Ising model on evolution networks and its application on opinion formation*

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Many phenomena show that in a favorable circumstance an agent still has an updating possibility, and in an unfavorable circumstance an agent also has a possibility of holding its own state and reselecting its neighbors. To describe this kind of phenomena an Ising model on evolution networks was presented and used for consensus formation and separation of opinion groups in human population. In this model the state-holding probability p and selection-rewiring probability q were introduced. The influence of this mixed dynamics of spin flips and network rewiring on the ordering behavior of the model was investigated. p hinders ordering of opinion networks and q accelerates the dynamical process of networks. Influence of q on the ordering and separating stems from its effect on average path length of networks.

Keywords: opinion formation, Ising model, evolution networks

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1. Introduction

The emergence of social behavior at the global population level has been widely studied.[1–9] The application of the statistical physics models to social dynamics is called sociophysics.[10,11] With statistical analysis of networks, the physics of networks[12] becomes a new field, and supplies important clues about how networks function and even offers predictions of their future behavior. As one extremely simple but very important model, the Ising model has been investigated on networks in recent years. The Ising model on a one-dimensional small-world (SW) network undergoes phase transition, which shows mean-field behavior.[13–19] An effective phase transition with a variety of temperature is also found on scale-free (SF) networks.[20,21] Being suited to investigate the order–disorder transition, the Ising model has been used on many real systems,[22–27] particularly, on the opinion formation systems.[28]

Most of these studies focus on the interaction among agents and the effect of the topology upon the evolution of the dynamical processes of networks.[29] However, in some real systems, the dynamical processes also modify the topology of networks.[30–38] Up to now, some studies have considered that complex feedback affects opinion formation.[39–42] Kozma and Barrat studied consensus formation in the Deffuant model on adaptive networks,[39] and Nardini *et al.* studied the co-evolving opinion formation in the Voter model.[40] Wang and Cai explored the continuous Sznajd opinion model on an adaptive network.[41] Benczik *et al.* investigated voter dynamics on an adaptive random network.[42]

These models mentioned above describe many physical and social phenomena, but some complex evolution phenomena are still not included. In reality, there exists a kind of phenomena: in a favorable circumstance an agent not only shows tendency to hold its own opinion but also has an updating possibility; and in an unfavorable circumstance an agent also has a possibility of holding its own state and reselecting its neighbors. The consensus formation and opinion group separation of human population are typical examples. To describe this kind of phenomena the Ising model on evolution networks is presented in this paper. The present model is suitable for describing co-evolution of the state and topological structure of networks, such as the process of consensus formation and opinion group separation on opinion networks.

2. Model and method

The present model is built on the basis of evolution networks and the Ising model. The agent system is described by an evolving network. Each agent corresponds to a node of the network and the coupling between two agents is represented by using the edge linking corresponding nodes. The Ising model is used for describing the state of agents (nodes in networks). The network can evolve in both states of nodes and topology of the network with time.

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Firstly, small-world (SW) networks are used for representing the agent system. In fact, scale-free networks or hierarchical networks can also be used. As the initial network the average degree $\langle k\rangle = 10$ and rewiring probability $\phi = 0.05$ are taken. The states of agents are described by the Ising model, thus each agent (node) has two permitted states: $+1$ and -1, which represent spin-up and spin-down, yes and no, or positive and negative, respectively. In the initial network the state of each node is taken from +1 and -1 randomly.

As time goes on, the network evolves in both topological structure and states of nodes. The state evolution is caused by the coupling between neighboring nodes in the network. The coupling strength between the i-th and j-th nodes can be expressed by the coupling energy $-J_{ij}s_is_j$, where $s_i = +1$ or -1 is the state of the i-th node. For nearest neighboring nodes the coupling constant has the value $J_{ij} = J$ and otherwise $J_{ij} = 0$. The state evolution of each node can be carried out by the Monte Carlo (MC) simulation based on the Metropolis algorithm.

The evolution of state and topological structure goes as follows. At the beginning, a node i is randomly chosen. The coupling energy can be calculated as

$$\varepsilon_i = -Js_i\sum_{j=1}^{k_i} s_j, \tag{1}$$

where k_i is the number of nearest neighboring nodes for the i-th node. Smaller ε_i implies that more nearest neighboring nodes possess the same state as that of the i-th node. After updating the state of the i-th node from s_i to $-s_i$ the coupling energy is symbolized as ε_i'. If $\varepsilon_i' > \varepsilon_i$, the number of the nearest neighboring nodes with the same state is greater than that with the opposite state. In this case the i-th node is situated in a local favorable circumstance and does not want to change the circumstance (i.e. does not want to replace the nearest neighboring nodes by other nodes), so only the state evolution can take place and the topology evolution does not happen. It makes the i-th node tend to keep the present state and update its state with a small probability. In this case ($\varepsilon_i' > \varepsilon_i$), the node updates the state from s_i to $-s_i$ with a Boltzmann-type probability

