Impulsive effect on exponential synchronization of neural networks with leakage delay under sampled-data feedback control*

S. Lakshmanan[a)b)], Ju H. Park[b)†], Fathalla A. Rihan[a)], and R. Rakkiyappan[c)]

[a)] *Department of Mathematical Sciences, College of Science, UAE University,* 17551, *Al Ain, UAE*

[b)] *Nonlinear Dynamics Group, Department of Electrical Engineering, Yeungnam University,* 280 *Daehak-Ro, Kyongsan* 712-749, *Republic of Korea*

[c)] *Department of Mathematics, Bharathiar University, Coimbatore* - 641 046, *Tamilnadu, India*

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We consider the impulsive effect on the exponential synchronization of neural networks with leakage delay under the sampled-data feedback control. We use an appropriate Lyapunov–Krasovskii functional combined with the input delay approach and some inequality techniques to derive sufficient conditions that ensure the exponential synchronization of the delayed neural network. The conditions are formulated in terms of the leakage delay, the sampling period, and the exponential convergence rate. Numerical examples are given to demonstrate the usefulness and the effectiveness of the results.

Keywords: exponential synchronization, impulsive effect, leakage delay, sampled-data feedback

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1. Introduction

In the past two decades, considerable attention has been devoted to the stability analysis of time-delay systems. The time delay is an import source of dynamical systems and it can distabilize the networks (systems) as a whole and create an oscillatory behavior. Therefore, many researchers have paid attention to analyze time-delay systems, for example, the fuzzy model with time delay,[1] the FitzHugh–Nagumo neuronal networks with time delay,[2] the passivity analysis with discrete time delay,[3] the adaptive control and H^{∞} filtering problem with time delay.[4–7] On the other hand, the chaotic synchronization has become one of the major properties of dynamical systems. It also possesses many applications in diverse areas such as secure communication, image processing, and harmonic oscillation generation.[8–11] Since the pioneering works of Pecora and Carroll,[12,13] many researchers paid attention to the synchronization of dynamical systems.[14–25]

Neural networks (NNs) have been widely studied, as they have massive applications in signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization. The occurrence of unpredictable behaviors including stable equilibria, periodic oscillations, bifurcation, and chaotic attractors mainly depends on the network's parameters and time delays. Accordingly, many researchers gave attention to the study of the synchronization and chaotic behaviors of NNs.[26–33] Meanwhile, the impulsive effects exist in a wide variety of developmental processes that subject to instantaneous perturbations at certain moments of time. These processes provide a natural description of observed evolution processes prevailing in several real world problems occurring in many fields such as biology, physics, and engineering.[34,35] Hence, it is necessary to investigate the impulsive effects to address the synchronization problem of delayed NNs.[36–41] For instance, the exponential synchronization of a class of NNs with mixed time-varying delays and impulsive effects has been studied in Ref. [14]. The authors in Refs. [36] and [37] discussed the exponential synchronization of delayed NNs with stochastic perturbations and impulsive effect.

Until now, most of the stabilization and synchronization problems for delayed NNs are based on the state feedback control and the adaptive control. These control methods are highly enhanced to achieve appropriate stabilization or synchronization. However, the control strategies must be continuous in time and often do not adapt well to real time situations. Since the middle of 1990s, due to the development of modern computer and communication networks, continuous time controllers have been replaced with digital controllers. The sampled-data control method, in which the control signals are kept constant during the sampling period and are allowed to change only at the sampling instant, has become more important than other control approaches.[42–45] The synchronization of delayed NNs based on the sampled-data control are addressed in Refs. [42] and [46]–[49].

On the other hand, the time delay is called as the leakage delay when it exists in the negative feedback terms of the NN. The leakage delay has a great impact on the dynamic behaviors of delayed NNs.[50–54] The synchronization of chaotic NNs with time delay in the leakage term and parametric un-

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†Corresponding author. E-mail: jessie@ynu.ac.kr

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certainties based on the sampled-data control has been considered in Ref. [49]. The exponential state estimation for impulsive NNs with time delay in the leakage term has been investigated in Ref. [55]. However, to the best of our knowledge, there are no reports on the impulsive effect on the exponential synchronization analysis of delayed NNs with leakage delay under the sampled-data feedback control. Inspired by the above discussion and to fill this gap, we try to perform a synchronization analysis of delayed NNs with leakage delay under the sampled-data feedback control. In this work, we investigate the impulsive effect on the exponential synchronization analysis of delayed NNs with leakage delay under the sampled-data feedback control. By choosing an appropriate Lyapunov–Krasovskii functional that involves double and triple integral terms, combined with an input delay approach along with some inequality techniques, sufficient conditions that ensure the exponential synchronization of delayed NNs have been derived. The conditions are expressed in terms of the linear matrix inequality (LMI) that can be easily solved by standard packages in the MATLAB LMI control toolbox. The proposed double and triple integral terms in the Lyapunov–Krasovskii functional play an important role in the obtained results that have been confirmed by numerical simulations.

Notation We introduce the following notations. Let $\mathbb{R}$ and $\mathbb{Z}_+$ denote the set of real numbers and the set of positive integers, respectively, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript T stands for the transposition. The notation $X \geq Y$ ($X > Y$) means that $X-Y$ is positive semi-definite (positive definite), where X and Y are symmetric matrices. The $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$, I denotes the identity matrix with appropriate dimensions, $\widetilde{\Lambda} = \{1,2,\ldots,n\}$. For any interval $J \subseteq \mathbb{R}$, set $S \subseteq \mathbb{R}^k$ $(1 \leq k \leq n)$, $C(J,S) = \Big\{\varphi : J \to V \text{ is continuously}\Big\}$, and $PC^1(J,S) = \Big\{\psi : J \to S \text{ is continuously differentiable everywhere except of a finite number of points } t, \text{ at which } \psi(t^+), \psi(t^-), \dot{\psi}(t^+), \dot{\psi}(t^-) \text{ exist and } \psi(t^+) = \psi(t), \dot{\psi}(t^+) = \dot{\psi}(t), \dot{\psi} \text{ denotes the derivative of } \psi\Big\}$. For $\psi(\cdot) = (\psi_1,\ldots,\psi_n)^{\mathrm{T}} \in PC^1([-\tau^*,0],\mathbb{R}^n)$, the norm is defined by $\|\psi\|_{\tau^*} = \max\Big\{\sup_{-\tau^*\leq s\leq 0}\|\psi(s)\|, \sup_{-\tau^*\leq s\leq 0}\|\dot{\psi}(s)\|\Big\}$, where $\tau^* = \max\{\bar{\tau},\sigma\}$ and diag$\{\cdots\}$ stands for a block diagonal matrix. The symbol $*$ always denotes the symmetric block in a matrix, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalues of a given matrix, respectively.

2. Problem description and preliminaries

Consider the following NN system with time-varying delay and impulsive effect:

$$\begin{aligned}
\dot{x}(t) &= -Ax(t-\sigma) + Bf(x(t)) \\
&\quad + Wf(x(t-\tau(t))) + J(t),\ t > 0, \\
x(t_k) - x(t_k^-) &= -C_k\Big\{x(t_k^-) - A\int_{t_k-\sigma}^{t_k} x(s)\,\mathrm{d}s\Big\}, \\
&\quad t \neq t_k,\ t_k \in \mathbb{Z}_+, \\
x(s) &= \phi(s),\ s \in [-\tau^*, 0].
\end{aligned} \tag{1}$$

Here

$$\begin{aligned}
x(t) &= [x_1(t)\ x_2(t)\cdots x_n(t)]^{\mathrm{T}}; \\
f(x(t)) &= [f_1(x_1(t))\ f_2(x_2(t))\cdots f_n(x_n(t))]^{\mathrm{T}};
\end{aligned}$$

$x_i(t)$ is the state of the i-th neuron at time t; $f_i(x_i(t))$ denotes the neuron activation function; $A = \mathrm{diag}\{a_1,\ldots,a_n\}$ is a diagonal matrix with positive entries; $B = (b_{ij})_{n\times n}$ and $W = (w_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively; $J(t) = [J_1(t)\,J_2(t)\cdots J_n(t)]^{\mathrm{T}}$ is an external input vector; $C_k \in \mathbb{R}^{n\times n}, k \in \mathbb{Z}_+$ denotes the impulsive matrix, and $\tau(t)$ denotes the time varying delay and satisfies

$$0 \leq \tau(t) \leq \bar{\tau}, \quad \dot{\tau}(t) \leq \mu,$$

in which $\bar{\tau}, \mu$ are real constants. The following assumptions are imposed on the network.

H1 The neuron activation functions $f_i(\cdot)$ in Eq. (1) satisfy

$$l_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq l_i^+,\ i = 1,2,\ldots,n, \tag{2}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}$, $\alpha_1 \neq \alpha_2$, and l_i^-, l_i^+ are real.

H2 The leakage delay $\sigma \geq 0$ is a constant.

H3 The impulse time t_k satisfies $0 = t_0 < t_1 < \cdots < t_k \to \infty$ and $\inf_{k\in\mathbb{Z}_+}\{t_k - t_{k+1}\}$.

System (1) is considered as the master system, and a slave system for system (1) can be described by

$$\begin{aligned}
\dot{y}(t) &= -Ay(t-\sigma) + Bf(y(t)) \\
&\quad + Wf(y(t-\tau(t))) + u(t) + J(t),\ t > 0, \\
y(t_k) - y(t_k^-) &= -C_k\Big\{y(t_k^-) - A\int_{t_k-\sigma}^{t_k} y(s)\,\mathrm{d}s\Big\}, \\
&\quad t \neq t_k,\ t_k \in \mathbb{Z}_+, \\
y(s) &= \varphi(s),\ s \in [-\tau^*, 0],
\end{aligned} \tag{3}$$

where $y(t) = [y_1(t)\ \ y_2(t)\cdots y_n(t)]^{\mathrm{T}}$ is the neuron state vector of the slave system, and $u(t) \in \mathbb{R}^n$ is an appropriate control input that will be designed in order to obtain certain control objectives.

