Scattering of particles in wave beam based on series expansion*

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(Received 28 September 2013; revised manuscript received 17 December 2013; published online 10 April 2014)

Based on the orthogonalities of the vector wave functions, the expressions of scattering fields are developed as the particle in the zero order field and in the first order fields of x, y, and z, respectively. A general relation between the expansion coefficients of scattering field and incident field is presented. Taking the elliptical beam for example, the scattering property of a particle in beam is investigated. After analyzing the effects of the beam waist, irradiating distance, etc. on scattering property, the validity of the proposed algorithm is demonstrated. Results show that the beam waist may improve the particle identification property and the particle has a strong scattering both in the forward direction and in the backward direction. The method proposed is simple and is a new way of researching the scattering from particles in electromagnetic beams.

Keywords: scattering, electromagnetic beam, Gaussian beam

PACS: 33.20.Fb, 52.25.Os, 41.85.Ap, 42.25.Bs **DOI:** 10.1088/1674-1056/23/6/063301

1. Introduction

The technology of photoelectric beam detection has now become an indispensable means in the fields of space, defense, and civilian industry.[1–8] The laser beam detection, beam imaging system of laser radar, etc. are gradually becoming a popular research focus. The scattering property of the particle for an incident beam and the interaction between the particle and the beam are subjects that need to be well studied in these realms. The scattering characteristics for a homogeneous sphere irradiated by a plane electromagnetic wave have been studied in detail,[9] which provides an evaluation standard for the general particle scattering. The scattering characteristics of the spherical particles for a Gaussian beam and other beams have been investigated, respectively.[10–12] The effects of factors such as the beam waist on particle scattering property have been analyzed. Additionally, the internal field and the near field for a homogeneous sphere in the presence of a high order Gaussian beam have been investigated[13] by using the expansion of spherical wave function. In the presence of focused Laguerre–Gaussian beam, Zhao and Jiang[14] studied the scattering property of a particle and derived the scattering coefficients. It was found that expanding the beam with the spherical wave function and unifying the coordinate system were effective methods of studying the scattering property for a pair of spheres.[15] The method of expressing the beam in series to study Rayleigh scattering and rough surface scattering has a high accuracy of calculation.[16,17] Although there are two ways to study the particle scattering property, namely the expanding of spherical vector wave function and the complete numerical calculation: the former usually cannot give the expanding coefficients of the incident wave and uses numerical integration, while the latter is not economic and its result is confined by factors such as the beam shape and the concrete programming. In this work, the distribution of field and scattering property for a particle based on the references will be investigated by expanding the electromagnetic beam in a Taylor series. Firstly the items of the Taylor series will be expressed with spherical wave function, which has the generality and is not confined by the beam shape. The expanded coefficients of the incident wave x^n, y^n, z^n, etc. will be developed. It is found that these coefficients are only dependent on the particle radius and beam partial derivatives. Secondly, the scattered fields corresponding to the incident wave of x^n, y^n, and z^n will be, respectively, presented in the analytical expressions. Finally taking the elliptical Gaussian beam as an example, the one order scattering property of a particle will be investigated in detail and the validity of the proposed method will be demonstrated. The effects of beam waist and frequency, etc. on scattering characteristics will be simulated. In the following analysis the $e^{j\omega t}$ time dependence will be assumed and suppressed throughout.

2. Scattered field by a particle in photoelectric beam

2.1. General expression of scattered field

Under the irradiation of a plane electromagnetic wave, the scattered field by a spherical particle is acquired with the well-known Mie theory. In the photoelectric beam, the scattered field by a particle is related to the particle radius, dielectric constant, and the beam shape. In order to make the obtained result universal, we assume that the incident wave beam is ex-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 60971079, 61271110, and 61102018).

