The interaction between zonal flow and Rossby waves with scalar nonlinearity*

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The nonlinear interactions between zonal flow and Rossby waves are studied by numerical simulations with focus on the effects of scalar nonlinearity. The numerical results show that the scalar nonlinearity has an appreciable influence on the Rossby dipole evolution and can reduce the threshold of the disturbance energy increase.

Keywords: Rossby wave, sheared zonal flow, dipole, scalar nonlinearity

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1. Introduction

The Rossby waves can represent large-scale vortices in the atmosphere and ocean, which are responsible for the global changes of weather on the Earth and the creation of large eddies and zonal bands on other planets such as Jupiter's Great Red Spot. The nonlinear dynamics of Rossby waves is often described by the following dimensionless quasi-geostrophic potential vorticity (QGPV) equation[1]

$$\frac{\partial}{\partial t}\left(\nabla^2\psi - F\psi\right) + J(\psi, \nabla^2\psi) + \beta\frac{\partial\psi}{\partial x} = 0. \tag{1}$$

Here, the symbols are standard notations in geophysical fluid dynamics (GFD), for detials, see Ref. [1]. The same Eq. (1), which is also known as the Charney–Obukhov equation, and the same waves (from the mathematical point of view) also appear in various phenomena of different physical natures. For example, they arise when one considers plasma in a strong magnetic field, and then these waves are called drift waves.[2–4] Equation (1) is also the well-known Charney–Hasegawa–Mima (CHM) equation in plasma physics.

The key features of many large-scale geophysical flows can be captured by Eq. (1); extensive studies have been made with different methods within the framework of Eq. (1). In recent years, there has been renewed interest in zonal flows and the mechanisms to generate zonal flow by Rossby waves.[4–15] Along with a variety of zonal jets in rotating fluids, zonal flows are also important in magnetized plasmas.[2–4] More recently, Kaladze and Pokhotelov[16] found that zonal flow interaction with Rossby waves will trigger the increase of disturbance energy and the new generated vortex structures will arise to produce the structural turbulence, but the effects of the scalar nonlinearity are not considered in their research.

The further investigation of reduced models involving the interaction of zonal jets and Rossby waves is still of great importance to our overall understanding of the GFD. The main purpose of the present study is to provide a numerical investigation of the QGPV Eq. (1) modified by the presence of a sheared zonal flow, with emphasis on the effects of the scalar nonlinearity. The model equation is presented in Section 2 and numerical simulations are carried out in Section 3. Our conclusions are found in Section 4.

2. Model equations

It is well known that equation (1) conserves the energy E and the enstrophy M, which are defined by[17]

$$E = \frac{1}{2}\iint\left[(\nabla\psi)^2 + F\psi^2\right]\mathrm{d}x\mathrm{d}y, \tag{2a}$$

$$M = \frac{1}{2}\iint\left[(\nabla^2\psi)^2 + F(\nabla\psi)^2\right]\mathrm{d}x\mathrm{d}y. \tag{2b}$$

In several applications, people encounter Rossby waves with a different form of nonlinear interaction. In particular, it is sometimes described by the equation with the scalar nonlinearity (see e.g. Refs. [2], [18]–[20] and the references cited therein).

To investigate the nonlinear interactions between zonal flow and Rossby waves, we consider a steady and horizontal parallel shear flow $\bar{u}(y)$ in a Cartesian frame (x, y) centered at a latitude whose directions are east for x and north for y. We suppose that the fluid is confined between two rigid walls y_1, y_2. We may write the total streamfunction as

$$\psi(x, y, t) = -\int\bar{u}(y)\mathrm{d}y + \psi'(x, y, t), \tag{3}$$

where ψ' is the perturbative streamfunction, and the corresponding velocity components for the disturbance are $u' = -\partial\psi/\partial y$ and $v' = \partial\psi/\partial x$. Substituting Eq. (3) into Eq. (1)

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and dropping the primes, we obtain

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\nabla^2\psi - F\frac{\partial\psi}{\partial t}+J(\psi,\nabla^2\psi) + \left(\beta-\frac{\partial^2\bar{u}}{\partial y^2}\right)\frac{\partial\psi}{\partial x}+\alpha\psi\frac{\partial\psi}{\partial x}=0, \tag{4}$$

where α is a positive constant associated with the β-effect.[12,19–21] It should be pointed out that, besides retaining the vector nonlinearity $J(\psi,\nabla^2\psi)$ in Eq. (4), we also add the scalar nonlinearity $\psi\partial\psi/\partial x$, which is also known as KdV-type nonlinearity. In Refs. [19] and [20], the scalar nonlinearity was considered by using the Petviashvili model, which has been invoked as an ageostrophic extension of Eq. (1). However, in the above mentioned research works,[2,12,18–20] the backgroud flows or zonal jets are not considered.