By defining the error signal as

$$e(t) = y(t) - x(t),$$

then, the error system can be represented as

$$\begin{aligned}
\dot{e}(t) &= -Ae(t-\sigma) + Bg(e(t)) \\
&\quad + Wg(e(t-\tau(t))) + u(t), \quad t > 0, \\
e(t_k) - e(t_k^-) &= -C_k\Big\{e(t_k^-) - A\int_{t_k-\sigma}^{t_k} e(s)\,\mathrm{d}s\Big\},
\end{aligned}$$

$$t \neq t_k,\ t_k \in \mathbb{Z}_+,$$
$$e(s) = \varphi(s) - \phi(s),\ s \in [-\tau^*, 0], \tag{4}$$

where $g(e(t)) = f(y(t)) - f(x(t))$. It can be found from Eq. (2) that functions $g_i(\cdot)$ satisfy the following condition:

$$l_i^- \leq \frac{g_i(\alpha)}{\alpha} \leq l_i^+, \quad i = 1, 2, \ldots, n, \tag{5}$$

where $\alpha \in \mathbb{R}$ and $\alpha \neq 0$.

The control signal is assumed to be generated by using a zero-order-hold (ZOH) function with a sequence of hold times $0 = \Delta_0 < \Delta_1 < \Delta_2 < \cdots < \Delta_\nu < \cdots$,

$$u(t) = Ke(\Delta_\nu), \qquad \Delta_\nu \leq t < \Delta_{\nu+1}, \tag{6}$$

where Δ_ν denotes the sampling instant and satisfies $\lim_{\nu\to\infty} \Delta_\nu = \infty$. Further, the sampling period under consideration is assumed to be bound by a known constant $d > 0$ and satisfies $\Delta_{\nu+1} - \Delta_\nu \leq d$ for all $\nu \geq 0$. Define a sawtooth function as follows:

$$d(t) = t - \Delta_\nu, \quad \Delta_\nu \leq t < \Delta_{\nu+1}.$$

It is easily seen that $0 \leq d(t) < d$. Therefore, the state feedback controller takes the form

$$u(t) = Ke(t - d(t)). \tag{7}$$

Then, the error system becomes

$$\begin{aligned}
\dot{e}(t) &= -Ae(t-\sigma) + Bg(e(t)) \\
&\quad + Wg(e(t-\tau(t))) + Ke(t-d(t)),\ t > 0, \\
e(t_k) - e(t_k^-) &= -C_k\Big\{e(t_k^-) - A\int_{t_k-\sigma}^{t_k} e(s)\,\mathrm{d}s\Big\}, \\
&\quad t \neq t_k,\ t_k \in \mathbb{Z}_+.
\end{aligned} \tag{8}$$

Before proceeding further, the following definition and lemma are introduced.

Definition 1[38] Systems (1) and (2) are said to be exponentially synchronized if there exist constants $\mathfrak{M} \geq 1$ and $\eta > 0$ such that

$$\|y(t) - x(t)\| \leq \mathfrak{M}\|\varphi(s) - \phi(s)\|_{\tau^*} \mathrm{e}^{-\eta t}$$

for any $t \geq 0$. Constant η is said to be the degree or decay rate of the exponential synchronization.

Lemma 1[56] For any positive symmetric constant matrix $M \in \mathbb{R}^{n\times n}$, a scalar $\tau > 0$, and a vector function $\omega(s) \in \mathbb{R}^n$, the integrations concerned are well defined, then

$$\begin{aligned}
&-\int_{t-\tau}^{t} \omega^{\mathrm{T}}(s) M \omega(s)\,\mathrm{d}s \\
&\leq -\frac{1}{\tau}\Big(\int_{t-\tau}^{t} \omega(s)\,\mathrm{d}s\Big)^{\mathrm{T}} M \Big(\int_{t-\tau}^{t} \omega(s)\,\mathrm{d}s\Big), \\
&-\int_{-\tau}^{0}\int_{t+\theta}^{t} \omega^{\mathrm{T}}(s) M \omega(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\leq -\frac{2}{\tau^2}\Big(\int_{-\tau}^{0}\int_{t+\theta}^{t} \omega(s)\,\mathrm{d}s\,\mathrm{d}\theta\Big)^{\mathrm{T}} M \\
&\quad \times \Big(\int_{-\tau}^{0}\int_{t+\theta}^{t} \omega(s)\,\mathrm{d}s\,\mathrm{d}\theta\Big).
\end{aligned}$$

The main objective of this paper is to design a feedback controller with the form (8) to ensure that the master system (1) synchronizes with the slave system (3). In other words, we are interested in finding a feedback gain matrix K such that the error system (8) is exponentially stable.

3. Main results

In this section, we investigate the exponential synchronization of NNs with impulsive effect and leakage delay. The analysis is based on the Lyapunov–Krasovskii functionals and the LMI framework. We arrive at the following Theorem.

Theorem 1 For given scalars $\bar{\tau} > 0$ and $\mu \geq 0$, assume that assumptions (H1)–(H3) hold. Then, master system (1) and slave system (3) are exponentially synchronous with decay rate η if there exist $P > 0$, $Q_k > 0$, $(k = 1, 2, 3, 4)$, $T_1 > 0$, $\begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} > 0$, $\begin{bmatrix} R_4 & R_5 \\ R_5^{\mathrm{T}} & R_6 \end{bmatrix} > 0$, $\begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3 \end{bmatrix} > 0$, $\begin{bmatrix} S_4 & S_5 \\ S_5^{\mathrm{T}} & S_6 \end{bmatrix} > 0$, $N > 0$, X, and diagonal matrices $T > 0$, $\Lambda > 0$, $\bar{\alpha} > 0$, $\bar{\beta} > 0$, $\Gamma_i > 0$ $(i = 1, 2)$ such that the following LMIs hold:

$$\begin{bmatrix} P & (I - C_k)^{\mathrm{T}} P \\ * & P \end{bmatrix} \geq 0, \quad k \in \mathbb{Z}_+, \tag{9}$$
$$\Xi < 0, \tag{10}$$

where $\Xi = (\Xi_{m,l})_{14\times 14}$ with

$$\begin{aligned}
\Xi_{1,1} &= 2\eta P - PA - A^{\mathrm{T}}P - 4\eta(L^-T - L^+\Lambda) \\
&\quad + \sum_{i=1}^{4} Q_i + 2\bar{\tau}\bar{\alpha}[L^+ - L^-] + \bar{\tau}R_1 - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3 \\
&\quad + \sigma^2 R_4 - \mathrm{e}^{-2\eta\sigma}R_6 + \frac{\bar{\tau}^2}{2}S_1 - 2\mathrm{e}^{-2\eta\bar{\tau}}S_3 \\
&\quad + \frac{\sigma^4}{2}S_4 - 2\sigma^2\mathrm{e}^{-2\eta\sigma}S_6 - \mathrm{e}^{-2\eta d}T_1 - 2L^-\Gamma_1 L^+, \\
\Xi_{1,2} &= \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3, \quad \Xi_{1,4} = PA + \mathrm{e}^{-2\eta\sigma}R_6 - NA, \\
\Xi_{1,5} &= \mathrm{e}^{-2\eta d}T_1 + X, \\
\Xi_{1,7} &= P - L^-T + L^+\Lambda + \bar{\tau}R_2 + \sigma^2 R_5 \\
&\quad + \frac{\bar{\tau}^2}{2}S_2 + \frac{\sigma^4}{2}S_5 - N, \\
\Xi_{1,8} &= 2\eta T - 2\eta\Lambda + \Gamma_1(L^+ + L^-) + NB, \\
\Xi_{1,9} &= NW, \quad \Xi_{1,10} = -\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2 + 2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}S_3, \\
\Xi_{1,11} &= 2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}S_3, \\
\Xi_{1,12} &= -2\eta PA + A^{\mathrm{T}}PA + 2\sigma\mathrm{e}^{-2\eta\sigma}S_6 - \mathrm{e}^{-2\eta\sigma}R_5, \\
\Xi_{1,13} &= -2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}S_2, \quad \Xi_{1,14} = -2\sigma\mathrm{e}^{-2\eta\sigma}S_5, \\
\Xi_{2,2} &= -(1-\mu)\mathrm{e}^{-2\eta\bar{\tau}}Q_1 - 2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3 - 2[L^+\Gamma_2 L^-],
\end{aligned}$$

$$
\begin{aligned}
\Xi_{2,3} &= \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3, \quad \Xi_{2,9} = \Gamma_2[L^+ + L^-],\\
\Xi_{2,10} &= \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2, \quad \Xi_{2,11} = -\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2,\\
\Xi_{3,3} &= -\mathrm{e}^{-2\eta\bar{\tau}}Q_2 - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3, \quad \Xi_{3,11} = \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2,\\
\Xi_{4,4} &= -\mathrm{e}^{-2\eta\sigma}Q_3 - \mathrm{e}^{-2\eta\sigma}R_6,\\
\Xi_{4,7} &= -\gamma A^{\mathrm{T}}N^{\mathrm{T}}, \quad \Xi_{4,12} = -A^{\mathrm{T}}PA + \mathrm{e}^{-2\eta\sigma}R_5,\\
\Xi_{5,5} &= -2\mathrm{e}^{-2\eta d}T_1, \quad \Xi_{5,6} = \mathrm{e}^{-2\eta d}T_1,\\
\Xi_{5,7} &= \gamma X^{\mathrm{T}}, \quad \Xi_{6,6} = -\mathrm{e}^{-2\eta d}Q_4 - \mathrm{e}^{-2\eta d}T_1,\\
\Xi_{7,7} &= \bar{\tau}R_3 + \sigma^2 R_6 + \frac{\bar{\tau}^2}{2}S_3 + \frac{\sigma^4}{2}S_6 + d^2T_1 - \gamma(N + N^{\mathrm{T}}),\\
\Xi_{7,8} &= T + \gamma NB - \Lambda, \quad \Xi_{7,9} = \gamma NW,\\
\Xi_{7,12} &= -PA, \quad \Xi_{8,8} = -2\Gamma_1, \quad \Xi_{9,9} = -2\Gamma_2,\\
\Xi_{10,10} &= -\frac{2}{\bar{\tau}}\bar{\alpha}[L^+ - L^-] - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_1 - \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_3,\\
\Xi_{10,11} &= -\frac{2}{\bar{\tau}}\bar{\alpha}[L^+ - L^-] - \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_3,\\
\Xi_{10,13} &= \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_2,\\
\Xi_{11,11} &= -\frac{2}{\bar{\tau}}\bar{\alpha}[L^+ - L^-] - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_1 - \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_3,\\
\Xi_{11,13} &= \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_2,\\
\Xi_{12,12} &= 2\eta A^{\mathrm{T}}PA - \mathrm{e}^{-2\eta\sigma}(R_4 + 2S_6), \quad \Xi_{12,14} = 2\mathrm{e}^{-2\eta\sigma}S_5,\\
\Xi_{13,13} &= \frac{8\eta}{\bar{\tau}^2}\bar{\alpha}[L^+ - L^-] - \frac{2}{\bar{\tau}^2}\mathrm{e}^{-2\eta\bar{\tau}}S_1, \ \Xi_{14,14} = -2\mathrm{e}^{-2\eta\sigma}S_4,
\end{aligned}
$$

and other entries of Ξ are zero. Moreover, the sampled-data controller gain matrix is given by $K = N^{-1}X$.