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pressed in the particle coordinate system, polarized in the x direction and propagating along the z direction, namely

$$\boldsymbol{E} = f(x,y,z)\hat{x}\,\mathrm{e}^{-\mathrm{j}kz}.$$

When the above expression is expanded at the origin of the system, it is obtained as

$$\begin{aligned}\boldsymbol{E} = \hat{x}\Bigg[& f(0,0,0) \\ &+ \frac{1}{1!}\left[x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+z\frac{\partial}{\partial z}\right]f(x,y,z)+\cdots\Bigg]\mathrm{e}^{-\mathrm{j}kz}. \quad (1)\end{aligned}$$

The first item on the right-hand side in Eq. (1) is a plane electromagnetic wave with an amplitude of $f(0,0,0)$, which reduces to

$$\boldsymbol{E}_0 = \hat{x}f_0\,\mathrm{e}^{-\mathrm{j}kz}.$$

This item may be expanded with the spherical vector wave function[17]

$$\begin{aligned}\boldsymbol{M}^i_{mn} &= \frac{1}{\sin\theta}\mathrm{z}_n(kr)\frac{\partial Y^i_{mn}}{\partial\phi}\hat{u}_\theta - \mathrm{z}_n(kr)\frac{\partial Y^i_{mn}}{\partial\theta}\hat{u}_\varphi, \\ \boldsymbol{N}^i_{mn} &= \frac{n(n+1)}{kr}\mathrm{z}_n(kr)Y^i_{mn}\hat{u}_r + \frac{1}{kr}\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{z}_n(kr)]\frac{\partial Y^i_{mn}}{\partial\theta}\hat{u}_\theta \\ &\quad + \frac{1}{kr\sin\theta}\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{z}_n(kr)]\frac{\partial Y^i_{mn}}{\partial\varphi}\hat{u}_\varphi, \\ Y^i_{mn} &= P^m_n(\cos\theta)\begin{bmatrix}\cos m\varphi, & i=1\\ \sin m\varphi, & i=2\end{bmatrix},\end{aligned}$$

where $\mathrm{z}_n(kr)$ may be the spherical Bessel function $\mathrm{j}_n(kr)$, spherical Hankel function $\mathrm{h}^{(1)}_n(kr)$ or $\mathrm{h}^{(2)}_n(kr)$. On the surface of sphere with radius a, it is proved that the other coefficients of expanding expression are zero. Thus, the electromagnetic field corresponding to the first item can be written as

$$\begin{aligned}\boldsymbol{E} &= \sum_n\left[a^2_{1n}\boldsymbol{M}^2_{1n} + b^1_{1n}\boldsymbol{N}^1_{1n}\right], \\ \boldsymbol{H} &= \frac{j}{\eta}\sum_n\left[a^2_{1n}\boldsymbol{N}^2_{1n} + b^1_{1n}\boldsymbol{M}^1_{1n}\right], \quad (2)\end{aligned}$$

where $\mathrm{z}_n(kr)$ denotes spherical Bessel function $\mathrm{j}_n(kr)$. The nonzero coefficients are, respectively, as follows:

$$\begin{aligned}a^2_{1n} &= \pi k_{1n}f_0\mathrm{j}_n(ka)s^2_n, \\ s^2_n &= n\int_{-1}^{1}\frac{\left[P^1_{n+1}(t)-tP^1_n(t)\right]}{(1-t^2)^{1/2}} \\ &\quad\times\left[1-\mathrm{j}kat-\frac{k^2a^2}{2}t^2+\frac{\mathrm{j}k^3a^3}{6}t^3+\cdots\right]\mathrm{d}t, \\ b^1_{1n} &= \pi l_{1n}\frac{f_0}{kr}\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{j}_n(kr)]s^1_n, \\ s^1_n &= \int_{-1}^{1}\left[\frac{P^1_n(t)\left(1-(n+1)t^2\right)+ntP^1_{n+1}(t)}{(1-t^2)^{1/2}}\right] \\ &\quad\times\left[1-\mathrm{j}kat-\frac{k^2a^2}{2}t^2+\cdots\right]\mathrm{d}t.\end{aligned}$$

