Exponential synchronization of chaotic Lur'e systems with time-varying delay via sampled-data control

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In this paper, we study the exponential synchronization of chaotic Lur'e systems with time-varying delays via sampled-data control by using sector nonlinearties. In order to make full use of information about sampling intervals and interval time-varying delays, new Lyapunov–Krasovskii functionals with triple integral terms are introduced. Based on the convex combination technique, two kinds of synchronization criteria are derived in terms of linear matrix inequalities, which can be efficiently solved via standard numerical software. Finally, three numerical examples are provided to demonstrate the less conservatism and effectiveness of the proposed results.

Keywords: exponential synchronization, chaotic Lur'e system, time-varying delay, sampled-data control

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1. Introduction

During the last few decades, a lot of effort has been paid to synchronization of chaotic systems because they have a wide range of applications in secure communication, chemical reaction, biological systems, and information science. Chaotic systems are very sensitive to initial conditions and system parameter variations, which makes it difficult to achieve synchronization between chaotic systems. Pecora and Carroll[1] first introduced the master-slave concept for achieving the synchronization of two identical chaotic systems. Afterwards, various kinds of synchronization schemes have been proposed by the authors in Refs. [2]–[12].

The Lur'e system is an important class of nonlinear system. The master-slave synchronization scheme for chaotic Lur'e systems has become an active research area in recent years[13–24] because many chaotic systems, including Chua's circuit, *n*-scroll attractors, and hyperchaotic attractors, can be modeled in the form of a Lur'e system. The Lur'e system is represented as a continuum in a linear system and a feedback nonlinearity satisfying sector bound constraints. On the other hand, the time delay is unavoidable in a real situation and it may cause undesirable dynamic behaviors such as oscillation, instability, and poor performance of the dynamical systems. For this reason, Yalcin *et al.*[14] introduced the effect of time delay in the chaotic synchronization of a Lur'e system and presented sufficient conditions for stability. Many synchronization schemes for the chaotic Lur'e systems with time delay have since been given.[18–21,25–29] Moreover, Zhang *et al.*[30] have presented global anti-synchronization for chaotic memristive neural networks with time-varying delays.

The controllers that are used in synchronization of chaotic Lur'e systems can be divided into two types. The first is based on feedback signals that are continuous and it has been found in analog circuits, including state-feedback control,[15] delayed output-feedback control,[16] dynamic output-feedback control,[18] adaptive control,[20] and proportional derivative (PD) control.[21] The second is based on feedback signals that are discrete at instant times, including impulsive control[13,22] and sampled-data control.[23,24] Because of the rapid growth of modern high-speed computers, microelectronics, and communication networks, continuous controllers may find it difficult to obtain an accurate feedback signal in real time.[25] In such a situation, discrete (digital) controllers are more preferable than continuous controllers. The main advantage of using sampled-data controller is to exploit the digital control techniques in real-time. A digital controller allows synchronization of chaotic systems only by using the samples of the state variables of the master system and the slave system at discrete time instants. These samples are used to control the synchronization between master and slave systems and they are termed a sampled-data controller.

However, the authors in Refs. [26] and [27] have presented the master-slave synchronization of Lur'e systems with sector and slope restricted nonlinearities based on a feedback controller. The absolute stability of Lur'e systems with time-varying delay and sector bounded nonlinearity has been addressed in Ref. [28]. Furthermore, the delay-dependent stability analysis of neutral type Lur'e systems with interval time-varying state delays and sector-bounded nonlinearities have been investigated in Refs. [29]–[31], while H_∞ control and H_∞ synchronization for chaotic Lur'e systems have been considered in Refs. [32] and [34]. It is worth mentioning that the

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authors in Refs. [35] and [36] have addressed robust absolute stability and observer design for Lur'e differential inclusion systems, respectively.

Most approaches to achieve chaotic synchronization are based on asymptotical stability.[37] It should be noted that the definition of asymptotical stability does not quantify the rate of convergence. Exponential stability is a more strengthened stability than asymptotical stability. Consequently, the exponential synchronization of a Lur'e system with time delays is an active area of research, see for example, Refs. [38]–[41]. In Refs. [39] and [40], the exponential synchronization of chaotic Lur'e system with time delays-based delayed feedback control has been considered. Recently, Wu *et al.*[41] have presented the exponential synchronization of a chaotic Lur'e system with time delays by using sampled-data control. However, to the best of the author's knowledge, the problem of exponential synchronization for Lur'e systems with interval time-varying delays via sampled-data control has not been fully investigated and remains open.