Proof Consider the following Lyapunov–Krasovskii functional for system (8):

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{11}$$

where

$$
\begin{aligned}
V_1(t) &= \mathrm{e}^{2\eta t}\Big[e(t) - A\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big]^{\mathrm{T}} P\Big[e(t) - A\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big],\\
V_2(t) &= 2\mathrm{e}^{2\eta t}\sum_{i=1}^{n}\Big\{t_i\int_0^{e_i}\Big[g_i(s) - l_i^- s\Big]\mathrm{d}s\\
&\quad + \lambda_i\int_0^{e_i}\Big[l_i^+ s - g_i(s)\Big]\mathrm{d}s\Big\}\\
&\quad + \int_{t-\tau(t)}^{t}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_1e(s)\mathrm{d}s\\
&\quad + \int_{t-\bar{\tau}}^{t}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_2e(s)\mathrm{d}s\\
&\quad + \int_{t-\sigma}^{t}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_3e(s)\mathrm{d}s\\
&\quad + \int_{t-d}^{t}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_4e(s)\mathrm{d}s,\\
V_3(t) &= 2\mathrm{e}^{2\eta t}\sum_{i=1}^{n}\Big\{\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}\alpha_i e_i(s)\Big[l_i^+e_i(s) - l_i^-e_i(s)\Big]\mathrm{d}s\mathrm{d}\theta\Big\},\\
V_4(t) &= \int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}\mathrm{e}^{2\eta s}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}R_1 & R_2\\ R_2^{\mathrm{T}} & R_3\end{bmatrix}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta\\
&\quad + \sigma\int_{-\sigma}^{0}\int_{t+\theta}^{t}\mathrm{e}^{2\eta s}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}R_4 & R_5\\ R_5^{\mathrm{T}} & R_6\end{bmatrix}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta\\
&\quad + \int_{-\bar{\tau}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\mathrm{e}^{2\eta s}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\\
&\quad \times\begin{bmatrix}S_1 & S_2\\ S_2^{\mathrm{T}} & S_3\end{bmatrix}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\lambda\,\mathrm{d}\theta\\
&\quad + \sigma^2\int_{-\sigma}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\mathrm{e}^{2\eta s}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\\
&\quad \times\begin{bmatrix}S_4 & S_5\\ S_5^{\mathrm{T}} & S_6\end{bmatrix}\begin{bmatrix}e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\lambda\,\mathrm{d}\theta,\\
V_5(t) &= d\int_{-d}^{0}\int_{t+\theta}^{t}\mathrm{e}^{2\eta s}\dot{e}^{\mathrm{T}}(s)T_1\dot{e}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned}
$$

Calculating the time derivative of $V(t)$ along the solution of Eq. (8) at the continuous interval $[t_{k-1},\ t_k)$, $k \in \mathbb{Z}_+$ gives

$$
\begin{aligned}
D^+V(t) &= D^+V_1(t) + D^+V_2(t) + D^+V_3(t)\\
&\quad + D^+V_4(t) + D^+V_5(t),
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
&D^+V_1(t)\\
&= 2\eta\,\mathrm{e}^{2\eta t}\Big[e(t) - A\int_{t-\sigma}^{t}e(s)\mathrm{d}s\Big]^{\mathrm{T}}P\Big[e(t) - A\int_{t-\sigma}^{t}e(s)\mathrm{d}s\Big]\\
&\quad + 2\mathrm{e}^{2\eta t}\Big[e(t) - A\int_{t-\sigma}^{t}e(s)\mathrm{d}s\Big]P\Big[\dot{e}(t) - Ae(t) + Ae(t-\sigma)\Big],\\
&D^+V_2(t)\\
&\le 4\eta\,\mathrm{e}^{2\eta t}\sum_{i=1}^{n}\Big\{t_i\int_0^{e_i}\Big[g_i(s) - l_i^-s\Big]\mathrm{d}s\\
&\quad + \lambda_i\int_0^{e_i}\Big[l_i^+s - g_i(s)\Big]\mathrm{d}s\Big\}\\
&\quad + 2\mathrm{e}^{2\eta t}\Big[g^{\mathrm{T}}(e(t)) - e^{\mathrm{T}}(t)L^-\Big]T\dot{e}(t)\\
&\quad + 2\mathrm{e}^{2\eta t}\Big[e^{\mathrm{T}}(t)L^+ - g^{\mathrm{T}}(e(t))\Big]\Lambda\dot{e}(t)\\
&\quad + \mathrm{e}^{2\eta t}\Big\{e^{\mathrm{T}}(t)Q_1e(t) - (1-\mu)\mathrm{e}^{-2\eta\bar{\tau}}e^{\mathrm{T}}(t-\tau(t))\\
&\quad \times Q_1e(t-\tau(t)) + e^{\mathrm{T}}(t)Q_2e(t)\\
&\quad - \mathrm{e}^{-2\eta\bar{\tau}}e^{\mathrm{T}}(t-\bar{\tau})Q_2e(t-\bar{\tau}) + e^{\mathrm{T}}(t)Q_3e(t)\\
&\quad - \mathrm{e}^{-2\eta\sigma}e^{\mathrm{T}}(t-\sigma)Q_3e(t-\sigma)\\
&\quad + e^{\mathrm{T}}(t)Q_4e(t) - \mathrm{e}^{-2\eta d}e^{\mathrm{T}}(t-d)Q_4e(t-d)\Big\},
\end{aligned}
$$

with $T = \mathrm{diag}\{t_1,\ldots,t_n\}$, $\Lambda = \mathrm{diag}\{\lambda_1,\ldots,\lambda_n\}$, $L^+ = \mathrm{diag}\{l_1^+,\ldots,l_n^+\}$, and $L^- = \mathrm{diag}\{l_1^-,\ldots,l_n^-\}$. Using Lemma 1 in Ref. [57], we have

$$
\begin{aligned}
&D^+V_2(t)\\
&\le 4\eta\,\mathrm{e}^{2\eta t}\Big\{\Big[g^{\mathrm{T}}(e(t)) - e^{\mathrm{T}}(t)L^-\Big]Te(t)\\
&\quad + \Big[e^{\mathrm{T}}(t)L^+ - g^{\mathrm{T}}(e(t))\Big]\Lambda e(t)\Big\}\\
&\quad + 2\mathrm{e}^{2\eta t}\Big\{\Big[g^{\mathrm{T}}(e(t)) - e^{\mathrm{T}}(t)L^-\Big]T\dot{e}(t)\\
&\quad + \Big[e^{\mathrm{T}}(t)L^+ - g^{\mathrm{T}}(e(t))\Big]\Lambda\dot{e}(t)\Big\}\\
&\quad + \mathrm{e}^{2\eta t}\Big\{e^{\mathrm{T}}(t)Q_1e(t) - \mathrm{e}^{-2\eta\bar{\tau}}e^{\mathrm{T}}(t-\tau(t))
\end{aligned}
$$

$$
\begin{aligned}
&\times Q_1 e(t-\tau(t)) + e^{\mathrm{T}}(t) Q_2 e(t) \\
&- \mathrm{e}^{-2\eta\bar{\tau}} e^{\mathrm{T}}(t-\bar{\tau}) Q_2 e(t-\bar{\tau}) + e^{\mathrm{T}}(t) Q_3 e(t) \\
&- \mathrm{e}^{-2\eta\sigma} e^{\mathrm{T}}(t-\sigma) Q_3 e(t-\sigma) \\
&+ e^{\mathrm{T}}(t) Q_4 e(t) - \mathrm{e}^{-2\eta d} e^{\mathrm{T}}(t-d) Q_4 e(t-d) \Big\}, \\
&D^+ V_3(t) \\
=\ & 4\eta\,\mathrm{e}^{2\eta t} \sum_{i=1}^{n} \Big\{ \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \alpha_i e_i(s) \Big[l_i^+ e_i(s) - l_i^- e_i(s) \Big] \mathrm{d}s \mathrm{d}\theta \Big\} \\
&+ 2\mathrm{e}^{2\eta t} \Big\{ \sum_{i=1}^{n} \Big\{ \alpha_i e_i(t) \Big[l_i^+ e_i(t) - l_i^- e_i(t) \Big] \bar{\tau} \Big\} \\
&- \sum_{i=1}^{n} \int_{t-\bar{\tau}}^{t} \alpha_i e_i(s) \Big[l_i^+ e_i(s) - l_i^- e_i(s) \Big] \mathrm{d}s \Big\} \\
=\ & 4\eta\,\mathrm{e}^{2\eta t} \Big\{ \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \bar{\alpha} e^{\mathrm{T}}(s) \Big[L^+ - L^- \Big] e(s) \mathrm{d}s \mathrm{d}\theta \Big\} \\
&+ 2\mathrm{e}^{2\eta t} \Big\{ e^{\mathrm{T}}(t) \Big[\bar{\alpha}\bar{\tau}(L^+ - L^-) \Big] e(t) \\
&- \int_{t-\bar{\tau}}^{t} e^{\mathrm{T}}(s) \bar{\alpha} [L^+ - L^-] e(s) \mathrm{d}s \Big\},
\end{aligned}
$$

