Gaussian form of its WF in Eq. (16). Using Eqs. (24) and (25), the WFs of the ACTS and the CATS are depicted in Fig. 4 for different values m and $\bar{n}$. There are some partially negative regions of the WFs for ACTS, which is evidence of the nonclassicality of the ACTS. While for CATS, although there are no negative regions, the WF presents the characteristics of the non-Gaussian. Therefore, the result shows that the nonclassical states generated by the AC operation can present more clear nonclassicality than those generated by the CA operation.

5. Non-Gaussianity of the ACTS and the CATS

From the above section, we know that the ACTS and the CATS are all non-Gaussian states. It is just the AC and CA operations that lead to losing the Gaussian character of the TS. In order to compare the non-Gaussianity properties for the ACTS and the CATS, in this section, we use the Hilbert–Schmidt degree of non-Gaussianity[18–20].

Here we just recall the degree of non-Gaussianity written for a Fock-diagonal state ρ

$$\rho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n|, \quad (26)$$

with $\sum_{n=0}^{\infty} p_n = 1$. In this case, the associate Gaussian state is a thermal one with the same average photon number occupancy $\langle n\rangle = \sum_{n=0}^{\infty} n p_n$,

$$\tau_{\mathrm{G}} = \sum_{n=0}^{\infty} s_n |n\rangle\langle n|, \quad (27)$$

with $s_n = \langle n\rangle^n/[(\langle n\rangle+1)^{n+1}]$. The Hilbert–Schmidt degree of non-Gaussianity is defined as

$$\delta_{\mathrm{HS}}[\rho] = \frac{1}{2}\left[1+\frac{\sum_n\left(s_n^2-2s_np_n\right)}{\sum_n p_n^2}\right] = \frac{1}{2}+\frac{1}{2\mathrm{Tr}\left(\rho^2\right)}\left[\frac{1}{2\langle n\rangle+1} - \frac{2}{\langle n\rangle+1}\sum_n p_n\frac{\langle n\rangle^n}{(\langle n\rangle+1)^n}\right]. \quad (28)$$

Substituting the respective average photon number, PND and purity for the ACTS and the CATS into Eq. (28), we obtain the analytical expressions of $\delta_{\mathrm{HS}}^{\mathrm{AC}}$ and $\delta_{\mathrm{HS}}^{\mathrm{CA}}$ and analyze them numerically. Plots of the Hilbert–Schmidt measure $\delta_{HS}[\rho]$ versus the number m of the ACTS and the CATS for different average photon numbers $\bar{n}$ are shown in Fig. 5. As one can seen from Fig. 5, the non-Gaussianity of the ACTS and the CATS increase with the number of operations m. In addition, for the same values m and $\bar{n}$, the value of the non-Gaussianity of the ACTS is slightly bigger than that of the CATS. Therefore, the result shows that the ACTS can present a slightly bigger non-Gaussianity than the CATS.

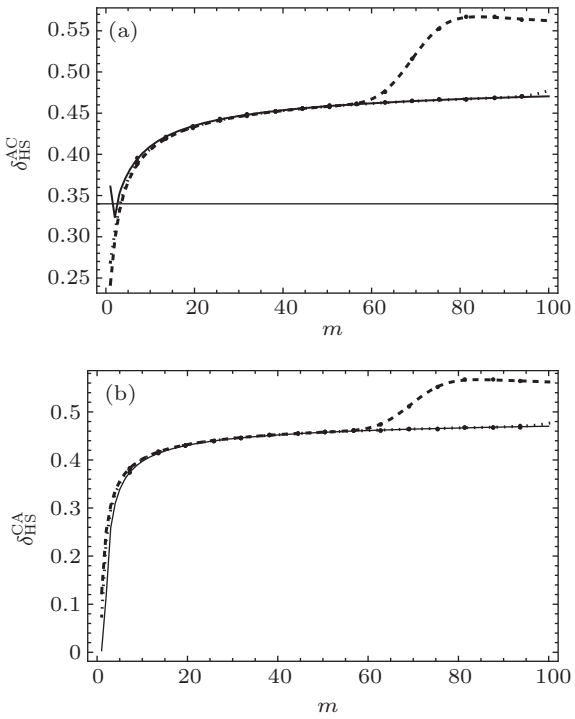


Fig. 5. HS non-Gaussianity of the ACTS (a) and the CATS (b) with different $\bar{n}=0.1$ (solid), $\bar{n}=0.5$ (dotted), and $\bar{n}=1$ (dashed).

6. Conclusion

In this work, we introduce two kinds of new quantum states (the ACTS and the CATS) by repeatedly operating AC and CA operation on the TS (mixed and classical) and then investigate their nonclassicality and non-Gaussianity. The analytical expression of the normalization factor is derived, which turns out to be a polylogarithm function, which is a compact expression. Furthermore, some quantum characters of the ACTS and the CATS are discussed according to the analytical expressions of the PND, average photon number, Mandel Q-parameter, and purity. In addition, we derive the WFs of the ACTS and the CATS and compare their nonclassicality by the negativity of the WFs. Additionally, we compare their non-Gaussianity according to the criterion of the Hilbert–Schmidt degree. It is shown that nonclassicality and non-Gaussianity are more strong after AC than after CA. Finally, it is worth mentioning that Sun *et al.*[29] presented an experimental scheme to realize AC and CA using a cavity system.

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