Stability analysis of Markovian jumping stochastic Cohen–Grossberg neural networks with discrete and distributed time varying delays*

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In this paper, the global asymptotic stability problem of Markovian jumping stochastic Cohen–Grossberg neural networks with discrete and distributed time-varying delays (MJSCGNNs) is considered. A novel LMI-based stability criterion is obtained by constructing a new Lyapunov functional to guarantee the asymptotic stability of MJSCGNNs. Our results can be easily verified and they are also less restrictive than previously known criteria and can be applied to Cohen–Grossberg neural networks, recurrent neural networks, and cellular neural networks. Finally, the proposed stability conditions are demonstrated with numerical examples.

Keywords: Cohen–Grossberg neural networks, global asymptotic stability, linear matrix inequality, Lyapunov functional, time-varying delays

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1. Introduction

Cohen and Grossberg[1] proposed a class of neural networks in 1983, which has received increasing interest[2,3] due to its promising potential for applications in classification, parallel computing, associative memory, especially in solving some optimization problems.[4] For Cohen–Grossberg neural networks (CGNNs), many sufficient conditions for global stability have been obtained by several authors. By using a Lyapunov functional, some criteria for the globally asymptotic stability of this model were established in Ref. [5]. Recently, a great deal of work has been carried out in the literature, see for example Refs. [6]–[8]. Although discrete time delays in the delayed feedback neural networks having a small number of cells usually serve as good approximation of the prime models, a real system is usually affected by external perturbations. Therefore, it is significant and of prime importance to consider stochastic effects on the stability property of the neural networks with delays (see Refs. [9]–[11]).

Neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, and hence there is a distribution of propagation delays over a period of time. It is worth noting that, although the signal propagation is sometimes instantaneous and can be modeled with discrete delays, it may also be distributed during a certain time period so that the distributed delays should be incorporated in the model. In other words, it is often the case that the neural network model possesses both discrete and distributed delays.[12] In recent years, the stabilities of Hopfield neural networks, cellular neural networks, and recurrent neural networks with distributed delays have been discussed in Refs. [13]–[37].

When the neural network incorporates abrupt changes in its structure, the Markovian jump linear system is very appropriate to describe its dynamics. This class of system is a hybrid system with two components in the state. The first one refers to the mode, which is described by a continuous time finite-state Markovian process, and the second one refers to a state that is represented by a system of differential equations. Markovian jump systems have the advantage of modeling the dynamic systems subject to abrupt variation in their structures, such as component failures or repairs, sudden environmental disturbance, changing subsystem interconnections, and operating in different points of a nonlinear plant.[38] Recently, there has been a growing interest in the study of neural networks with Markovian jumping parameters (See Refs. [39]–[44]).

Motivated by the above discussion, we shall study a class of stochastic CGNNs with discrete and distributed time-varying delays and Markovian jumping parameters. The main purpose of this paper is to study the global asymptotic stability of MJSCGNNs. To the best of the author's knowledge, there are no results for global asymptotic stability problem for MJSCGNNs in terms of linear matrix inequalities (LMIs). The main advantage of the LMI-based approach is that the LMI stability conditions can be solved numerically using MATLAB LMI toolbox,[45] which implements the state of art interior-point algorithms.[46] We also provide numerical examples to demonstrate the effectiveness of the proposed stability results.

Notations: Throughout the manuscript the following notations are used. $A > 0$ (or $A < 0$) to denote that the matrix A is a symmetric and positive-definite (or negative definite) matrix.

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Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (it is right continuous and $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets); $\mathcal{L}^p_{\mathcal{F}_0}([-\tau,0];\mathbf{R}^n)$ be the family of all bounded, $\mathcal{F}_0$- measurable $\mathcal{C}([-\tau,0];\mathbf{R}^n)$-valued random variables $\xi=\{\xi(\theta):-\tau\leq\theta\leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0}\mathbf{E}|\xi(\theta)|^p<\infty$. The mathematical expectation operator with respect to the given probability measure $\mathcal{P}$ is denoted by $\mathbf{E}\{\cdot\}$. The shorthand diag $\{\cdots\}$ denotes the block diagonal matrix. $\|\cdot\|$ stands for the Euclidean norm. Moreover, the notation $*$ always denotes the symmetric block in one symmetric matrix. $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ is probability space, where Ω is the sample space. $\{\eta_t,t\geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in finite set $S=\{1,2,\ldots,s\}$ with given a probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ and the initial model η_0. Let $\Gamma=[\pi_{kj}]$, $k,j\in S$, which denotes the transition rate matrix with transition probability

$$\Pr(\eta_{t+\Delta t}=j|\eta_t=i)=\begin{cases}\pi_{ij}\,\Delta t+o(\Delta t),\ i\neq j,\\ 1+\pi_{ij}\Delta t+o(\Delta t),\ i=j,\end{cases}$$

where $\Delta t>0$,

$$\lim_{\Delta t\to 0}\Big(\frac{o(\Delta t)}{\Delta t}\Big)=0,$$

and $\pi_{ij}\geq 0$ is the transition rate from mode i to mode j satisfying $\pi_{ij}\geq 0$ for $i\neq j$ with

$$\pi_{ii}=-\sum_{j=1,\ j\neq i}^{s}\pi_{ij},\quad i,j\in S.$$

2. System description and preliminaries

Delayed stochastic Cohen–Grossberg neural networks can be described by the following delay differential equation