where $\bar{\alpha} = \mathrm{diag}\{\alpha_1, \ldots, \alpha_n\}$. By using Lemma 1, we have

$$
\begin{aligned}
&D^+ V_3(t) \\
\le\ & 4\eta\,\mathrm{e}^{2\eta t} \frac{2}{\bar{\tau}^2} \Big\{ \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} e^{\mathrm{T}}(s) \mathrm{d}s \mathrm{d}\theta \Big\} \bar{\alpha} \\
&\Big[L^+ - L^- \Big] \Big\{ \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} e(s) \mathrm{d}s \mathrm{d}\theta \Big\} \\
&+ 2\mathrm{e}^{2\eta t} \Big\{ e^{\mathrm{T}}(t) \Big[\bar{\alpha}\bar{\tau}(L^+ - L^-) \Big] e(t) \\
&- \frac{1}{\bar{\tau}} \Big[\int_{t-\tau(t)}^{t} e^{\mathrm{T}}(s) \mathrm{d}s + \int_{t-\bar{\tau}}^{t-\tau(t)} e^{\mathrm{T}}(s) \mathrm{d}s \Big] \\
&\times (\bar{\alpha}[L^+ - L^-]) \Big[\int_{t-\tau(t)}^{t} e(s) \mathrm{d}s + \int_{t-\bar{\tau}}^{t-\tau(t)} e(s) \mathrm{d}s \Big] \Big\}, \\
&D^+ V_4(t) \\
=\ & \bar{\tau}\,\mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&- \int_{t-\bar{\tau}}^{t} \mathrm{e}^{2\eta s} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix} \mathrm{d}s \\
&+ \sigma^2 \mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_4 & R_5 \\ R_5^{\mathrm{T}} & R_6 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&- \sigma \int_{t-\sigma}^{t} \mathrm{e}^{2\eta s} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_4 & R_5 \\ R_5^{\mathrm{T}} & R_6 \end{bmatrix} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix} \mathrm{d}s \\
&+ \frac{\bar{\tau}^2}{2} \mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&- \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \mathrm{e}^{2\eta s} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3 \end{bmatrix} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix} \mathrm{d}s \mathrm{d}\theta \\
&+ \frac{\sigma^4}{2} \mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_4 & S_5 \\ S_5^{\mathrm{T}} & S_6 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&- \sigma^2 \int_{-\sigma}^{0} \int_{t+\theta}^{t} \mathrm{e}^{2\eta s} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_4 & S_5 \\ S_5^{\mathrm{T}} & S_6 \end{bmatrix} \begin{bmatrix} e(s) \\ \dot{e}(s) \end{bmatrix} \mathrm{d}s \mathrm{d}\theta, \\
\le\ & \bar{\tau}\,\mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&- \mathrm{e}^{2\eta(t-\bar{\tau})} \Big\{ \frac{1}{\tau(t)} \begin{bmatrix} \int_{t-\tau(t)}^{t} e(s) \mathrm{d}s \\ \varpi_1(t) \end{bmatrix}^{\mathrm{T}} \\
&\times \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} \int_{t-\tau(t)}^{t} e(s) \mathrm{d}s \\ \varpi_1(t) \end{bmatrix} + \frac{1}{\bar{\tau} - \tau(t)} \\
&\times \begin{bmatrix} \int_{t-\bar{\tau}}^{t-\tau(t)} e(s) \mathrm{d}s \\ \varpi_2(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} \int_{t-\bar{\tau}}^{t-\tau(t)} e(s) \mathrm{d}s \\ \varpi_2(t) \end{bmatrix} \Big\} \\
&+ \sigma^2 \mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_4 & R_5 \\ R_5^{\mathrm{T}} & R_6 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&- \mathrm{e}^{2\eta(t-\sigma)} \begin{bmatrix} \int_{t-\sigma}^{t} e(s) \mathrm{d}s \\ \varpi_4(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_4 & R_5 \\ R_5^{\mathrm{T}} & R_6 \end{bmatrix} \begin{bmatrix} \int_{t-\sigma}^{t} e(s) \mathrm{d}s \\ \varpi_4(t) \end{bmatrix} \\
&+ \frac{\bar{\tau}^2}{2} \mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&+ \frac{\sigma^4}{2} \mathrm{e}^{2\eta t} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_4 & S_5 \\ S_5^{\mathrm{T}} & S_6 \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \\
&- \frac{2}{\tau^2} \mathrm{e}^{2\eta(t-\bar{\tau})} \begin{bmatrix} \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} e(s) \mathrm{d}s \mathrm{d}\theta \\ \varpi_3(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3 \end{bmatrix} \\
&\times \begin{bmatrix} \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} e(s) \mathrm{d}s \mathrm{d}\theta \\ \varpi_3(t) \end{bmatrix} \\
&- 2\mathrm{e}^{2\eta(t-\sigma)} \begin{bmatrix} \int_{-\sigma}^{0} \int_{t+\theta}^{t} e(s) \mathrm{d}s \mathrm{d}\theta \\ \sigma e(t) - \int_{t-\sigma}^{t} e(s) \mathrm{d}s \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_4 & S_5 \\ S_5^{\mathrm{T}} & S_6 \end{bmatrix} \\
&\times \begin{bmatrix} \int_{-\sigma}^{0} \int_{t+\theta}^{t} e(s) \mathrm{d}s \mathrm{d}\theta \\ \sigma e(t) - \int_{t-\sigma}^{t} e(s) \mathrm{d}s \end{bmatrix},
\end{aligned}
$$

where $\varpi_1(t) = e(t) - e(t-\tau(t))$, $\varpi_2(t) = e(t-\tau(t)) - e(t-\bar{\tau})$, $\varpi_3(t) = \bar{\tau} e(t) - \int_{t-\tau(t)}^{t} e(s) \mathrm{d}s - \int_{t-\bar{\tau}}^{t-\tau(t)} e(s) \mathrm{d}s$, and $\varpi_4(t) = e(t) - e(t-\sigma)$. Finally, we have

$$
\begin{aligned}
&D^+ V_5(t) \\
=\ & d^2 \mathrm{e}^{2\eta t} \dot{e}(t) T_1 \dot{e}(t) - d \int_{t-d}^{t} \mathrm{e}^{-2\eta s} \dot{e}^{\mathrm{T}}(s) T_1 \dot{e}(s) \mathrm{d}s \\
\le\ & \mathrm{e}^{2\eta t} \Big\{ d^2 \dot{e}^{\mathrm{T}}(t) T_1 \dot{e}(t) - \mathrm{e}^{-2\eta d} [e(t) - e(t-d(t))]^{\mathrm{T}} \\
&\times T_1 [e(t) - e(t-d(t))] \\
&- \mathrm{e}^{-2\eta d} [e(t-d(t)) - e(t-d)]^{\mathrm{T}} \\
&\times T_1 [e(t-d(t)) - e(t-d)] \Big\}.
\end{aligned}
$$

For positive diagonal matrices Γ_1 and Γ_2 and with Assumption (H1), we have

$$
\begin{aligned}
&-2\mathrm{e}^{2\eta t} \Big\{ g^{\mathrm{T}}(e(t)) \Gamma_1 g(e(t)) + 2e^{\mathrm{T}}(t) \Gamma_1 (L^+ + L^-) \\
&\times g(e(t)) - 2e^{\mathrm{T}}(t) L^- \Gamma_1 L^+ e(t) \Big\} \ge 0, \qquad (13) \\
&-2\mathrm{e}^{2\eta t} \Big\{ g^{\mathrm{T}}(e(t-\tau(t))) \Gamma_2 g(e(t-\tau(t))) \\
&+ 2e^{\mathrm{T}}(t-\tau(t)) \Gamma_2 (L^+ + L^-) g(e(t-\tau(t))) \\
&- 2e^{\mathrm{T}}(t-\tau(t)) L^- \Gamma_2 L^+ e(t-\tau(t)) \Big\} \ge 0. \qquad (14)
\end{aligned}
$$

Furthermore, for a positive definite matrix N with appropriate dimensions and a scalar γ, the following equation holds:

$$
\begin{aligned}
&2\mathrm{e}^{2\eta t} [e^{\mathrm{T}}(t) N + \dot{e}^{\mathrm{T}}(t) \gamma N] \Big[-Ae(t-\sigma) + Bg(e(t)) \\
&+ Wg(e(t-\tau(t)) + Ke(t-d(t)) - \dot{e}(t) \Big] = 0. \qquad (15)
\end{aligned}
$$

By adding Eqs. (13)–(15) into Eq. (12) and using $X = NK$, we have

$$D^+V(t) \le \mathrm{e}^{2\eta t}\xi^{\mathrm{T}}(t)\Xi\xi(t), \tag{16}$$

where

$$\begin{aligned}\xi^{\mathrm{T}}(t) = \Big[&e^{\mathrm{T}}(t)\ \ e^{\mathrm{T}}(t-\tau(t))\ \ e^{\mathrm{T}}(t-\bar{\tau})\\ &e^{T}(t-\sigma)\ \ e^{\mathrm{T}}(t-d(t))\ \ e^{\mathrm{T}}(t-d)\\ &\dot{e}^{\mathrm{T}}(t)\ \ g^{\mathrm{T}}(e(t))\ \ g^{\mathrm{T}}(e(t-\tau(t)))\\ &\int_{t-\tau(t)}^{t} e^{\mathrm{T}}(s)\mathrm{d}s \int_{t-\bar{\tau}}^{t-\tau(t)} e^{\mathrm{T}}(s)\mathrm{d}s\ \ \int_{t-\sigma}^{t} e^{\mathrm{T}}(s)\mathrm{d}s\\ &\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} e^{\mathrm{T}}(s)\mathrm{d}s\mathrm{d}\theta\ \ \int_{-\sigma}^{0}\int_{t+\theta}^{t} e^{\mathrm{T}}(s)\mathrm{d}s\mathrm{d}\theta\Big].\end{aligned}$$

