A further study on the spreading and directionality of Gaussian array beams in non-Kolmogorov turbulence*

Lu Lu(陆 璐), Ji Xiao-Ling(季小玲)†, Deng Jin-Ping(邓金平), and Li Xiao-Qing(李晓庆)

Department of Physics, Sichuan Normal University, Chengdu 610066, *China*

(Received 21 September 2013; revised manuscript received 4 November 2013; published online 10 April 2014)

It is found that in free space, the curves of the mean-squared beam width may each have a cross point at a certain propagation distance z_c. For Gaussian array beams, the analytical expressions of z_c are derived. For the coherent combination, z_c is larger than that for the incoherent combination. However, in non-Kolmogorov turbulence, the cross point disappears, and the Gaussian array beams will have the same directionality in terms of the angular spread. Furthermore, a short propagation distance is needed to reach the same directionality when the generalized exponent is equal to 3.108. In particular, it is shown that the condition obtained in previous studies is not necessary for laser beams to have the same directionality in turbulence, which is explained physically. On the other hand, the relative average intensity distributions at the position where the Gaussian array beams have the same mean-squared beam width are also examined.

Keywords: cross point of curves of the mean-squared beam width, spreading and directionality, non-Kolmogorov turbulence, Gaussian array beams

PACS: 42.68.Bz, 42.25.Bs, 42.25.Dd **DOI:** 10.1088/1674-1056/23/6/064209

1. Introduction

The turbulence-induced spatial broadening of laser beams is a limiting factor in most applications. In recent years, much work has been carried out concerning the spreading and directionality of different laser beams in atmospheric turbulence.[1–11] It has been shown that the turbulence results in broadening and partially coherent beams are less sensitive to the turbulence than fully coherent ones.[1–7] In 2003, Shirai *et al.*[8] pointed out that under a certain condition there exist equivalent Gaussian Schell-model (GSM) beams that may have the same directionality (i.e., the same angular spread) as a fully coherent Gaussian laser beam in Kolmogorov turbulence. In 2008, we[11] extended this result from GSM beams to general beams, i.e., general beams may generate the same angular spread as a fully coherent Gaussian laser beam both in free space and in Kolmogorov turbulence if the diffractive coefficients of the mean-squared beam width are the same. In this paper, we show that this condition is necessary in free space but not in turbulence.

It has been experimentally shown that generally atmospheric turbulence might possess a different structure from the conventional Kolmogorov one.[12–14] The atmospheric turbulence may exhibit non-homogeneity and/or anisotropy.[15] The atmosphere was shown to be layered in terms of the power spectra at different altitudes. Toselli *et al.*[16,17] introduced a non-Kolmogorov power spectrum by using a generalized exponent and a generalized amplitude factor. The conventional Kolmogorov spectrum can be treated as a special case of this non-Kolmogorov power spectrum when the generalized exponent is equal to 11/3. Recently, the spreading of GSM beams propagating through non-Kolmogorov turbulence was examined.[18] The coherence and polarization of stochastic electromagnetic beams in non-Kolmogorov turbulence were studied.[19] The intensity fluctuation of flat-topped beams in non-Kolmogorov weak turbulence was studied.[20] The effective radius of curvature of spatially partially coherent beams and the dynamic evolutions of composite coherence vortices in non-Kolmogorov atmospheric turbulence were studied in Refs. [21] and [22], respectively. Very recently, we found that polychromatic laser beams are less sensitive to turbulence than monochromatic ones, and decentred laser beams are less sensitive to turbulence than centred ones.[23]

On the other hand, in the past decades laser array beams have attracted a great deal of attention[24–27] because of their wide applications in high-power system, inertial confinement fusion, high-energy weapons, and so on. Until now, only very few papers have dealt with the propagation of laser array beams in non-Kolmogorov turbulence. Recently, the average intensities of the truncated laser beam arrays with beam distortions in non-Kolmogorov turbulence were investigated in Ref. [28]. The spreading of a radial Gaussian beam array in non-Kolmogorov turbulence was studied in Ref. [29], in which the influences of the beam number, the generalized exponent, and the ring radius on the mean-squared beam width were examined.

In this paper, some new propagation properties of Gaussian array beams through non-Kolmogorov turbulence are dis-

*Project supported by the National Natural Science Foundation of China (Grant No. 61178070) and the Construction Plan for Scientific Research Innovation Teams of Universities in Sichuan Province, China (Grant No. 12TD008).

†Corresponding author. E-mail: jiXL100@163.com

© 2014 Chinese Physical Society and IOP Publishing Ltd

covered. In Section 2, we find out that in free space there may exist a cross point of curves of the mean-squared beam width at a certain propagation distance, and the condition derived in previous studies[8–11] is not necessary for laser beams to have the same directionality in turbulence. In Section 3, we give the physical reasons for the results obtained in Section 2, and examine the relative average intensity distributions at the position where Gaussian array beams have the same mean-squared beam width. In Section 4, the main results obtained in this paper are summarized.

2. Cross point of curves of the mean-squared beam width versus propagation distance

2.1. Theory model and formulae

Assuming that the one-dimensional array beam consists of N individual off-axis Gaussian beams which are correlated with perfect replicas of each other. For such a case, the off-axis Gaussian beams are combined coherently. Specifically let N be an odd number. Thus, the cross-spectral density function of the Gaussian array beam in the source plane $z=0$ is expressed as

$$\begin{aligned} &W^{(0)}(x_1',x_2',z=0)\\ &=\sum_{m=-(N-1)/2}^{(N-1)/2}\sum_{n=-(N-1)/2}^{(N-1)/2}\\ &\times\exp\left[-\frac{(x_1'-mx_\mathrm{d})^2+(x_2'-nx_\mathrm{d})^2}{w_0^2}\right], \end{aligned} \tag{1}$$

where w_0 is the waist width of individual off-axis Gaussian beams, and x_d is the separation between adjacent off-axis Gaussian beams.

