# Ultrashort pulse breaking in optical fiber with third-order dispersion and quintic nonlinearity*

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The optical wave breaking (OWB) characteristics in terms of the pulse shape, spectrum, and frequency chirp, in the normal dispersion regime of an optical fiber with both the third-order dispersion (TOD) and quintic nonlinearity (QN) are numerically calculated. The results show that the TOD causes the asymmetry of the temporal- and spectral-domain, and the chirp characteristics. The OWB generally appears near the pulse center and at the trailing edge of the pulse, instead of at the two edges of the pulse symmetrically in the case of no TOD. With the increase of distance, the relation of OWB to the TOD near the pulse center increases quickly, leading to the generation of ultra-short pulse trains, while the OWB resulting from the case of no TOD at the trailing edge of the pulse disappears gradually. In addition, the positive (negative) QN enhances (weakens) the chirp amount and the fine structures, thereby inducing the OWB phenomena to appear earlier (later). Thus, the TOD and the positive (negative) QN are beneficial (detrimental) to the OWB and the generation of ultra-short pulse trains.

**Keywords:** optical wave breaking, third-order dispersion, quintic nonlinearity

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## 1. Introduction

Owing to the interplay of the dispersion effects and self-phase or cross-phase modulation, many interesting and useful nonlinear optical phenomena, such as modulation instability,[1,2] optical solitons,[3] supercontinuum generation,[4–6] pulse compression,[7,8] etc., can occur in optical fibers. Typically, optical wave breaking, as an important phenomenon, is thought to occur in a normal dispersion optical fiber and generally manifests itself in the appearance of oscillation structures in the wings of the pulse and, concomitantly, in the sidelobes on the pulse spectrum. It has aroused special and increasing interest for its theoretical and practical significances. Theoretically, researchers have already developed several approximate analytical methods to qualitatively and semi-quantitatively analyze its physical mechanism and the condition for avoiding wave breaking, and to predict the wave breaking distance, the temporal evolution of an optical pulse, etc. Typical analytical methods include variable substitution,[9] chirp analysis,[10,11] and the moment method.[12] These analytical methods are also valuable and adaptable for studying the generation of similariton. On the other hand, however, the optical wave breaking phenomenon must be taken into consideration seriously in many practical applications, because many previous investigations have revealed its detriment to the maximal and high efficiency pulse compression,[13,14] generating similaritons in fiber amplifiers,[11] obtaining high-quality wave-breaking-free pulses in fiber lasers,[15–18] etc. On the other hand, previous reports also indicate that it is beneficial for coherent continuum formation.[19]

So far, optical wave breaking has been investigated theoretically,[9] numerically,[10–12,14,20] experimentally,[13,14] and theoretically accompanied by numerical simulation[10–12] or by experimental study.[13] In particular, some studies have even taken into account the influences of the third-order dispersion on optical wave breaking in fiber lasers,[21,22] fiber amplifiers,[23] and silicon optical waveguides.[24] It is discovered that the third-order dispersion can trigger asymmetric profiles in both the temporal and spectral domains of the pluses but may not reduce the pulse quality considerably.[21,22] While in fiber amplifiers, the results indicate that the third-order dispersion can compensate for nonlinear phase shifts.[23] In passive optical fibers, the influence of quintic nonlinearity on optical wave breaking has been investigated[25,26] analytically and numerically. As many previous studies have revealed, when the ultra-short optical pulse has high incident intensities or when it propagates in a material with high nonlinear coefficients such as semiconductor-doped glass fibers, both the third-order dispersion and the quintic nonlinearity

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will take effect and influence its propagation considerably. To the best of our knowledge, however, the combined influence of the third-order dispersion and the quintic nonlinearity on the optical wave breaking of the picosecond or femtosecond pulse has not been explored yet, which is also the task of the present work.