From Eq. (10), we can obtain

$$\begin{aligned}&\mathrm{e}^{-2\eta t}D^+V(t) \le \xi^{\mathrm{T}}(t)\Xi\xi(t) < 0,\\ &t \in [t_{k-1},\ t_k),\quad k \in \mathbb{Z}_+.\end{aligned} \tag{17}$$

The following proof is similar to that in Ref. [55]. From system (8), we know

$$\begin{aligned}&e(t_k) - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\\ =\ &e(t_k^-) - C_k\Big[e(t_k^-) - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\Big] - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\\ =\ &(I-C_k)\Big[e(t_k^-) - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\Big].\end{aligned} \tag{18}$$

Moreover, it follows from Eq. (9) that

$$\begin{aligned}&\begin{bmatrix} P & (I-C_k)^{\mathrm{T}}P\\ \star & P\end{bmatrix} \ge 0,\\ \Leftrightarrow\ &\begin{bmatrix} I & -(I-C_k)^{\mathrm{T}}\\ 0 & I\end{bmatrix}\begin{bmatrix} P & (I-C_k)^{\mathrm{T}}P\\ \star & P\end{bmatrix}\\ &\times\begin{bmatrix} I & 0\\ -(I-D_k)^{\mathrm{T}} & I\end{bmatrix} \ge 0\end{aligned} \tag{19}$$

$$\begin{aligned}\Leftrightarrow\ &\begin{bmatrix} P-(I-C_k)^{\mathrm{T}}P(I-C_k) & 0\\ \star & P\end{bmatrix} \ge 0\\ \Leftrightarrow\ &P-(I-C_k)^{\mathrm{T}}P(I-C_k) \ge 0.\end{aligned} \tag{20}$$

From Eqs. (18) and (19), we have

$$\begin{aligned}V_1(t_k) =\ &\Big[e(t_k) - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\Big]^{\mathrm{T}}\\ &\times P\Big[e(t_k) - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\Big]\\ =\ &\Big[e(t_k^-) - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\Big]^{\mathrm{T}}(I-C_k)^{\mathrm{T}}\\ &\times P(I-C_k)\Big[e(t_k^-) - A\int_{t_k-\sigma}^{t_k} e(s)\mathrm{d}s\Big]\\ =\ &V_1(t_k^-),\end{aligned}$$

which implies

$$V(t_k) \le V(t_k^-),\ \ k \in \mathbb{Z}_+. \tag{21}$$

By using Eqs. (18) and (21), we obtain

$$V(t) \le V(0),\ \ t \ge 0. \tag{22}$$

Hence, using Lemma 1, we have

$$\begin{aligned}&\Big\|\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big\|^2\\ =\ &\Big[\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big]^{\mathrm{T}}\Big[\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big] \le \sigma\int_{t-\sigma}^{t} e^{\mathrm{T}}(s)e(s)\mathrm{d}s\\ \le\ &\frac{\sigma}{\lambda_{\min}(Q_3)}\int_{t-\sigma}^{t} e^{\mathrm{T}}(s)Q_3e(s)\mathrm{d}s\\ \le\ &\frac{\sigma}{\lambda_{\min}(Q_3)}\mathrm{e}^{-2\eta(t-\sigma)}\int_{t-\sigma}^{t} \mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_3e(s)\mathrm{d}s\\ \le\ &\frac{\sigma}{\lambda_{\min}(Q_3)}\mathrm{e}^{-2\eta(t-\sigma)}V(0),\end{aligned}$$

which implies

$$\Big\|\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big\| \le \sqrt{\frac{\sigma}{\lambda_{\min}(Q_3)}V(0)}\ \ \mathrm{e}^{-\eta(t-\sigma)}. \tag{23}$$

Similarly, considering the definition of $V_1(t)$, we can obtain

$$\Big\|e(t) - A\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big\| \le \sqrt{\frac{1}{\lambda_{\min}(P)}V(0)}\ \ \mathrm{e}^{-\eta(t-\sigma)},$$

which together with the definition of $V_1(t)$ yields

$$\begin{aligned}&\big\|e(t)\big\|\\ \le\ &\Big\|A\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big\| + \Big\|e(t) - A\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big\|\\ \le\ &\max_{i\in\tilde{\Lambda}}a_i\Big\|\int_{t-\sigma}^{t} e(s)\mathrm{d}s\Big\| + \sqrt{\frac{1}{\lambda_{\min}(P)}V(0)}\ \ \mathrm{e}^{-\eta(t-\sigma)}\\ \le\ &\max_{i\in\tilde{\Lambda}}a_i\sqrt{\frac{\sigma}{\lambda_{\min}(Q_3)}V(0)}\ \ \mathrm{e}^{-\eta(t-\sigma)}\\ &+\sqrt{\frac{1}{\lambda_{\min}(P)}V(0)}\ \ \mathrm{e}^{-\eta(t-\sigma)}\\ \le\ &\Big\{\max_{i\in\tilde{\Lambda}}a_i\sqrt{\frac{\sigma}{\lambda_{\min}(Q_3)}}\\ &+\sqrt{\frac{1}{\lambda_{\min}(P)}}\Big\}\sqrt{V(0)}\ \ \mathrm{e}^{-\eta(t-\sigma)}.\end{aligned} \tag{24}$$

Here, we note

$$\begin{aligned}&V(0)\\ =\ &\Big[e(0) - A\int_{-\sigma}^{0} e(s)\mathrm{d}s\Big]^{\mathrm{T}}P\Big[e(0) - A\int_{-\sigma}^{0} e(s)\mathrm{d}s\Big]\\ &+2\sum_{i=1}^{n}\Big\{t_i\int_{0}^{e_i}\Big[g_i(s) - l_i^-s\Big]\mathrm{d}s + \lambda_i\int_{0}^{e_i}\Big[l_i^+s - g_i(s)\Big]\mathrm{d}s\Big\}\\ &+\int_{-\tau(0)}^{0}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_1e(s)\mathrm{d}s\\ &+\int_{-\bar{\tau}}^{0}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_2e(s)\mathrm{d}s + \int_{-\sigma}^{0}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_3e(s)\mathrm{d}s\end{aligned}$$

$$
\begin{aligned}
&+\int_{-d}^{0} \mathrm{e}^{2\eta s} e^{\mathrm{T}}(s) Q_4 e(s)\,\mathrm{d}s \\
&+2\sum_{i=1}^{n}\left\{\int_{-\bar{\tau}}^{0}\int_{\theta}^{0} \alpha_i e_i(s)\left[l_i^{+}e_i(s)-l_i^{-}e_i(s)\right]\mathrm{d}s\mathrm{d}\theta\right\} \\
&+\int_{-\bar{\tau}}^{0}\int_{\theta}^{0} \mathrm{e}^{2\eta s}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} R_1 & R_2\\ R_2^{\mathrm{T}} & R_3\end{bmatrix}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta \\
&+\sigma\int_{-\sigma}^{0}\int_{\theta}^{0} \mathrm{e}^{2\eta s}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} R_4 & R_5\\ R_5^{\mathrm{T}} & R_6\end{bmatrix}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta \\
&+\int_{-\bar{\tau}}^{0}\int_{\theta}^{0}\int_{\lambda}^{0} \mathrm{e}^{2\eta s}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} S_1 & S_2\\ S_2^{\mathrm{T}} & S_3\end{bmatrix}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\lambda\,\mathrm{d}\theta \\
&+\sigma^2\int_{-\sigma}^{0}\int_{\theta}^{0}\int_{\lambda}^{0} \mathrm{e}^{2\eta s}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} S_4 & S_5\\ S_5^{\mathrm{T}} & S_6\end{bmatrix}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\lambda\,\mathrm{d}\theta \\
&+d\int_{-d}^{0}\int_{\theta}^{0} \mathrm{e}^{2\eta s}\dot{e}^{\mathrm{T}}(s) T_1 \dot{e}(s)\,\mathrm{d}s\mathrm{d}\theta \\
\le\ &\Big\{2\lambda_{\max}(P)(1+\sigma^2\max_{i\in\tilde{\Lambda}}a_i) \\
&+2\lambda_{\max}(L^{+}-L^{-})\lambda_{\max}(T+\Lambda)+\bar{\tau}[\lambda_{\max}(Q_1) \\
&+\lambda_{\max}(Q_2)]+\sigma\lambda_{\max}(Q_3)+d\lambda_{\max}(Q_4) \\
&+2\bar{\tau}^2\lambda_{\max}(L^{+}-L^{-})+\bar{\tau}^2\lambda_{\max}(\mathfrak{R}_1) \\
&+\sigma^3\lambda_{\max}(\mathfrak{R}_2)+\bar{\tau}^3\lambda_{\max}(\mathfrak{S}_1)+\sigma^5\lambda_{\max}(\mathfrak{S}_2) \\
&+d^2\lambda_{\max}(T_1)\Big\}\|\varphi-\phi\|_{\tau^*}^2<\infty,
\end{aligned}
$$

where

$$
\mathfrak{R}_1=\begin{bmatrix} R_1 & R_2\\ R_2^{\mathrm{T}} & R_3\end{bmatrix},\quad \mathfrak{R}_2=\begin{bmatrix} R_4 & R_5\\ R_5^{\mathrm{T}} & R_6\end{bmatrix},
$$
$$
\mathfrak{S}_1=\begin{bmatrix} S_1 & S_2\\ S_2^{\mathrm{T}} & S_3\end{bmatrix},\quad \mathfrak{S}_2=\begin{bmatrix} S_4 & S_5\\ S_5^{\mathrm{T}} & S_6\end{bmatrix}.
$$

Substituting the above inequality into Eq. (24) gives

$$
\|e(t)\|\le \mathfrak{M}\|\varphi-\phi\|_{\tau^*}\mathrm{e}^{-\eta t},\quad t>0, \tag{25}
$$

where

$$
\begin{aligned}
\mathfrak{M}\ =\ &\mathrm{e}^{\eta\sigma}\left[\max_{i\in\tilde{\Lambda}}a_i\sqrt{\frac{\sigma}{\lambda_{\min}(Q_3)}}+\sqrt{\frac{1}{\lambda_{\min}(P)}}\right] \\
&\times\Big\{2\lambda_{\max}(P)(1+\sigma^2\max_{i\in\Lambda}a_i) \\
&+2\lambda_{\max}(L^{+}-L^{-})\lambda_{\max}(T+\Lambda) \\
&+\bar{\tau}[\lambda_{\max}(Q_1)+\lambda_{\max}(Q_2)]+\sigma\lambda_{\max}(Q_3) \\
&+d\lambda_{\max}(Q_4)+2\bar{\tau}^2\lambda_{\max}(L^{+}-L^{-}) \\
&+\bar{\tau}^2\lambda_{\max}(\mathfrak{R}_1)+\sigma^3\lambda_{\max}(\mathfrak{R}_2) \\
&+\bar{\tau}^3\lambda_{\max}(\mathfrak{S}_1)+\sigma^5\lambda_{\max}(\mathfrak{S}_2)+d^2\lambda_{\max}(T_1)\Big\}^{\frac{1}{2}}\ge 1.
\end{aligned}
$$

By Definition 1, this implies that system (8) is globally exponential stable with a convergence rate η. This completes the proof.

