Co-evolution of the brand effect and competitiveness in evolving networks*

Guo Jin-Li(郭进利)†

Business School, University of Shanghai for Science and Technology, Shanghai 200093, *China*

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The principle that 'the brand effect is attractive' underlies the preferential attachment. Here we show that the brand effect is just one dimension of attractiveness. Another dimension is competitiveness. We firstly introduce a general framework that allows us to investigate the competitive aspect of real networks, instead of simply preferring popular nodes. Our model accurately describes the evolution of social and technological networks. The phenomenon that more competitive nodes become richer can help us to understand the evolution of many competitive systems in nature and society. In general, the paper provides an explicit analytical expression of degree distributions of the network. In particular, the model yields a nontrivial time evolution of nodes' properties and the scale-free behavior with exponents depending on the microscopic parameters characterizing the competition rules. Secondly, through theoretical analyses and numerical simulations, we reveal that our model has not only the universality for the homogeneous weighted network, but also the character for the heterogeneous weighted network. Thirdly, we also develop a model based on the profit-driven mechanism. It can better describe the observed phenomenon in enterprise cooperation networks. We show that the standard preferential attachment, the growing random graph, the initial attractiveness model, the fitness model, and weighted networks can all be seen as degenerate cases of our model.

Keywords: complex network, weighted network, scale-free network, competitive network

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1. Introduction

In the past decade, sociologists, physicists, and computer scientists have empirically studied networks in such diverse areas as the World Wide Web (WWW), email networks, social networks, citation networks of academic publications, router networks, etc. It is remarkable that Watts *et al.* proposed the WS model and Barabási *et al.* proposed an evolving model named the BA model.[1] Their research began a new era in the study of complex networks. So far, complex networks has been a subject attracting increasing interest in the scientific community.[1–14] However, numerous examples convincingly indicate that in real systems, a node's connectivity and growth rate do not depend on its age alone. Therefore, the BA model has been improved by many scientists in order to describe real networks better. Bianconi and Barabási have proposed a fitness model in view of the competition phenomenon in the evolving process.[6] In addition, Dorogovtsev *et al.* investigated an initial attractiveness model of directed networks.[7] In the model of Dorogovtsev *et al.*, the attractiveness is a constant. However, different nodes in real networks usually have different attractiveness. For instance, some webpages on the WWW may attract considerably more links than others.

The principle that 'the brand effect is attractive' underlies the preferential attachment.[1,12] The simplest proxy to the brand effect is a node's connectivity. If new connections are made preferentially to more popular nodes, then the degree distribution of nodes follows a power law.[12] However, the brand effect is just one ingredient of attractiveness; another ingredient is the competitiveness, both the initial attractiveness model and the Barrat–Barthelemy–Vespignani (BBV) model[15,16] have difficulty describing the mechanism of competition networks. Some documents on the WWW through a combination of good content and marketing acquire a large number of links in a very short time, easily overtaking older websites. In a networked community where users establish links to others, indicating their "trust" for the link receiver's opinion, over time, reviewers in the online review community may receive new incoming trust links and also contribute new reviews, both of which increase their competitiveness to other members of the community.[4] This leads to the formation of a competitive network, with high in-degree individuals being the opinion leaders. If two countries have diplomatic relations, an edge between them can be established, which forms a national network of relationships. However, this network cannot reflect well the competition of the economic and the military strengths of the two countries. The comprehensive national strength reflects a country's competitiveness. In the evolution of the national network, degrees of nodes are considered together with the comprehensive national strengths, based on which a competitive network is formed. How can the evolution

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†Corresponding author. E-mail: phd5816@163.com

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mechanism of this type of competitive network be described?

In the paper, we try to combine the fitness of nodes with their competitiveness, such as the social skills of an individual, the content of a web page, or the content of a scientific article. We develop one simple model that allows us to investigate the competitive effect of the evolution of a real network. It is found through theoretical analyses and numerical simulations that the degree distribution of the competitive model is universal for weighted networks. Although there have been a large number of models in complex networks, the problem concerning which kind of networks has a broader universality has not been discussed in the study of complex networks. This is an important problem. This study will show that the standard preferential attachment,[1] the growing random graph,[1] the initial attractiveness model, the fitness model, and the weighted network can all be seen as degenerate cases of our model.

The rest of this paper is organized as follows. A model with the co-evolution of the brand effect and the competitiveness is firstly proposed to estimate the competitive aspect of real networks in quantitative terms. Assuming that the existence of the competitiveness modifies the preferential attachment to compete for links, we find that the time dependence of a node's connectivity depends on the competitiveness of the node. An analytical expression for the connectivity distribution of the model is derived by motivating an integral equation. The theoretical results are verified by computer simulations. In Section 3, it is analytically deduced that the homogeneous coupling weighted network is not only a special case of the competitive network,[3] but the heterogeneous coupling weighted network is also a special case of that. The theoretical results are verified through numerical simulations. In Section 4, we develop a model based on the profit-driven mechanism. It describes that the income of an enterprise cooperation network follows Pareto's distribution, which is claimed to appear in several different aspects relevant to entrepreneurs and business managers. Finally, a conclusion is given in Section 5.