$$\mu = \exp[-(\varepsilon_i' - \varepsilon_i)/k_{\rm B}T], \tag{2}$$

and holds the state s_i with the probability $1-\mu$ [see Fig. 1(a)]. In Eq. (2) $k_{\rm B}$ is Boltzmann constant and T is the dimensionless temperature in units of $J/k_{\rm B}$. The parameter T reflects on the influence of the thermal fluctuation.[43] It is an overall approximation for all random events which influence the state-selection of nodes. The larger T is, the larger the randomness of state-selection is. If $\varepsilon_i' \le \varepsilon_i$, the number of the nearest neighboring nodes with the same state is less than that with the opposite state. In this case the i-th node is situated in a local unfavorable circumstance. The node can make a choice: updating its state to meet the circumstance or keeping the present state and changing the circumstance. Thus, when $\varepsilon_i' \le \varepsilon_i$, the i-th node holds the present state with the state-holding probability p or updates the state from s_i to $-s_i$ with the probability $1-p$, as shown in Fig. 1(b). Along with taking state holding, the i-th node rewires the links of the nearest neighboring nodes with the opposite state by the selection-rewiring probability q. The latter operation leads to the network topology evolution.

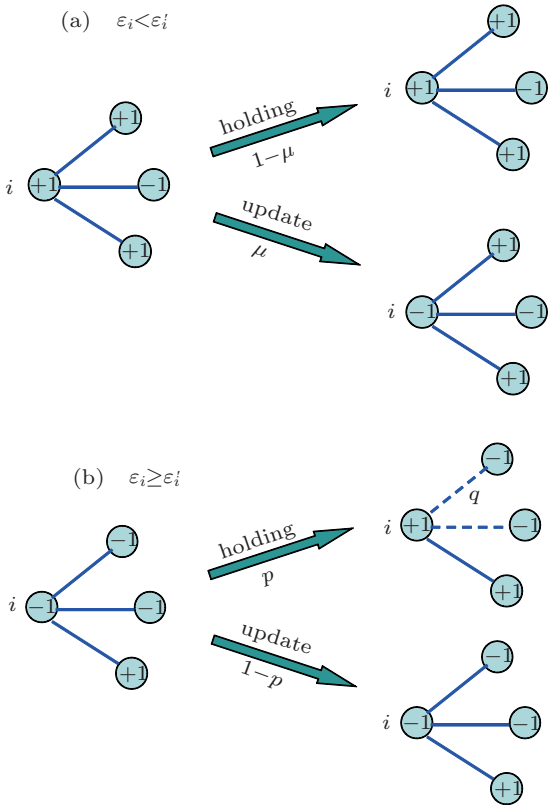


Fig. 1. (color online) Illustration of the Ising model on evolution networks. The ε_i' and ε_i are the coupling energy of the i-th node after and before the state is updated, respectively. (a) If $\varepsilon_i' > \varepsilon_i$, the state of the i-th node updates from s_i to $-s_i$ with a Boltzmann-type probability μ and holds with the probability $1-\mu$. (b) If $\varepsilon_i' \le \varepsilon_i$, the state updates from s_i to $-s_i$ with the probability $1-p$ and holds with p, p is the state-holding probability. Along with state holding, the i-th node rewires the links of nearest neighboring nodes with the opposite state by the selection rewiring probability q.

3. Results and discussion

3.1. Ising model on static networks

When the selection-rewiring probability $q = 0$, the present model degenerates to the model on static networks, but it is still different from the known Ising model on static networks. In the present case the states of nodes can change, but the links of nodes keep unvaried, i.e. only state evolution takes place and topology evolution does not occur. If not only $q = 0$ but also the state-holding probability $p = 0$, the present model degenerates to the previous Ising model on static networks.

In the present Ising model on static networks, the state evolution is dependent on the state-holding probability p. The parameter p describes this possibility: in a local unfavorable circumstance a node still holds its own state with the probability p, e.g., a person holds on to one's own views even he is in the minority at the local area. To represent the state evolution the order parameter $M = |\frac{1}{N}\sum_i s_i|$ is introduced for statistically describing the average state of nodes in a system, where N is the total number of nodes. For a ferromagnetic system without applied field, M expresses the spontaneous magnetization. To check the influence of the state-holding probability p on the state-ordering behavior (magnetization) of nodes, the relation of the order parameter M and the temperature T was calculated on SW networks with size $N = 1000$ at a series of p [see Fig. 2(a)]. Figure 2(a) indicates that there exists a critical transition as the order parameter M varies with temperature. It shows when the temperature is larger than the critical transition temperature T_C ($T > T_C$), the thermal fluctuation makes the system become completely random and the order parameter M reaches zero. To get the transition temperature T_C, the Binder's fourth-order cumulant U_N and $[\langle M\rangle]N^{1/4}$ were calculated as a function of the temperature [see Fig. 2(b)]. The Binder cumulant has the form[14,15]

