Rectification effect in asymmetric Kerr nonlinear medium*

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Based on the transfer matrix method, the recursion of an electromagnetic wave propagating in an asymmetric Kerr nonlinear medium is analytically formulated, from which the rectification effect is clearly presented. The effects on the rectification region of the linear part and nonlinear coefficient of permittivity are both studied, and the energy densities before and after rectification are discussed. We use a rectifying factor to describe the intensity of the rectification effect. The result shows that every transmission peak is divided into two parts when the symmetry is broken, and nonlinear asymmetry has a more significant effect on the rectification effect than the linear asymmetry. The rectification intensity and area will be enlarged when the asymmetry factor is increased in a certain range.

Keywords: rectification, transfer matrix method, Kerr nonlinearity, asymmetry

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1. Introduction

Since the nonlinear physics was put forward in the late 20th century,[1,2] there has been a growing interest in the research of the nonlinear medium in the past years.[3–12] The nonlinear science has reached into multiple branches of physics such as mechanics, electromagnetism, optics, and solid theory. Waves propagating in a nonlinear medium can lead to some complex phenomena such as shock, soliton, and frequency doubling. The movement of electrons in a solid satisfies the Schrödinger equation, while the propagations of mechanical waves and EM waves obey the wave equation. The difference is that the former is the first derivative of wave function with respect to time while the later is the second one. In spite of this, correlational studies show that under the condition of a stationary state, the propagations of waves in phononic systems and photonic systems have high degree of similarity.[13–17] In electromagnetism and wave optics, a medium whose permittivity is determined by square module of the electric field, called the Kerr nonlinear medium, is applied to a periodic structure such as a photonic system, in order to obtain the adjustable energy band structure and transmission area,[5,7] and also the phononic system in quantum mechanics.[6]

Recently, more and more research reports about asymmetric nonlinear systems have been springing up.[18–21] An optical device capable of unidirectional light transmission is a direct analogy to an electronic diode and has great application potential in optical communication and information processing.[18] Such as a device called the "wave diode" or "optical diode", which is similar to a semiconductor diode, presents distinct transmission properties when the incident wave is propagating in two opposite directions.[21] As is well known, the implementation of a nonlinear photonic multilayer-based optical diode depends on two conditions. First, the system should be strongly nonlinear to ensure a highly transmissive state which is very sensitive to parameter variation. Second, the structure must be spatially asymmetric, so that when the wave is impinging from one side, the transmission coefficient is greater than that when the wave is impinging from the other side to make the system single-direction conductive. That is the so-called rectification effect. Although this effect is quite similar to the PN junction used for a semiconductor diode, the principle is very different.

The rectification effect of electrons can be obtained by solving the discrete nonlinear Schrödinger (DNLS) equation.[22–24] The DNLS equation is proved to be a perfect approximation for the evolution of a longitudinal Bloch wave in a muti-layer photonic or phononic crystal.[24] In this paper, we will show that the rectification effect can also occur on an EM wave in an asymmetric photonic system with Kerr nonlinearity. The recursion of the electric field is obtained from the solution of the wave equation by applying the transfer matrix method, which is widely used in photonic systems.[25] Similar to the DNLS equation, the difference equation of an electric field on lattice points can be also used for solving the one-dimensional (1D) scattering problem. When the mirror symmetry is broken, the transmission peak will be divided into two peaks, one is for the right-traveling wave and the other is for the left-traveling wave. The significant rectification effect will be observed when asymmetry is considered. We will discuss the intensity and region of the rectification effect.

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2. Theoretical model of photonic system with local asymmetric Kerr nonlinearity and recursion relation of wave function

As mentioned above, a linear and space-reversal symmetric system cannot present the rectification effect. So the mirror symmetry and linearity must be broken. Most photonic systems are constructed by the high refractivity material filling in the background medium. If the optical thickness of the filling material is constant, the thickness will be inversely proportional to the refractivity, which means that a thin filling material will have a higher refractivity. This can be viewed as a comb-like structure. So we consider a photonic system whose permittivity has the form of summation of δ-functions, which is similar to the δ potential barrier in quantum mechanics. The dielectric function can be written as $\bar{\varepsilon}(1+\varepsilon(z))$, where $\bar{\varepsilon}$ is the permittivity of the background and $\varepsilon(z)$ can be given as follows:

$$\varepsilon(z) = \sum_{n=-\infty}^{0} \alpha\delta(z-na) + \sum_{n=1}^{N} (\alpha_n - \beta_n |E(z)|^2)\delta(z-na) + \sum_{n=N+1}^{\infty} \alpha\delta(z-na). \quad (1)$$