2. Competitive networks with the brand effect and the competitiveness

In the last few years, a property frequently identified in complex networks was the scale-free property.[4] A network is said to be scale-free if its degree distribution follows a power law at least asymptotically.[1] The most widely accepted network growth phenomenon that produces a scale-free network is the preferential attachment.[1] Interestingly, Yuan *et al.* found that the Sina microblog network is also a scale-free network.[5] Lu *et al.* found that whereas network structure-based factors such as preferential attachment and reciprocity are significant drivers of the network growth, intrinsic node characteristics such as the number of reviews and textual characteristics such as objectivity, readability, and comprehensiveness of reviews are also significant drivers of the network growth.[4]

Some individuals in social networks acquire more social links than others, or on the WWW, some webpages attract considerably more links than others. The rate at which nodes in a network increase their connectivity depends on their competitiveness to compete for links. Although our model considers a network that grows through the addition of new nodes such as the creation of new webpages or the emergence of new companies, a node in our model has its own competitiveness. The evolution of the network is not only related to the degree of the node, but also related to the competitiveness of that node.

Motivated by the above arguments, our model combines the brand effect with the competitiveness. To incorporate the competitiveness of nodes, we assign a competitiveness parameter ξ to each node, chosen from a distribution $G(x)$, accounting for the comprehensive strengths of companies. In other words, the competitive model is simply constructed as follows.

(i) **Random growth** The network starts from an initial one with m_0 nodes. Suppose that nodes arrive at the system in accordance with a Poisson process having rate λ. Each node entering the network is tagged with its own competitiveness ξ_i and fitness y_i, where ξ_i and y_i are taken from given distributions $G(x)$ and $F(y)$, respectively. When a new node is added to the system at time t, this new site is connected to m $(m \leq m_0)$ previously existing vertices. Here $c = \int x \mathrm{d}G(x)$ and $E[y] = \int x \mathrm{d}F(x)$ are finite.

(ii) **Preferential attachment** We assume that the probability that a new node will connect to node i already present in the network depends on the connectivity $k_i(t)$ and the competitiveness ξ_i of that node, such that

$$\Pi(k_i(t)) = \frac{(by_i + d)k_i(t) + a\xi_i}{\sum_i ((by_i + d)k_i(t) + a\xi_i)}, \tag{1}$$

where $b, d \geq 0$, t_i denotes the time at which the i-th node is added to the system, and $k_i(t)$ denotes the degree of the i-th node at time t. We call the above model model$_1$.

If $b = 0$, $d = 1$, and $a = 1$, the mode degenerates to the model in Ref. [3]. If $b = d = 0$, $a \neq 0$, and the probability density function $G(x)$ is $\delta(x-1)$, i.e., the competitiveness of all nodes equals to 1, the mode degenerates to a growing random graph. Without loss of generality, we assume that $b + d \neq 0$. If $k_i(t)$ is a continuous real variable, the rate at which $k_i(t)$ changes is expected to be proportional to degree $k_i(t)$. Consequently, $k_i(t)$ satisfies the dynamical equation

$$\frac{\partial k_i(t)}{\partial t} = m\lambda \frac{(by_i + d)k_i(t) + a\xi_i}{\sum_i ((by_i + d)k_i(t) + a\xi_i)}. \tag{2}$$

Let

$$B=\lim_{t\to\infty}\frac{b}{\lambda t}\sum_i y_i k_i(t)+2md+ac. \tag{3}$$

Assume that $N(t)$ is the total number of nodes that occur by time t. By the Poisson process theory, we know $E[N(t)]\approx\lambda t$. Thus for sufficiently large t, we have

$$B\approx\frac{1}{\lambda t}\sum_i((by_i+d)k_i(t)+a\xi_i). \tag{4}$$

Let $A=B/m$. Substituting Eq. (4) into Eq. (2), we can rewrite Eq. (2) as

$$\frac{\partial k_i(t)}{\partial t}=\frac{(by_i+d)k_i(t)+a\xi_i}{At}. \tag{5}$$

Since $k_i(t_i)=m$, from Eq. (5), we have

$$k_i(t,y,\xi)=\left(m+\frac{a\xi_i}{by_i+d}\right)\left(\frac{t}{t_i}\right)^{\beta(y_i)}-\frac{a\xi_i}{by_i+d}, \tag{6}$$

where $\beta(y)=(by+d)/A$.

The dynamic exponent $\beta(y)$ is bounded, i.e., $0<\beta(y)<1$, because a node always increases the number of links in time ($\beta(y)>0$) and $k_i(t)$ cannot increase faster than $N(t)$ ($\beta(y)<1$). Since

$$2m\lambda t=\sum_i k_i(t)=\int \mathrm{d}F(y)\int \mathrm{d}G(\xi)\int_0^t k_s(t,y,\xi)\lambda\,\mathrm{d}s,$$

we obtain that A can be determined by the following integral equation:

$$\begin{aligned}&\int\left(m+\frac{ac}{by+d}\right)\frac{x}{x-(by+d)}\mathrm{d}F(y)\\&=ac\int\frac{1}{by+d}\mathrm{d}F(y)+2m.\end{aligned} \tag{7}$$

Equation (7) is equivalent to

$$\begin{aligned}&m\int\frac{x}{x-(by+d)}\mathrm{d}F(y)\\&+ac\int\frac{1}{x-(by+d)}\mathrm{d}F(y)-2m=0.\end{aligned} \tag{8}$$

Equation (8) is said to be a characteristic equation of the degrees of the competitive network.