$$\begin{aligned}\mathrm{d}x(t)=&\Big\{-\alpha(\eta_t)(x(t))\Big[\beta(\eta_t)(x(t))-A(\eta_t)f_1(x(t))\\&-B(\eta_t)f_2(x(t-h(t)))-C(\eta_t)\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big]\Big\}\\&\times\mathrm{d}t+\sigma(t,x(t),x(t-h(t),(\eta_t)))\,\mathrm{d}\omega(t),\end{aligned}\tag{1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^{\mathrm{T}}\in R^n$ is the state variable and $\alpha(\eta_t)(x(t))=\mathrm{diag}(\ \alpha_1(\eta_t)(x_1(t)),\ \ldots,\alpha_n(\eta_t)(x_n(t)))^{\mathrm{T}}$ represents the amplification function, $\beta(\eta_t)(x(t))=\mathrm{diag}(\beta_1(\eta_t)(x_1(t)),\ldots,\beta_n(\eta_t)(x_n(t)))^{\mathrm{T}}$ is the behaved function, and $f_i(x)=[f_{i1}(x_1(t)),\ldots,f_{in}(x_n(t))]^{\mathrm{T}}$, $(i=1,2,3)$ are the neuron activation functions. The matrices $A(\eta_t)\in R^{n\times n}$, $B(\eta_t)\in R^{n\times n}$, and $C(\eta_t)\in R^{n\times n}$ are, respectively, the connection weight matrix, the discretely delayed connection weight matrix, and the distributively delayed connection weight matrix. $h(t)$ and $\tau(t)$ are the discrete delay and distributed delay, respectively, and satisfies $0\leq\tau(t)\leq\bar{\tau}$, $0\leq h(t)\leq\bar{h}$, $\dot{h}(t)\leq d<1$. $w(t)=(w_1(t),w_2(t),\ldots,w_m(t))^{\mathrm{T}}$ is an m-dimensional Brownian motion defined on $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$ and $\sigma(t,x(t),x(t-h(t)))$: $R_+\times R^n\times R^n\to R^{n\times m}$ is locally Lipchitz-continuous and also satisfies the linear growth conditions. Moreover, $\sigma(t,x(t),x(t-h(t)))$ satisfies,

$$\begin{aligned}&\mathrm{tr}\Big(\sigma^{\mathrm{T}}(t,x(t),x(t-h(t)))\sigma(t,x(t),x(t-h(t)))\Big)\\&\leq x^{\mathrm{T}}(t)Xx(t)+x^{\mathrm{T}}(t-h(t))Yx(t-h(t)),\end{aligned}\tag{2}$$

where X and Y are positive-diagonal matrices with appropriate dimensions.

In the above equation (1) for $(\eta_t)=k$, $k\in S$, $A(\eta_t)=A_k$, $B(\eta_t)=B_k$, $C(\eta_t)=C_k$,

$$\begin{aligned}\mathrm{d}x(t)=&\Big\{-\alpha_k(x(t))\Big[\beta_k(x(t))-A_kf_1(x(t))-B_kf_2(x(t-h(t)))\\&-C_k\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big]\Big\}\mathrm{d}t\\&+\sigma_k(t,x(t),x(t-h(t)))\,\mathrm{d}\omega(t),\end{aligned}\tag{3}$$

Definition 1[47] A R^n-valued stochastic process $\{x(t)\}_{t_0\leq t\leq T}$ is called a solution of equation

$$\mathrm{d}x(t)=y(t)\,\mathrm{d}t+g(t)\,\mathrm{d}\omega(t),$$

if it has the following properties:

(i) $\{x(t)\}_{t_0\leq t\leq T}$ is continuous and $\mathcal{F}_t$ adapted;

(ii) $\{y(t)\}_{t_0\leq t\leq T}\in\mathcal{L}^1([t_0,T];R^n)$ while $\{g(t)\}_{t_0\leq t\leq T}\in\mathcal{L}^2([t_0,T];R^{n\times m})$;

(iii) for any $t\in[t_0,T]$ equation

$$x(t)=x(t_0)+\int_{t_0}^{t}y(s)\,\mathrm{d}s+\int_{t_0}^{t}g(s)\,\mathrm{d}w(s)$$

holds with probability 1.

Fact 1[48] The trivial solution of a stochastic differential equation,

$$\begin{aligned}&\mathrm{d}x(t)=F(t,x(t))\,\mathrm{d}t+G(t,x(t))\,\mathrm{d}w(t),\\&x(t)=\phi_x(t)\ \text{ for every }\ t\in[-\tau,0],\end{aligned}$$

on $t\in[t_0,T]$, where $F:R_+\times R^n\to R^n$ and $G:R_+\times R^n\to R^{n\times m}$, are globally asymptotically stable in probability if there exists a function $V(t,x(t))\in\mathbb{R}_+\times\mathbb{R}^n$ which is positive-definite in the Lyapunov sense and satisfies,

$$\mathcal{L}V(t,x(t))=\frac{\partial V}{dt}+\mathrm{grad}(V)F+\frac{1}{2}\mathrm{tr}(GG^{\mathrm{T}})\mathrm{Hess}(V)<0.$$

The matrix $\mathrm{Hess}(V)$ is the Hessian matrix of second-order partial derivatives.