For simplicity, we set the lattice constant $a=1$, the background permittivity $\bar{\varepsilon}=1$, and the light velocity $c=1$. α_n represents the linear part while $-\beta_n|E(z)|^2$ denotes the negative Kerr nonlinearity. The model is shown in Fig. 1, which is similar to the one in Ref. [5] but not identical at all. When the asymmetry in the nonlinear region is considered, there exists at least one n such that $\alpha_n \neq \alpha_{N-n+1}$ or $\beta_n \neq \beta_{N-n+1}$ in the region $1 \leq n \leq N$.

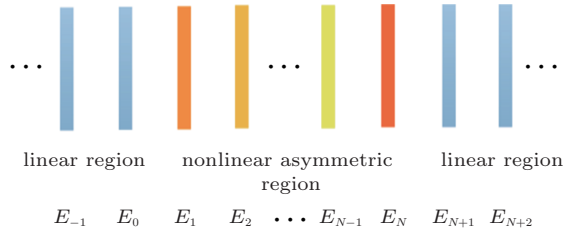


Fig. 1. (color online) Schematic diagram of 1D photonic system with local asymmetric Kerr nonlinearity. E_n represents the electric field at lattice point n. The two linear regions are separated by the Kerr nonlinear region with asymmetry.

In the following, we will deduce the difference equation of an EM wave propagating in the system with local asymmetric nonlinearity by applying the transfer matrix method. The electric field obeys the wave equation:

$$E''(z) + \omega^2(1+\varepsilon(z))E(z) = 0. \quad (2)$$

From Eq. (1), it is obvious that between the two adjacent lattice points $(n < z < n+1)$ $\varepsilon(z) = 0$. One can solve the electric field and its derivative in the middle area, and they are given as follows:

$$E(z) = A_n e^{i\omega(z-n)} + B_n e^{-i\omega(z-n)}, \quad (3)$$

$$E'(z) = i\omega(A_n e^{i\omega(z-n)} - B_n e^{-i\omega(z-n)}). \quad (4)$$

At every lattice point, $E''(z)$ does not exist because $\omega^2(1+\varepsilon(z))E(z)$ is infinite here. This can also be seen from the jump condition of $E'(z)$. Performing integrals on both sides of Eq. (2) in the interval $[n-\Delta, n+\Delta]$ and let $\Delta \to 0^+$, one can obtain the jump condition of $E'(z)$

$$E'_{n+} - E'_{n-} = \begin{cases} -\omega^2\alpha E_n, & n<1, n>N, \\ -\omega^2(\alpha_n - \beta_n|E_n|^2)E_n, & 1 \leq n \leq N, \end{cases} \quad (5)$$

where the subscript n represents the corresponding value at the lattice point n. Nevertheless, $E'(z)$ exists except for a set of countable discontinuity points of the first kind, which means that $E(z)$ should be continuous at all lattice points.

By defining column vector $\psi_n^{\pm} = (E_n \quad E'_{n\pm}/\omega)^{\mathrm{T}}$, equation (5) can be rewritten as

$$\psi_n^+ = S_n\psi_n^-, \quad S_n = \begin{pmatrix} 1 & 0 \\ -b_n\omega & 1 \end{pmatrix}, \quad (6)$$

where $b_n = \alpha$ in the linear region while $b_n = \alpha_n - \beta_n|E_n|^2$ in the nonlinear region. For the free space between two adjacent lattice points, we have

$$\psi_{n+1}^- = M\psi_n^+, \quad M = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix}. \quad (7)$$

From Eqs. (6) and (7), one can obtain the following recursion relations:

$$\psi_{n+1}^- = G_n\psi_n^-,$$
$$G_n = MS_n = \begin{pmatrix} \cos\omega - b_n\omega\sin\omega & \sin\omega \\ -\sin\omega - b_n\omega\cos\omega & \cos\omega \end{pmatrix}, \quad (8)$$
$$\psi_n^+ = T_{n+1}\psi_{n+1}^+,$$
$$T_{n+1} = M^{-1}S_{n+1}^{-1} = \begin{pmatrix} \cos\omega - b_{n+1}\omega\sin\omega & -\sin\omega \\ \sin\omega + b_{n+1}\omega\cos\omega & \cos\omega \end{pmatrix}. \quad (9)$$