In this paper, the exponential synchronization of chaotic Lur'e systems with interval time-varying delays by using sampled-data control is investigated. Note that nonlinear function satisfies the sector bound constraints. The new triple integral term

$$\frac{d_{12}^2}{2}\int_{t-d_2}^{t-d_1}\int_{\theta}^{t-d_1}\int_{\lambda}^{t-d_1}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta$$

in Lyapunov–Krasovskii functional was constructed in Ref. [42], which makes the available information about the lower bound of the time-varying delay and leads to less conservatism. Motivated by the above-mentioned facts, a new Lyapunov–Krasovskii functional with triple integrals are considered in this paper. By using a convex combination technique, new synchronization criteria are given in terms of linear matrix inequalities (LMIs) that can be efficiently solved via standard numerical software. Finally, we give three numerical examples to demonstrate that the derived results are less conservative than previous results.

Notation: Throughout this paper, the superscripts T and -1 mean the transpose and the inverse of a matrix, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices, $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm, $P>0$ ($P\geq 0$) means that P is a real symmetric and positive-definite (semi-positive-definite) matrix, I stands for an appropriately dimensioned identity matrix, and $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix, and symmetric term in a symmetric matrix is denoted by $\star$.

2. Problem description

Consider the following master–slave synchronization scheme with sampled data control

$$\begin{aligned}
\mathcal{M}:&\begin{cases}\dot{x}(t) &= Ax(t)+Bx(t-d(t))+W\varphi(Dx(t)),\\ p(t) &= Cx(t),\end{cases}\\
\mathcal{S}:&\begin{cases}\dot{z}(t) &= Az(t)+Bz(t-d(t))+W\varphi(Dz(t))+u(t),\\ q(t) &= Cz(t),\end{cases}\\
\mathcal{C}:&\ u(t)=K(p(t_k)-q(t_k)),\ t_k\leq t\leq t_{k+1}
\end{aligned}\tag{1}$$

with master system $\mathcal{M}$, the slave system $\mathcal{S}$, and the controller $\mathcal{C}$. $\mathcal{M}$ and $\mathcal{S}$ with $u(t)=0$ are identical chaotic time-varying delayed Lur'e systems with state vectors $x(t), z(t)\in\mathbb{R}^n$, outputs of subsystems $p(t), q(t)\in\mathbb{R}^l$, respectively. $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times n}$, $C\in\mathbb{R}^{l\times n}$, $D\in\mathrm{R}^{n_h\times n}$, and $W\in\mathrm{R}^{n\times n_h}$ are known constant matrices. $u(t)\in\mathrm{R}^n$ is the slave system control input and $K\in\mathrm{R}^{n\times n_h}$ is the sampled-data controller gain matrix to be designed. It is assumed that $\varphi(\cdot):\mathbb{R}^{n_h}\to\mathbb{R}^{n_h}$ is a diagonal nonlinearity with $\varphi(\cdot)$ belonging to the sector $[0,s_i]$ for $i=1,2,\ldots,n_h$. Throughout this paper the following assumptions are made:

Assumption 1 The time-varying delay satisfies the following conditions

$$0\leq d_1\leq d(t)\leq d_2,\ \dot{d}(t)\leq d_\mu \tag{2}$$

and denote $d_{12}=d_2-d_1$.

Assumption 2 The sampling instants t_k ($k=0,1,2,\ldots$) satisfy

$$\begin{aligned}
&0\leq t_0<t_1<\cdots<t_k<\cdots<\lim_{k\to+\infty}t_k=+\infty,\\
&h_k=t_{k+1}-t_k\leq h,
\end{aligned}\tag{3}$$

for all $k\geq 0$, where $h>0$ represents the upper bound of the sampling periods.