$$U_N = 1 - \frac{[\langle M^4\rangle]}{3[\langle M^2\rangle]^2}, \tag{3}$$

where $\langle\cdots\rangle$ and $[\cdots]$ denote the thermal average and the average over different network realizations, respectively. In computations over 2×10^4 MC steps after discarding 2×10^4 MC steps were taken for the thermal average at each temperature, and over 100 configurations were taken for the average over different network realizations. It can be seen from the lower left inset of Fig. 2(b) that U_N–T curves show a unique crossing point at $T_C \approx 6.2$ (in units of J/k_B) for networks with various sizes. On the other hand, $[\langle M\rangle]N^{\beta/\nu}$ has been also used for determining the critical transition temperature.[16] It is well known that the phase transition of the Ising model on SW networks shows mean-field behavior,[17] and corresponding critical exponent $\alpha = 0$, $\beta = 1/2$, and $\nu = (2-\alpha)/d = 2$, where the dimension $d = 1$ for one-dimensional SW networks. Thus, $\beta/\nu = 1/4$. The $[\langle M\rangle]N^{1/4}$–$T$ curves have been calculated and plotted in the upper right inset of Fig. 2(b). It also shows a unique crossing point $T_C \approx 6.2$ for networks with various sizes.

Using the plots of the Binder cumulant U_N and $[\langle M\rangle]N^{1/4}$ against the temperature, the critical transition temperature T_C was obtained at a series of the state-holding probability p [see Fig. 2(b)]. Figure 2(b) indicates that T_C decreases with increment of p. It can be explained as follows. In an Ising system the coupling between the nearest neighboring nodes (denoted by J) makes the system ordered and the thermal fluctuation (denoted by T) makes the system random. These two opposite actions compete with each other. At the critical transition temperature T_C, they attain to equilibrium. When $\varepsilon_i' \le \varepsilon_i$, for the previous Ising model on static networks the coupling action tries hard to make the state update, but for the present Ising model on static networks the state holds with the state-holding probability p. Therefore, introduction of p corresponds to reducing the coupling action and promoting random. As a result the critical transition temperature T_C decreases with the increase of the state-holding probability p.

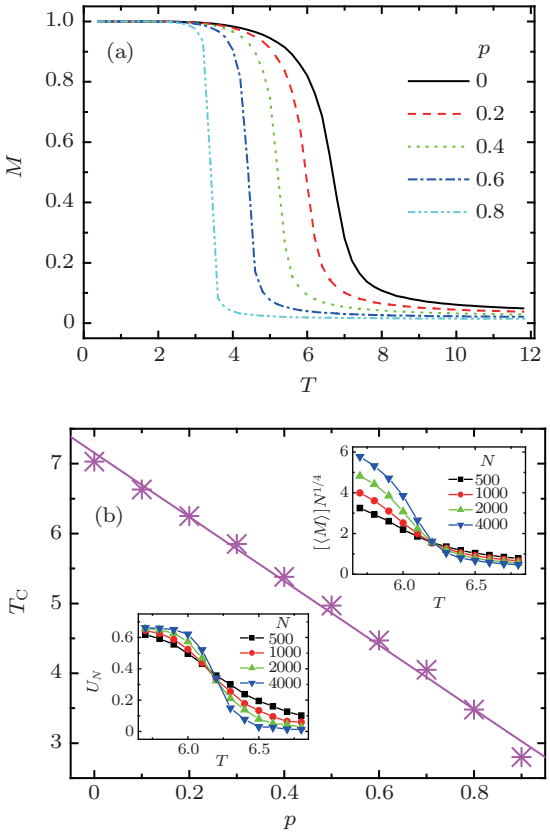


Fig. 2. (color online) Influence of the state-holding probability p upon the ordering behavior for the Ising model on static networks. (a) Order parameter M versus the temperature T for the system on the SW networks with size $N = 1000$. The state-holding probability $p = 0$, 0.2, 0.4, 0.6, and 0.8, respectively. (b) T_C versus p. Lower left inset: when $p = 0.2$, dynamic Binder's cumulant U_N has a unique crossing point at $T_C \approx 6.2$ (in units of J/k_B). Upper right inset: when $p = 0.2$, $[\langle M\rangle]N^{1/4}$ has a unique crossing point at $T_C \approx 6.2$ also. The simulations are averaged over 100 realizations.

Moreover, we investigated the influence of p on the ordering rate. The half-ordering time $t_{0.5}$ is used for the ordering rate in the Ising model on static networks. The half-ordering time $t_{0.5}$ is defined as the time at which the order parameter $M = 0.5$. Figure 3 shows the variation of the order parameter M with t and the variation of $t_{0.5}$ with p. It can be seen that the half-ordering time $t_{0.5}$ increases with the state-holding probability p, i.e., state-holding retards the progress of ordering. The reason is that introduction of p reduces the coupling action.