Remark 1 In Refs. [36]–[39], the authors studied the exponential synchronization of chaotic NNs and complex dynamical networks based on the impulsive effect. The impulsive controller design for the exponential synchronization of chaotic NNs with mixed delays has also been addressed in Ref. [40]. At the same time, the synchronization of chaotic NNs under the sampled-data control was widely investigated in Refs. [42], [46], and [49]. Inspired by the above work, we have investigated the impulsive effect on the exponential synchronization of NNs with leakage delay under the sampled-data feedback control. Here, we would like to point out that the impulsive effect and the sampled-data feedback control for the exponential synchronization of NNs are taken into account in this paper for the first time.

Remark 2 The authors in Ref. [58] considered the stability analysis of delayed NNs by using the triple-integral method and a new type of triple-integral term has been included in the Lyapunov–Krasovskii functional to derive delay-dependent criteria for the considered NNs. In the present paper, we introduce the double integral term $2\mathrm{e}^{2\eta t}\sum_{i=1}^{n}\left\{\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}\alpha_i e_i(s)\left[l_i^{+}e_i(s)-l_i^{-}e_i(s)\right]\mathrm{d}s\mathrm{d}\theta\right\}$ to the Lyapunov–Krasovskii functional (11), which is different from that in the existing literature. In additional, the term

$$
-\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}\mathrm{e}^{2\eta s}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} S_1 & S_2\\ S_2^{\mathrm{T}} & S_3\end{bmatrix}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta
$$

is handled as

$$
\begin{aligned}
&-\frac{2}{\tau^2}\mathrm{e}^{2\eta(t-\bar{\tau})}\begin{bmatrix}\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}e(s)\mathrm{d}s\mathrm{d}\theta\\ \varpi_3(t)\end{bmatrix}^{\mathrm{T}} \\
&\times\begin{bmatrix} S_1 & S_2\\ S_2^{\mathrm{T}} & S_3\end{bmatrix}\begin{bmatrix}\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}e(s)\mathrm{d}s\mathrm{d}\theta\\ \varpi_3(t)\end{bmatrix}
\end{aligned}
$$

with the help of Lemma 1 in $D^{+}V_4(t)$. It can been seen that the leakage delay term has a great impact on the dynamical behavior of NNs.

Now, let us consider the special case of leakage term $\sigma=0$ in systems (1) and (3), then the error system (8) is simplified as

$$
\begin{aligned}
&\dot{e}(t)=-Ae(t)+Bg(e(t))+Wg(e(t-\tau(t))) \\
&\quad+Ke(t-d(t)),\quad t>0, \\
&e(t_k)-e(t_k^{-})=-C_k e(t_k^{-}),\ t\ne t_k,\ t_k\in\mathbb{Z}_{+}.
\end{aligned}\tag{26}
$$

We introduce the Lyapunov–Krasovskii functional for system (26)

$$
V(t)=\tilde{V}_1(t)+\tilde{V}_2(t)+V_3(t)+\tilde{V}_4(t)+V_5(t),
$$

where

$$
\begin{aligned}
\tilde{V}_1(t)\ &=\ \mathrm{e}^{2\eta t}e(t)^{\mathrm{T}}Pe(t), \\
\tilde{V}_2(t)\ &=\ 2\mathrm{e}^{2\eta t}\sum_{i=1}^{n}\Big\{t_i\int_{0}^{e_i}\left[g_i(s)-l_i^{-}s\right]\mathrm{d}s \\
&\quad+\lambda_i\int_{0}^{e_i}\left[l_i^{+}s-g_i(s)\right]\mathrm{d}s\Big\} \\
&\quad+\int_{t-\tau(t)}^{t}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_1e(s)\,\mathrm{d}s \\
&\quad+\int_{t-\bar{\tau}}^{t}\mathrm{e}^{2\eta s}e^{\mathrm{T}}(s)Q_2e(s)\,\mathrm{d}s
\end{aligned}
$$

$$+\int_{t-d}^{t} \mathrm{e}^{2\eta s} e^{\mathrm{T}}(s) Q_4 e(s)\,\mathrm{d}s,$$

$$\tilde{V}_4(t) = \int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} \mathrm{e}^{2\eta s}\begin{bmatrix} e(s) \\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3\end{bmatrix}\begin{bmatrix} e(s) \\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\theta + \int_{-\bar{\tau}}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \mathrm{e}^{2\eta s}\begin{bmatrix} e(s) \\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}} \times \begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3\end{bmatrix}\begin{bmatrix} e(s) \\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\mathrm{d}\lambda\,\mathrm{d}\theta.$$

By using a similar method to the one employed in Theorem 1, we can easily obtain the following corollary for system (26).

Corollary 1 For given scalars $\bar{\tau} > 0$ and $\mu \geq 0$, assume that assumptions H1 and H3 hold. Then, master system (1) with $\sigma = 0$ and slave system (3) with $\sigma = 0$ are exponentially synchronous with decay rate η if there exist $P > 0$, $Q_k > 0$, $(k = 1,2,4)$, $T_1 > 0$, $\begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3\end{bmatrix} > 0$, $\begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3\end{bmatrix} > 0$, $N > 0$, X, and diagonal matrices $T > 0$, $\Lambda > 0$, $\bar{\alpha} > 0$, $\Gamma_i > 0$ $(i = 1,2)$ such that the following LMIs hold:

$$\begin{bmatrix} P & (I - C_k)^{\mathrm{T}} P \\ * & P\end{bmatrix} \geq 0, \quad k \in \mathbb{Z}_+, \tag{27}$$

$$\Psi < 0, \tag{28}$$

where $\Psi = (\Psi_{m,l})_{11\times 11}$ with

$$\begin{aligned}
\Psi_{1,1} &= 2\eta P - NA - A^{\mathrm{T}}N^{\mathrm{T}} - 4\eta(L^- T - L^+\Lambda) + Q_1 + Q_2 + Q_4 + 2\bar{\tau}\bar{\alpha}[L^+ - L^-] + \bar{\tau}R_1 - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3 + \frac{\bar{\tau}^2}{2}S_1 - 2\mathrm{e}^{-2\eta\bar{\tau}}S_3 - \mathrm{e}^{-2\eta d}T_1 - 2L^-\Gamma_1 L^+,\\
\Psi_{1,2} &= \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3, \quad \Psi_{1,4} = \mathrm{e}^{-2\eta d}T_1 + X,\\
\Psi_{1,6} &= P - L^- T + L^+\Lambda + \bar{\tau}R_2 + \frac{\bar{\tau}^2}{2}S_2 - N - \gamma A^{\mathrm{T}}N^{\mathrm{T}},\\
\Psi_{1,7} &= 2\eta T - 2\eta\Lambda + \Gamma_1(L^+ + L^-) + NB,\\
\Psi_{1,8} &= NW, \quad \Xi_{1,9} = -\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2 + 2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}S_3,\\
\Psi_{1,10} &= 2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}S_3, \quad \Psi_{1,11} = -2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}S_2,\\
\Psi_{2,2} &= -(1-\mu)\mathrm{e}^{-2\eta\bar{\tau}}Q_1 - 2\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3 - 2[L^+\Gamma_2 L^-],\\
\Psi_{2,3} &= \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3, \quad \Psi_{2,8} = \Gamma_2[L^+ + L^-],\\
\Psi_{2,9} &= \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2, \quad \Psi_{2,10} = -\frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2,\\
\Psi_{3,3} &= -\mathrm{e}^{-2\eta\bar{\tau}}Q_2 - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_3, \quad \Psi_{3,10} = \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_2,\\
\Psi_{4,4} &= -2\mathrm{e}^{-2\eta d}T_1, \quad \Xi_{4,5} = \mathrm{e}^{-2\eta d}T_1,\\
\Psi_{4,6} &= \gamma X^{\mathrm{T}}, \quad \Psi_{5,5} = -\mathrm{e}^{-2\eta d}Q_4 - \mathrm{e}^{-2\eta d}T_1,\\
\Psi_{6,6} &= \bar{\tau}R_3 + \frac{\bar{\tau}^2}{2}S_3 + d^2T_1 - \gamma(N + N^{\mathrm{T}}),\\
\Psi_{6,7} &= T - \Lambda + \gamma NB, \quad \Psi_{6,8} = \gamma NW,\\
\Psi_{7,7} &= -2\Gamma_1, \quad \Psi_{8,8} = -2\Gamma_2,\\
\Psi_{9,9} &= -\frac{2}{\bar{\tau}}\bar{\alpha}[L^+ - L^-] - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_1 - \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_3,\\
\Psi_{9,10} &= -\frac{2}{\bar{\tau}}\bar{\alpha}[L^+ - L^-] - \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_3,\\
\Psi_{9,11} &= \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_2,\\
\Psi_{10,10} &= -\frac{2}{\bar{\tau}}\bar{\alpha}[L^+ - L^-] - \frac{\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}}R_1 - \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_3,\\
\Psi_{10,11} &= \frac{2\mathrm{e}^{-2\eta\bar{\tau}}}{\bar{\tau}^2}S_2,\\
\Psi_{11,11} &= \frac{8\eta}{\bar{\tau}^2}\bar{\alpha}[L^+ - L^-] - \frac{2}{\bar{\tau}^2}\mathrm{e}^{-2\eta\bar{\tau}}S_1,
\end{aligned}$$

and the other entries of Ψ are zero. Moreover, the sampled-data controller gain matrix is given by $K = N^{-1}X$.