The above integration is independent of the particle parameter. Functions k_{mn} and l_{mn} can be found from Ref. [18]. In application, the series is truncated by k^4a^4, this is accurate enough for the use here. Partial simulations of the integration are given as

$$\begin{aligned}s^2_1 &= 0.2k^2a^2 + 0.0119k^4a^4, \\ s^2_2 &= \mathrm{j}0.8ka - \mathrm{j}0.0571k^3a^3, \\ s^2_3 &= 0.7286k^2a^2 - 0.0377k^4a^4, \\ s^2_4 &= -\mathrm{j}0.0423k^3a^3, \\ s^2_5 &= -1.125k^2a^2 + 0.0255k^4a^4, \\ s^1_1 &= 2.6667 - 0.8\mathrm{j}ka + 0.0825\mathrm{j}k^3a^3, \\ s^1_2 &= -0.2143k^2a^2 + 0.0139k^4a^4, \\ s^1_3 &= 0.9143\mathrm{j}ka - 0.0762\mathrm{j}k^3a^3, \\ s^1_4 &= 0.4894k^2a^2 - 0.0274k^4a^4, \\ s^1_5 &= -0.0346\mathrm{j}k^3a^3.\end{aligned}$$

According to expression (2), the scattered field and the internal field may be written, respectively, as

$$\begin{aligned}\boldsymbol{E}_s &= \sum_n\left[a^2_{1ns}\boldsymbol{M}^2_{1n} + b^1_{1ns}\boldsymbol{N}^1_{1n}\right], \\ \boldsymbol{H}_s &= \frac{j}{\eta}\sum_n\left[a^2_{1ns}\boldsymbol{N}^2_{1n} + b^1_{1ns}\boldsymbol{M}^1_{1n}\right], \quad (3) \\ \boldsymbol{E}_i &= \sum_n\left[a^2_{1ni}\boldsymbol{M}^2_{1n} + b^1_{1ni}\boldsymbol{N}^1_{1n}\right], \\ \boldsymbol{H}_i &= \frac{\mathrm{j}m}{\eta}\sum_n\left[a^2_{1ni}\boldsymbol{N}^2_{1n} + b^1_{1ni}\boldsymbol{M}^1_{1n}\right], \quad (4)\end{aligned}$$

where $\mathrm{z}_n(kr)$ represents $\mathrm{h}^{(2)}_n(kr)$, and $\mathrm{z}_n(kr)$ will be the spherical Bessel function $\mathrm{j}_n(mkr)$ in expression (4). From expressions (2)–(4), the expanding coefficients of the scattered field are obtained as

$$a^2_{1ns} = a^2_{1n}f_1(n,a), \quad (5)$$

where

$$f_1(n,a) = \frac{\mathrm{j}_n(ka)\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{j}_n(kmr)] - \mathrm{j}_n(kma)\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{j}_n(kr)]}{\mathrm{j}_n(kma)\frac{\mathrm{d}}{\mathrm{d}r}\left[r\mathrm{h}^{(2)}_n(kr)\right] - \mathrm{h}^{(2)}_n(ka)\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{j}_n(kmr)]},$$

$$b^1_{1ns} = b^1_{1n}f_2(n,a), \quad (6)$$

where

$$f_2(n,a) = \frac{\mathrm{j}_n(ka)\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{j}_n(kmr)] - m^2\mathrm{j}_n(kma)\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{j}_n(kr)]}{m^2\mathrm{j}_n(kma)\frac{\mathrm{d}}{\mathrm{d}r}\left[r\mathrm{h}^{(2)}_n(kr)\right] - \mathrm{h}^{(2)}_n(ka)\frac{\mathrm{d}}{\mathrm{d}r}[r\mathrm{j}_n(kmr)]}.$$

For simplicity, the expression

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[r\mathrm{h}^{(2)}_n(kr)\right]$$

in Eq. (5) or (6) denotes the derivative at point $r=a$. It is shown that the relation between the coefficient of the scattered

field and that of the incident field is only determined by the summation index n.