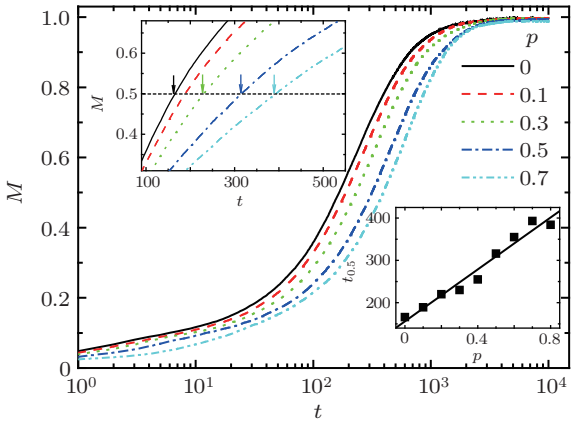


Fig. 3. (color online) Order parameter M versus time t for the Ising model on static networks with $N = 2000$ at $T = 3$ and $p = 0$, 0.1, 0.3, 0.5, and 0.7, respectively. Upper left inset: arrow indicates the position of the half-ordering time $t_{0.5}$, which is defined as the time at which $M = 0.5$. Lower right inset: the half-ordering time $t_{0.5}$ versus the state-holding probability p. The simulations are averaged over 100 realizations.

3.2. Ising model on evolution networks

When the selection-rewiring probability $q \neq 0$ and $\varepsilon_i' \leq \varepsilon_i$, the i-th node holds the present state with the state-holding probability p and rewires the links of the nearest neighboring nodes with the opposite state by the selection-rewiring probability q. In this case not only the states of nodes change, but also the topological structure of the network varies, i.e., the state and topology co-evolve with time. To show the influence of the selection-rewiring probability q upon the ordering behavior, figure 4 plots the relation of the order parameter M and the temperature T for the Ising model on evolution networks with $N = 1000$ and $p = 0.2$ at a series of q. It can be seen that the selective reconnection enhances ability against thermal agitation, and this influence reaches maximum very fast (at about $q = 0.01$). The influence of the selection-rewiring probability q upon the ordering can be explained by using the variation of the average path length D with q in evolution networks. The inset of Fig. 4 plots the relation of the average path length D and the selection-rewiring probability q at $T = 6$ and $N = 1000$. It can be found that D rapidly decreases with increment of q and then tends to a stable value at about $q = 0.01$. Small average path length is favorable for coupling among nodes on the networks, thus it promotes the system ordering.

The selection-rewiring probability q also has influence upon the ordering rate for the Ising model on evolution networks. Figure 5(a) shows the order parameter M versus time t at $T = 6$ for $q = 0.005$, 0.01, 0.1, and 0.8, respectively. It can be seen that the selection-rewiring accelerates the system to an ordering state. To quantify the ordering rate of the Ising model on evolution networks we introduce the ordering time t_c, which is defined as the time at which the order parameter M achieves essentially stable value [see inset of Fig. 5(a)]. Figure 5(b) shows that t_c decreases with increasing q. The influence of the selection-rewiring probability q upon the ordering time t_c can be also explained with the behavior of the average path length D. In the inset of Fig. 5(b) the average path length D decreases with the increment of q. It means that the increment of q leads to D decreasing, so the coupling increases and the ordering time t_c decreases.

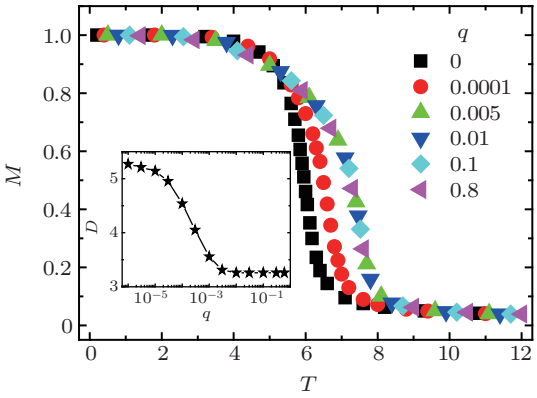


Fig. 4. (color online) Order parameter M versus the temperature T for the Ising model on evolution networks with $N = 1000$ and $p = 0.2$ for $q = 0$, 0.005, 0.01, 0.1, and 0.8, respectively. Inset: average path length D versus the selection-rewiring probability q at $p = 0.2$ and $T = 6$. The simulations are averaged over 100 realizations.