Note that the recursion relations (8) and (9) are irreversible in the nonlinear region because the transfer matrices G_n and T_{n+1} are dependent on ψ_n^- and ψ_{n+1}^+ (or rather, E_n and E_{n+1}). But we can still obtain a useful recursion relation about the electric field. From Eqs. (6)–(9), one can eliminate E'_{n+} and E'_{n-} for obtaining a simpler difference equation about E_n, i.e.,

$$E_{n+1} - E_{n-1} = (2\cos\omega - b_n\omega\sin\omega)E_n. \quad (10)$$

Because the rectification only exists in a local nonlinear asymmetric region and the magnetic field does not need to be considered, it is strongly suggested that equation (10), instead Eqs. (8) and (9), should be used to simplify the calculation of transmittance.

3. Rectification effect of electromagnetic wave in asymmetric Kerr medium

Consider the following scattering problem ($k > 0$):

$$E_n = \begin{cases} R_0 \mathrm{e}^{\mathrm{i}kn} + R\mathrm{e}^{-\mathrm{i}kn}, & n \leq 1, \\ T\mathrm{e}^{\mathrm{i}kn}, & n \geq N, \end{cases} \tag{11}$$

where R_0, R, and T represent the amplitudes of incident, reflected, and transmitted waves, respectively. At the beginning of our discussion, in order to observe the rectification effect, the symmetric system should be partially or totally eliminated. For simplicity, we set $N = 2$, and introduce two asymmetry factors λ_1, λ_2 to represent the asymmetric features of linear and nonlinear parts respectively. The coefficients α_1, α_2, β_1, β_2 are related by $\alpha_{1,2} = \alpha(1 \pm \lambda_1)$ and $\beta_{1,2} = \beta(1 \pm \lambda_2)$. From Eq. (11), it is easy to determine the amplitudes of incident and reflected waves, which are as follows:

$$R_0 = \frac{\mathrm{e}^{-\mathrm{i}k}E_0 - E_1}{\mathrm{e}^{-\mathrm{i}k} - \mathrm{e}^{\mathrm{i}k}},$$

and

$$R = \frac{\mathrm{e}^{\mathrm{i}k}E_0 - E_1}{\mathrm{e}^{\mathrm{i}k} - \mathrm{e}^{-\mathrm{i}k}}.$$

What we are interested in is the transmission through the nonlinear asymmetric region. By iterating Eq. (10) twice (let $n = 2, 3$), one can obtain the transmittance for the case $k > 0$ as follows:

$$\begin{aligned} t &= \left|\frac{T}{R_0}\right|^2 \\ &= \left|\frac{E_2(\mathrm{e}^{-\mathrm{i}k} - \mathrm{e}^{\mathrm{i}k})}{\mathrm{e}^{-\mathrm{i}k}E_0 - E_1}\right|^2 \\ &= \left|\frac{2\sin k}{1 + (\nu - \mathrm{e}^{\mathrm{i}k})(\mathrm{e}^{\mathrm{i}k} - \mu)}\right|^2, \end{aligned} \tag{12}$$

where $\mu = 2\cos\omega - \omega\sin\omega(\alpha_2 - \beta_2|T|^2)$, and $\nu = 2\cos\omega - \omega\sin\omega[\alpha_1 - \beta_1|T|^2(\mu^2 - 2\mu\cos k + 1)]$. For the case $k < 0$, one only needs to exchange subscripts 1 and 2 to reverse the model.

For a conventional 1D photonic crystal, the dispersion relation has been obtained by applying a transfer matrix method based on the Bloch theorem in a previous study,[25] and given as

$$\cos k = \cos\omega - \frac{1}{2}\alpha\omega\sin\omega. \tag{13}$$

Equation (13) is not suitable for a nonlinear situation, but for incident, reflected, and transmitted waves propagating in a linear region, it is still workable. It can be viewed as a constraint relationship among frequencies and wave vectors of incident, reflected, and transmitted waves.