Defining the following state variables for the MJSCGNNs,

$$y(t)=\alpha_k(x(t))y_1(t),\tag{4}$$

$$\begin{aligned}y_1(t)=-\Big[&\beta_k(x(t))-A_kf_1(x(t))-B_kf_2(x(t-h(t)))\\&-C_k\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big],\end{aligned}\tag{5}$$

$$g(t)=\sigma_k(t,x(t),x(t-h(t))).\tag{6}$$

The MJSCGNNs can then be represented as

$$\mathrm{d}x(t) = y(t)\,\mathrm{d}t + g(t)\,\mathrm{d}\omega(t). \tag{7}$$

Moreover, the following equality holds,

$$\begin{aligned} x(t) - x(t-h(t)) &= \int_{t-h(t)}^{t} \mathrm{d}x(s) \\ &= \int_{t-h(t)}^{t} y(s)\,\mathrm{d}s + \int_{t-h(t)}^{t} g(s)\,\mathrm{d}\omega(s). \end{aligned} \tag{8}$$

Throughout this paper, we assume that

(H1) $\alpha_{ki}(x(t)) > 0$, α_{ki} are bounded; that is, there exists $\underline{\alpha}_{ki}$, $\overline{\alpha}_{ki} > 0$ such that $\underline{\alpha}_{ki} \le \alpha_{ki} \le \overline{\alpha}_{ki}$, $i = 1,2\ldots,n$, $\overline{\alpha}_k = \max\{\overline{\alpha}_{ki}\}$, $\underline{\alpha}_k = \min\{\underline{\alpha}_{ki}\}$, $\Theta_k = \mathrm{diag}\{\overline{\alpha}_{k1}, \overline{\alpha}_{k2}, \ldots, \overline{\alpha}_{kn}\}$.

(H2) $x_i(t)\ \beta_{ki}\ x_i(t) \ge \mu_{ki}\ x_i^2(t)$, $\mu_{ki} > 0$ and $\Delta_k = \mathrm{diag}\{\mu_{k1}, \ldots, \mu_{kn}\}$.

(H3) Each of the neuron activation functions in system (1) $f_{1i}(\cdot)$, $f_{2i}(\cdot)$, and $f_{3i}(\cdot)$ $(i = 1,2,\ldots,n)$ satisfies the following conditions

$$0 \le \frac{f_{ki}(y_1) - f_{ki}(y_2)}{y_1 - y_2} \le L_{ki},$$

where L_{ki} $(k = 1,2,3)$, $(i = 1,2,\ldots n)$ are some constants and they can be positive. So, it is less restrictive than the descriptions on both the sigmoid activation functions and the Lipschitz-type activation functions. Denote $L_k = \mathrm{diag}\{L_{k1}, \ldots, L_{kn}\}$.

3. Global stability results

In this section, some sufficient conditions of stability criterion for system (3) are obtained.

Theorem 1 Given scalars $\bar{\tau}$, $\bar{h}$, and d, then the equilibrium point of system (3) is globally asymptotically stable, if there exist symmetric positive-definite matrices $R > 0$, $S > 0$, $U > 0$, $V > 0$, $W > 0$, positive-diagonal matrices $P_k > 0$, $Q > 0$, $T_1 > 0$, $T_2 > 0$, symmetric matrices N_1, N_2, N_3, N_4, N_5, and scalars $\rho_k > 0$, $\mu > 0$, $\nu > 0$ such that the following conditions hold for $k = 1,2,\ldots,s$

$$\begin{bmatrix} \Omega_k & \hat{\Omega}_k & \tilde{\Omega}_k \\ * & -V & * \\ * & * & -W \end{bmatrix} < 0, \tag{9}$$

$$\Omega_k = \begin{bmatrix} \Omega_{k11} & \Omega_{k12} & \Omega_{k13} & \Omega_{k14} & \Omega_{k15} & \Omega_{k16} \\ * & \Omega_{k22} & \Omega_{k23} & \Omega_{k24} & \Omega_{k25} & \Omega_{k26} \\ * & * & \Omega_{k33} & \Omega_{k34} & \Omega_{k35} & \Omega_{k36} \\ * & * & * & \Omega_{k44} & \Omega_{k45} & \Omega_{k46} \\ * & * & * & * & \Omega_{k55} & \Omega_{k56} \\ * & * & * & * & * & \Omega_{k66} \end{bmatrix},$$

$$P_k < \rho I, \tag{10}$$

$$Q < \mu I, \tag{11}$$

$$W < \nu I, \tag{12}$$

where

$$\begin{aligned} \Omega_{k11} &= -P_k\Delta_k - \Delta_k^{\mathrm{T}}P_k + R + L_2^{\mathrm{T}}SL_2 + \bar{\tau}^2\, L_3^{\mathrm{T}}\, UL_3 \\ &\quad + (\rho + \mu + h\nu)X + N_1^{\mathrm{T}} + N_1 - \Delta_k^{\mathrm{T}}N_1 - N_1^{\mathrm{T}}\Delta_k \\ &\quad + \sum_{j=1}^{s} \pi_{ij}\, P_j, \\ \Omega_{k12} &= -N_1 + N_2^{\mathrm{T}} - \Delta_k^{\mathrm{T}}N_2^{\mathrm{T}}, \\ \Omega_{k13} &= P_kA_k - \Delta_k^{\mathrm{T}}Q + N_3^{\mathrm{T}} + N_1A_k - \Delta_k^{\mathrm{T}}N_3^{\mathrm{T}} + L_1T_1, \\ \Omega_{k14} &= P_kB_k + N_4^{\mathrm{T}} + N_1B_k - \Delta_k^{\mathrm{T}}N_4^{\mathrm{T}}, \\ \Omega_{k15} &= P_kC_k + N_1C_k, \\ \Omega_{k16} &= N_5^{\mathrm{T}} - N_1 - \Delta_k^{\mathrm{T}}N_5^{\mathrm{T}}, \\ \Omega_{k22} &= (\rho + \mu + h\nu)Y - (1-d)R - N_2^{\mathrm{T}} - N_2, \\ \Omega_{k23} &= -N_3^{\mathrm{T}} + N_2A_k, \\ \Omega_{k24} &= -N_4^{\mathrm{T}} + N_2B_k + L_2T_2, \\ \Omega_{k25} &= N_2C_k,\ \Omega_{k26} = -N_2 - N_5^{\mathrm{T}}, \\ \Omega_{k33} &= QA_k + A_k^{\mathrm{T}}Q^{\mathrm{T}} + N_3A_k + A_k^{\mathrm{T}}N_3^{\mathrm{T}} - 2T_1, \\ \Omega_{k34} &= QB_k + N_3B_k + A_k^{\mathrm{T}}N_4^{\mathrm{T}},\ \Omega_{k35} = QC_k + N_3C_k, \\ \Omega_{k36} &= A_k^{\mathrm{T}}N_5^{\mathrm{T}} - N_3, \\ \Omega_{k44} &= N_4B_k + B_k^{\mathrm{T}}N_4^{\mathrm{T}} - (1-d)S - 2T_2, \\ \Omega_{k45} &= N_4C_k,\ \ \Omega_{k46} = B_k^{\mathrm{T}}N_5^{\mathrm{T}} - N_4, \\ \Omega_{k55} &= -U, \\ \Omega_{k56} &= C_k^{\mathrm{T}}N_5^{\mathrm{T}}, \\ \Omega_{k66} &= h^2\Theta_k^{\mathrm{T}}\, V\, \Theta_k - N_5^{\mathrm{T}} - N_5, \\ \hat{\Omega}_k &= \tilde{\Omega}_k = [N_1^{\mathrm{T}}\ N_2^{\mathrm{T}}\ N_3^{\mathrm{T}}\ N_4^{\mathrm{T}}\ 0\ N_5^{\mathrm{T}}]^{\mathrm{T}}. \end{aligned}$$