When the incident wave is

$$\boldsymbol{E}_x = \hat{\boldsymbol{x}} f_x x \mathrm{e}^{-\mathrm{j}kz},$$

where f_x denotes the partial derivative at origin with respect to x coordinate. With the available method, we may also obtain the corresponding electromagnetic field as

$$\begin{aligned} \boldsymbol{E}_x &= \sum_n \left[a^2_{2nx}\boldsymbol{M}^2_{2n} + b^1_{2nx}\boldsymbol{N}^1_{2n}\right], \\ \boldsymbol{H}_x &= \frac{\mathrm{j}}{\eta}\sum_n \left[a^2_{2nx}\boldsymbol{N}^2_{2n} + b^1_{2nx}\boldsymbol{M}^1_{2n}\right]. \end{aligned} \tag{7}$$

The coefficients are, respectively,

$$\begin{aligned} a^2_{2nx} &= \pi f_x k_{2n} a \mathrm{j}_n(ka) s^2_{nx}, \\ s^2_{nx} &= \left(1+\frac{n}{2}\right)\int_{-1}^{1}\left[tP^2_n(t) - P^2_{n-1}(t)\right] \\ &\quad \times \left[1 - \mathrm{j}kat - \frac{k^2a^2}{2}t^2 + \cdots\right]\mathrm{d}t, \\ b^1_{2nx} &= \frac{\pi l_{2n} f_x}{2k}\left\{\frac{\mathrm{d}\left[r\mathrm{j}_n(kr)\right]}{\mathrm{d}r}\right\}_{r=a} s^1_{nx}, \\ s^1_{nx} &= \int_{-1}^{1}\left[P^2_n(t)(2+nt^2) - t(n+2)P^2_{n-1}(t)\right] \\ &\quad \times \left[1 - \mathrm{j}kat - \frac{k^2a^2}{2}t^2 + \cdots\right]\mathrm{d}t. \end{aligned}$$

Partial simulations of the integration are given as

$$\begin{aligned} &s^2_{2x} = 2\left[-0.8\mathrm{j}ka + 0.0571\mathrm{j}k^3a^3\right], \\ &s^2_{3x} = 2.5\left[-0.4572k^2a^2 + 0.0254k^4a^4\right], \\ &s^2_{4x} = 0.381\mathrm{j}k^3a^3, \\ &s^2_{5x} = 0.0808k^4a^4, \\ &s^2_{1x} = 0, \\ &s^2_{nx} = 0, \quad n \geq 6, \\ &s^1_{2x} = 9.6 - 3.6571\mathrm{j}ka + 0.3556\mathrm{j}k^3a^3, \\ &s^1_{3x} = -1.7381k^2a^2 + 0.1019k^4a^4, \\ &s^1_{4x} = 3.8095\mathrm{j}ka - 0.2424\mathrm{j}k^3a^3, \\ &s^1_{5x} = 3.1572k^2a^2 - 0.1591k^4a^4, \\ &s^1_{1x=0} = 0. \end{aligned}$$

According to expression (7), we can write the scattered field and the internal field with the spherical vector wave function, and using the boundary condition of electromagnetic field on the surface of the particle, the coefficients of scattered field are obtained as

$$a^2_{2nxs} = a^2_{2nx} f_1(n,a), \tag{8}$$

$$b^1_{2nxs} = b^1_{2nx} f_2(n,a). \tag{9}$$

When the incident waves are, respectively,

$$\boldsymbol{E}_y = \hat{x} f_y y \mathrm{e}^{-\mathrm{j}kz},$$

$$\boldsymbol{E}_z = \hat{x} f_z z \mathrm{e}^{-\mathrm{j}kz},$$

where f_y and f_z are, respectively, the partial derivatives with respect to y and z at the origin. E_y denotes an incident wave polarized in the x direction with a amplitude of $f_y y$, and E_z represents an incident wave polarized in the x direction with an amplitude of $f_z z$. The scattering fields corresponding to the above two formulae may be developed by the same method as that used in developing the scattering field of particles in field of expression (7). Thus, they are not studied here in detail. The following are the relations between the coefficients of scattered fields and those of the incidents fields:

$$a^1_{2nys} = a^1_{2ny} f_1(n,a), \tag{10}$$

$$b^2_{2nys} = b^2_{2ny} f_2(n,a), \tag{11}$$

$$a^2_{1nzs} = a^2_{1nz} f_1(n,a), \tag{12}$$

$$b^1_{1nzs} = b^1_{1n} f_2(n,a). \tag{13}$$

When a general electromagnetic wave beam irradiates the particle, from expressions (8)–(13), we may obtain the total scattered field as

$$\begin{aligned} \boldsymbol{E}_s &= \sum_n \left[a^2_{1ns}\boldsymbol{M}^2_{1n} + b^1_{1ns}\boldsymbol{N}^1_{1n} + a^2_{2nxs}\boldsymbol{M}^2_{2n} + b^1_{2nxs}\boldsymbol{N}^1_{2n}\right. \\ &\quad \left. + a^1_{2nys}\boldsymbol{M}^1_{2n} + b^2_{2nys}\boldsymbol{N}^2_{2n} + a^2_{1nzs}\boldsymbol{M}^2_{1n} + b^1_{1nzs}\boldsymbol{N}^1_{1n}\right]. \end{aligned} \tag{14}$$

In expression (14), the summation index n starts from 1. For the case of the index n starting from 2, we should fill a zero coefficient in front of the coefficients starting from 2. Utilizing the orthogonality of the spherical vector wave functions, we may obtain the scattered power by a surface of sphere with radius r.

$$P_{\mathrm{s}} = \frac{1}{2\eta}\sum_n (t_1 + t_2 + t_3 + t_4), \tag{15}$$

where

$$\begin{aligned} t_1 &= \left(\left|a^2_{1ns}\right|^2 + \left|a^2_{1nzs}\right|^2\right) r^2 \mathrm{h}^{(2)}_n(kr)\mathrm{h}^{(2)*}_n(kr)\frac{2n^2\pi(n+1)^2}{2n+1}, \\ t_2 &= \left(\left|a^2_{2nxs}\right|^2 + \left|a^1_{2nys}\right|^2\right) r^2 \mathrm{h}^{(2)}_n(kr)\mathrm{h}^{(2)*}_n(kr) \\ &\quad \times \frac{2n\pi(n+1)}{2n+1}\frac{(n+2)!}{(n-2)!}, \\ t_3 &= \left(\left|b^2_{2nys}\right|^2 + \left|b^1_{2nxs}\right|^2\right)\frac{1}{k^2}\left\{\frac{\mathrm{d}}{\mathrm{d}r}\left[r\mathrm{h}^{(n)}_n(kr)\right]\right\} \\ &\quad \times \left\{\frac{\mathrm{d}}{\mathrm{d}r}\left[r\mathrm{h}^{(n)}_n(kr)\right]\right\}^* \frac{2n\pi(n+1)}{2n+1}\frac{(n+2)!}{(n-2)!}, \\ t_4 &= \left(\left|b^1_{1nzs}\right|^2 + \left|b^1_{1ns}\right|^2\right)\frac{1}{k^2}\left\{\frac{\mathrm{d}}{\mathrm{d}r}\left[r\mathrm{h}^{(n)}_n(kr)\right]\right\} \\ &\quad \times \left\{\frac{\mathrm{d}}{\mathrm{d}r}\left[r\mathrm{h}^{(n)}_n(kr)\right]\right\}^* \frac{2n^2\pi(n+1)^2}{2n+1}. \end{aligned}$$

The scattering cross section σ is easily acquired, both by the scattered power and by the power density of the incident wave beam. Therefore, there is no need to discuss it here.