From Eq. (6), we obtain

$$\begin{aligned}&P\{k_i(t,y,\xi)\ge k\}\\&=P\left\{t_i\le\left(\frac{(by_i+d)m+a\xi_i}{(by_i+d)k+a\xi_i}\right)^{\frac{A}{by_i+d}}t\right\},\quad t\gg t_i.\end{aligned}$$

Notice that the node arrival process is the Poisson process having rate λ, time t_i follows a gamma distribution with parameter (i,λ), therefore, the probability $P\{k_i(t,y,\xi)<k\}$ can be written as

$$\begin{aligned}&P\{k_i(t,y,\xi)<k\}\\&=\exp\left[-\lambda t\left(\frac{(by_i+d)m+a\xi_i}{(by_i+d)k+a\xi_i}\right)^{\frac{A}{by_i+d}}\right]\\&\times\sum_{l=0}^{i-1}\frac{1}{l!}\left(\lambda t\left(\frac{(by_i+d)m+a\xi_i}{(by_i+d)k+a\xi_i}\right)^{\frac{A}{by_i+d}}\right)^l.\end{aligned} \tag{9}$$

From Eq. (9), we have

$$\begin{aligned}&P\{k_i(t,y,\xi)=k\}\\&\approx\frac{\lambda tA}{(by_i+d)m+a\xi_i}\left(\frac{(by_i+d)m+a\xi_i}{(by_i+d)k+a\xi_i}\right)^{\frac{A}{by_i+d}+1}\\&\times\exp\left(-\lambda t\left(\frac{(by_i+d)m+a\xi_i}{(by_i+d)k+a\xi_i}\right)^{\frac{A}{by_i+d}}\right)\\&\times\frac{1}{(i-1)!}\left[\lambda t\left(\frac{(by_i+d)m+a\xi_i}{(by_i+d)k+a\xi_i}\right)^{\frac{A}{by_i+d}}\right]^{i-1}.\end{aligned} \tag{10}$$

From Eq. (10), we obtain the stationary average degree distribution

$$\begin{aligned}P(k)&\approx A\int\mathrm{d}F(y)\int\frac{1}{(by+d)m+a\xi}\\&\times\left(\frac{(by+d)m+a\xi}{(by+d)k+a\xi}\right)^{\frac{A}{by+d}+1}\mathrm{d}G(\xi),\end{aligned} \tag{11}$$

where A is a solution of the characteristic Eq. (8).

When $b=0$ and $d=1$,

$$P(k)\approx\int\frac{2+ac/m}{m+a\xi}\left(\frac{m+a\xi}{k+a\xi}\right)^{3+ac/m}\mathrm{d}G(\xi). \tag{12}$$

When $a=d=0$ and $b=1$, from Eq. (1), model$_1$ reduces to the fitness model.[6] Using Eq. (11) and the characteristic Eq. (8), we obtain

$$P(k)\approx\frac{C}{m}\int\frac{1}{y}\left(\frac{m}{k}\right)^{\frac{C}{y}+1}\mathrm{d}F(y), \tag{13}$$

where C is a solution of the following integral equation:

$$\int\frac{x}{x-y}\mathrm{d}F(y)=2.$$

We consider a simple version of model$_1$.

(i) **Random growth** The network starts from an initial one with m_0 nodes. Suppose that nodes arrive at the system in accordance with a Poisson process having rate λ. Each node entering the network is tagged with its own η_i, and assume that η_i are independent random variables taken from a given distribution $F(y)$. When a new node is added to the system at time t, this new site is connected to m ($m\le m_0$) previously existing vertices. Here $c=\int y\mathrm{d}F(y)$ is finite.

(ii) **Preferential attachment** We assume that the probability with which a new node will connect to node i already present in the network depends on the connectivity $k_i(t)$ and the competitiveness η_i of that node, such that

$$\Pi(k_i(t))=\frac{(b\eta_i+d)k_i(t)+a\eta_i}{\sum_i((b\eta_i+d)k_i(t)+a\eta_i)}, \tag{14}$$

where $b, d \geq 0$ and $b+d \neq 0$. We call this model model$_2$.

Similarly to model$_1$, we have

$$\begin{aligned} & P\{k_i(t,\eta) < k\} \\ &= \exp\left(-\lambda t\left(\frac{(b\eta_i+d)m+a\eta_i}{(b\eta_i+d)k+a\eta_i}\right)^{\frac{A}{b\eta_i+d}}\right) \\ &\quad \times \sum_{l=0}^{i-1}\frac{1}{l!}\left(\lambda t\left(\frac{(b\eta_i+d)m+a\eta_i}{(b\eta_i+d)k+a\eta_i}\right)^{\frac{A}{b\eta_i+d}}\right)^l. \end{aligned} \tag{15}$$

From Eq. (15), we obtain

$$\begin{aligned} P(k) &\approx \int \frac{A}{(b\eta+d)m+a\eta} \\ &\quad \times \left(\frac{(b\eta+d)m+a\eta}{(b\eta+d)k+a\eta}\right)^{\frac{A}{b\eta+d}+1} \mathrm{d}F(\eta), \end{aligned} \tag{16}$$

where A is a solution of the characteristic Eq. (8).