First we break the symmetry of the linear part of the permittivity. Let $\alpha = \beta = 1$, and the asymmetry factors are set to be $\lambda_1 = 0.05$, $\lambda_2 = 0$. Transmission–incidence curves and transmission spectra are presented in Figs. 2(a) and 2(b), where $k = 0.1$ represents the right traveling wave while $k = -0.1$ denotes the left one. The inflection points in Fig. 2(a) correspond to the transmission peaks in Fig. 2(b). In order to compare the symmetric case between the asymmetric one, the curve of $\lambda_1 = 0$ is also depicted. The behaviors of left and right traveling waves are identical at the symmetric system. But when asymmetry is considered, the transmission peaks of left and right traveling waves shift towards the opposite direction, and each peak is divided into two peaks. Near the transmission peak of the left traveling wave, the right traveling wave has an extremely low transmittance, and so does the right traveling wave. This is exactly the so-called rectification effect. In the regions marked with D_1, D_2, D_3 in Fig. 2(a), the system is single-direction conductive and the rectification effect can remarkably occur. To take advantage of this effect, one always chooses the highest single-direction transmittance region like the D_2 and D_3 marked regions, while D_1 should not be taken into consideration because of its low transmittance.

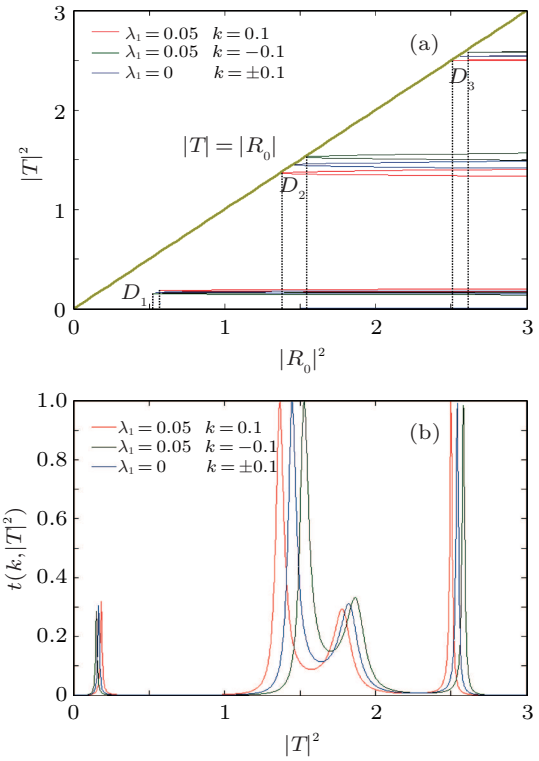


Fig. 2. (color online) Transmission properties of 1D photonic system with asymmetry of linearity, $\lambda_2 = 0$. (a) Transmission–incidence curves. Line $|T| = |R_0|$ represents total transmission. In the regions D_1, D_2, D_3, the rectification effects can be remarkably observed. (b) Dependences of transmittance on transmitted amplitude.

Now we investigate the rectification effect caused by the asymmetry of a nonlinear coefficient. The asymmetry factors are reset to be $\lambda_1 = 0$, $\lambda_2 = 0.05$. Transmission–incidence curves and transmission spectra are presented in Figs. 3(a) and 3(b). It can be seen that compared with the asymmetric transmittance of the linear part, the transmittance is slightly

reduced, but the regions with a rectification effect D_2, D_3 are evidently enlarged. This means that breaking the symmetry of a nonlinear coefficient can obtain a larger rectification effect region than that in the linear part. Because the nonlinearity is negative, the moving direction of the transmission peak is different from that in the linear case.

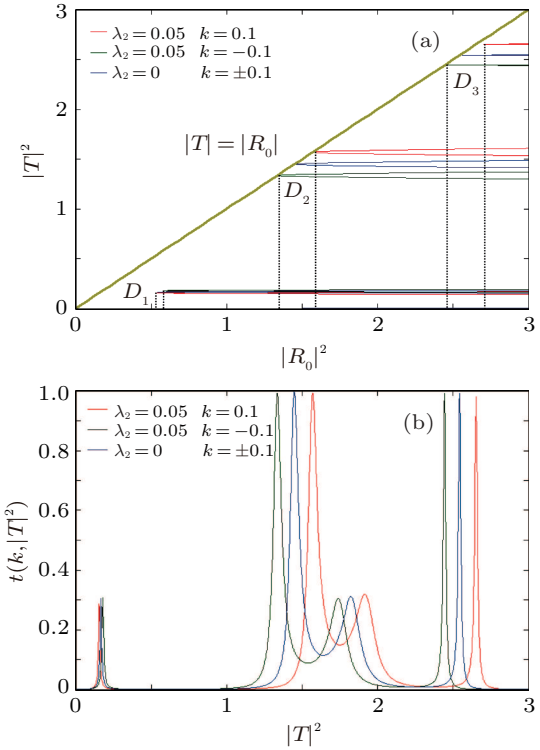


Fig. 3. (color online) Transmission properties of 1D photonic system with asymmetry of Kerr nonlinearity, $\lambda_1 = 0$. (a) Transmission–incidence curves. Line $|T| = |R_0|$ represents the total transmission. In the regions D_1, D_2, and D_3 the rectification effect can be remarkably observed. (b) Dependences of transmittance on transmitted amplitude.