Based on the extended Huygens–Fresnel principle, the average intensity of the coherent combined Gaussian array beams represented by Eq. (1) propagating through non-Kolmogorov turbulence reads[30]

$$\begin{aligned} \langle I(x,z)\rangle &= \frac{k}{2\pi z}\iint \mathrm{d}x_1'\mathrm{d}x_2' W^{(0)}(x_1',x_2',z=0)\\ &\times\exp\left\{\frac{\mathrm{i}k}{2z}\left[\left(x_1'^2-x_2'^2\right)-2\left(x_1'-x_2'\right)x\right]\right\}\\ &\times\left\langle\exp\left[\psi^*\left(x,x_1',z\right)+\psi\left(x,x_2',z\right)\right]\right\rangle_m, \end{aligned} \tag{2}$$

where k is the wave number related to the wavelength λ by $k=2\pi/\lambda$, $\psi(x',x,z)$ represents the random part of the complex phase of a spherical wave due to the turbulence, $\langle\bullet\rangle_m$ denotes the average over the ensemble of the turbulent medium, and[30]

$$\begin{aligned} &\left\langle\exp\left[\psi^*\left(x,x_1',z\right)+\psi\left(x,x_2',z\right)\right]\right\rangle_m\\ &=\exp\left\{-4\pi^2k^2z\int_0^1\int_0^\infty \kappa\Phi_n(\kappa,\alpha)\right.\\ &\left.\times\left[1-\mathrm{J}_0\left(\kappa\xi\left|x_2'-x_1'\right|\right)\mathrm{d}\kappa\mathrm{d}\xi\right]\right\}. \end{aligned} \tag{3}$$

Here, $\mathrm{J}_0(\bullet)$ is the Bessel function of the first kind and order zero, $\Phi_n(\kappa,\alpha)$ is the power spectrum of fluctuations in the refractive index of the turbulent medium, κ is the magnitude of the spatial wave number, and α is the fractal constant of the atmospheric turbulence power spectrum (i.e., the generalized exponent). In this paper, we assume that the turbulence is governed by non-Kolmogorov statistics, and that the power spectrum has the van Karman form, i.e.,[16]

$$\begin{aligned} &\Phi_n(\kappa,\alpha)=A(\alpha)\tilde{C}_n^2\frac{\exp[-(\kappa^2/\kappa_m^2)]}{(\kappa^2+\kappa_0^2)^{\alpha/2}},\\ &0\le\kappa<\infty,\ \ 3<\alpha<4, \end{aligned} \tag{4}$$

where $\tilde{C}_n^2$ is a generalized refractive-index structure parameter in units of $m^{3-\alpha}$; $\kappa_0=2\pi/L_0$, with L_0 being the turbulence outer scale; $\kappa_m=c(\alpha)/l_0$, with l_0 being the turbulence inner scale, and[16]

$$\begin{aligned} A(\alpha)&=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left(\frac{\alpha\pi}{2}\right),\\ c(\alpha)&=\left[\Gamma\left(\frac{5-\alpha}{2}\right)A(\alpha)\frac{2}{3}\pi\right]^{1/(\alpha-5)}, \end{aligned} \tag{5}$$

with $\Gamma(\bullet)$ being the Gamma function. When $\alpha=11/3$, $A(11/3)=0.033$, $\tilde{C}_n^2=C_n^2$, the spectrum reduces to the conventional Kolmogorov spectrum.

The mean-squared beam width is defined as

$$w^2(z)=\frac{4\int x^2\langle I(x,z)\rangle\,\mathrm{d}x}{\int\langle I(x,z)\rangle\,\mathrm{d}x}. \tag{6}$$

Substituting Eqs. (2) and (3) into Eq. (6) and following the treatment of deriving Eq. (22) in Ref. [11], we obtain the mean-squared beam width of the coherent combined Gaussian array beams propagating in non-Kolmogorov turbulence, i.e.,

$$w^2(z)\,|_\mathrm{co}=A+Bz^2+Fz^3, \tag{7}$$

where

$$A=\frac{1}{G}\sum_{m=-(N-1)/2}^{(N-1)/2}\sum_{n=-(N-1)/2}^{(N-1)/2}S[w_0^2+(m+n)^2x_\mathrm{d}^2], \tag{8}$$

$$\begin{aligned} B&=\frac{1}{G}\sum_{m=-(N-1)/2}^{(N-1)/2}\sum_{n=-(N-1)/2}^{(N-1)/2}\frac{4S}{w_0^2k^2}\\ &\times\left[1-\frac{1}{w_0^2}(m-n)^2x_\mathrm{d}^2\right], \end{aligned} \tag{9}$$

$$S=\exp[-\frac{1}{2w_0^2}(m-n)^2x_\mathrm{d}^2], \tag{10}$$

$$G=\sum_{m=-(N-1)/2}^{(N-1)/2}\sum_{n=-(N-1)/2}^{(N-1)/2}S, \tag{11}$$

$$F=\frac{8}{3}\pi^2\int_0^\infty\kappa^3\Phi_n(\kappa,\alpha)\,\mathrm{d}\kappa. \tag{12}$$

From Eq. (7) it can be seen that A denotes the mean-squared beam width in the source plane, and B is the diffractive coefficient in free space. The third term on the right-hand

side of Eq. (7) represents the effect of spreading due to turbulence, which is independent of the array beam parameters. When $F = 0$, equation (7) reduces to the mean-squared beam width of the coherent combined Gaussian array beams in free space.