When $b=0$, $d=1$, and the probability density function $F(x)$ is $\delta(x-1)$, i.e., all nodes have the same competitiveness, the network reduces to the initial attractiveness model. Therefore, from Eq. (16), the initial attractiveness model is a scale-free network with the degree distribution

$$P(k) \approx \frac{2m+a}{m(m+a)}\left(\frac{m+a}{k+a}\right)^{3+\frac{a}{m}}, \tag{17}$$

and the degree exponent

$$\gamma = 3+\frac{a}{m}. \tag{18}$$

If $F(x)$ is a uniform distribution on [0, 1], the characteristic equation (8) reduces to

$$(mA+ac)\ln\left|\frac{A-d}{A-(b+d)}\right| - 2bm = 0. \tag{19}$$

Equation (19) is equivalent to

$$(mA+ac)\ln\left(\frac{A-d}{A-(b+d)}\right)^2 - 4bm = 0. \tag{20}$$

The simulation result of the degree distribution of model$_2$ with the uniform distribution is shown in Fig. 1. The simulation result is in good agreement with the analytical one.

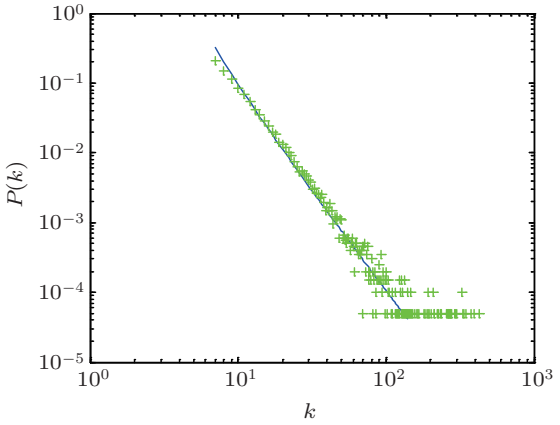


Fig. 1. (color online) The degree distribution of model$_2$ with the uniform distribution. The blue line is the theoretical prediction, the symbols represent the simulation result. The number of network nodes is 20000, $m_0 = 12$, $m = 7$, $a = 5$, $b = 2$, $d = 1$.

3. The universality of the competitive network for the weighted network

In the Internet, it is easy to realize that the introduction of a new connection to a router corresponds to an increase in the traffic handled by the other routers.[16] Indeed, in many technological, large infrastructure, and social networks, it is commonly believed that a reinforcement of the weights is due to the network's growth. In this spirit, Barrat *et al.* developed a model for a growing weighted network.[16] They took into account the coupled evolution in time of the topology and weights.

The definition of the BBV model is based on two coupled mechanisms: the topological growth and the weights' dynamics. Weighted networks are usually described by an adjacency matrix w_{ij} which represents the weight on the edge connecting nodes i and j, with $i, j = 1, 2, \ldots, N$, where N is the size of the network. Here only undirected networks are considered, in which the weight matrix is symmetric, that is $w_{ij} = w_{ji}$. The BBV weighted network model is defined as follows.[15,16]

(i) **Random growth** The network starts from an initial seed of m_0 nodes connected by edges with assigned weight w_0. Suppose that nodes arrive at the system in accordance with a Poisson process having rate λ. A new node n is added at time t. This new site is connected to m ($m \leq m_0$) previously existing nodes (i.e., each new node will have initially exactly m edges, all with equal weight w_0).

(ii) **Strength driven attachment** The new node n preferentially chooses sites with large strengths, i.e., node i is chosen with the probability

$$\Pi_{n\to i} = \frac{s_i}{\sum_j s_j}, \tag{21}$$

where $s_i = \sum_{j\in\Omega(i)} w_{ij}$ is the strength of node i, and the sum runs over the set $\Omega(i)$ of the neighbors of i.

(iii) **Weights' dynamics** The weight of each new edge (n,i) is initially set to a given value w_0. A new edge on node i will trigger only local rearrangements of the weights on the existing neighbors $j \in \Omega(i)$ according to the simple rule

$$w_{ij} \to w_{ij} + \Delta w_{ij}, \tag{22}$$

where $\Delta w_{ij} = \delta_i w_{ij}/s_i$, and δ_i is defined as the updating coefficient and is independent of time t.

This rule yields a total strength increase for node i of $w_0+\delta_i$, implying that $s_i \to s_i + w_0 + \delta_i$. After the weights have been updated, the growth process is iterated by introducing a new node, i.e., going back to step (i) until the desired size of the network is reached.

The changed strength is composed of three parts: the original strength, the new edge weight brought by the new node, and the increment of the old edge weight.