When we left the impulsive effect in systems (1) and (3), then the master and the slave systems can be described as follows:

$$\begin{aligned}
\dot{x}(t) &= -Ax(t-\sigma) + Bf(x(t)) + Wf(x(t-\tau(t))) + J(t),\\
\dot{y}(t) &= -Ay(t-\sigma) + Bf(y(t)) + Wf(y(t-\tau(t))) + u(t) + J(t).
\end{aligned} \tag{29}$$

Therefore, the error system becomes

$$\begin{aligned}
\dot{e}(t) &= -Ae(t-\sigma) + Bg(e(t)) + Wg(e(t-\tau(t))) + Ke(t-d(t)).
\end{aligned} \tag{30}$$

From error system (30), we arrive at the following Corollary.

Corollary 2 For given scalars $\bar{\tau} > 0$ and $\mu \geq 0$, assume that assumptions H1 and H2 hold. Then, master system and slave system in Eq. (29) are exponentially synchronous with decay rate η if there exist $P > 0$, $Q_k > 0$, $(k = 1,2,3)$, $T_1 > 0$, $\begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3\end{bmatrix} > 0$, $\begin{bmatrix} R_4 & R_5 \\ R_5^{\mathrm{T}} & R_6\end{bmatrix} > 0$, $\begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3\end{bmatrix} > 0$, $\begin{bmatrix} S_4 & S_5 \\ S_5^{\mathrm{T}} & S_6\end{bmatrix} > 0$, $N > 0$, X, and diagonal matrices $T > 0$, $\Lambda > 0$, $\bar{\alpha} > 0$, $\Gamma_i > 0$ $(i = 1,2)$ such that the following LMI holds:

$$\Xi < 0, \tag{31}$$

where Ξ is defined in Theorem 1.

When we remove the impulsive effect and the leakage term in systems (1) and (3), the master and the slave systems can be described as follows:

$$\begin{aligned}
\dot{x}(t) &= -Ax(t) + Bf(x(t)) + Wf(x(t-\tau(t))) + J(t),\\
\dot{y}(t) &= -Ay(t) + Bf(y(t)) + Wf(y(t-\tau(t))) + u(t) + J(t).
\end{aligned} \tag{32}$$

Hence, the error system becomes

$$\begin{aligned}
\dot{e}(t) &= -Ae(t) + Bg(e(t)) + Wg(e(t-\tau(t))) + Ke(t-d(t)).
\end{aligned} \tag{33}$$

Then, the following corollary is derived.

Corollary 3 For given scalars $\bar{\tau} > 0$ and $\mu \geq 0$, assume that assumption H1 holds. Then, master system and slave system in Eq. (32) are exponentially synchronous with decay rate η if there exist $P > 0$, $Q_k > 0$, $(k = 1,2,4)$, $T_1 > 0$, $\begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} > 0$, $\begin{bmatrix} S_1 & S_2 \\ S_2^{\mathrm{T}} & S_3 \end{bmatrix} > 0$, $N > 0$, X, and diagonal matrices $T > 0$, $\Lambda > 0$, $\bar{\alpha} > 0$, $\Gamma_i > 0$ $(i = 1,2)$ such that the following LMI holds:

$$\Psi < 0, \tag{34}$$

where Ψ is defined in Corollary 1.

Remark 3 The authors in Ref. [59] studied robust $\mathscr{H}_\infty$ filtering for uncertain impulsive stochastic systems under sampled measurements, and have designed a stochastically stable filter using the locally sampled measurements, which ensures both robust stochastic stability and a prescribed level of $\mathscr{H}_\infty$ performance for the filtering error dynamics for all admissible uncertainties. Meanwhile, the exponential stability of impulsive systems with application to uncertain sampled-data systems was discussed in Ref. [60]. It should be noted that there are no results available in the existing literature related to the dynamics of impulsive systems with sampled data interval delays other than Refs. [59] and [60]. Motivated by the above discussion, we have introduced the impulsive effects on the exponential synchronization of delayed NNs with leakage delays and sampled data interval delays. The sampled data control has been assimilated as a particular delay $d(t)$ in the considered delayed NNs where the impulsive effects affect the system dynamics. Since there are no results published with the combination of impulsive effects and sampling interval delays for delayed NNs, this is a motivation of this research work.

4. Numerical examples

Here, we provide two numerical examples to illustrate the benefits and effectiveness of the suggested methodology for the synchronization of delayed NNs.

Example 1 Consider the delayed NN (1) with the following parameters:

$$A = \begin{bmatrix} 1.1 & 0 \\ 0 & 1.2 \end{bmatrix}, \quad B = \begin{bmatrix} 1.1 & -0.6 \\ 1.1 & -1.2 \end{bmatrix},$$
$$W = \begin{bmatrix} -2.6 & -0.43 \\ -0.5 & -1.4 \end{bmatrix}, \quad C_k = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The activation functions are taken as follows:

$$f_1(x_1(t)) = f_2(x_2(t)) = \tanh x(t).$$

It is clear that assumption H1 is satisfied, with $l_1^- = l_2^- = 0$ and $l_1^+ = l_2^+ = 1$. Thus, we can obtain the following parameters:

$$L_1 = \mathrm{diag}\{0,0,0\}, \quad L_2 = \mathrm{diag}\{1,1,1\}.$$

By using MATLAB LMI control toolbox, from Theorem 1 and Corollary 1, we calculate the maximum sampling period and the maximum exponential convergence rate for fixed $\gamma = \mu = 0.25$, $\bar{\tau} = 1$ and different values of σ, which are listed in Tables 1 and 2. It describes the relationship between the maximum sampling period and the maximum exponential convergence rate. In the simulation, the time-varying delay is $\tau(t) = \frac{\mathrm{e}^t}{\mathrm{e}^t+1}$, which gives $\bar{\tau} = 1$ and $\mu = 0.25$. The numerical simulation results of master system (1) and slave system (3) with $u(t) = 0$ are displayed for different values σ in Figs. 1–4 with the initial states $x(t) = [0.4\ 0.25]^{\mathrm{T}}$ and $y(t) = [0.35\ 0.25]^{\mathrm{T}}$ for $t \in [-1,0]$.

Let $\bar{\tau} = 1$, $\sigma = d = 0.1$, $\mu = \gamma = 0.25$, $\eta = 0.1$. Then, we obtain the gain matrix from Theorem 1 as follows:

$$K = \begin{bmatrix} -5.2429 & -0.7919 \\ -1.9066 & -2.7773 \end{bmatrix}.$$

The simulation result of error system (8) is given in Fig. 5. From Tables 1 and 2, it is clear that Corollary 1 provides less conservative results than Theorem 1. This is due to the maximum sampling period and the exponential convergence rate strongly affected by the leakage delay. It has been seen that the dynamical behavior is affected by the increased value of the leakage delay, as shown in Tables 1 and 2 and Figs. 1–5.

Table 1. Maximum value of d.

Method η	0.01	0.03	0.07	0.15	0.2
Theorem 1 $\sigma = 0$	0.15	0.14	0.13	0.12	0.11
Theorem 1 $\sigma = 0.1$	0.12	0.12	0.11	0.10	0.09
Theorem 1 $\sigma = 0.2$	0.10	0.10	0.09	0.08	0.07
Theorem 1 $\sigma = 0.3$	0.09	0.08	0.08	0.07	0.07
Corollary 1	0.15	0.14	0.13	0.12	0.11

Table 2. Maximum value of η.

Method d	0.1	0.09	0.05	0.03	0.01
Theorem 1 $\sigma = 0$	0.29	0.36	0.73	1.01	1.54
Theorem 1 $\sigma = 0.1$	0.17	0.24	0.60	0.87	1.28
Theorem 1 $\sigma = 0.2$	0.04	0.10	0.45	0.72	1.19
Corollary 1	0.29	0.36	0.73	1.01	1.54

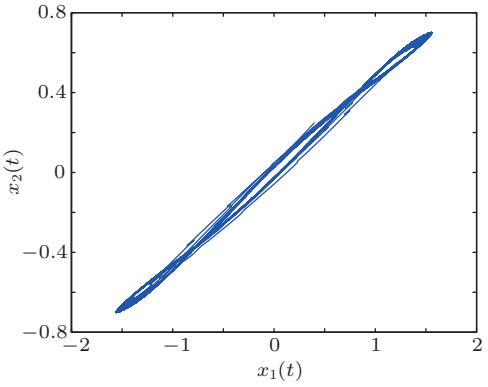


Fig. 1. (color online) The dynamic behavior of master system (1) with $\sigma = 0$.

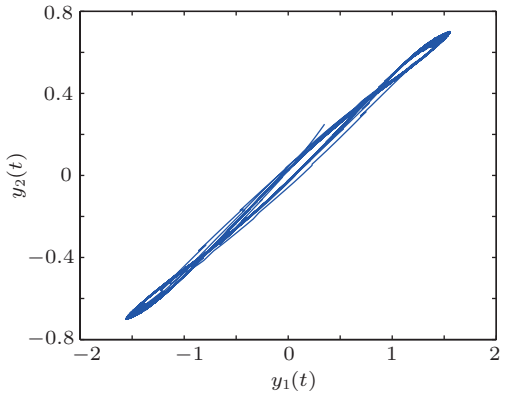


Fig. 2. (color online) The dynamic behavior of slave system (3) with $u(t) = \sigma = 0$.

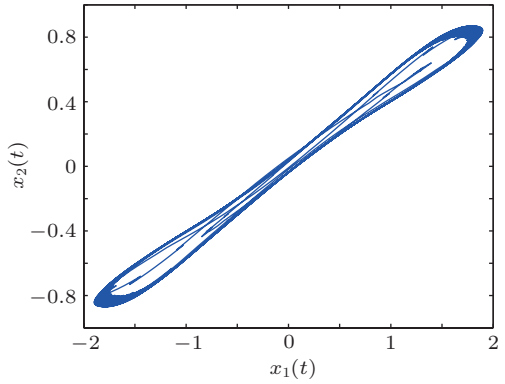


Fig. 3. (color online) The dynamic behavior of master system (1) with $\sigma = 0.2$.