From Eq. (7), we can see that for the coherent combination, in free space the i-th and the j-th curves of $w(z)$ versus z will have a cross point at the position $z_c|_{co}$, which is given by

$$z_c|_{co} = \sqrt{\frac{A_j - A_i}{B_i - B_j}}, \quad (i, j = 1, 2, 3\ldots), \tag{13}$$

where A_i and B_i are the corresponding parameters of A and B in Eq. (7) for the i-th curve $w(z)$ versus z, and A_j and B_j are the corresponding parameters of A and B in Eq. (7) for the j-th curve $w(z)$ versus z. It is noted that equation (13) is valid only when $B_i \neq B_j$ because there does not exist a cross point between two curves of $w(z)$ versus z when $B_i = B_j$.

The smaller A means that Gaussian array beams have the smaller waist width in the source plane $z = 0$. In physics, the smaller waist width results in the larger diffraction (i.e., the larger diffractive coefficient B). It must be $A_j - A_i > 0$ and $B_i - B_j > 0$, or $A_j - A_i < 0$ and $B_i - B_j < 0$. Therefore, $(A_j - A_i)/(B_i - B_j)$ is always non-negative in Eq. (13). On the other hand, equation (7) shows that the curve of $w(z)$ versus z is a hyperbola. In mathematics, $(A_j - A_i)/(B_i - B_j) > 0$ must be satisfied if there exists a cross point between two hyperbola curves of $w(z)$ versus z.

Similarly, for the incoherent combination, we can obtain the mean-squared beam width of Gaussian array beams propagating in non-Kolmogorov turbulence, i.e.,

$$w^2(z)|_{inco} = A' + B'z^2 + Fz^3, \tag{14}$$

where

$$A' = w_0^2 + (N^2 - 1)x_d^2/3, \tag{15}$$

$$B' = 4/(w_0^2 k^2). \tag{16}$$

For the coherent combination, from Eq. (14) we can also obtain the position $z_c|_{inco}$ of the cross point between different curves of $w(z)$ versus z in free space, i.e.,

$$z_c|_{inco} = \frac{1}{2\sqrt{3}} k_i k_j w_{0i} w_{0j} \times \sqrt{\frac{3(w_{0i}^2 - w_{0j}^2) + (N_i^2 - 1)x_{di}^2 - (N_j^2 - 1)x_{dj}^2}{k_i^2 w_{0i}^2 - k_j^2 w_{0j}^2}}. \tag{17}$$

From Eq. (17) we have $z_c|_{inco} = w_{0i} w_{0j}/2$, if $k_i = k_j$, $N_i = N_j$, and $x_{di} = x_{dj}$. In addition, $z_c|_{inco} \to \infty$ if $w_{0i} = w_{0j}$ and $k_i = k_j$.

2.2. Numerical examples and analysis

In the numerical calculation examples in this paper we take $L_0 = 1$ m, $l_0 = 0.01$ m, and $\lambda = 1.06$ μm. The changes of w with α are shown in Fig. 1. From Fig. 1 it can be seen that for the coherent combination w is smaller than for the incoherent combination, and for both the coherent combination and incoherent combination the values of w reach their maxima when $\alpha = 3.108$. Figure 2 shows the changes of w/w_{free} with α, where w_{free} is the corresponding mean-squared beam width in free space. It is known that the large w/w_{free} means that the spreading is more affected by turbulence. Figure 2 indicates that for the coherent combination, the spreading is more affected by turbulence than for the incoherent combination, and for both the coherent combination and incoherent combination, the influences of turbulence on the spreading reach their maxima when $\alpha = 3.108$.

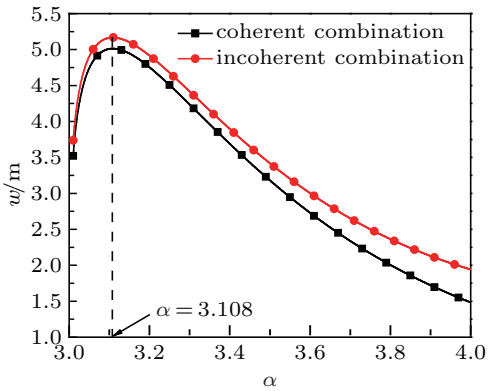


Fig. 1. (color online) Changes of w with α, under $w_0 = x_d = 5$ mm, $N = 7$, $z = 20$ km, and $\tilde{C}_n^2 = 5 \times 10^{-14}$ m$^{3-\alpha}$.

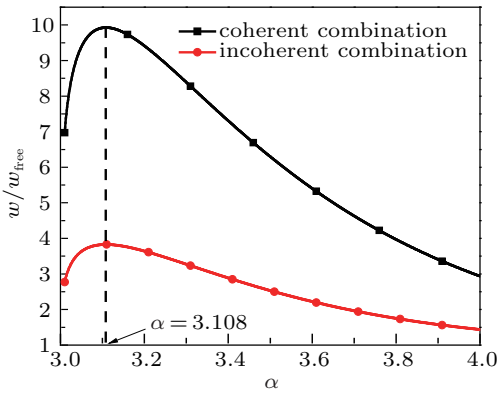


Fig. 2. (color online) Changes of w/w_{free} with α under $w_0 = x_d = 5$ mm, $N = 7$, $z = 20$ km, and $\tilde{C}_n^2 = 5 \times 10^{-14}$ m$^{3-\alpha}$.

Figure 3 displays three curves of w versus z for the coherent combination case. The three curves have different values of w_0 and N, but satisfy Eq. (13). From Fig. 3(a) it can be seen that in free space, the three curves cross at the same point, $z_c|_{co} = 5396$ m. However, in Kolmogorov turbulence (see Fig. 3(b), $\alpha = 11/3$) and in non-Kolmogorov turbulence

(see Fig. 3(c), $\alpha = 3.108$) the cross point of the three curves disappears, and the three curves are close to each other as the propagation distance z increases, namely, they may have the same directionality. Comparing Fig. 3(b) with Fig. 3(c), we can see that the three Gaussian array beams reach the same directionality earlier when $\alpha = 3.108$ than when $\alpha = 11/3$. The physical reason is that the influence of turbulence on the spreading reaches its maximum when $\alpha = 3.108$ (see Fig. 2).