2.2. Scattering property of a particle in the elliptical Gaussian beam

From Ref. [3], we may find the expression of the elliptical Gaussian beam in the particle's coordinate system. When this beam is polarized in the x direction and propagates along the z direction, it can be written as

$$f(0,0,0) = E_0\sqrt{-Q_{x0}Q_{y0}}\,\mathrm{e}^{-\mathrm{i}kd}, \tag{16}$$

$$f_z = E_0 \frac{\dfrac{1}{kW_{0y}^2} + \dfrac{1}{kW_{0x}^2} - \mathrm{i}\dfrac{4d}{k^2W_{0y}^2W_{0x}^2}}{\left(\mathrm{i}\left(\dfrac{2\mathrm{d}}{kW_{0y}^2} + \dfrac{2\mathrm{d}}{kW_{0x}^2}\right) + \dfrac{4d^2}{k^2W_{0y}^2W_{0x}^2} - 1\right)^{3/2}}\mathrm{e}^{-\mathrm{i}kd},$$

$$f_x = 0, \quad f_y = 0, \tag{17}$$

where

$$Q_{y0} = \left(\mathrm{i} + \frac{2\mathrm{d}}{kW_{0y}^2}\right)^{-1}$$

and

$$Q_{x0} = \left(\mathrm{i} + \frac{2\mathrm{d}}{kW_{0x}^2}\right)^{-1}.$$

We can obtain the scattered field from a particle in the elliptical Gaussian beam by combining the expressions (2), (7), (16), and (17). The following are the partial simulations.

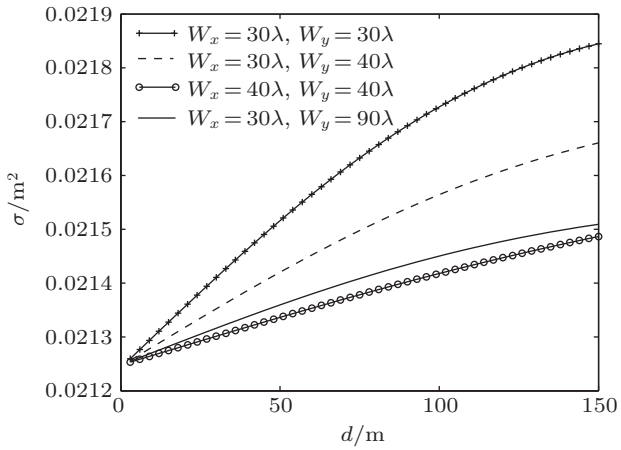


Fig. 1. Variations of RCS with irradiating distance.

The radar cross section (RCS) change with irradiating distance is shown in Fig. 1. The frequency is 3 GHz, the size of the particle is 0.3 mm and the relative dielectric constant is $5.23 - \mathrm{j}\,0.26$ in the computation. The result shows that the RCS is proportional to the irradiating distance d. Since the space between the beam and the particles is greater, the Gaussian beam is closer to the plane wave. When a plane wave irradiates the particle, its RCS is largest, which is in agreement with that in Ref. [18]. In a case where other conditions remain unchanged, a smaller beam waist width will improve its focus property. An excellent focus property will make the particle have more electromagnetic powers from the exposure beam, thus the scattering field becomes strong, as shown in Fig. 1. In Fig. 2, according to Ref. [18], the operating wave length is 0.6328 μm and radius of the particle is 30 μm. This figure shows that the dust particle has a stronger scattering in the forward and backward direction. In Fig. 3, the following parameters are assumed: the beam waist is 10 μm, the radius is 9.1929 μm, and the relative refractive index is 1.33. The dotted line represents the results given by the present study and the solid line denotes the results cited from Ref. [18]. These two results are in agreement with each other. This demonstrates the validity of the algorithm used in this paper. Figure 4 shows a relation between the relative scattered intensity and the beam waist. It is seen that the scattered intensity is nonlinearly increased for the particle as the beam waist increases. It is well known that a bigger beam waist will move the beam closer to the plane electromagnetic wave. Particles have the strongest scattered property in the plane electromagnetic wave.