3.1. Heterogeneous coupling

In this section, we focus on the heterogeneous coupling with δ_i that are some sample observations individually taken from a population of X with a distribution of $F(x)$, and $E[X] = \int x\mathrm{d}F(x) = c$ is finite.

When a new node n is added to the network, an already present node i can be affected in two ways. (i) It is chosen with probability (21) to be connected to n; then its connectivity increases by 1, and its strength increases by $w_0 + \delta_i$. (ii) One of its neighbors $j \in \Omega(i)$ is chosen to be connected to n. Then the connectivity of i is not modified, but w_{ij} is increased according to the rule (22), and thus s_i is increased by $\delta_j w_{ij}/s_j$. This dynamical process modulated by the respective occurrence probabilities $\frac{s_i(t)}{\sum_l s_l(t)}$ and $\frac{s_j(t)}{\sum_l s_l(t)}$ is thus described by the following evolution equations for $s_i(t)$ and $k_i(t)$:

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = m(w_0+\delta_i)\frac{s_i}{\sum\limits_j s_j} + \sum_{j\in\Omega(i)} m\frac{s_j}{\sum\limits_l s_l}\delta_j\frac{w_{ij}}{s_j}, \tag{23}$$

$$\frac{\mathrm{d}k_i}{\mathrm{d}t} = m\frac{s_i}{\sum\limits_j s_j}. \tag{24}$$

Since node j is the neighbor of node i, and in the renewal process of the network, the weights of i and j are renewed simultaneity, without loss of generality, we can use $\delta_j \approx \delta_i$ in $\sum\limits_{j\in\Omega(i)} m\frac{s_j}{\sum_l s_l}\delta_j\frac{w_{ij}}{s_j}$, therefore,

$$\sum_{j\in\Omega(i)} m\frac{s_j}{\sum\limits_l s_l}\delta_j\frac{w_{ij}}{s_j} = m\delta_i\frac{s_i}{\sum\limits_l s_l}.$$

We obtain

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = m(w_0+2\delta_i)\frac{s_i}{\sum\limits_j s_j}. \tag{25}$$

Substituting Eq. (24) into Eq. (25) yields

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = (w_0+2\delta_i)\frac{\mathrm{d}k_i}{\mathrm{d}t}.$$

Since node i arrives at the network at time t_i, we have $k_i(t_i) = m$ and $s_i(t_i) = mw_0$. Then the above equation is integrated from t_i to t. We obtain

$$s_i = (w_0+2\delta_i)k_i - 2\delta_i m, \tag{26}$$

and probability (21) is modified as

$$\Pi_{n\to i} = \frac{(w_0+2\delta_i)k_i - 2\delta_i m}{\sum\limits_j[(w_0+2\delta_j)k_j - 2\delta_j m]}. \tag{27}$$

By comparing probability (27) and probability (14), we know that the weighted network is model$_2$ with $b = 2, d = w_0, a = -2m$. From Eq. (16), we obtain the stationary average degree distribution of the BBV model

$$P(k) \approx \frac{A}{w_0 m}\int\left(\frac{w_0 m}{(2\eta+w_0)k - 2m\eta}\right)^{\frac{A}{2\eta+w_0}+1}\mathrm{d}F(\eta), \tag{28}$$

where A is a solution of the following integral equation

$$\int\left(1-\frac{2c}{2y+w_0}\right)\frac{A}{A-(2y+w_0)}\mathrm{d}F(y) = 2-2c\int\frac{1}{2y+w_0}\mathrm{d}F(y). \tag{29}$$

According to Eqs. (26) and (15), we obtain the density function of the strength as

$$f(x) \approx \frac{A}{w_0 m}\int\frac{1}{2y+w_0}\left(\frac{w_0 m}{x}\right)^{\frac{A}{2y+w_0}+1}\mathrm{d}F(y). \tag{30}$$

If $F(x)$ is a uniform distribution on [0,1], equation (29) reduces to

$$(A-1)\ln\left|\frac{A-w_0}{A-2-w_0}\right| - 4 = 0. \tag{31}$$

Equation (28) reduces to

$$P(k) \approx \frac{A}{w_0 m}\int_0^1\left(\frac{w_0 m}{(2\eta+w_0)k - 2m\eta}\right)^{\frac{A}{2\eta+w_0}+1}\mathrm{d}\eta. \tag{32}$$

Equation (30) reduces to

$$f(x) \approx \frac{A}{w_0 m}\int_0^1\frac{1}{2y+w_0}\left(\frac{w_0 m}{x}\right)^{\frac{A}{2y+w_0}+1}\mathrm{d}y. \tag{33}$$

If $w_0 = 1$, equation (31) reduces to

$$(A-1)\ln\left|\frac{A-1}{A-3}\right| - 4 = 0. \tag{34}$$

Let

$$\varphi(x) = (x-1)\ln\left|\frac{x-1}{x-3}\right| - 4. \tag{35}$$

A solution of Eq. (34) is equivalent to seeking an intersection point of $y = \varphi(x)$ with the x axis.