## 2. Calculations and discussion

As is well known, the standard nonlinear Schrödinger equation which governs the picosecond optical pulse propagating in an ordinary passive optical fiber only includes the second-order dispersion and cubic nonlinearity.[10,11] While in fiber lasers, this equation should be modified by supplementing the gain and gain dispersion terms.[16–18,27] However, as mentioned in many reports, when the subpicosecond optical pulse has a high incident intensity or when it propagates in material with a high nonlinear coefficient such as semiconductor-doped glass fibers, the nonlinear Schrödinger equation even includes the fourth-order dispersion and cubic-quintic nonlinearity.[28] To emphasize the influence of the third-order dispersions, we ignore the fourth-order dispersion, which is generally small. Then, the extended nonlinear Schrödinger equation can be obtained in the following form:

$$\frac{\partial A}{\partial z}+\mathrm{i}\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2}-\frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3}=\mathrm{i}\gamma_1|A|^2A+\mathrm{i}\gamma_2|A|^4A, \quad (1)$$

where $A$ is the slowly varying envelope of the optical field, the parameters $\beta_2$ and $\beta_3$ are the second- and the third-order dispersions, respectively, $\gamma_1$ and $\gamma_2$ are cubic and quintic nonlinear coefficients, respectively, $z$ is the propagating distance, and $T$ is the retarded time. In order to numerically calculate Eq. (1) conveniently, one usually transforms Eq. (1) into the following normalized form:

$$\frac{\partial U}{\partial \xi}=\mathrm{i}a_0\frac{\partial^2 U}{\partial \tau^2}+a_3\frac{\partial^3 U}{\partial \tau^3}+\mathrm{i}a_1|U|^2U+\mathrm{i}a_2|U|^4U. \quad (2)$$

Here, $a_0=-0.5\mathrm{sgn}(\beta_2)L_{\mathrm{D2}}/L_{\mathrm{D1}}=-0.5\mathrm{sgn}(\beta_2)\mu$ with $L_{\mathrm{D1}}=T_0^2/|\beta_2|$ and $L_{\mathrm{D2}}=T_0^3/|\beta_3|$ being the second- and the third-order dispersion lengths of the fiber, respectively. $T_0$ is the pulse width. $a_1=L_{\mathrm{D2}}/L_{\mathrm{NL1}}=\mu N^2$ and $a_2=\mathrm{sgn}(\gamma_2)L_{\mathrm{D2}}/L_{\mathrm{NL2}}=\mu N^2R$ with $L_{\mathrm{NL1}}=1/(|\gamma_1|P_0)$ and $L_{\mathrm{NL2}}=1/\left(|\gamma_2|P_0^2\right)$ being the cubic and quintic nonlinear lengths of the fiber, respectively. $a_3=\mathrm{sgn}(\beta_3)/6$. $U=A/\sqrt{P_0}$ and $\xi=z/L_{\mathrm{D2}}$ are the normalized envelope of the optical field and distance, respectively. $N=\sqrt{L_{\mathrm{D1}}/L_{\mathrm{NL1}}}$ is the soliton order. $\tau=T/T_0$ is the normalized time. The parameters $\mu$ and $R$ are respectively defined as $\mu=L_{\mathrm{D2}}/L_{\mathrm{D1}}$ and $R=\mathrm{sgn}(\gamma_2)L_{\mathrm{NL1}}/L_{\mathrm{NL2}}$. $P_0$ is the incident optical power, and sgn denotes the symbol function.

According to Eq. (2), one can numerically calculate both the temporal- and spectral-domain evolutions of optical pulses by adopting the split-step Fourier method. During the following calculation, in order to conveniently partially compare with those of the previous reports,[10,23,24] the common parameters are set to be $\mu=5$ and $N=8$. Correspondingly, the matched parameters should be $\lambda_0=1.55$ μm, $\beta_2=0.2$ ps$^2$·km$^{-1}$, $\beta_3=0.01$ ps$^3$·km$^{-1}$, $\gamma_1=5.1$ W$^{-1}$·km$^{-1}$, $P_0=40.42$ W, and $T_0=0.25$ ps. Besides, the values of parameter $\gamma_2$ should be $\pm4.5132\times10^{-2}$ W$^{-2}$·km$^{-1}$, $-2.2566\times10^{-2}$ W$^{-2}$·km$^{-1}$, and $-6.0177\times10^{-2}$ W$^{-2}$·km$^{-1}$, for $R=\pm0.36$, $-0.18$, and $-0.48$, respectively. Moreover, without loss of generality, we assume that the initial optical pulse is of a Gaussian profile.