Proof We use the following Lyapunov functional to derive the stability result

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t),$$

where

$$\begin{aligned} V_1(t) &= 2\sum_{k=1}^{s}\sum_{i=1}^{n} p_{ki} \int_0^{x_i(t)} \frac{s}{\alpha_{ki}(s)}\,\mathrm{d}s, \\ V_2(t) &= 2\sum_{i=1}^{n} q_i \int_0^{x_i(t)} \frac{f_{1i}(s)}{\alpha_{ki}(s)}\,\mathrm{d}s, \\ V_3(t) &= \int_{t-h(t)}^{t} \left[x^{\mathrm{T}}(s)Rx(s) + f_2^{\mathrm{T}}(x(s))Sf_2(x(s))\right]\mathrm{d}s, \\ V_4(t) &= \bar{\tau}\int_{-\bar{\tau}}^{0}\int_{t+s}^{t} f_3^{\mathrm{T}}(x(\theta))Uf_3(x(\theta))\,\mathrm{d}\theta\,\mathrm{d}s, \\ V_5(t) &= \bar{h}\int_{-\bar{h}}^{0}\int_{t+s}^{t} y^{\mathrm{T}}(\theta)Vy(\theta)\,\mathrm{d}\theta\,\mathrm{d}s, \\ V_6(t) &= \int_{-h}^{0}\int_{t+s}^{t} \mathrm{tr}\left(g^{\mathrm{T}}(\theta)Wg(\theta)\right)\mathrm{d}\theta\,\mathrm{d}s, \end{aligned}$$

in which R, S, U, V,W are positive matrices, p_i, q_i are positive scalars to be determined.

We can calculate the derivative of $V(t)$ along the trajectories of the system (3), it can then be obtained by Itô's formula that

$$\begin{aligned}
\mathcal{L}V_1(t) \le\ & 2x^{\mathrm{T}}(t)P_k\Big[-\beta_k(x(t))+A_kf_1(x(t))\\
&+B_kf_2(x(t-h(t)))+C_k\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big]\\
&+\mathrm{tr}\left(\sigma_k^{\mathrm{T}}(t,x(t),x(t-h(t)))\,P_k\sigma_k(t,x(t),x(t-h(t)))\right)\\
&+x^{\mathrm{T}}(t)\sum_{j=1}^{s}\pi_{ij}\,P_j\,x(t), \qquad (13)\\
\mathcal{L}V_2(t) \le\ & 2f_1^{\mathrm{T}}(x(t))Q\Big[-\beta_k(x(t))+A_kf_1(x(t))\\
&+B_kf_2(x(t-h(t)))+C_k\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big]\\
&+\mathrm{tr}\left(\sigma_k^{\mathrm{T}}(t,x(t),x(t-h(t)))\right.\\
&\left.\times Q\,\sigma_k(t,x(t),x(t-h(t)))\right), \qquad (14)\\
\mathcal{L}V_3(t) \le\ & \left[x^{\mathrm{T}}(t)\,R\,x(t)+f_2^{\mathrm{T}}(x(t))Sf_2(x(t))\right]\\
&-(1-d)\left[x^{\mathrm{T}}(t-h(t))Rx(t-h(t))\right.\\
&\left.+f_2^{\mathrm{T}}(x(t-h(t)))Sf_2(x(t-h(t)))\right], \qquad (15)\\
\mathcal{L}V_4(t) \le\ & \bar{\tau}^2\,f_3^{\mathrm{T}}(x(t))Uf_3(x(t))\\
&-\bar{\tau}\int_{t-\bar{\tau}}^{t}f_3^{\mathrm{T}}(x(s))Uf_3(x(s))\,\mathrm{d}s, \qquad (16)\\
\mathcal{L}V_5(t) \le\ & \bar{h}^2\,y_1^{\mathrm{T}}(t)\Theta_k^{\mathrm{T}}\,V\,\Theta_k y_1(t)-\bar{h}\int_{t-\bar{h}}^{t}y^{\mathrm{T}}(s)Vy(s)\,\mathrm{d}s, \qquad (17)\\
\mathcal{L}V_6(t) \le\ & h\,\mathrm{tr}\left(g^{\mathrm{T}}(t)\,W\,g(t)\right)-\int_{t-h}^{t}\mathrm{tr}\left(g^{\mathrm{T}}(s)Wg(s)\right)\mathrm{d}s. \qquad (18)
\end{aligned}$$