From Fig. 2, we know that a positive solution of Eq. (34) exists. Then we use a numerical integral method to calculate Eq. (32) by solving Eq. (34). The degree distribution of the heterogeneous coupling weighted network is shown in Fig. 3. The degree distributions for both the weighted network and the competitive network corresponding to the weighted network are clearly shown in Fig. 3. The strength distribution of the weighted network is shown in Fig. 4. The validity of the above analysis is verified.

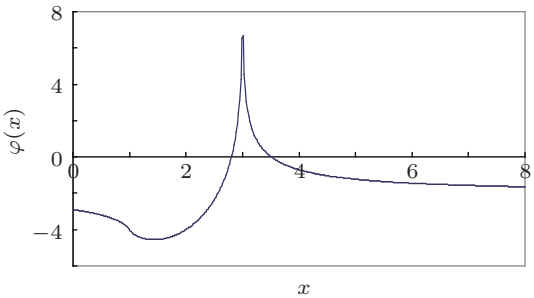


Fig. 2. An image of $y = \varphi(x)$.

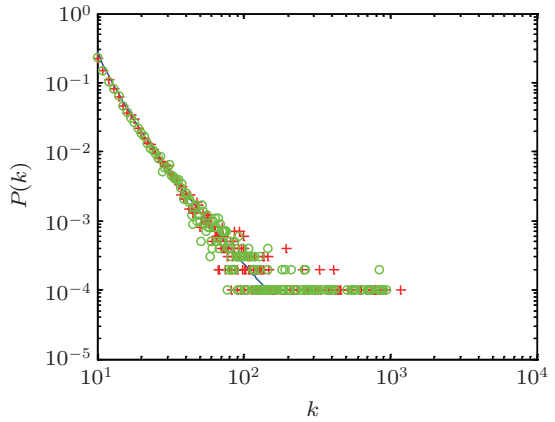


Fig. 3. (color online) The degree distributions of the heterogeneous coupling weighted network and the competitive network corresponding to this weighted network. The blue line is the theoretical prediction, the plus symbols denote the simulation result of the heterogeneous coupling weighted network with $w_0 = 1$. The circles are the simulation result of the competitive network. The number of network nodes is 10000, $m_0 = 12$, and $m = 10$.

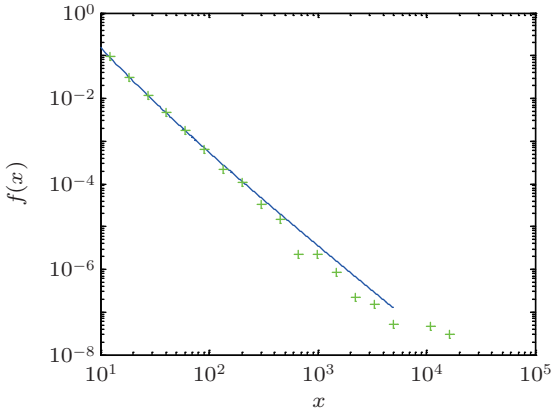


Fig. 4. (color online) The density function of the strength of the heterogeneous coupling weighted network using the logarithmic binning method. The blue line is the theoretical prediction; the plus symbols are the simulation result. The number of network nodes is 10000, $m_0 = 12$, $m = 10$, $w_0 = 1$.

3.2. Homogeneous coupling

In this section, we will focus on the simplest form of coupling with $\delta_i = \delta = \text{const}$. This case corresponds to a very homogeneous system in which all the vertices have an identical coupling between the addition of new edges and the corresponding weights' increase.

Similar to the heterogeneous coupling, we have

$$s_i = (w_0 + 2\delta)\, k_i - 2\delta m, \tag{36}$$

and probability (21) is modified as

$$\Pi_{n\to i} = \frac{k_i - \dfrac{2\delta}{(w_0+2\delta)} m}{\sum\limits_j \left[k_j - \dfrac{2\delta}{(w_0+2\delta)} m\right]}. \tag{37}$$

By comparing probability (37) and probability (14) with $b = 0$, $d = 1$, and $F(y) = \delta(y-1)$, it can be inferred that

$$a = -\frac{2\delta}{w_0 + 2\delta} m. \tag{38}$$

The probability of the preferential attachment can be modified as

$$\Pi_{n\to i} = \frac{k_i + a}{\sum\limits_j (k_j + a)},$$

which is in accord with that of the initial attractiveness model. That is to say, if the updating coefficient δ is a constant, the corresponding competitiveness a will also be a constant, which verifies the universality of the competitive network for the weighted network.[3]

Moreover, from Eq. (17), the degree distribution of the BBV weighted network behaves as $P(k) \propto k^{-\gamma}$, where

$$\gamma = 3 + \frac{a}{m} = 2 + \frac{w_0}{2\delta + w_0}. \tag{39}$$

Therefore, the degree distribution of the weighted network can be obtained directly from the results of the competitive network.