The numerically calculated temporal domain shapes of an initial Gaussian pulse for different values of the quintic nonlinearity parameters $R$ are shown in Fig. 1. It can be clearly seen from Fig. 1 that the positive quintic nonlinearity can effectively enhance optical wave breaking by developing more oscillation structures of the pulses and making optical wave breaking appear earlier, while the negative quintic nonlinearity leads to the opposite scenario. Furthermore, with the increase of the absolute value of the negative quintic nonlinearity parameter $R$, the optical wave breaking weakens in terms of the delayed appearance of and the decreasing of the oscillation structures. Therefore, appropriate negative quintic nonlinearity can be utilized to suppress and retard optical wave breaking. Moreover, in comparison with the case of no third-order dispersion, the pulse obviously becomes asymmetric and the optical wave breaking generated oscillation structures appear near the pulse center and on the trailing edge of the pulse, instead of on the trailing and leading edge of the pulse symmetrically. Concretely, with the increase of distance, the pulse is distorted first with the sharpening of the leading edge. Afterwards, oscillation structures begin to appear at the trailing edge of the pulse instead of at the leading and trailing edges of the pulse symmetrically in the case of no third-order dispersion. However, with the further increase of distance, oscillation structures also begin to appear near the pulse center. Meanwhile, the pulse broadens asymmetrically and stretches to its trailing edge. Then, the oscillation structures at the trailing edge will increase before decreasing and finally disappear, whereas the ones near the pulse center will become more and more apparent. Thus, one can readily conclude that the optical wave breakings at the trailing edge and near the pulse center respectively result from the cases of the second- and the third-order dispersions.