By using Jensen's inequality,[49] we have

$$\begin{aligned}
&-\bar{h}\int_{t-\bar{h}}^{t}y^{\mathrm{T}}(s)Vy(s)\,\mathrm{d}s\\
&\le-\Big[\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s\Big]^{\mathrm{T}}V\Big[\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s\Big], \qquad (19)\\
&-\bar{\tau}\int_{t-\bar{\tau}}^{t}f_3^{\mathrm{T}}(x(s))Uf_3(x(s))\,\mathrm{d}s\\
&\le-\Big[\int_{t-\tau(t)}^{t}f_3(x((s)))\,\mathrm{d}s\Big]^{\mathrm{T}}U\Big[\int_{t-\tau(t)}^{t}f_3(x((s)))\,\mathrm{d}s\Big]. \qquad (20)
\end{aligned}$$

From expressions (2), (13), and (14) we have

$$\begin{aligned}
&\mathrm{tr}\left(\sigma_k^{\mathrm{T}}(t,x(t),x(t-h(t)))P_k\sigma_k(t,x(t),x(t-h(t)))\right)\\
&\le\lambda_M(P_k)[x^{\mathrm{T}}(t)Xx(t)+x^{\mathrm{T}}(t-h(t))Yx(t-h(t))]\\
&\le\rho[x^{\mathrm{T}}(t)Xx(t)+x^{\mathrm{T}}(t-h(t))Yx(t-h(t))], \qquad (21)\\
&\mathrm{tr}\left(\sigma_k^{\mathrm{T}}(t,x(t),x(t-h(t)))Q\sigma_k(t,x(t),x(t-h(t)))\right)\\
&\le\lambda_M(Q)[x^{\mathrm{T}}(t)Xx(t)+x^{\mathrm{T}}(t-h(t))Yx(t-h(t))]\\
&\le\mu[x^{\mathrm{T}}(t)Xx(t)+x^{\mathrm{T}}(t-h(t))Yx(t-h(t))], \qquad (22)\\
&\mathrm{tr}\left(\sigma_k^{\mathrm{T}}(t,x(t),x(t-h(t)))W\sigma_k(t,x(t),x(t-h(t)))\right)\\
&\le\lambda_M(W)[x^{\mathrm{T}}(t)Xx(t)+x^{\mathrm{T}}(t-h(t))Yx(t-h(t))]\\
&\le\nu[x^{\mathrm{T}}(t)Xx(t)+x^{\mathrm{T}}(t-h(t))Yx(t-h(t))]. \qquad (23)
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\mathcal{L}V(t) \le\ & x^{\mathrm{T}}(t)\big[-P_k\Delta_k-\Delta_k^{\mathrm{T}}P_k+R+L_2^{\mathrm{T}}SL_2+(\rho+\mu+h\nu)X\\
&+\bar{\tau}^2L_3^{\mathrm{T}}UL_3+\sum_{j=1}^{s}\pi_{ij}\,P_j\big]x(t)+2x^{\mathrm{T}}(t)PA_kf_1(x(t))\\
&+2x^{\mathrm{T}}(t))PB_kf_2(x(t-h(t)))\\
&+2x^{\mathrm{T}}(t)PC_k\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\\
&-2f_1^{\mathrm{T}}(x(t))Q\Delta_kx(t)+2f_1^{\mathrm{T}}(x(t))QA_kf_1(x(t))\\
&+2f_1^{\mathrm{T}}(x(t))QB_kf_2(x(t-h(t)))\\
&+2f_1^{\mathrm{T}}(x(t))QC_k\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\\
&+x^{\mathrm{T}}(t-h(t))\big[(\rho+\mu+h\nu)Y-(1-d)R\big]x(t-h(t))\\
&+\bar{h}^2y_1^{\mathrm{T}}(t)\Theta_k^{\mathrm{T}}V\Theta_ky_1(t)\\
&-(1-d)f_2^{\mathrm{T}}(x(t-h(t)))Sf_2(x(t-h(t)))\\
&-\Big(\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s\Big)^{\mathrm{T}}V\Big(\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s\Big)\\
&-\Big(\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big)^{\mathrm{T}}U\Big(\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big)\\
&-\int_{t-\bar{h}}^{t}\mathrm{tr}\left(g^{\mathrm{T}}(s)Wg(s)\right)\mathrm{d}s\Big\}. \qquad (24)
\end{aligned}$$

From expressions (4)–(6), we can see that the following equations are true for any matrices N_1, N_2, N_3, N_4, and N_5 with appropriate dimensions,

$$\begin{aligned}
&2\Big[x^{\mathrm{T}}(t)\,N_1+x^{\mathrm{T}}(t-h(t))\,N_2+f_1^{\mathrm{T}}(x(t))\,N_3\\
&+f_2^{\mathrm{T}}(x(t-h(t)))\,N_4+y_1^{\mathrm{T}}(t)N_5\Big]^{\mathrm{T}}\Big[x(t)-x(t-h(t))\\
&-\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s-\int_{t-h(t)}^{t}g(s)\,\mathrm{d}\omega(s)\Big]=0, \qquad (25)\\
&2\Big[x^{\mathrm{T}}(t)\,N_1+x^{\mathrm{T}}(t-h(t))\,N_2+f_1^{\mathrm{T}}(x(t))\,N_3\\
&+f_2^{\mathrm{T}}(x(t-h(t)))\,N_4+y_1^{\mathrm{T}}(t)N_5\Big]^{\mathrm{T}}\Big[-y_1(t)-\beta_k(x(t))\\
&+A_kf_1(x(t))+B_kf_2(x(t-h(t)))+C_k\int_{t-\tau(t)}^{t}f_3(x(s))\,\mathrm{d}s\Big]\\
&=0. \qquad (26)
\end{aligned}$$