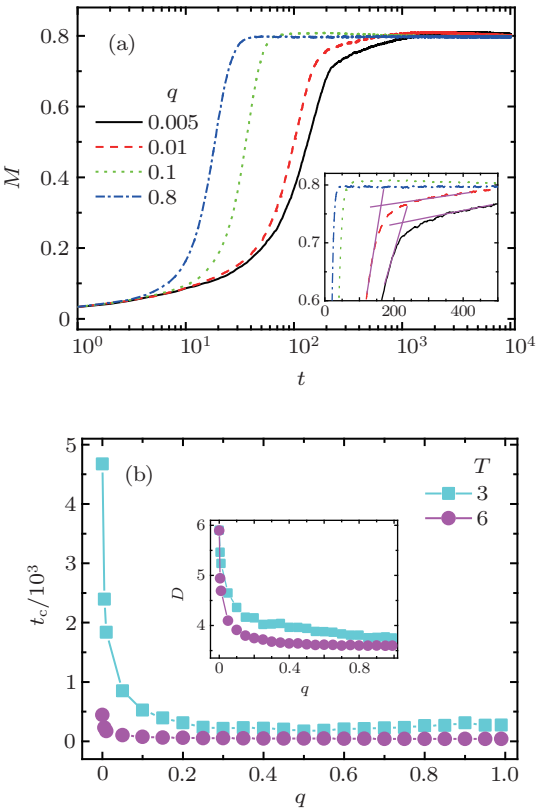


Fig. 5. (color online) Influence of the selection-rewiring probability q upon the ordering for the Ising model on evolution networks with $N = 2000$. (a) Order parameter M versus time t at $T = 6$ for $q = 0.005$, 0.01, 0.1, and 0.8, respectively. Inset: the intersecting point of two extended lines indicates the position of the ordering time t_c, which is defined as the time at which the order parameter M achieves essentially stable value. (b) Ordering time t_c versus q for $T = 3$ and 6, respectively. Inset: average path length D versus the selection-rewiring probability q at $p = 0.2$ for $T = 3$ and 6, respectively. The simulations are averaged over 100 realizations.

The selection-rewiring probability q also has influence upon the rate of consensus formation. Figure 5 shows order parameter M versus time t at $T = 6$ for $q = 0.005$, 0.01, 0.1, and 0.8, respectively. We find that the selection-rewiring can accelerate the convergence to a consensus state. Defining t_c to be the time in which the order parameter M becomes mainly stable [see inset of Fig. 5(a)], we find that t_c declines with increasing q [see Fig. 5(b)]. It can be found that the average path length D decreases with increment of q [see inset of Fig. 5(b)].

In conclusion, the interplay and feedback mechanisms between dynamical processes and network topology on evolution networks not only influence the system order, but also promote formation of the consensus state.

3.3. Application to consensus formation

The Ising model on evolution networks can be used for describing the process of the consensus formation of human population. In opinion networks a node represents an agent (individual), and the state of the node expresses the opinion (positive or negative) of the agent. The link between two nodes indicates that corresponding agents are neighboring and have influence on each other. The opinion (state) of each agent is not only dominated by the communication with the neighbors (through the coupling constant J), but also is affected by the whole society (through the thermal agitation $k_B T$). Thus, the temperature T is introduced and considered as a measurement of the social influence on people making decisions. The behaviors dominated by coupling-agitation competition are described with a Boltzmann-type probability μ. The social circumstance, particularly the local circumstance, affects the opinion of an agent. In a local favorable circumstance an agent not only keeps their own opinion with the probability $1-\mu$, but also changes their present opinion to the opposite one with the probability μ. In a local unfavorable circumstance, the agent can keep their own opinion with the state-holding probability p and reselect the neighbors with the opposite opinion by the selection rewiring probability q. Interaction among individuals and social influence makes most people tend to share the same opinion and leads to consensus formation.

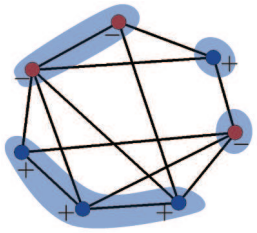


Fig. 6. (color online) Illustration of dividing the opinion clusters on a 7-node opinion network. In each cluster the nodes possess the same opinion and each of them has at least one nearest neighbor with the same opinion. As a special example, a node which has no nearest neighbor with the same opinion is also regarded as one cluster.

The opinion networks can be divided into opinion clusters.[39] For simplicity, a 7-node network is taken as an illustration of dividing the opinion clusters (see Fig. 6). In this network the nodes are divided into four opinion clusters. The nodes belonging to the same cluster possess the same opinion, namely, there is a channel of communication among them to share ideas. When the opinion network evolves, the size of each cluster and the total cluster number also vary with time. Therefore, one can investigate the process of consensus formation through the evolution of the opinion clusters on networks.