For the coherent combination case, changes of w with z for different values of x_d and N, and changes of w with z for different values of w_0 and x_d are depicted in Figs. 4 and 5, respectively, where equation (13) is satisfied. Comparing Figs. 4 and 5 with Fig. 3 we can see that similar results can be obtained. It is noted that in free space, the beam spreading is smaller before the cross point and it becomes larger after the cross point.

For the incoherent combination case, the changes of w with z for different values of w_0 and N, the changes of w with z for different values of x_d and N, and the changes of w with

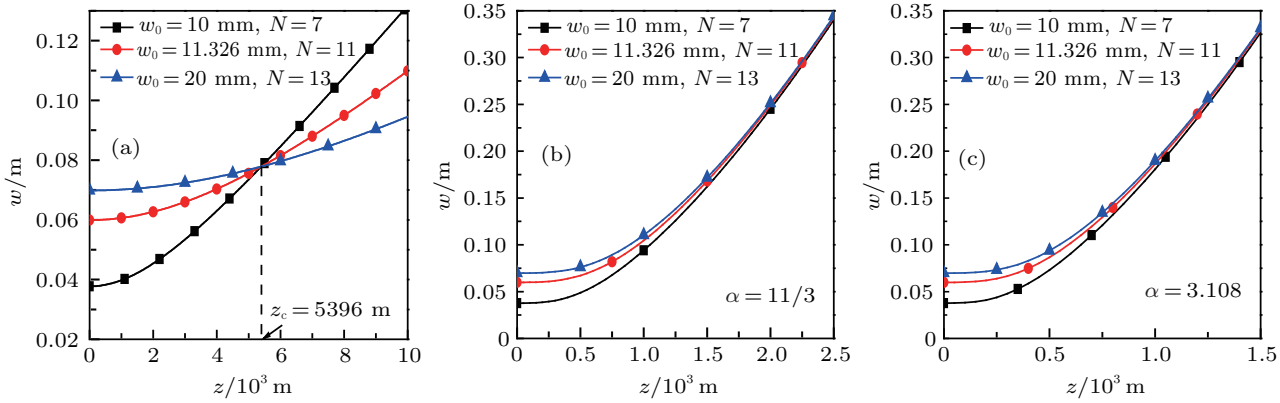


Fig. 3. (color online) For the coherent combination case (a) in free space and (b) and (c) in turbulence, changes of w with z for different values of w_0 and N, $x_d = 10$ mm, and $\tilde{C}_n^2 = 5 \times 10^{-13}$ m$^{3-\alpha}$.

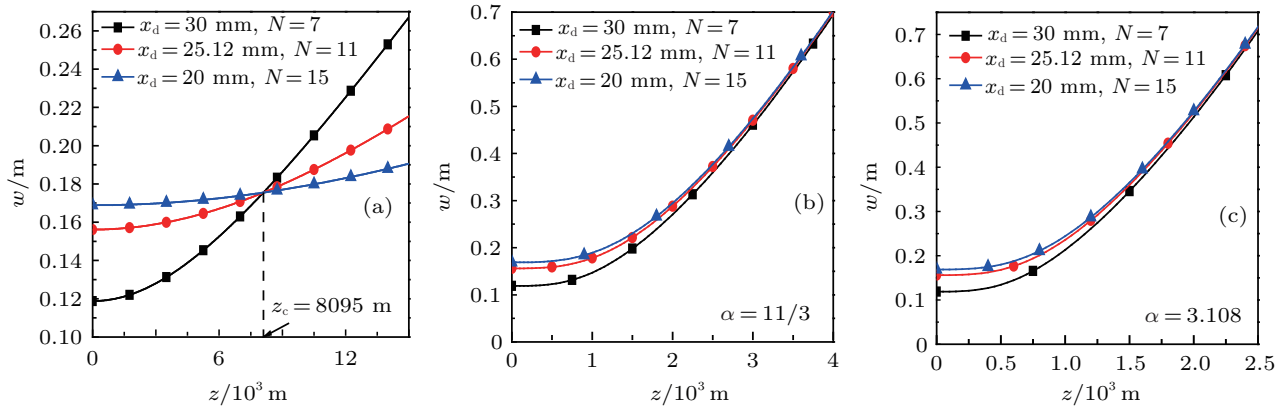


Fig. 4. (color online) For the coherent combination case (a) in free space and (b) and (c) in turbulence, changes of w with z for different values of x_d and N, $w_0 = 13.3$ mm, and $\tilde{C}_n^2 = 5 \times 10^{-13}$ m$^{3-\alpha}$.