It follows from Lemma in Ref. [50] that

$$\begin{aligned}
&-2\Big[x^{\mathrm{T}}(t)\,N_1+x^{\mathrm{T}}(t-h(t))\,N_2+f_1^{\mathrm{T}}(x(t))\,N_3\\
&+f_2^{\mathrm{T}}(x(t-h(t)))\,N_4+y_1^{\mathrm{T}}(t)N_5\Big]\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s\\
&\le\xi^{\mathrm{T}}(t)\,N\,V^{-1}\,N^{\mathrm{T}}\,\xi(t)\\
&+\Big(\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s\Big)^{\mathrm{T}}\,V\,\Big(\int_{t-h(t)}^{t}y(s)\,\mathrm{d}s\Big), \qquad (27)\\
&-2\Big[x^{\mathrm{T}}(t)\,N_1+x^{\mathrm{T}}(t-h(t))\,N_2+f_1^{\mathrm{T}}(x(t))\,N_3\\
&+f_2^{\mathrm{T}}(x(t-h(t)))\,N_4+y_1^{\mathrm{T}}(t)N_5\Big]\int_{t-h(t)}^{t}g(s)\,\mathrm{d}\omega(s)\\
&\le\xi^{\mathrm{T}}(t)\,N\,W^{-1}\,N^{\mathrm{T}}\,\xi(t)\\
&+\Big(\int_{t-h(t)}^{t}g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}}\,W\,\Big(\int_{t-h(t)}^{t}g(s)\,\mathrm{d}w(s)\Big), \qquad (28)
\end{aligned}$$

where

$$\xi^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\,x^{\mathrm{T}}(t-h(t))\,f_1^{\mathrm{T}}(x(t))\,f_2^{\mathrm{T}}(x(t-h(t)))\,y_1^{\mathrm{T}}(t)],$$

$N^{\mathrm{T}} = [N_1^{\mathrm{T}}\ N_2^{\mathrm{T}}\ N_3^{\mathrm{T}}\ N_4^{\mathrm{T}}\ N_5^{\mathrm{T}}]$.

From assumption (H3) we know that

$$f_1(x(t))[f_1(x(t)) - L_1x(t)] \le 0, \tag{29}$$

$$f_2(x(t-h(t))[f_2(x(t-h(t)) - L_2x(t-h(t))] \le 0. \tag{30}$$

Then, for $T_1 = \mathrm{diag}\{t_{11},\ldots,t_{1n}\} \ge 0$, $T_2 = \mathrm{diag}\{t_{21},\ldots,t_{2n}\} \ge 0$ we have

$$\begin{aligned}\mathcal{L}V(t) \le\ & \mathcal{L}V(t) - 2T_1f_1(x(t))[f_1(x(t)) - L_1x(t)] \\ & - 2T_2f_2(x(t-h(t)))[f_2(x(t-h(t))) \\ & - L_2x(t-h(t))].\end{aligned} \tag{31}$$

From expressions (13)–(31) we obtain

$$\begin{aligned}\mathcal{L}V(t) \le\ & \psi^{\mathrm{T}}(t)\,\Omega_k\,\psi(t) \\ & + \Big(\int_{t-\bar{h}}^{t} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}} W \Big(\int_{t-\bar{h}}^{t} g(s)\,\mathrm{d}w(s)\Big) \\ & - \int_{t-\bar{h}}^{t} \mathrm{tr}\left(g^{\mathrm{T}}(s)Wg(s)\right)\mathrm{d}s,\end{aligned} \tag{32}$$

where

$$\begin{aligned}\psi(t) =\ & \Big[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-h(t))\ f_1^{\mathrm{T}}(x(t))f_2^{\mathrm{T}}(x(t-h(t))) \\ & \times \Big(\int_{t-\tau(t)}^{t} f_3(x(s))\,\mathrm{d}s\Big)^{\mathrm{T}}\ y_1^{\mathrm{T}}(t)\Big]^{\mathrm{T}}.\end{aligned}$$

Then, there must exist a scalar $\alpha > 0$ such that

$$\Omega_k + \begin{bmatrix} \alpha I & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} < 0.$$

Noting that,[51]

$$\begin{aligned}& E\Big\{\Big(\int_{t-\bar{h}}^{t} g(s)\,\mathrm{d}w(s)\Big)^{\mathrm{T}} W \Big(\int_{t-\bar{h}}^{t} g(s)\,\mathrm{d}w(s)\Big)\Big\} \\ & = E\Big\{\int_{t-\bar{h}}^{t} \mathrm{tr}\left(g^{\mathrm{T}}(s)Wg(s)\right)\mathrm{d}s\Big\}.\end{aligned} \tag{33}$$

Taking the mathematical expectation on both sides of inequality (32), one can obtain that

$$\mathbf{E}\mathcal{L}V(t) \le \mathbf{E}[\psi^{\mathrm{T}}(t)\Omega_k\psi(t)] \le \alpha\mathbf{E}|x(t)|^2,$$

which indicates from the Lyapunov stability theory, that the dynamics of the neural network (3) is robustly, globally, asymptotically stable in the mean square, which completes the proof.