It is obvious that for a certain incident wave, there may exist several modes of reflected wave and transmitted wave. For example, for $|R_0|^2 = 1.5$ in Fig. 3(a), one can find five solutions of the transmitted amplitude of the left traveling wave and $|T|^2 = 1.345$, 1.324, 0.187, 0.168, 0.0004 and three solutions of the transmitted amplitude of the right traveling wave and $|T|^2 = 0.162$, 0.146, 0.0004, respectively. The energy density (square module of the electric field) before and after rectification is presented in Fig. 4. There exist two relatively high energy transmission states for the left traveling wave but none for the right traveling wave, which means that the system can be conductive only for the left traveling wave in this case. Because of the interference of incident wave and reflected wave, the oscillation of the amplitude occurs on the side of incidence. The amplitude may be reduced to zero due to destructive interference and increased inversely. After rectification, the energy densities are the same at all lattice points. It can also be seen that the wave that has a higher energy density before rectification becomes weaker. This is because the incident, reflected, and transmitted waves should obey the energy concentration law.

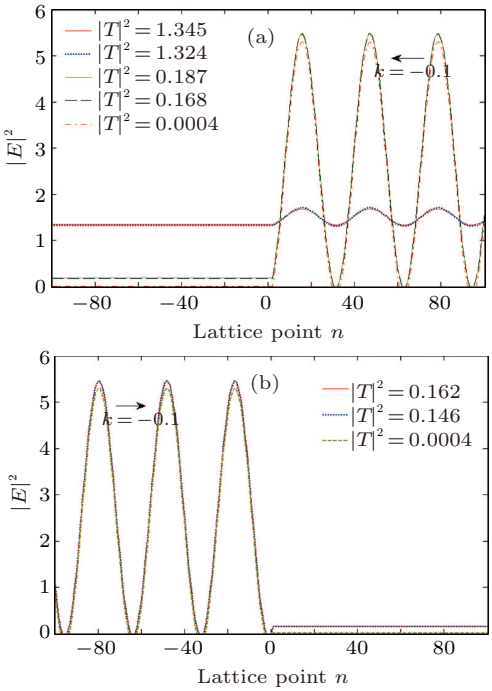


Fig. 4. (color online) Dependences of energy density (square module of electric field) on lattice points, $|R_0|^2 = 1.5$. The parameters are the same as those in Fig. 3. Oscillation is caused by interference of incident and reflected waves. (a) Left traveling wave, $|T|^2 = 1.345$, 1.324, 0.187, 0.168, 0.0004. (b) Right traveling wave, $|T|^2 = 0.162$, 0.146, 0.0004.

To measure the efficiency of rectification, one can introduce the rectifying factor. Using a concept similar to the contrast ratio in optics, the rectifying factor is defined as

$$f = \frac{t(k,|T|^2) - t(-k,|T|^2)}{t(k,|T|^2) + t(-k,|T|^2)}. \tag{14}$$

Obviously $|f| \leq 1$. When $t(k,|T|^2) = t(-k,|T|^2)$, it means that the system has space-reversal symmetry and $f = 0$. The system is double-direction conductive and no rectification effect exists. When $t(k,|T|^2) = 0$ or $t(-k,|T|^2) = 0$, $|f| = 1$, the system is totally single-direction conductive, and the ideal rectification effect can be present. The dependences of rectifying factor f on wave vector k and transmitted amplitude T are depicted in Fig. 5, $\lambda_1 = 0$, $\lambda_2 = 0.05$, 0.1, 0.2, 0.4. In the red region $f = 1$ while in the blue region $f = -1$. These two regions represent the positive and negative rectification effects respectively. It can be seen that when the nonlinear asymmetry factor is positive, the positive rectification effect can ensure a transmitted wave with a higher energy, while for the area in which the negative rectification effect can occur, the transmission amplitude is relatively small. When the asymmetry is enhanced from 0.05 to 0.2, the double-direction conductive region is shrunk rapidly, meanwhile the intensity and area of the positive rectification effect are enlarged all at once. Most of the area becomes totally single-direction conductive. But when the asymmetry factor continues to increase to 0.4, the rectification intensity increases slightly and the area almost remains unchanged. However, in the middle of two positive rectification areas, a weak negative rectification effect can be observed.