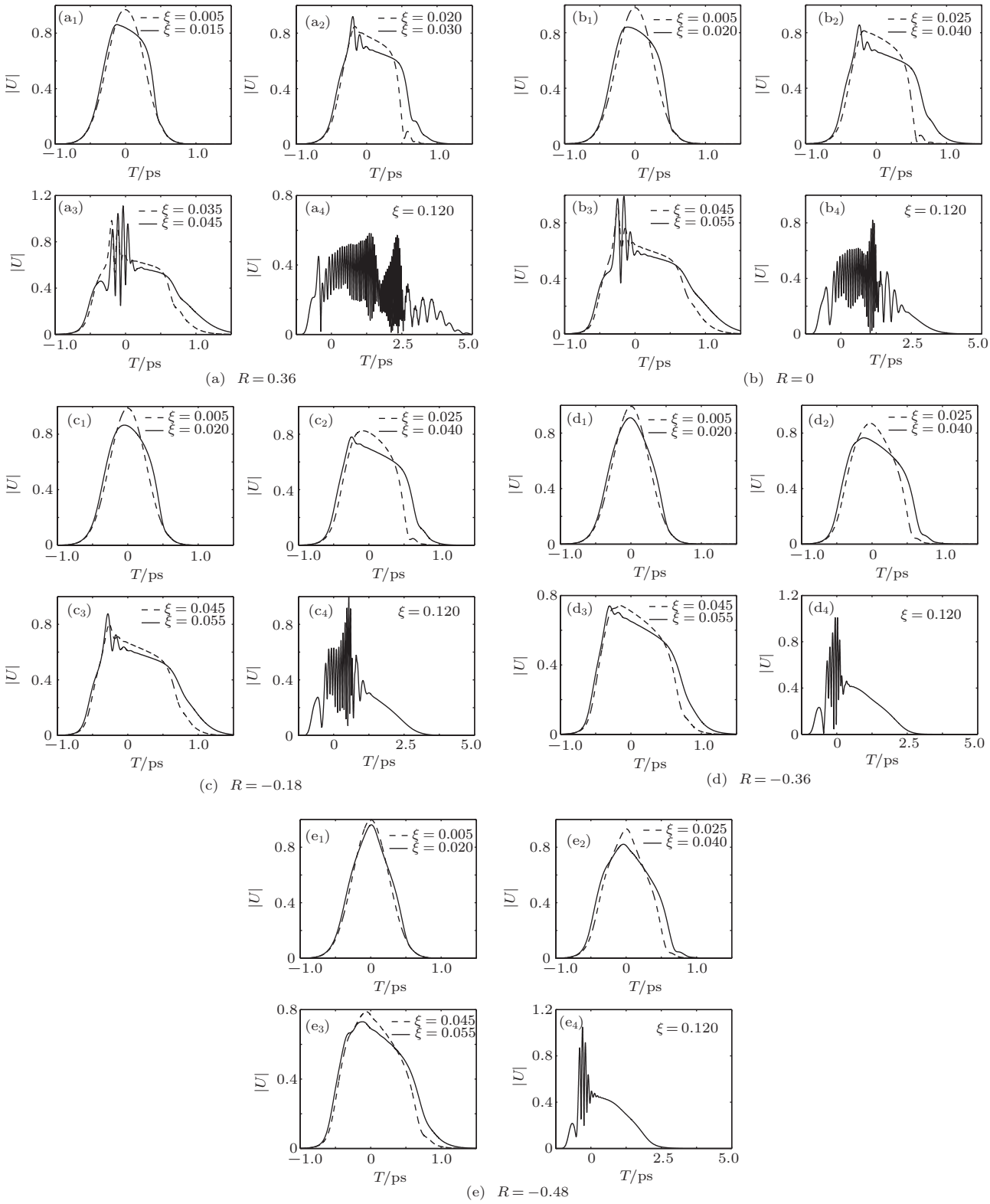



**Fig. 1.** Numerically calculated temporal domain evolutions of optical pulses for different values of quintic nonlinearity parameter *R*.

Corresponding to the temporal domain evolutions discussed above, the spectral evolutions are numerically calculated for different values of quintic nonlinearity parameter *R* as shown in Fig. 2. Obviously the spectra become asymmetric and exhibit multi-peak structures as well, which is quite different from the case of no third-order dispersions.[25,26] Moreover, the spectral maximum is red-shifted whereas the whole spectrum stretches toward the anti-Stokes direction. More (less) multi-peak spectral structures and a wider (narrower) frequency range can be observed for positive (negative) quintic nonlinearity than in the case of cubic nonlinearity. In other words, the spectrum for the positive (negative) quintic nonlinearity will broaden more rapidly (slowly).