We now consider the following stochastic CGNNs with discrete and distributed time varying delays, as in Ref. [52]

$$\begin{aligned}\mathrm{d}x(t) =\ & -\alpha(x(t))\Big[\beta(x(t)) - Af_1(x(t)) - Bf_2(x(t-h(t))) \\ & - C\int_{t-\tau(t)}^{t} f_3(x(s))\,\mathrm{d}s\Big]\,\mathrm{d}t \\ & + \sigma(t,x(t),x(t-h(t)))\,\mathrm{d}\omega(t),\end{aligned} \tag{34}$$

when $s = 1$ it is a special form of Eq. (3). For convenience, we have deleted the subscript "1".

Theorem 2 Given scalars $\bar{\tau}$, $\bar{h}$, and d, the equilibrium point of system (34) is globally asymptotically stable, if there exist symmetric positive-definite matrices $R > 0$, $S > 0$, $U > 0$, $V > 0$, $W > 0$, positive-diagonal matrices $P > 0$, $Q > 0$, $T_1 > 0$, $T_2 > 0$, symmetric matrices N_1, N_2, N_3, N_4, N_5, and scalars $\rho > 0$, $\mu > 0$, $\nu > 0$ such that the following conditions hold

$$\begin{bmatrix} \Omega & \hat{\Omega} & \tilde{\Omega} \\ * & -V & * \\ * & * & -W \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} \\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} \\ * & * & * & * & \Omega_{55} & \Omega_{56} \\ * & * & * & * & * & \Omega_{66} \end{bmatrix}, \tag{36}$$

$$P < \rho I, \tag{37}$$

$$Q < \mu I, \tag{38}$$

$$W < \nu I, \tag{39}$$

where

$$\begin{aligned}\Omega_{11} =\ & -P\Delta - \Delta^{\mathrm{T}}P + R + L_2^{\mathrm{T}}SL_2 + \bar{\tau}^2\,L_3^{\mathrm{T}}\,UL_3 \\ & + (\rho+\mu+h\nu)X + N_1^{\mathrm{T}} + N_1 - \Delta^{\mathrm{T}}N_1 - N_1^{\mathrm{T}}\Delta, \\ \Omega_{12} =\ & -N_1 + N_2^{\mathrm{T}} - \Delta^{\mathrm{T}}N_2^{\mathrm{T}}, \\ \Omega_{13} =\ & PA - \Delta^{\mathrm{T}}Q + N_3^{\mathrm{T}} + N_1A - \Delta^{\mathrm{T}}N_3^{\mathrm{T}} + L_1T_1, \\ \Omega_{14} =\ & PB + N_4^{\mathrm{T}} + N_1B - \Delta^{\mathrm{T}}N_4^{\mathrm{T}}, \\ \Omega_{15} =\ & PC + N_1C,\ \ \Omega_{16} = N_5^{\mathrm{T}} - N_1 - \Delta^{\mathrm{T}}N_5^{\mathrm{T}}, \\ \Omega_{22} =\ & (\rho+\mu+h\nu)Y - (1-d)R - N_2^{\mathrm{T}} - N_2, \\ \Omega_{23} =\ & -N_3^{\mathrm{T}} + N_2A, \\ \Omega_{24} =\ & -N_4^{\mathrm{T}} + N_2B + L_2T_2, \\ \Omega_{25} =\ & N_2C,\ \ \Omega_{26} = -N_2 - N_5^{\mathrm{T}}, \\ \Omega_{33} =\ & QA + A^{\mathrm{T}}Q^{\mathrm{T}} + N_3A + A^{\mathrm{T}}N_3^{\mathrm{T}} - 2T_1, \\ \Omega_{34} =\ & QB + N_3B + A^{\mathrm{T}}N_4^{\mathrm{T}}, \\ \Omega_{35} =\ & QC + N_3C, \\ \Omega_{36} =\ & A^{\mathrm{T}}N_5^{\mathrm{T}} - N_3, \\ \Omega_{44} =\ & N_4B + B^{\mathrm{T}}N_4^{\mathrm{T}} - (1-d)S - 2T_2, \\ \Omega_{45} =\ & N_4C, \\ \Omega_{46} =\ & B^{\mathrm{T}}N_5^{\mathrm{T}} - N_4, \\ \Omega_{55} =\ & -U,\ \ \Omega_{56} = C^{\mathrm{T}}N_5^{\mathrm{T}}, \\ \Omega_{66} =\ & h^2\Theta^{\mathrm{T}}\,V\,\Theta - N_5^{\mathrm{T}} - N_5, \\ \hat{\Omega}_k =\ & \tilde{\Omega}_k = [N_1^{\mathrm{T}}\ N_2^{\mathrm{T}}\ N_3^{\mathrm{T}}\ N_4^{\mathrm{T}}\ 0\ N_5^{\mathrm{T}}]^{\mathrm{T}}.\end{aligned}$$

Proof The proof of the theorem is similar to that in the proof of Theorem 1.

Remark 1 Theorem 2 depends on the size of time delay element, which means that it is less conservative than those delay-independent ones, especially when the delay value is very small. It is realized that the problems studied in many

other papers, such as Refs. [52] and [53], are also special classes of this paper.

4. Numerical example

In this section, we present numerical examples to illustrate the effectiveness of the proposed results.