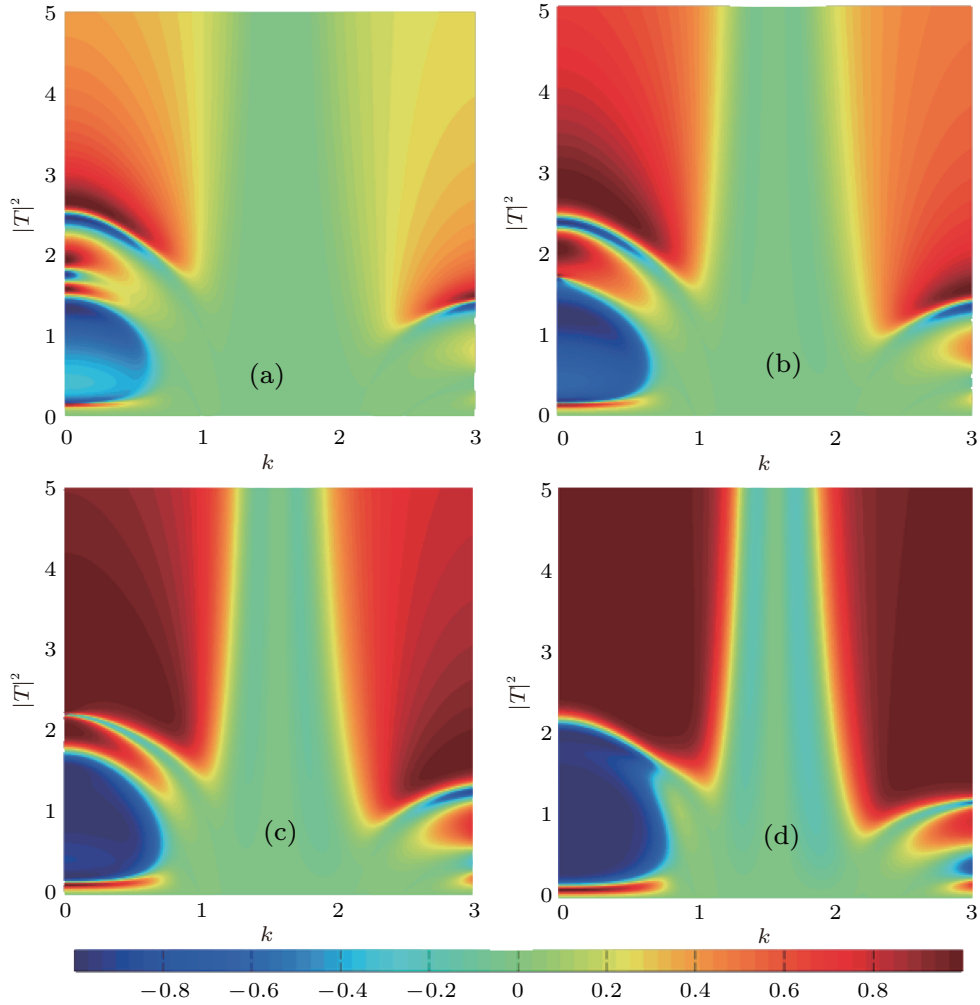


Fig. 5. (color online) Dependences of rectifying factor on wave vector and transmitted amplitude, $\lambda_1 = 0$. (a) $\lambda_2 = 0.05$, (b) $\lambda_2 = 0.1$, (c) $\lambda_2 = 0.2$, (d) $\lambda_2 = 0.4$.

4. Conclusions

A photonic system with asymmetric Kerr nonlinearity is investigated. We analytically formulate the recursion relation of an electric field by using the transfer matrix method. From the difference equation of the wave function, the scattering problem is solved at $N = 2$. The transmission peak can be divided into two when the symmetry is broken. The rectification effect is clearly presented in this system. Compared with linear asymmetry, nonlinear asymmetry will cause a larger area of the rectification effect. The increase of asymmetry factor within a certain range can remarkably enlarge the intensity and area of the rectification effect. But when it continues to increase beyond this range, the rectification does not have significant change.

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