According to Eqs. (36) and (15), we obtain

$$\begin{aligned} P\{s_i < x\} &= P\left\{k_i < \frac{x + 2\delta m}{w_0 + 2\delta}\right\} \\ &= \exp\left(-\lambda t \left(\frac{m w_0}{x}\right)^{2+a/m}\right) \\ &\quad \times \sum_{l=0}^{i-1} \frac{1}{l!} \left(\lambda t \left(\frac{m w_0}{x}\right)^{2+a/m}\right)^l. \end{aligned} \tag{40}$$

Hence, the density function of s_i is

$$\begin{aligned} f_{s_i}(x) \approx \left(2 + \frac{a}{m}\right) \frac{(w_0 m)^{2+a/m} \lambda t}{x^{3+a/m}} \exp\left[-\lambda t \left(\frac{m w_0}{x}\right)^{2+a/m}\right] \\ \times \frac{1}{(i-1)!} \left[\lambda t \left(\frac{m w_0}{x}\right)^{2+a/m}\right]^{i-1}. \end{aligned} \tag{41}$$

Then the density function of the stationary average node strength distribution can be deduced from Eq. (41) as

$$f(x) \approx \left(2 + \frac{a}{m}\right) \frac{(w_0 m)^{2+a/m}}{x^{3+a/m}}. \tag{42}$$

By instituting Eq. (38) into the above equation, the density function $f(x)$ can be analogously calculated, yielding the power-law behavior

$$f(x) = (\gamma - 1)(w_0 m)^{\gamma - 1} \frac{1}{x^\gamma}, \quad \gamma = 2 + \frac{w_0}{2\delta + w_0}. \tag{43}$$

We take $N = 10000$, $m_0 = 10$, and $m = 6$ for both networks. The updating coefficient δ of the weighted network equals 1, the corresponding competitiveness a equals $-8/3$. The degree distributions are shown in Fig. 5. The strength distribution of the weighted network is shown in Fig. 6

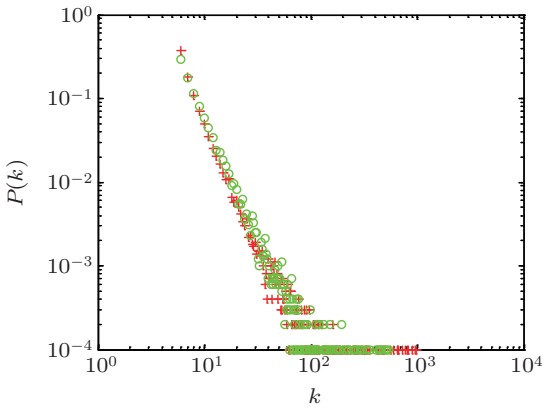


Fig. 5. (color online) The comparisons of degree distributions of the weighted network and the competitive model. The plus symbols denote the simulation results of the homogeneous coupling weighted network with $w_0 = 1$. The circles are the simulation result of the competitive network. The number of network nodes is 10000, $m_0 = 10$, $m = 6$, $w_0 = 1$.

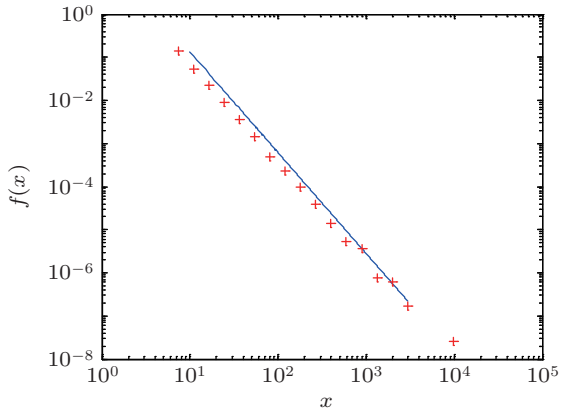


Fig. 6. (color online) The density function of the strength of the homogeneous coupling weighted network using the logarithmic binning method. The blue line is the theoretical prediction; the plus symbols denote the simulation result. The number of network nodes is 10000, $m_0 = 10$, $m = 6$, $w_0 = 1$.

4. Profit-driven evolution of networks

Hanaki *et al.* studied the cooperative behavior emerging in an environment where individual behaviors and interaction structures co-evolve.[17] If a company is regarded as a node, an edge between two companies can be established when they have a cooperative relationship, which forms an enterprise cooperation network. Each enterprise has an initial investment to join the network. Companies can make a profit when they cooperate. The network is characterized by profit-driven network growth. The definition of the model is based on the profit-driven mechanism.

(i) **Random growth** The network starts from an initial one with m_0 nodes. Suppose that nodes arrive at the system in accordance with a Poisson process having rate λ. Each node entering the system is tagged with its own investment ξ_i, and we suppose that ξ_i are independent random variables having a common distribution $G(x)$, with $c = \int x \mathrm{d}G(x)$ being finite. When a new node is added to the system at time t, this new site is connected to m ($m \leq m_0$) previously existing vertices. If node i gets a link, it makes profit η_i, and we also assume that η_i are independent random variables having a common distribution $F(x)$.

(ii) **Preferential attachment** We assume that the probability that a new node will connect to node i already present in the network depends on the total profit $f_i(t)$ of node i and its investment ξ_i of that node, such that

$$\Pi_{n\to i} = \frac{f_i(t)+\xi_i}{\sum\limits_i (f_i(t)+\xi_i)}. \tag{44}$$

We call the above model model$_3$.