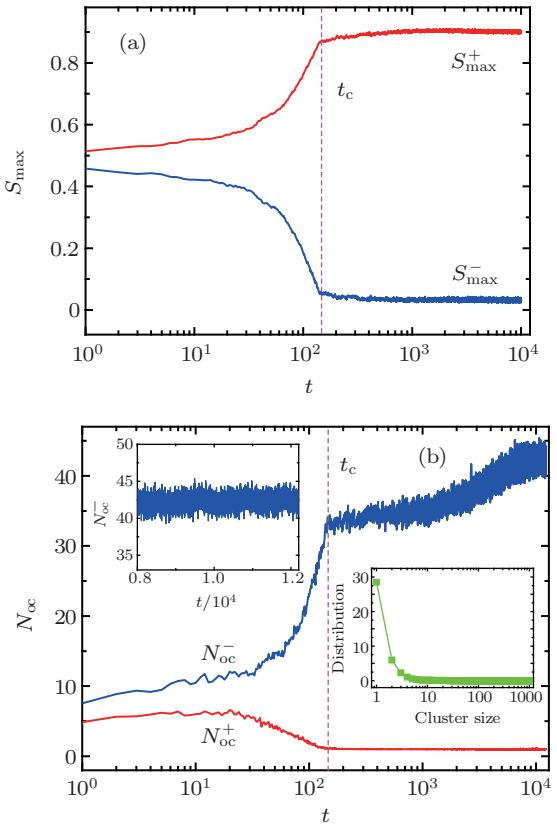


Fig. 7. (color online) Evolution of several characteristic quantities with time in the process of consensus formation for the Ising model on evolution networks with $N = 1000$ at the temperature $T = 6$ and the selection-rewiring probability $q = 0.01$. (a) The relative size $S^+_{\max}$ of the largest opinion cluster and that $S^-_{\max}$ of the second-largest opinion cluster. (b) The number N^+_{oc} of clusters with positive opinion and that N^-_{oc} with negative opinion. Upper left inset: N^-_{oc} as time varying from $t = 8.0 \times 10^3$ to $t = 1.2 \times 10^4$. Lower right inset: cluster-size distribution at $t = 1.0 \times 10^4$. The dashes show the ordering time (t_c).

The evolution of opinion clusters on networks can be described by means of the variation of the largest cluster with time. To do so, the variation of the relative size of the largest opinion cluster $S_{\max}$ and the number of opinion clusters N_{oc} with time has been calculated at the temperature $T = 6$ and selection-rewiring probability $q = 0.01$. The results are plotted in Fig. 7. Usually, the opinion of agents in the largest opinion cluster is opposite to that in the second-largest opinion cluster, so the relative size of the largest and the second-largest opinion clusters can be expressed as $S^+_{\max}$ and $S^-_{\max}$, respectively. Correspondingly, the number of clusters with positive

and negative opinions can be expressed as N_{oc}^{+} and N_{oc}^{-}, respectively. It can be seen from Fig. 7(a) that S_{max}^{+} increases and S_{max}^{-} decreases with time increasing. Meanwhile, as shown in Fig. 7(b), N_{oc}^{+} decreases and N_{oc}^{-} increases with time increasing.

According to Fig. 7, the process of consensus formation can be described as follows. In the beginning, the size and number of clusters with positive and negative opinions are close to each other and the system is in a polarized state. As the time increases, the size S_{max}^{+} of the largest opinion cluster increases and that S_{max}^{-} of the second-largest one decreases. At the same time, the number of the preponderant opinion clusters (N_{oc}^{+}) changes a little, but the number of the minority opinion clusters (N_{oc}^{-}) increases quickly. In this process, some agents belonging to negative opinion clusters reverse themselves about opinion and join positive opinion clusters; meanwhile some positive opinion clusters merge into larger clusters. After the ordering time t_c the system tends to the consensus state. Lastly, the system arrives at the stable state. In this case, there exists one very large preponderant opinion cluster and many small clusters with positive and negative opinions intersperse on the network. The insets of Fig. 7(b) show the characteristics of the opinion networks in the stable state (consensus formation). The upper left inset of Fig. 7(b) plots the number of negative opinion clusters N_{oc}^{-} between 8×10^{3} and 1.2×10^{4} time steps. It can be seen that in the stable state ($t > 8000$ steps) N_{oc}^{-} does not vary except thermal agitation. The lower right inset shows the cluster-size distribution at $t = 10^{4}$. It indicates that the largest cluster includes most of the nodes and many small clusters consist of one or a few nodes.

In order to reveal the variation of links joining nodes (agents) in the process of consensus formation, the average number of links per agent was measured at $T = 6$ and $q = 0.01$. The results are shown in Fig. 8. In this figure the average number of links joining agents with the positive, negative, and opposite opinions are symbolized as L^{++}, L^{--}, and $L^{-+} = L^{+-}$, respectively. Since the total number of links is preserved, only two of them are independent. By making a comparison between Figs. 7 and 8 it is found that in company with the increment of the relative size S_{max}^{+} of the largest opinion cluster, the links between positive opinion agents L^{++} increase, links between negative opinion agents L^{--} decrease, and links between opposite agents L^{-+} reduce. It implies that in the process of consensus formation a new link tends to take place between the selection-rewiring node and an old node in positive opinion clusters. i.e., nodes in preponderant opinion clusters have larger probability to gain new links.[40] Therefore, the opinion network can evolve into a consensus state due to the co-evolution of the states and topology of opinion networks.