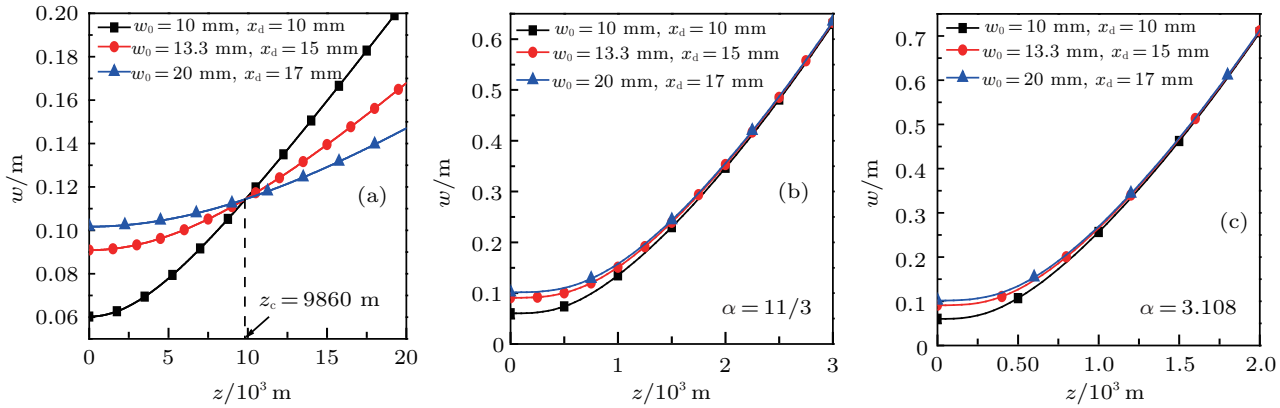


Fig. 5. (color online) For the coherent combination case (a) in free space and (b) and (c) in turbulence , changes of w with z for different values of w_0 and x_d, $N = 11$, and $\tilde{C}_n^2 = 5 \times 10^{-13}$ m$^{3-\alpha}$.

z for different values of w_0 and x_d are depicted in Figs. 6–8, respectively, where equation (17) is satisfied. Comparing Figs. 6–8 with Figs. 3–5, we can obtain that in free space $z_c|_{inco}$ is smaller than $z_c|_{co}$, and in turbulence the shorter propagation distance is needed to achieve the same directionality for the incoherent combination than for the coherent combination. It is mentioned that in Fig. 7(a) a cross point of the three curves does not exist because $w_{0i} = w_{0j}$ and $k_i = k_j$ (i.e., $B'_i = B'_j$), which is in agreement with the result obtained in Eq. (17). The reason is that the two hyperbolas (i.e., the two curves of w versus z) never meet when $B'_i = B'_j$ and $A'_i \neq A'_j$, which is different from the behavior for the coherent combination case (see Fig. 4(a)).

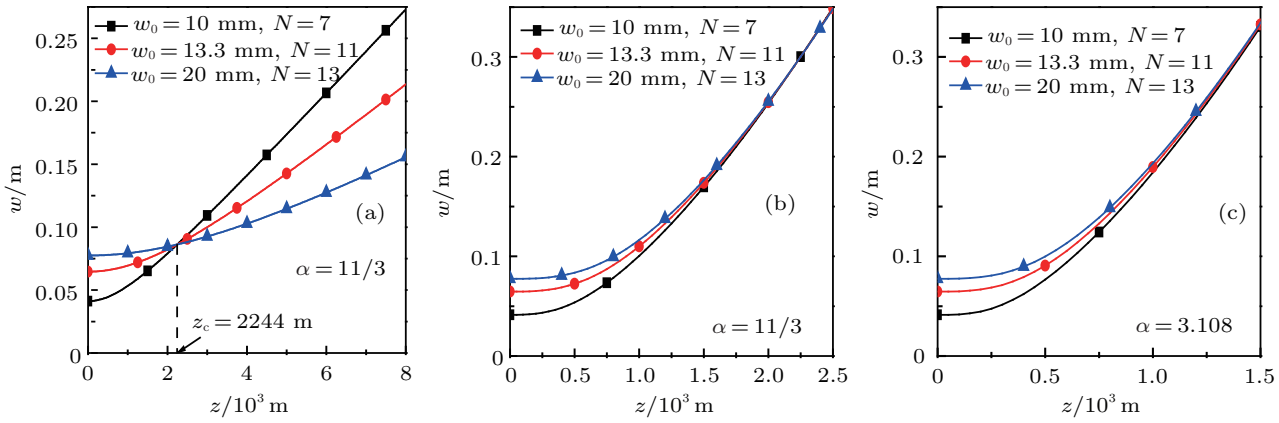


Fig. 6. (color online) For the incoherent combination case (a) in free space and (b) and (c) in turbulence, changes of w with z for different values of w_0 and N, under $x_d = 10$ mm and $\tilde{C}_n^2 = 5 \times 10^{-13}$ m$^{3-\alpha}$.

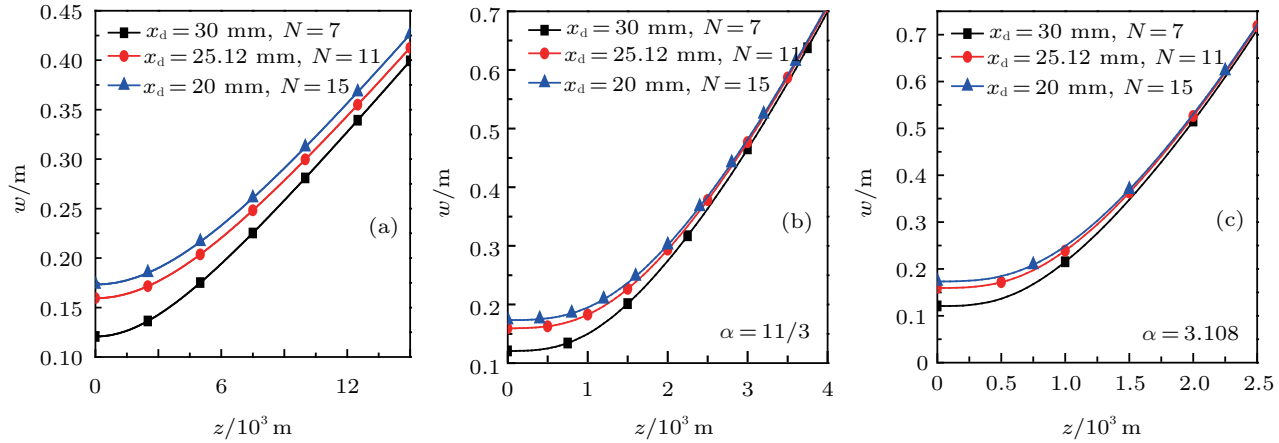


Fig. 7. (color online) For the incoherent combination case (a) in free space and (b) and (c) in turbulence, changes of w with z for different values of x_d and N, under $w_0 = 13.3$ mm and $\tilde{C}_n^2 = 5 \times 10^{-13}$ m$^{3-\alpha}$.