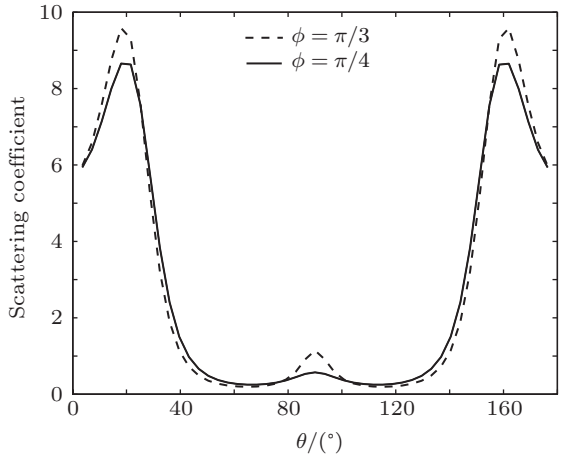


Fig. 2. Variations of normalized RCS with angle.

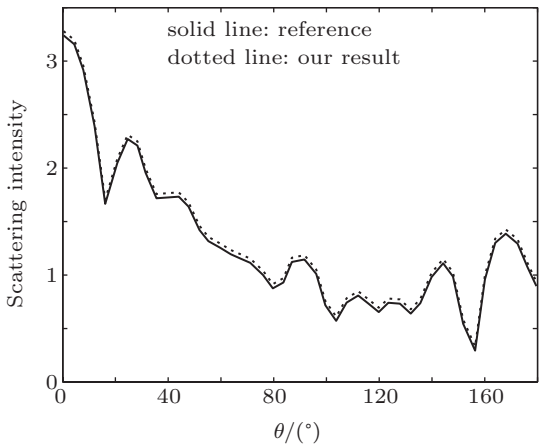


Fig. 3. A comparison of the results predicted in this paper with the results available from the literature.

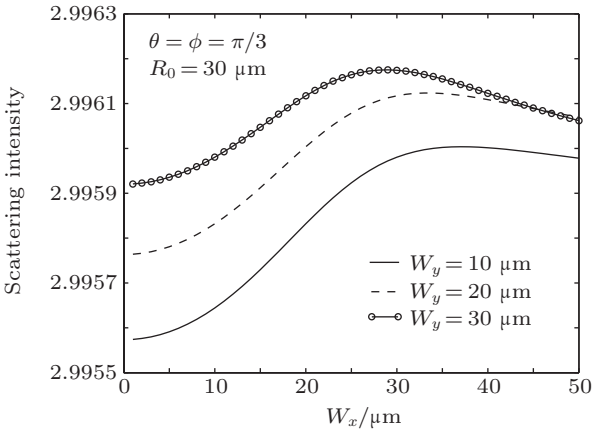


Fig. 4. Effects of the beam waist on scattering intensity.

3. Conclusions

Based on the orthogonalities of vector wave functions and the trigonometric functions, the scattered fields for the zero order incident field and the one order incident field for a general beam wave have been developed, respectively. A general expression between the coefficients of the incident wave expressions and those of scattered field expressions is presented. In a real application, only the derivatives of the beam are needed to entirely determine the scattered field. The cumbersome process of expanding the electromagnetic beam directly with the vector spherical wave function is avoided. The scattering characteristics of the particle in the elliptical Gaussian beam are investigated. The effects of the beam waist, irradiating distance, etc. on the scattered cross section are analyzed, and the scattered intensity is simulated. The validity of the proposed method is tested by comparing the results of this article with those in Ref. [18]. The obtained results are not confined by the size of the particle and frequency, which is of significance in the investigation of scattered property for the particle or the target in an electromagnetic wave beam.

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