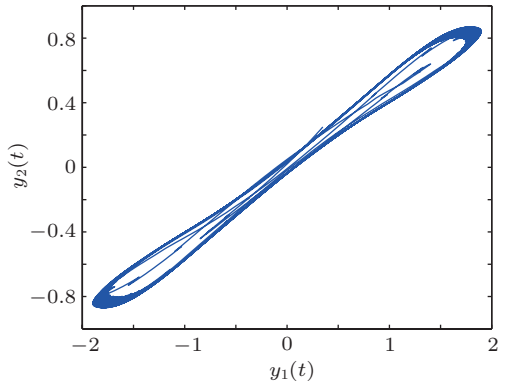


Fig. 4. (color online) The dynamic behavior of slave system (3) with $u(t) = 0$ and $\sigma = 0.2$.

Example 2 Consider the delayed NN (32) with the following parameters:[26]

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1+\frac{\pi}{4} & 20 \\ 0.1 & 1+\frac{\pi}{4} \end{bmatrix},$$
$$W = \begin{bmatrix} -\frac{1.3\sqrt{2}\,\pi}{4} & 0.1 \\ 0.1 & -\frac{1.3\sqrt{2}\,\pi}{4} \end{bmatrix}.$$

The activation functions are given by

$$f_1(x_1(t)) = f_2(x_2(t)) = \frac{1}{2}\Big[|x(t)+1| - |x(t)-1|\Big],$$

which gives $l_1^- = l_2^- = 0$ and $l_1^+ = l_2^+ = 1$, i.e.,

$$L_1 = \mathrm{diag}\{0,0,0\}, \quad L_2 = \mathrm{diag}\{1,1,1\}.$$

By Corollaries 2 and 3, we obtain the maximum sampling period and the maximum exponential convergence rate for fixed $\gamma = \mu = 0.25$ and $\bar{\tau} = 1$ as listed in Table 3. From Table 3, it is clear that Corollary 3 provides less conservative results than Corollary 2. The time-varying delay is chosen as $\tau(t) = \frac{e^t}{e^t+1}$ satisfying $\bar{\tau} = 1$ and $\mu = 0.25$ for the simulation. The chaotic behaviors of the master system and the slave system in Eq. (32) with $u(t) = 0$ are shown in Figs. 6 and 7, respectively, with the initial states chosen as $x(t) = [0.1 \;\; 0.1]^{\mathrm{T}}$ and $y(t) = [-0.3 \;\; -0.3]^{\mathrm{T}}$ for $t \in [-1, 0]$. In the case of the leakage term $\sigma = 0.1$, the chaotic behaviors of the master system and the slave system in Eq. (29) are given in Figs. 8 and 9.

Table 3. Maximum values of d and η.

Method η	0.01	0.08	0.15	0.2	0.25
Corollary 2: $\sigma = 0.1$	0.15	0.14	0.13	0.13	0.12
Corollary 2: $\sigma = 0.2$	0.13	0.12	0.11	0.10	0.10
Corollary 3: d	0.17	0.16	0.15	0.14	0.14
Method d	0.1	0.09	0.05	0.03	0.01
Corollary 2: $\sigma = 0.1$	0.43	0.51	0.93	1.23	1.66
Corollary 2: $\sigma = 0.2$	0.27	0.35	0.74	0.99	1.42
Corollary 3: η	0.55	0.64	1.07	1.37	1.93

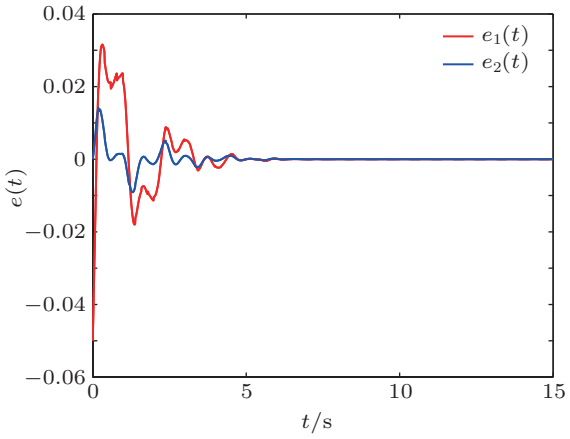


Fig. 5. (color online) The error trajectories are converging to zero.

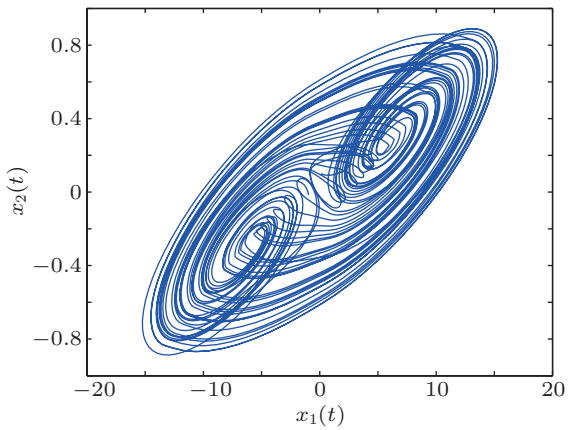


Fig. 6. (color online) The dynamic behavior of the master system in Eq. (32).

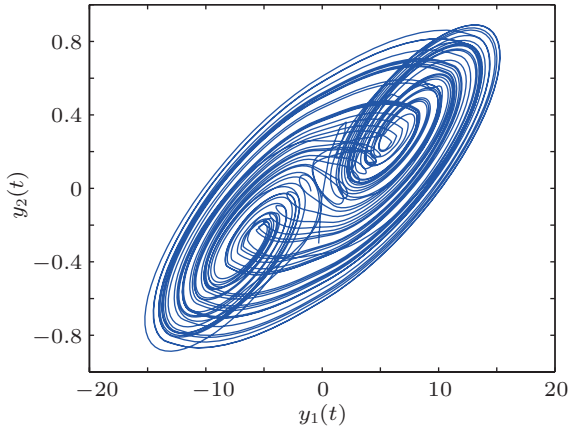


Fig. 7. (color online) The dynamic behavior of the slave system in Eq. (32) without $u(t)$.

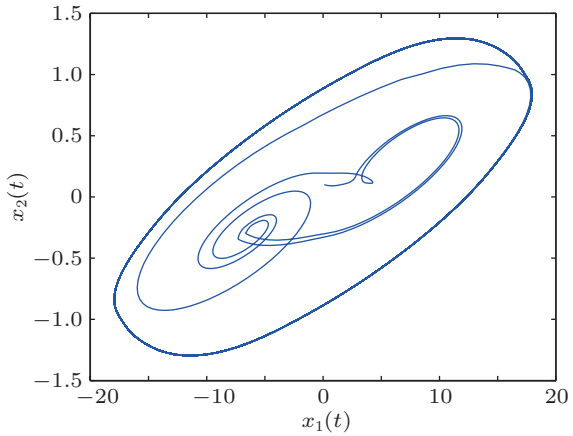


Fig. 8. (color online) The dynamic behavior of the master system in Eq. (29) with $\sigma = 0.1$.

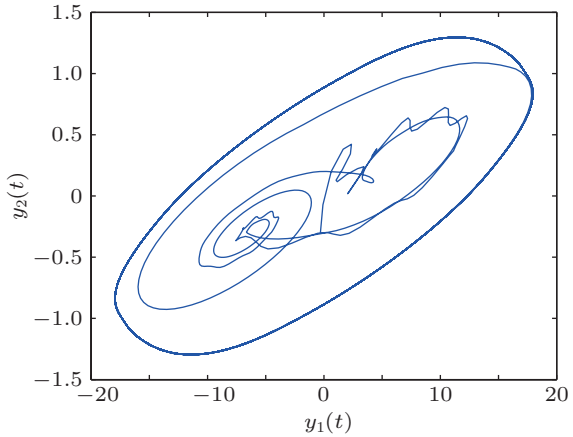


Fig. 9. (color online) The dynamic behavior of the slave system in Eq. (29) with $u(t) = 0$ and $\sigma = 0.1$.

Let $\bar{\tau} = 1$, $d = 0.1$, $\mu = \gamma = 0.25$, $\eta = 0.1$. Then, by Corollary 3, we obtain the gain matrix

$$K = \begin{bmatrix} -6.5703 & -13.3787 \\ -0.1638 & -5.1988 \end{bmatrix},$$

and the response of error system (33) is given in Fig. 10.

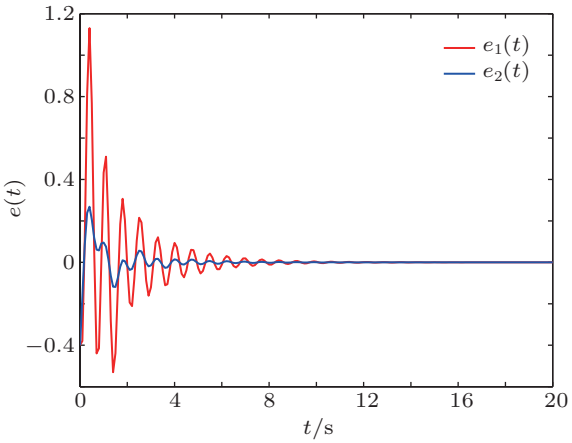


Fig. 10. (color online) The error trajectories are converging to zero.

Remark 4 Figures 5 and 10 depict the synchronization errors of state variables between the master system and the slave system. The numerical simulations clearly verify the effectiveness and feasibility of the developed feedback controller under sampled data to ensure the synchronization of delayed NNs with the leakage delay and impulsive effect.

5. Conclusion

The impulsive effect on the exponential synchronization of delayed NNs with leakage delay under the sampled-data feedback control has been considered. Based on an appropriate Lyapunov–Krasovskii functional and the input delay approach, sufficient conditions that ensure the exponential synchronicity of the delayed neural network have been derived. In the numerical simulations, it has been shown that the maximum sampling period and the exponential convergent rate are affected by the leakage delay with the impulsive effect.

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