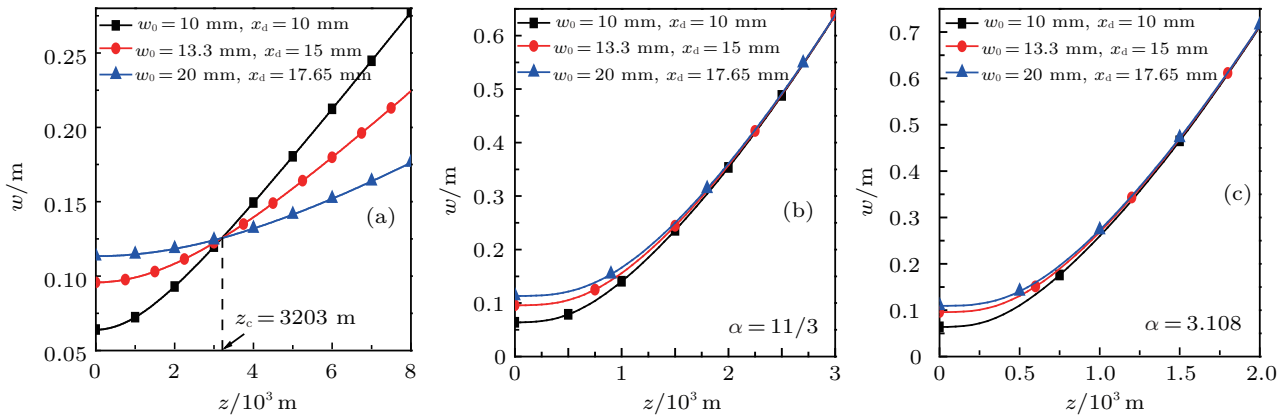


Fig. 8. (color online) For the incoherent combination case (a) in free space and (b) and (c) in turbulence, changes of w with z for different values of w_0 and x_d, under $N = 11$ and $\tilde{C}_n^2 = 5 \times 10^{-13}$ m$^{3-\alpha}$.

3. Further discussion

3.1. Angular spread

From Eqs. (7) and (14), for the coherent combination and the incoherent combination, their angular spreads of Gaussian array beams propagating through non-Kolmogorov turbulence are obtained as follows:

$$\theta\,|_{\rm co} = \lim_{z\to\infty}\frac{w(z)}{z} = \sqrt{B+Fz}, \tag{18}$$

and

$$\theta\,|_{\rm inco} = \sqrt{B'+Fz}, \tag{19}$$

respectively.

Equations (18) and (19) indicate that we have $\theta_i = \theta_j$ (or the same directionality) both in free space and in non-Kolmogorov turbulence if the condition $B_i = B_j$ (or $B'_i = B'_j$) is satisfied. It is mentioned that this condition has also been derived in Refs. [8]–[11] for laser beams having the same directionality both in free space and in Kolmogorov turbulence. A further discussion is given as follows.

In free space, equations (18) and (19) reduce to $\theta_{\rm free}\,|_{\rm co} = \sqrt{B}$ and $\theta_{\rm free}\,|_{\rm inco} = \sqrt{B'}$ respectively and the angular spread depends on w_0, k, and the type of beam combination. In addition, $\theta_{\rm free}\,|_{\rm co}$ depends on N and $x_{\rm d}$, but $\theta_{\rm free}\,|_{\rm inco}$ is independent of N and $x_{\rm d}$. Therefore, in free space, the angular spread (or the directionality) will be different for different values of the diffractive coefficient B or B', which is in agreement with the results obtained in Figs. 3(a)–8(a).

On the other hand, in Eqs. (18) and (19), the second term Fz caused by turbulence will dominate the angular spread when the propagation distance z is large enough. When z is large enough, we still have $\theta_i \approx \theta_j$ (or the same directionality) in turbulence even though $B_i \neq B_j$ (or $B'_i \neq B'_j$), which are consistent with the results obtained in Figs. 3(b)–8(b) and Figs. 3(c)–8(c). Therefore, in turbulence, the condition $B_i = B_j$ (or $B'_i = B'_j$) is not necessary for Gaussian array beams to have the same directionality. This result is also valid for other types of laser beams.

3.2. Average intensity

In this section, we study the average intensity distributions of the coherent and incoherent combined Gaussian array beams at the propagation distance where they have the same mean-squared beam width.