The classical QGPV equation (1) without scalar nonlinearity has dipole vortex Rossby solutions,[22] which describe the propagation of solitary dipole structures consisting of cyclones and anticyclones. However, the presence of KdV scalar nonlinearity leads to the formation of monopole vortical structures (see, e.g. Ref. [19]). Analogously, the modified QGPV equation (4) under the action of sheared zonal flow describes the evolution of such vortex structures and the final dynamics of the formation of solitary structures depends on the interaction of these two nonlinearities. Indeed, with the help of Eqs. (1), (2a), and (4), one easily obtains the dynamic equation

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \iint\frac{\partial\bar{u}}{\partial y}\left[\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y}\right]\mathrm{d}x\mathrm{d}y = -\iint\frac{\partial\bar{u}}{\partial y}\left[u'v'\right]\mathrm{d}x\mathrm{d}y. \tag{5}$$

We note that in the absence of zonal flow, the disturbance vortex energy is conserved. Therefore, the existence of sheared zonal flow can be viewed as the presence of an external energy source or sink for the Rossby wave disturbances or the large-scale zonal flows can also be excited due to the Reynolds stresses of short-scale Rossby waves. The configuration of the basic state shear and the disturbance momentum flux determines the energy exchanges between Rossby wave disturbances and zonal flows.

3. Numerical simulations

Equation (4) is analyzed with effective implicit difference schemes developed by the authors of Ref. [21] in a square domain $[-5,5]\times[-5,5]$ with the periodic boundary condition in the x direction and the fixed boundary condition in the y direction. The cross iterative process is adopted to solve the difference equations.

The well-known dipole-vortex solution of Larichev and Reznik[22] (also called a modon or dipole) is used as the initial perturbation field and the jet stream as sheared zonal flow $\bar{u}=\mathrm{sech}^2(y/d)$ ($-4d<y<4d$ represents the effective breadth of the jet), see Fig. 1. The unsheared zonal flows are $\bar{u}=1$ and $\bar{u}=-1$, which represent westerly and easterly in the Earth's atmosphere respectively. For simplicity, we set $\beta=0.4$, $F=0.6$, and $\alpha=1$ in Eq. (4), which are the typical values for the Earth's atmosphere (see Ref. [21] and the references cited there).

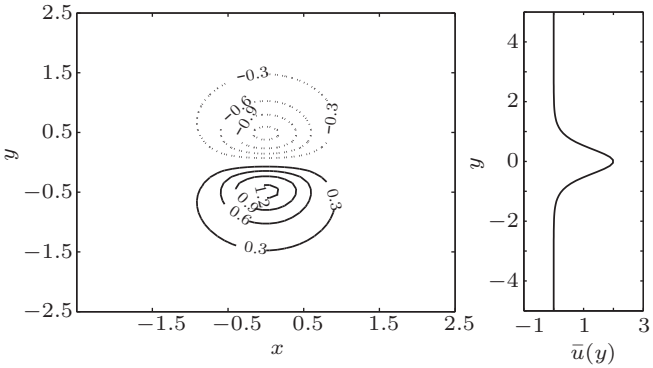


Fig. 1. Initial field and jet stream. Contour plots of ψ are shown (left) and the jet stream (right) (in order to get a clearer view of the initial structure, we only plot ψ in the domain $[-2.5,\ 2.5]\times[-2.5,\ 2.5]$ while the simulation domain is $[-5,\ 5]\times[-5,\ 5]$).

Firstly, we set $\bar{u}=\pm1$ to study the cases of unsheared zonal flows. Without the scalar nonlinearity, simulations show that the initially given dipole retains its shape and speed pretty well, and the disturbance energy does not change as predicted by Eq. (5). Without background flows, the presence of the scalar nonlinearity is known to introduce cyclone–anticyclone asymmetry. However, under the presence of zonal flows and the scalar nonlinearity, we find the dipole develops in a different way. In Fig. 2, when the disturbance structure moves eastward (lower panel), we see the initial dipole topples rapidly and disintegrates into two monopoles, a cyclone and an anticyclone, at the same time and after that the cyclone develops while the anticyclone decays all the way. In the upper panel, we find the westward propagating dipole develops like a seesaw: the cyclone develops at first, then the anticyclone develops and this cycle repeats itself, and they maintain approximately the dipole structure. In both cases, the disturbance energy stays almost unchanged as predicted by Eq. (5) without shear. In other words, there is no energy exchange between zonal flows and Rossby vortexes. Zonal flows and the scalar nonlinearity will lead to the uneven development of vortices with different polarities.

Secondly, we take the jet stream as the background zonal flow. Without the scalar nonlinearity, the initial dipole keeps its structure rather well, see the lower panel in Fig. 3, and just like the unsheared cases, the energy also stays almost unchanged, see Fig. 4. Then taking the effects of scalar nonlinearity into consideration, we see the initial dipole topples quickly and degenerates into two monopoles, then the two vortices turn into unregular shapes and gradually more dis-

turbance vortices are generated all over the simulation domain shown in the upper panel in Fig. 3 and the energy starts to grow, see Fig. 4. It is known that the scalar nonlinearity can lead to the uneven development of the two vortices in the dipole structure, so even when the initial configuration of the basic state shear and disturbance momentum flux is not in favor for the disturbance development, the disturbance may develop, that is to say the scalar nonlinearity can reduce the threshold for instability. However, the disturbance energy cannot grow all the time, and the disturbance energy decay is also observed, see Fig. 4. The phenomena can be viewed as the anisotropic accumulation of Rossby wave energy in a large-scale zonal flow. In other words, the large-scale zonal flows can also be excited by the short-scale Rossby waves.[4–7,11,12]