Example 1 Consider an MJSCGNN with time-varying delays together with the mode $s = 2$ given, as follows:

$$\begin{aligned} \mathrm{d}x(t) = & -\alpha_k(x(t))\Big[\beta_k(x(t)) - A_k f_1(x(t)) - B_k f_2(x(t-h(t))) \\ & - C_k \int_{t-\tau(t)}^{t} f_3(x(s))\,\mathrm{d}s\Big]\,\mathrm{d}t \\ & + \sigma_k(t, x(t), x(t-h(t)))\,\mathrm{d}\omega(t), \end{aligned}$$

with the following parameters,

$$\Delta_1 = \begin{bmatrix} 4.2 & 0 \\ 0 & 3.8 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & -1.7 \\ -1.6 & 1 \end{bmatrix},\ \Delta_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.1 & -1.1 \\ 1.1 & -1.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} -3 & -0.5 \\ 1 & 2.5 \end{bmatrix},\ \Theta_1 = \begin{bmatrix} 1.3 & 0 \\ 0 & 1.3 \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix},\ L_i = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$i = 1, 2, 3.\ p_{11} = -2,\ p_{12} = 2,\ p_{21} = -3,\ p_{22} = 3.$

We select $h = 1.5$, $\tau = 0.5$, and $d = 0.9$. By using the Matlab LMI toolbox, we solve the LMIs in Theorem 1 [LMIs (9)–(12)], the feasible solutions are

$$P_1 = \begin{bmatrix} 16.8462 & 0 \\ 0 & 17.3251 \end{bmatrix},\ P_2 = \begin{bmatrix} 16.7954 & 0 \\ 0 & 16.9821 \end{bmatrix},$$

$$R = \begin{bmatrix} 47.5819 & -29.9365 \\ -29.9365 & 58.3134 \end{bmatrix},\ S = 10^4 \begin{bmatrix} 4.9030 & 0.0070 \\ 0.0070 & 4.9064 \end{bmatrix},$$

$$V = \begin{bmatrix} 0.1566 & -0.1234 \\ -0.1234 & 0.1920 \end{bmatrix},\ W = \begin{bmatrix} 0.0750 & 0 \\ 0 & 0.0919 \end{bmatrix},$$

$$U = 10^4 \begin{bmatrix} 1.9677 & 0.0000 \\ 0.0000 & 1.9687 \end{bmatrix},\ Q = \begin{bmatrix} 14.7526 & 0 \\ 0 & 14.5358 \end{bmatrix}.$$

Therefore, the concerned neural network with time-varying delays is globally asymptotically stable. Figures 1 and 2 presents the sample path simulation of state.

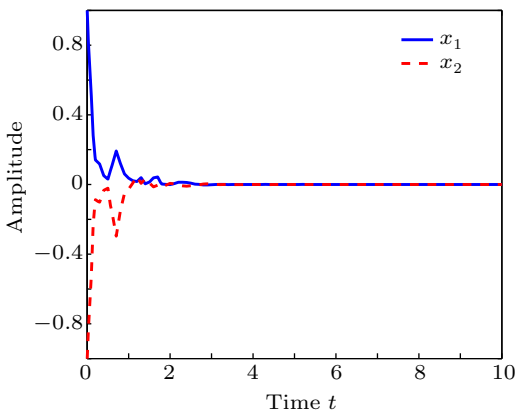


Fig. 1. (color online) State trajectory of the system in Example 1 when $s = 1$.

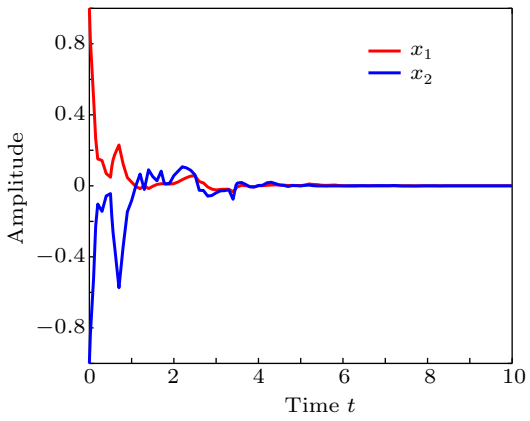


Fig. 2. (color online) State trajectory of the system in Example 1 when $s = 2$.

Example 2 Consider the Cohen–Grossberg neural network[52] with

$$\Delta_1 = a \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.3 & -1.8 & 0.5 \\ -1.1 & 1.6 & 1.1 \\ 0.6 & 0.4 & -0.3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.6 \\ -0.8 & 1.1 & -1.2 \end{bmatrix},\ C_1 = \begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0.3 & 0.7 & -0.3 \\ 1.2 & -1.1 & = 0.5 \end{bmatrix},$$

$$\Theta_1 = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}.$$

Therefore, the concerned neural network with time-varying delays is globally asymptotically stable. When $\bar{\alpha} > 1.2989$, the criterion given in Ref. [52] will fail. Applying Theorem 2 in this paper for any value of amplification function the system is asymptotically stable. If we choose $a = 1$, the system is asymptotically stable by applying Corollary 1 in Ref. [52]. While choosing $a > 0.31$ Corollary 1 in Ref. [52] is invalid. The allowable range of a is $[0.31, 0.6]$ in Ref. [54]. However, by applying Theorem 2 [LMIs (35)–(39)], the allowable range of a is $[0.31, 1.4429]$. Which may imply that the developed approach in this paper leads to less conservative results, to some extent.

5. Conclusion

In this paper, we have performed the stability analysis for a class of Markovian jumping Cohen–Grossberg neural network with discrete and distributed time varying delays. Some new stability criteria have been presented to guarantee that the MJSCGNNs are asymptotically stable. A LMI approach has been used to solve the underlying problem. The applicability of the derived results has been demonstrated through the numerical examples and the results are compared with some existing results to show the effectiveness and reduced conservativeness of the proposed method.

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