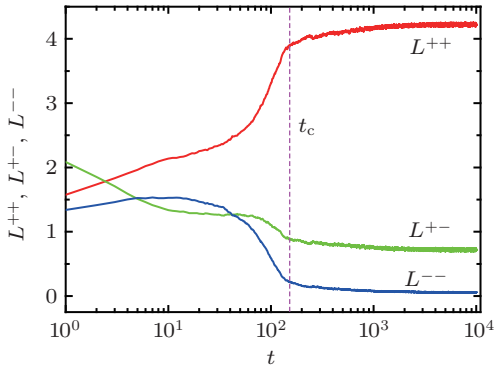


Fig. 8. (color online) Evolution of the average number of links per agent with time in the process of consensus formation for the Ising model on evolution networks with $N = 1000$ at the temperature $T = 6$ and for the selection-rewiring probability $q = 0.01$. L^{++}, L^{--}, and L^{+-} are symbols for the average number of links joining agents with the positive, negative, and opposite opinions, respectively. The dashes show the ordering time (t_c).

Now we turn to discussion of effect of the state-holding probability p and selection-rewiring probability q on the consensus formation. The consensus formation is an evolution process of the opinion system. Placing Ising spins on the nodes of the opinion network, the Hamiltonian of the system is given by

$$H = -J\sum_{i,j} a_{ij}s_i s_j, \tag{4}$$

where the matrix element $a_{ij} = 1$ if nodes i and j are connected, otherwise $a_{ij} = 0$. The consensus formation corresponds to the system energy minimizing, so the evolution of opinion networks inevitably leads to the heterogeneous connections reducing. There are two ways to reduce the heterogeneous connections. One is assimilating minority opinion nodes to preponderant opinion nodes, which corresponds to the state evolution caused by the couple interaction among nodes. The second is rewiring of heterogeneous connections, which corresponds to the topology evolution. Under disadvantageous circumstances, introduction of the state-holding probability p is unfavorable to agents updating the state and represses the assimilation. Therefore, the system with large state-holding probability p has low ordering ability, i.e., the consensus formation time is prolonged as the state-holding probability p increases. The selection-rewiring probability q has two functions: increasing long-distance links and decreasing heterogeneous connections. The former makes the average path length of the network decrease that is favorable for coupling among nodes and promotes the system ordering. The latter affords more opportunity to add homogeneous connection by topology evolution that speeds up the consensus formation.

However, evolution of the opinion networks leads to the consensus formation that takes place just at lower temperature (stable society). In this case the consensus opinion formed in a short term and the minority opinions exist simultaneously in society. At higher temperature (turbulent society), the order

parameter $M = 0$ (see Fig. 4). In consideration of the ordering parameter to be the average magnetization of the system, the zero-ordering parameter does not implicate the system to be completely random. To reveal the evolution process of the states of agents at higher temperature we plot the dependence of the average relative size $S^{+}_{\max}$ of the largest opinion cluster and that $S^{-}_{\max}$ of the second-largest one upon time at a higher temperature $T = 9$ [see Fig. 9(a)]. As mentioned above the state (opinion) of agents in the largest opinion cluster is opposite to that in the second-largest opinion one. Figure 9(b) shows the evolution of the number of the clusters with positive and negative opinion, N^{+}_{cluster} and N^{-}_{cluster}, respectively. Moreover, figure 9(c) plots the average number of links joining agents with the positive, negative, and opposite opinions, L^{++}, L^{--}, and L^{-+} (L^{+-}), respectively.

Figure 9 reveals the evolution process of the opinion system at higher temperature as follows. In the initial network the nodes with contrary opinions distribute randomly and the numbers of the nodes with contrary opinions are equal. As time goes, with the aid of selection-rewiring of links and coupling among nodes, long-distance links increase and heterogeneous connections decrease that promotes the positive and negative largest clusters forming and total number of clusters decreasing. After the phase separation time t_{p} the opinion cluster separation essentially completes, i.e., the system consists mainly of two largest clusters with contrary opinions, the number of clusters reaches minimum and the average number of links L^{++}, L^{--}, and L^{+-} are inclined to stability. After that, the coupling action among nodes in the clusters with contrary opinions is in equilibrium, so the evolution of the states of nodes and evolution of topological structure of the network are controlled by the thermal agitation. In this stage both isolated nodes and heterogeneous links increase until thermal equilibrium is reached. Figure 9 also shows that the phase separation time t_{p} decreases with increment of the selection-rewiring probability q. The reason is that increment of q leads to the average path length D of the network decreasing and affords more opportunity to add homogeneous connection. As a result selection-rewiring speeds the phase separation and makes the phase separation time t_{p} reduce. For comparison to the Ising model on static networks the phase separation behaviors at $q = 0$ are also shown in Fig. 9. Since no selection-rewiring takes place, the process of phase separation is dominated by the competition between thermal agitation and coupling action. The stage of minimizing the cluster number does not exist. However, the phase separation time is very long by lack of acceleration of selection-rewiring.