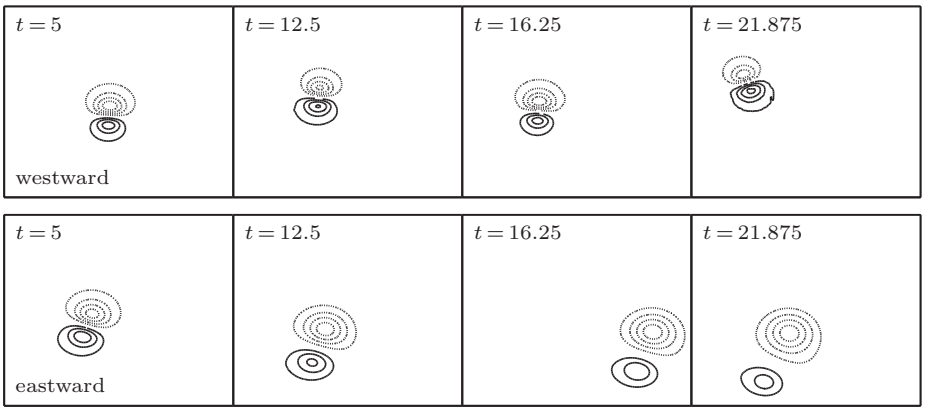


Fig. 2. The time evolution of the dipole structure considering the scalar nonlinearity under unsheared zonal flows. Upper and lower panels are westward and eastward moving cases respectively (Contour plots of ψ are shown with full lines denoting positive values and dashed lines denoting negative values. The interval between the adjacent isolines is 0.3).

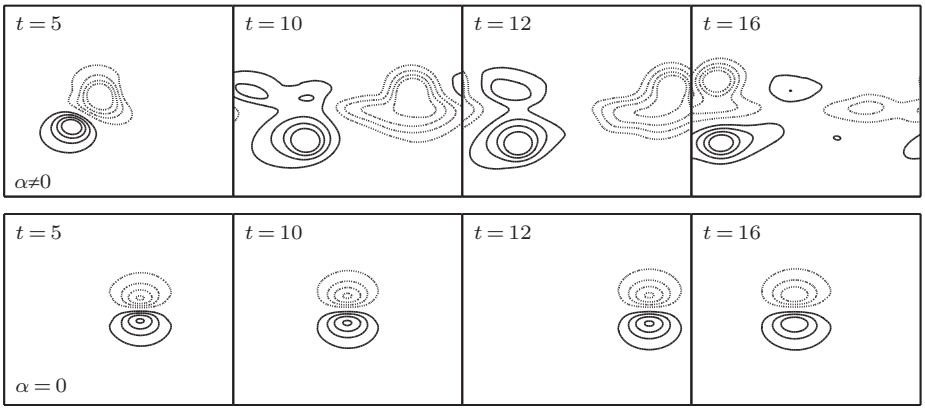


Fig. 3. The evolution of a dipole vortex in the presence of a jet stream. Upper and lower panels indicate with and without effects of the scalar nonlinearity respectively (Contour plots of ψ are shown with full lines denoting positive values and dashed lines denoting negative values. The interval between the adjacent isolines is 0.3).

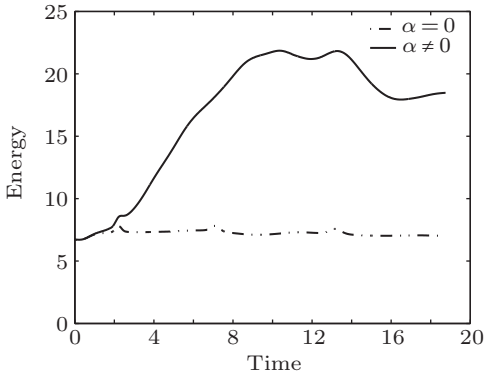


Fig. 4. The time evolution of the disturbance energy in the presence of a jet stream as a zonal flow. The full line shows the case considering the scalar nonlinearity, while the dashed line shows the case neglecting it.

4. Conclusions

In the present paper, we have adopted the jet stream as the mean zonal flow, which is common in the midlatitude atmosphere of Earth, and explored the nonlinear interaction between zonal flow and Rossby waves numerically within the framework of the QGPV equation modified by the zonal shear flow and the scalar nonlinearity. The numerical simulations have shown that the existence of zonal flows and the scalar nonlinearity will lead to uneven development of the Rossby vortices with different polarities, and the scalar nonlinearity can reduce the threshold of disturbance energy in-

crease. These conclusions are consistent with the theoretical results,[17] therefore, we can be more certain that the scalar nonlinearity makes the instability occur more easily but that it might not be the only important factor; more cases need to be investigated to explore the influence of shear type on disturbance energy growth.

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