The strong fluctuation condition of turbulence can be written as $\kappa\xi\,|x'_2 - x'_1| \ll 1$. The function $\mathrm{J}_0(\kappa\xi\,|x'_2 - x'_1|)$ in the last exponential function on the right-hand side of Eq. (3) may be replaced, to a good approximation, by the first two terms in the power series expansion of $\mathrm{J}_0(\kappa\xi\,|x'_2 - x'_1|)$ when $\kappa\xi\,|x'_2 - x'_1| \ll 1$, i.e.,

$$\mathrm{J}_0(\kappa\xi\,|x'_2 - x'_1|) \approx 1 - \frac{1}{4}(\kappa\xi\,|x'_2 - x'_1|)^2. \tag{20}$$

Taking into consideration Eq. (20), the average intensity of the coherent combined Gaussian array beams propagating through non-Kolmogorov turbulence is

$$\langle I(x,z)\rangle = H\sum_{m=-(N-1)/2}^{(N-1)/2}\sum_{n=-(N-1)/2}^{(N-1)/2} \times \exp(-Px^2 + Qx + R), \tag{21}$$

where

$$H = \frac{k/2z}{\sqrt{\frac{1}{w_0^4} + \frac{k^2zF}{4w_0^2} + \left(\frac{k}{2z}\right)^2}}, \tag{22}$$

$$P = \left(\frac{k}{2z}\right)^2\left[\left(\frac{2zHUT}{k}\right)^2 + \left(\frac{1}{U}\right)^2\right], \tag{23}$$

$$Q = \mathrm{i}\frac{kx_{\rm d}}{2z}\left(\frac{4z^2H^2U^2VT}{k^2} - \frac{2m}{w_0^2U^2}\right)^2, \tag{24}$$

$$R = x_{\rm d}^2\left[-\frac{m^2+n^2}{w_0^2} + \left(\frac{zHUV}{k}\right)^2 + \left(\frac{m}{w_0^2U}\right)^2\right], \tag{25}$$

$$U = \sqrt{\frac{1}{w_0^2} + \frac{k^2zF}{8} - \mathrm{i}\frac{k}{2z}}, \tag{26}$$

$$V = \frac{2n}{w_0^2} + \frac{mk^2zF}{4w_0^2U^2}, \quad \text{and} \quad T = 1 - \frac{k^2zF}{8U^2}. \tag{27}$$

Similarly, under the condition $\kappa\xi\,|x'_2 - x'_1| \ll 1$, we can obtain the average intensity of the incoherent combined Gaussian array beams propagating through non-Kolmogorov turbulence, i.e.,

$$\langle I(x,z)\rangle = H\sum_{m=-(M-1)/2}^{(M-1)/2} \times \exp\left[-2H^2\left(\frac{x}{w_0} - \frac{mx_{\rm d}}{w_0}\right)^2\right]. \tag{28}$$

Figures 9 and 10 give the relative average intensity distributions $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ at the propagation distance where the three Gaussian array beams have the same value of w, which correspond to Figs. 3 and 6 respectively. From Figs. 9 and 10, it can be seen that for both coherent combination and incoherent combination, in free space the relative average intensity distributions are not exactly the same even though the three Gaussian array beams have the same value of w. However, both in Kolmogorov turbulence and in non-Kolmogorov turbulence, the relative average intensity distributions are the same when the three Gaussian array beams have the same value of w.

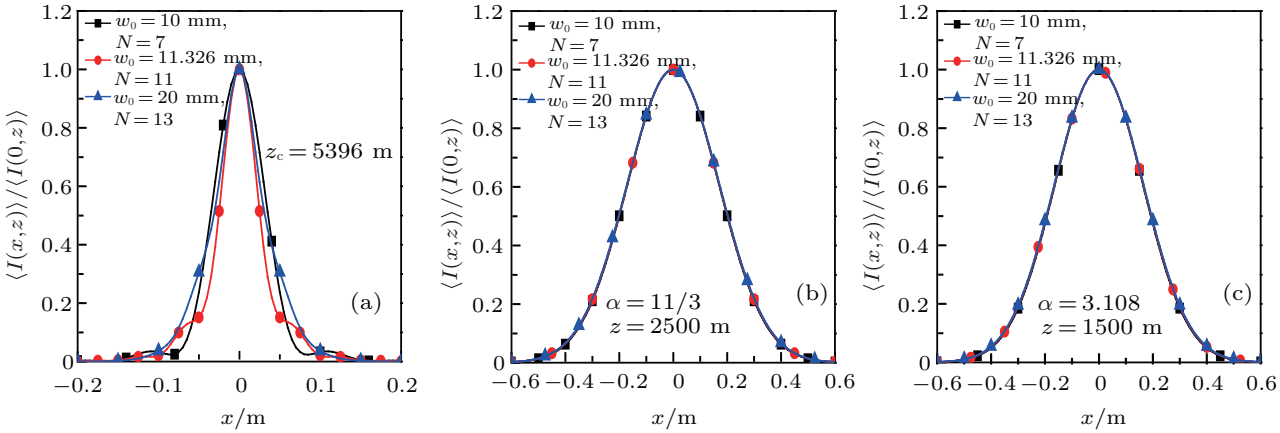


Fig. 9. (color online) Relative average intensity distributions $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ corresponding to Fig. 3 at the position where Gaussian array beams have the same w (a) in free space and (b) and (c) in turbulence under $\tilde{C}_n^2 = 5\times10^{-13}$ m$^{3-\alpha}$.