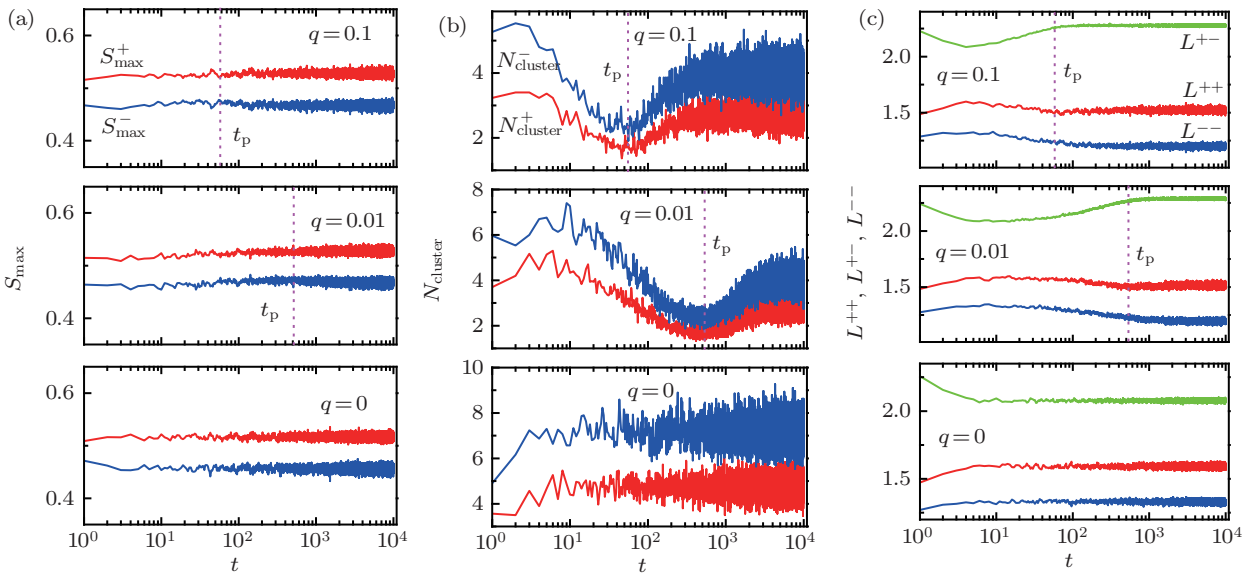


Fig. 9. (color online) Evolution of several characteristic quantities with time at higher temperature $T = 9$ and the selection-rewiring probability $q = 0$, 0.01, and 0.1 for the Ising model on evolution networks with $N = 1000$. (a) Evolution of the average relative size $S^{+}_{\max}$ of the largest opinion cluster (red lines) and that $S^{-}_{\max}$ of the second-largest one (blue lines). The agents possess the positive opinion in the largest cluster and they possess the negative opinion in the second-largest cluster. (b) Evolution of the number N^{+}_{oc} of the clusters with positive opinion (red lines) and that N^{-}_{oc} with negative opinion (blue lines). (c) Evolution of the average number of links per agent with time. L^{++}, L^{--}, and L^{+-} are symbols for the average number of links joining agents with the positive, negative, and opposite opinions, respectively. The dashes indicate the phase separation time t_{p}, at which phase separation essentially completes.

To sum up, the influences of the model parameters upon the consensus formation and phase separation of opinion networks can be described as follows. The state-holding probability p reduces ability to resist thermal agitation. It makes the ordering-order transition temperature T_{C} decrease and the ordering time t_{c} and phase separation time t_{p} increase. Small selection-rewiring probability q (< 0.1) has an obvious effect on consensus formation and opinion group separation. Incre-

ment of q makes the ordering time t_c and phase separation time t_p decrease.

4. Conclusions

There exists a kind of phenomena, in which the states of agents and topological structure of system co-evolve and interact with each other. The Ising model on evolution networks is suitable for describing this kind of phenomena. There are two parameters in this model. Increment of the state-holding probability p is equivalent to reducing the effective coupling and retards the progress of ordering (at lower temperature) and phase separating (at higher temperature). Increase of the selection-rewiring probability q speeds up ordering and phase separation. It can be explained as follows. The selection-rewiring probability q has two functions: increasing long-distance links and decreasing heterogeneous connections. The former makes the average path length of networks decrease that is favorable for coupling among nodes. The latter affords more opportunity to add homogeneous connection. These two functions promote the whole system ordering (at lower temperature) and local ordering (at higher temperature).

Using this model for opinion networks it is found that there exist two ways of opinion formation: consensus and polarization. In the case of lower temperature (stable society), the consensus opinion may be formed in a short term and the minority opinions also exist in society. In the case of higher temperature (turbulent society), the evolution of the opinion networks like phase separation and contrary opinions coexist. Increment of p retards the progress of the consensus formation and opinion cluster separation. Increase of q speeds up the consensus formation and opinion cluster separation.

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