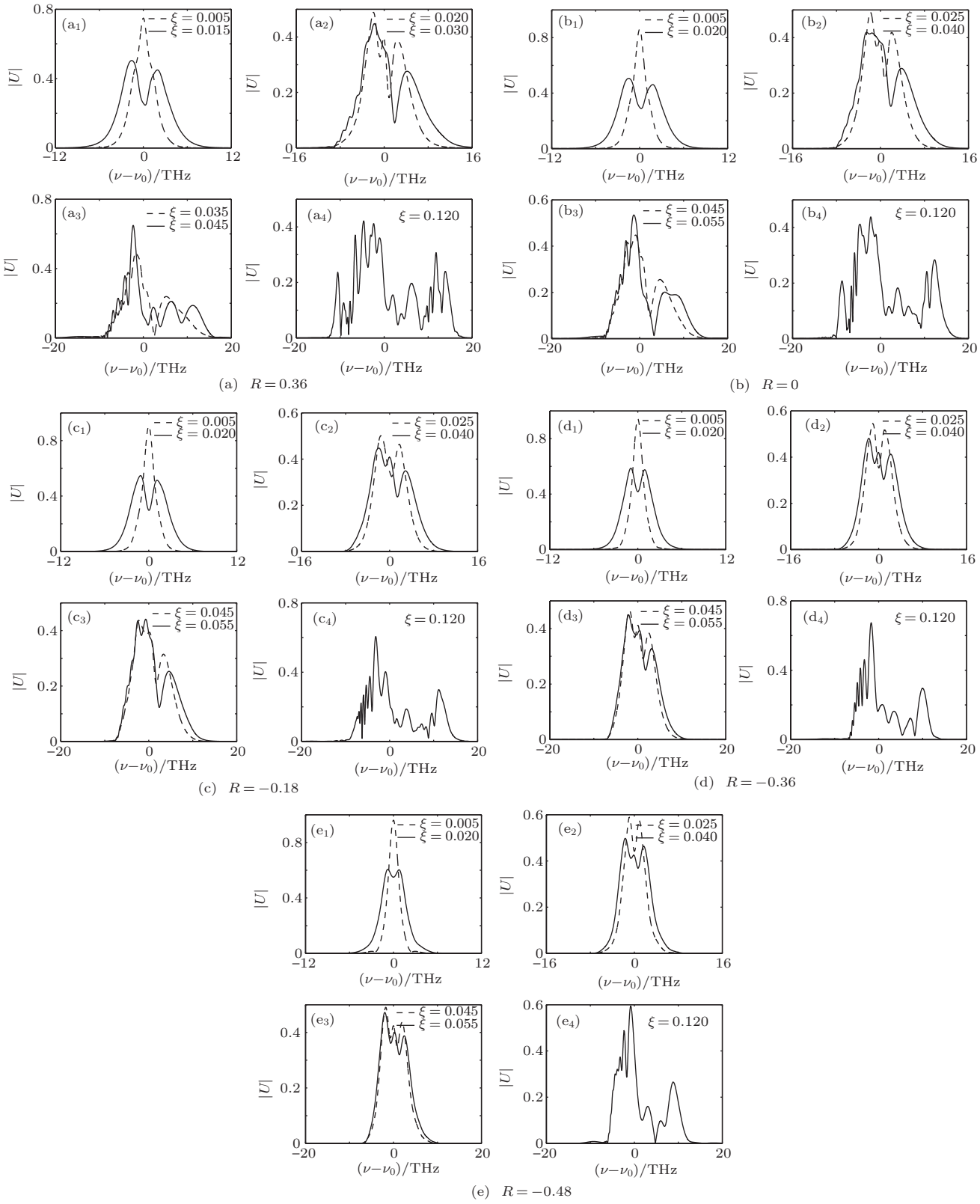



**Fig. 2.** Numerically calculated spectral evolutions of optical pulses for different values of quintic nonlinearity parameter $R$.

It is worth mentioning that, if neglecting the third-order dispersion, our numerical results accord well with those of Refs. [10], [25], and [26] for cubic and cubic-quintic nonlinearities, respectively.

Both the temporal and spectral domain evolutions are closely related to the frequency chirps developed during optical wave breaking. Thus, the numerically calculated frequency chirps of an initially Gaussian pulse for different values of quintic nonlinearity parameter $R$ are shown in Figs. 3–5. It can be seen that at the initial evolution stage, the chirp takes