Let the synchronization error system of system (1) be $e(t)=x(t)-z(t)$, then it can be written in the following form

$$\begin{aligned}
\dot{e}(t)=&Ae(t)+Be(t-d(t))+W\eta(De(t),z(t))-KCe(t-h(t)),\\
&t_k\leq t\leq t_{k+1},
\end{aligned}\tag{4}$$

where $\eta(De(t),z(t))=\varphi(D(e(t)+z(t))-\varphi(Dz(t))$ and $h(t)=t-t_k$. It can be seen that $0\leq h(t)\leq h$ and $\dot{h}(t)=1$ for $t\neq t_k$. Note that $\varphi_i(\cdot)$ belongs to sector $[0,s_i]$, it can be found that for any $i=1,2,\ldots,n_h$, and $\forall r,z$

$$0\leq\frac{\eta_i(d_i^{\mathrm{T}}e,z)}{d_i^{\mathrm{T}}e}=\frac{\varphi_i[d_i^{\mathrm{T}}(e+z)]-\varphi_i(d_i^{\mathrm{T}}e)}{d_i^{\mathrm{T}}}\leq s_i,\ d_i^{\mathrm{T}}\neq 0,\tag{5}$$

where d_i^{T} denotes the i-th row vector of D. From inequality (5), it can be easily found that for any $i=1,2,\ldots,n_h$ and $\forall e,z$

$$\eta_i(d_i^{\mathrm{T}}e,z)[\eta_i(d_i^{\mathrm{T}}e,z)-s_id_i^{\mathrm{T}}]\leq 0.\tag{6}$$

Based on inequality (6), the following inequality holds for any matrices $v_l = \mathrm{diag}\{v_{l1}, v_{l2}, \ldots, v_{ln_h}\} \geq 0,\ l = 1, 2,$

$$-\sum_{j=1}^{n_h} v_{lj}\eta_j(d_j^{\mathrm{T}}e(t), z(t))(\eta_j(d_j^{\mathrm{T}}e(t), z(t)) - s_j d_j^{\mathrm{T}}) \geq 0, \quad (7)$$

which are equivalent to

$$e^{\mathrm{T}}(t)D^{\mathrm{T}}SV_l\eta(De(t), z(t)) - \eta(De(t), z(t))^{\mathrm{T}}V_l\eta(De(t), z(t)) \geq 0, \quad (8)$$

where $S = \mathrm{diag}\{s_1, s_2, \ldots s_{n_h}\}$.

Before proceeding further, the following essential lemmas and definitions are introduced.

Lemma 1[43,44] For any constant matrix $X \in \mathbf{R}^{n\times n}$, $X = X^T > 0$, two scalars $h_2 \geq h_1 > 0$, such that the integrations concerned are well defined, then

$$\begin{aligned}
&-\frac{h_2^2 - h_1^2}{2}\int_{t-h_2}^{t-h_1}\int_{t+\theta}^{t} e^{\mathrm{T}}(s)Xe(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\leq -\Big(\int_{t-h_2}^{t-h_1}\int_{t+\theta}^{t} e(s)\,\mathrm{d}s\,\mathrm{d}\theta\Big)^{\mathrm{T}} X \Big(\int_{t-h_2}^{t-h_1}\int_{t+\theta}^{t} e(s)\,\mathrm{d}s\,\mathrm{d}\theta\Big), \\
&-(h_2 - h_1)\int_{t-h_2}^{t-h_1} e^{\mathrm{T}}(s)Xe(s)\,\mathrm{d}s \\
&\leq -\Big(\int_{t-h_2}^{t-h_1} e(s)\,\mathrm{d}s\Big)^{\mathrm{T}} X \Big(\int_{t-h_2}^{t-h_1} e(s)\,\mathrm{d}s\Big).
\end{aligned}$$

Lemma 2[45] For any vectors δ_1, δ_2, constant matrices R, S, and real scalars $\alpha \geq 0, \beta \geq 0$ satisfying that $\begin{bmatrix} R & S \\ * & R \end{bmatrix} \geq 0$ and $\alpha + \beta = 1$, then the following inequality holds:

$$-\frac{1}{\alpha}\delta_1^{\mathrm{T}}R\delta_1 - \frac{1}{\beta}\delta_2^{\mathrm{T}}R\delta_2 \leq -\begin{bmatrix}\delta_1 \\ \delta_2\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} R & S \\ * & R\end{bmatrix}\begin{bmatrix}\delta_1 \\ \delta_2\end{bmatrix}.$$