When a new node n is added to the network, an already present node i can be affected in the following way. It is chosen with probability (44) to be connected to n; then its connectivity increases by 1, and its profit increases by η_i. This dynamical process modulated by the respective occurrence probability (44) is thus described by the following evolution equations for $f_i(t)$ and $k_i(t)$:

$$\frac{\partial f_i(t)}{\partial t} = m\eta_i\lambda \frac{f_i(t)+\xi_i}{\sum\limits_i (f_i(t)+\xi_i)}, \tag{45}$$

$$\frac{\partial k_i(t)}{\partial t} = m\lambda \frac{f_i(t)+\xi_i}{\sum\limits_i (f_i(t)+\xi_i)}. \tag{46}$$

Substituting Eq. (45) into Eq. (46) yields

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} = \eta_i \frac{\mathrm{d}k_i}{\mathrm{d}t}.$$

Since node i arrives at the network at time t_i, we have $k_i(t_i) = m$ and $f_i(t_i) = m\eta_i$. Then the above equation is integrated from t_i to t, we obtain

$$f_i(t) = \eta_i k_i(t). \tag{47}$$

Probability (44) is modified to

$$\Pi_{n\to i} = \frac{\eta_i k_i+\xi_i}{\sum\limits_j (\eta_i k_j+\xi_j)}. \tag{48}$$

By comparing probability (48) and probability (1), we know that this network is model$_1$ with $b = 1$, $d = 0$, $a = 1$. From Eq. (11), we obtain the stationary average degree distribution of model$_3$

$$P(k) \approx A \int \mathrm{d}F(y) \int \frac{1}{my+\xi} \left(\frac{my+\xi}{ky+\xi}\right)^{\frac{A}{y}+1} \mathrm{d}G(\xi), \tag{49}$$

where A is a solution of the following characteristic equation

$$m\int \frac{x}{x-y}\mathrm{d}F(y) + c\int \frac{1}{x-y}\mathrm{d}F(y) - 2m = 0. \tag{50}$$

According to Eqs. (9) and (47), we obtain the density function of the profits

$$g(x) = A \int \mathrm{d}F(\eta) \int \frac{1}{\eta(m\eta+\xi)} \left(\frac{m\eta+\xi}{x+\xi}\right)^{\frac{A}{\eta}+1} \mathrm{d}G(\xi). \tag{51}$$

If $F(x)$ and $G(x)$ are $\delta(x-1)$ and $\delta(x-a)$, respectively, then, from Eqs. (49) and (51), the network is a scale-free network with the degree distribution

$$P(k) \approx \frac{2m+a}{m(m+a)}\left(\frac{m+a}{k+a}\right)^{3+a/m},$$

the density function of the profits is

$$g(x) \approx \frac{2m+a}{m(m+a)}\left(\frac{m+a}{x+a}\right)^{3+a/m}, \tag{52}$$

and the degree exponent is $\gamma = 3 + a/m$. Equation (52) is the generalized Pareto distribution, it shows that the financial wealth of the network follows Pareto's law (also known as the 80–20 rule, the law of the vital few).

5. Conclusion

In the past decade, the preferential attachment based on the degrees of nodes in networks have been considered. However, the co-evolution of networks and behavior has not received as much attention as it deserves. We show that whereas phenomena highlighted in the extant literature, such as preferential attachment and reciprocity, are important drivers of the network growth, intrinsic properties of nodes, such as the economic and military strength in the national network of relationships, and are also very significant drivers of the network growth. Our models admit that the competitiveness and the survival of the fittest are important drivers of the network growth. This paper not only provides theoretical proofs, but also a simulation verification. The results from our simulations show that the numerical simulations of the models agree with the analytical results well. The results from our theoretical analysis show that the characteristic equation can serve as a very effective method when dealing with multi-scale networks. We also notice the interesting work in Ref. [18]. However, the model and the characteristic equation in Ref. [18] are not all identical with ours.

The competitive networks and the weighted networks are two kinds of evolving models. The evolving mechanism of these models is discussed in the paper. The weighted network models can be divided into two categories according to whether the weight is assigned with fixed or variable values. The former includes the weighted scale-free (WSF) model[13,14] and the Zheng–Trimper–Zheng–Hui (ZTZH) model,[19] and the latter includes the BBV model[15,16] and the Dorogovtsev–Mendes (DM) model.[20] The topological structure of the WSF model or the ZTZH model completely agrees with that of the BA model. Therefore, the two models can be seen as a special case of our model. For the ZTZH expanded model discussed in Ref. [19], its preferential attachment mechanism is in accordance with the fitness model; therefore, it is also a special case of our model. The probability of preferential attachment in the BBV model depends on the node strength. This paper reveals that our model has not only the universality for the homogeneous weighted network, but also the character for the heterogeneous weighted network.

In the economic system, a famous wealth distribution is Pareto's law. In many countries and regions, a similar phenomenon has been found since Pareto developed the principle. The cooperation in the economic system is a common behavior. The profit-driven model can better describe the observed phenomenon in the economic system.

The paper also shows that the standard preferential attachment, the growing random graph, the initial attractiveness model, the fitness model, and weighted networks can all be seen as degenerate cases of our model.

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