on an s-shaped profile whether the quintic nonlinearity parameter is negative, positive, or zero. Afterwards, the oscillatory chirp structures for the positive (negative) quintic nonlinearity appear earliest (latest) at the trailing edge of the pulse. Owing to these nonmonotonic chirp structures, overtaking of different parts of the pulse occurs, which causes the optical wave to break ultimately at the trailing edge of the pulse. With an increase of distance, these oscillatory chirp structures increase before decreasing in terms of their intensities as well as numbers and then disappear finally. Correspondingly, the optical wave breaking at the trailing edge of the pulse will inevitably experience generation, increase, decrease, and disappearance. In comparison, the oscillatory chirp structures near the pulse center appear late but grow rapidly. Owing to these nonmonotonic chirps, more and more multi-peak structures appear and soon dominate the ones at the trailing edge. As a result, with the further increase of distance, these subsequent intense nonmonotonic chirps will inevitably result in optical wave breaking, which manifests itself in the generation of ultra-short pulse trains. Besides, one can still realize that after the initial evolution stage, the positive frequency chirps ultimately dominate the negative ones, which makes the whole spectrum stretch toward the anti-Stokes direction as described above. Moreover, the intense central asymmetry of chirp curves naturally causes the asymmetry of the pulse shapes and spectra. In addition, figures 3–5 also indicate that more (less) oscillatory chirp peaks can be observed for positive (negative) quintic nonlinearity at the same distance, which means that positive (negative) quintic nonlinearity is beneficial (detrimental) to the optical wave breaking and the generation of ultra-short pulse trains. However, the generated ultra-short pulse trains exhibit intense pedestals. Thus, further shaping of the pulse is required to improve the pulse quality before its practical application.