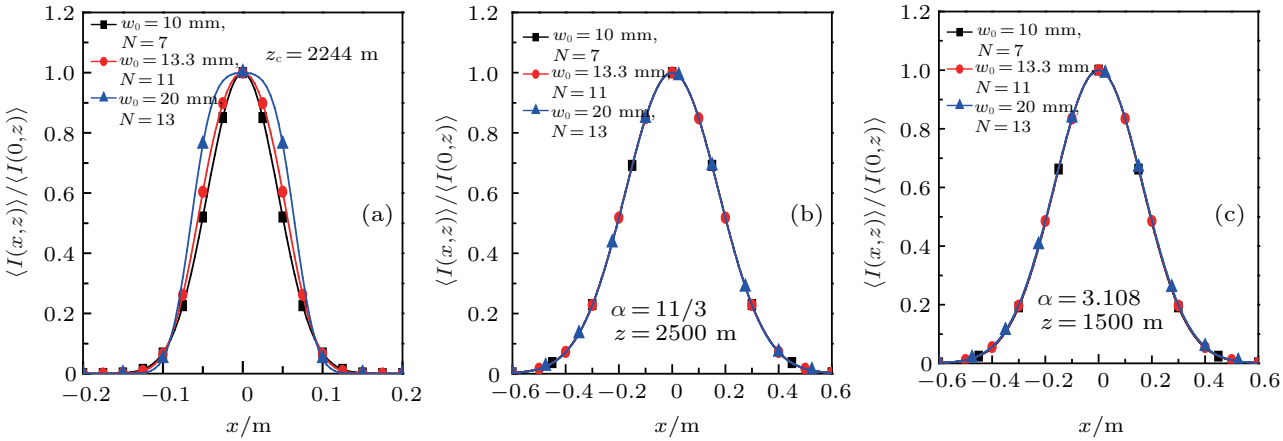


Fig. 10. (color online) Relative average intensity distributions $\langle I(x,z)\rangle/\langle I(0,z)\rangle$ corresponding to Fig. 6 at the position where Gaussian array beams have the same w (a) in free space and (b) and (c) in turbulence under $\tilde{C}_n^2 = 5\times10^{-13}$ m$^{3-\alpha}$.

4. Conclusions

In this paper, some new propagation properties of Gaussian array beams through non-Kolmogorov turbulence are discovered. It is found that in free space, the curves of the mean-squared beam width may cross at the same point at a certain propagation distance z_c, and analytical expressions of z_c are derived. For the coherent combination, the value of z_c is larger than that for the incoherent combination. However, in non-Kolmogorov turbulence, the cross point disappears, and the Gaussian array beams will have the same directionality in terms of the angular spread. Furthermore, a short propagation distance is needed to reach the same directionality when $\alpha = 3.108$. In particular, it is shown that in turbulence, the condition derived in previous studies[8–11] is not necessary for laser beams to have the same directionality in turbulence, which is explained physically. It is noted that the same directionality is exact in Refs. [8]–[11], but only approximate in this paper. On the other hand, it is found that in free space, the relative average intensity distributions are not exactly the same at the position z_c. However, in turbulence, the relative average intensity distributions are the same at the position where Gaussian array beams have the same mean-squared beam width. The results obtained in this paper will be useful for applications of array beams propagating through atmospheric turbulence.

References

[1] Gbur G and Wolf E 2002 *J. Opt. Soc. Am. A* **19** 1592
[2] Young C Y, Gilchrest Y V and Macon B R 2002 *Opt. Eng.* **41** 1097
[3] Ricklin J C and Davidson F M 2002 *J. Opt. Soc. Am. A* **19** 1794
[4] Dan Y Q, Zeng S G, Hao B Y and Zhang B 2010 *J. Opt. Soc. Am. A* **27** 426
[5] Zhou G Q, Cai Y J and Chu X X 2012 *Opt. Express* **20** 9897
[6] Li Y Q and Wu Z S 2012 *Chin. Phys. B* **21** 054203
[7] Qian X M, Zhu W Y and Rao R Z 2012 *Chin. Phys. B* **21** 094202
[8] Shirai T, Dogariu A and Wolf E 2003 *Opt. Lett.* **28** 610
[9] Ji X L and Li X Q 2009 *J. Opt. Soc. Am. A* **26** 236
[10] Chen X W and Ji X L 2008 *Opt. Commun.* **281** 4765
[11] Ji X L, Li X Q and Ji G M 2008 *Opt. Express* **16** 18850
[12] Kolmogorov A N and Acad C R 1941 *Sci. URSS* **30** 301
[13] Rao C H, Jiang W H and Ling N 2000 *J. Mod. Opt.* **47** 1111
[14] Zilberman A, Golbraikh E and Kopeika N S 2005 *Proc. SPIE* **5987** 598702
[15] Tatarski V I 1961 *Wave Propagation in a Turbulent Medium* (New York: McGraw-Hill)

[16] Toselli I, Andrews L C, Phillips R L and Ferrero V 2007 *Proc. SPIE* **6551** 65510E-1
[17] Toselli I, Andrews L C, Phillips R L and Ferrero V 2008 *Opt. Eng.* **47** 026003
[18] Wu G H, Guo H, Yu S and Luo B 2010 *Opt. Lett.* **35** 715
[19] Shchepakina E and Korotkova O 2010 *Opt. Express* **18** 10650
[20] Gerçekcioğlu H and Baykal Y 2012 *J. Opt. Soc. Am. A* **29** 169
[21] Huang Y P, Zhao G P, Xiao X and Wang F H 2012 *Acta Phys. Sin.* **61** 144202 (in Chinese)
[22] He X M and Lü B D 2012 *Acta Phys. Sin*. **61** 054201 (in Chinese)
[23] Deng J P, Ji X L, Pu Z C, Li X Q and Jia X H 2013 *Opt. Commun.* **301–302** 19
[24] Yelden E F, Seguin H J J, Capjack C E and Nikumb S K 1991 *Appl. Phys. Lett.* **58** 693
[25] Bilida W D, Strohschein J D and Seguin H J J 1997 *Proc. SPIE* **2987** 13
[26] Cai Y J, Lin Q, Baykal Y and Eyyuboğlu H T 2007 *Opt. Commun.* **278** 157
[27] Zhu Y B, Zhao D M and Du X Y 2008 *Opt. Express* **16** 18437
[28] Tao R M, Si L, Ma Y X, Zhou P and Liu Z J 2012 *Appl. Opt.* **51** 5609
[29] Tang H, Ou B L, Luo B, Guo H and Dang A H 2011 *J. Opt. Soc. Am. A* **28** 1016
[30] Andrews L C and Phillips R L 2005 *Laser Beam Propagation through Random Media*, 2nd edn. (Bellingham: SPIE Optical Engineering Press)