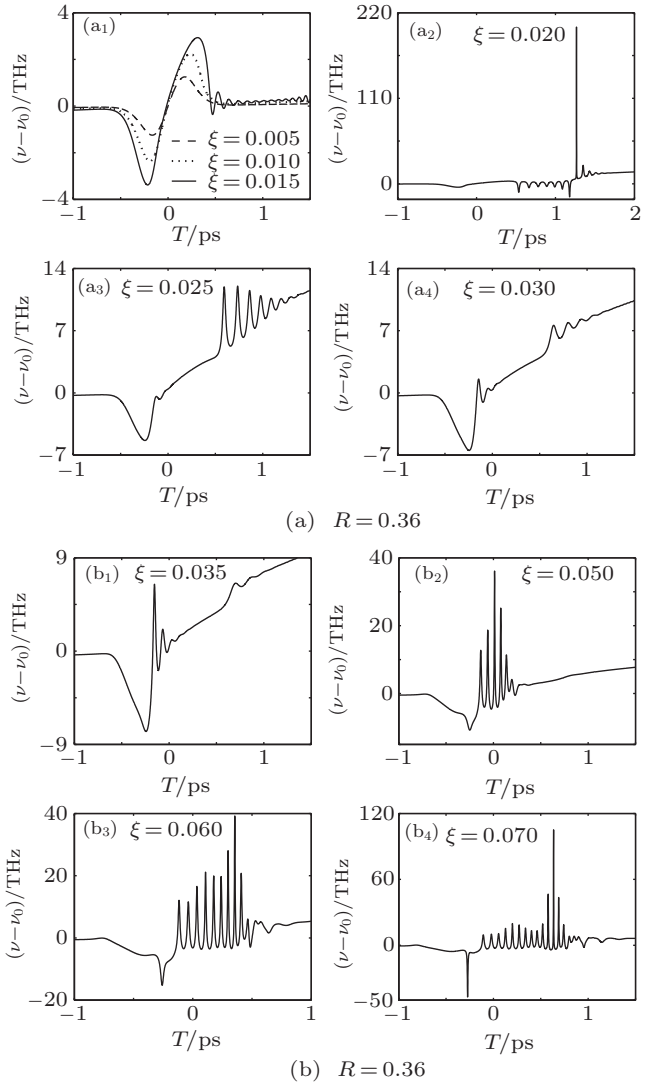



**Fig. 3.** Numerically calculated evolutions of frequency chirps of optical pulses for $R = 0.36$.

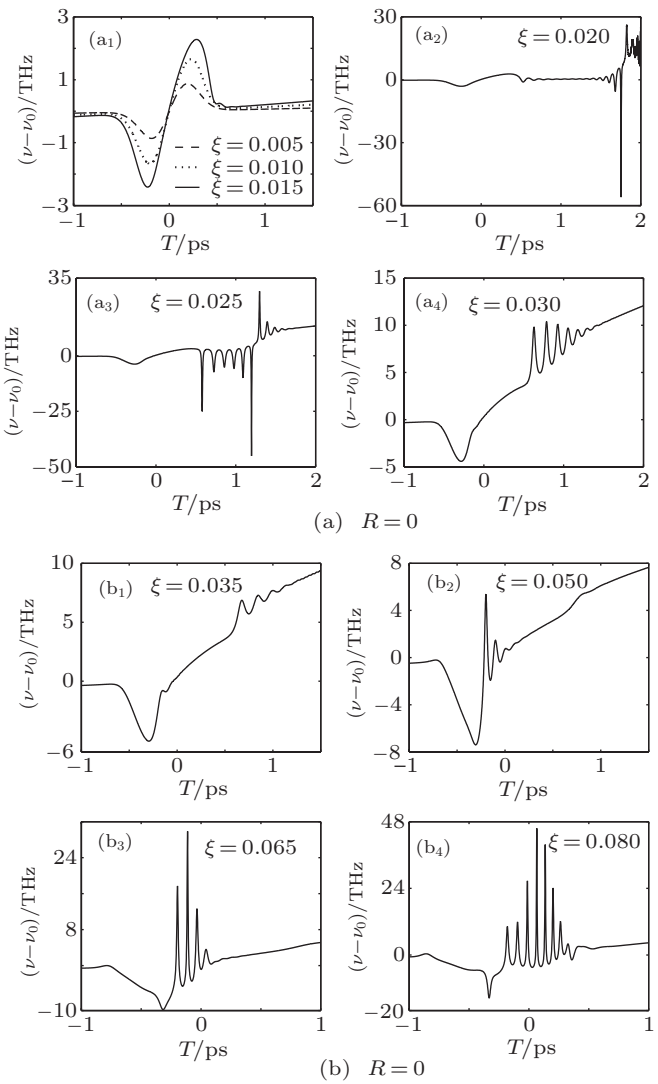



**Fig. 4.** Numerically calculated evolutions of frequency chirps of optical pulses for $R = 0$.

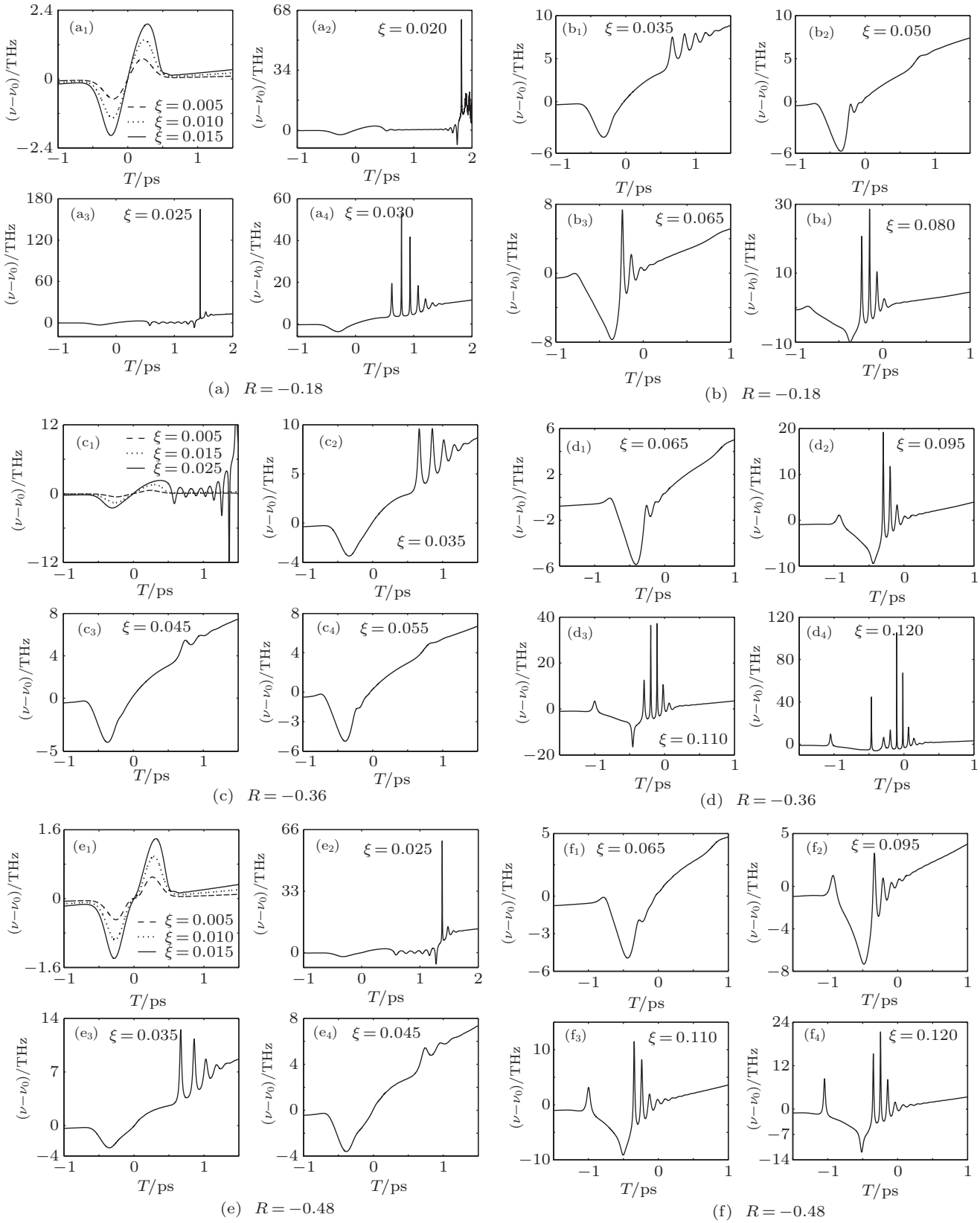



**Fig. 5.** Numerically calculated evolutions of frequency chirps of optical pulses for some values of negative quintic nonlinearity $R=-0.18$, $-0.36$, $-0.48$.

In comparison, in the case of Ref. [17], the frequency chirp in the central region of the pulse is linear and very low, which means that the frequency components of different pulse positions can travel at approximately the same speed. Thus, wave-breaking-free pulses can be generated. In our work, however, the frequency chirps are obviously highly nonlinear

and very large, which means that the frequency components of different pulse positions will travel at different speeds. Therefore, optical wave breaking inevitably occurs more easily here.

## 3. Conclusions

In this paper, the temporal- and spectral-domain evolutions of initial Gaussian optical pulses towards the optical wave breaking and the generation of ultra-short pulse trains are numerically investigated on the basis of an extended nonlinear Schödinger equation with both the third-order dispersion and cubic-quintic nonlinearity. The properties of optical wave breaking and the generated ultra-short pulse trains are qualitatively explained from the viewpoint of chirp evolutions. The results indicate that the third-order dispersion results in the intense central asymmetry of chirp curves, which correspondingly causes the asymmetry of the pulse shape and its spectrum. Moreover, the third-order dispersion-caused frequency chirp will quickly and ultimately dominate the one caused by the case of no third-order dispersion and is obviously beneficial to the optical wave breaking and the generation of ultra-short optical pulse trains. The positive (negative) quintic nonlinearity can enhance (weaken) the chirp amount as well as the nonmonotonic chirp structures and then make the optical wave breaking phenomena appear earlier (later) and more easily. Accordingly, the positive (negative) quintic nonlinearity is beneficial (detrimental) to the optical wave breaking and the generation of ultra-short optical pulse trains. In other words, from the viewpoint of engineering applications, the third-order dispersion and positive quintic nonlinearity can be utilized to generate coarse ultra-short optical pulse trains which may be applicable to the occasion where the requirement is not too high. On the contrary, the negative quintic nonlinearity can be utilized to suppress optical wave breaking and may be applicable to obtaining high-quality wave-breaking-free ultra-short pulses in fiber lasers or to high-efficiency pulse compression.

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