Lemma 3 Consider the synchronization error system (4), then the following inequality holds

$$\|e(t)\|^2 \leq \theta_1\|e(t_k)\|^2 + \theta_2\int_{t_k - d(t_k)}^{t_k - d_1}\|e(\alpha)\|^2\,\mathrm{d}\alpha,\ t_k \leq t_{k+1}, \quad (9)$$

where

$$\begin{aligned}
\theta_1 &= 5(1 + \|KC\|^2h^2)e^{5(\|A\|^2 + \|W\|^2\|SD\|^2)h^2}, \\
\theta_2 &= 5\|B\|^2 h e^{5(\|A\|^2 + \|W\|^2\|SD\|^2)h^2}.
\end{aligned}$$

Proof For any $t \in [t_k, t_{k+1})$, it follows from expression (4) that

$$\begin{aligned}
\|e(t)\| \leq\ & \|e(t_k)\| + \Big\|\int_{t_k}^{t} Ae(\alpha)\,\mathrm{d}\alpha\Big\| + \Big\|\int_{t_k}^{t} Be(\alpha - d(\alpha))\,\mathrm{d}\alpha\Big\| \\
&+ \Big\|\int_{t_k}^{t} W\eta(De(\alpha), z(\alpha))\,\mathrm{d}\alpha\Big\| \\
&- \Big\|\int_{t_k}^{t} KCe(t_k)\,\mathrm{d}\alpha\Big\|. \quad (10)
\end{aligned}$$

Applying the Cauchy–Schwartz inequity to inequality (10), we then have

$$\begin{aligned}
\|e(t)\|^2 \leq\ & 5\|e(t_k)\|^2 + \Big\|\int_{t_k}^{t} Ae(\alpha)\,\mathrm{d}\alpha\Big\|^2 \\
&+ 5\Big\|\int_{t_k}^{t} Be(\alpha - d(\alpha))\,\mathrm{d}\alpha\Big\|^2 \\
&+ 5\Big\|\int_{t_k}^{t} W\eta(De(\alpha), z(\alpha))\,\mathrm{d}\alpha\Big\|^2 \\
&- 5\Big\|\int_{t_k}^{t} KCe(t_k)\,\mathrm{d}\alpha\Big\|^2. \quad (11)
\end{aligned}$$

Again using Cauchy–Schwartz inequity, we obtain from above

$$\begin{aligned}
\|e(t)\|^2 \leq\ & 5\|e(t_k)\|^2 + \int_{t_k}^{t}\|Ae(\alpha)\|^2\,\mathrm{d}\alpha \\
&+ 5\int_{t_k}^{t}\|Be(\alpha - d(\alpha))\|^2\,\mathrm{d}\alpha \\
&+ 5\int_{t_k}^{t}\|W\eta(De(\alpha), z(\alpha))\|^2\,\mathrm{d}\alpha \\
&- 5\int_{t_k}^{t}\|KCe(t_k)\|^2\,\mathrm{d}\alpha. \quad (12)
\end{aligned}$$

On the other hand, it can be found from expression (5) that

$$\|W\eta(De(\alpha), z(\alpha))\|^2 \leq \|W\|^2\|SD\|^2\|e(t)\|^2. \quad (13)$$

Substituting inequality (13) into inequality (12), we obtain

$$\begin{aligned}
\|e(t)\|^2 \leq\ & 5(\|A\|^2 + \|W\|^2\|SD\|^2h)\int_{t_k}^{t}\|e(\alpha)\|^2\,\mathrm{d}\alpha \\
&+ 5\|e(t_k)\|^2 + 5\|KC\|^2 h\int_{t_k}^{t}\|e(t_k)\|^2\,\mathrm{d}\alpha \\
&+ 5\|B\|^2 h\int_{t_k}^{t}\|e(\alpha - d(\alpha))\|^2\,\mathrm{d}\alpha, \\
\|e(t)\|^2 \leq\ & 5(1 + \|KC\|^2h^2)\|e(t_k)\|^2 \\
&+ 5(\|A\|^2 + \|W\|^2\|SD\|^2h)\int_{t_k}^{t}\|e(\alpha)\|^2\,\mathrm{d}\alpha \\
&+ 5\|B\|^2 h\int_{t_k - d(t_k)}^{t_k - d_1}\|e(\alpha)\|^2\,\mathrm{d}\alpha. \quad (14)
\end{aligned}$$

By applying the Gronwall–Bellman Lemma to inequality (14), we get inequality (9) immediately. This completes the proof.

Remark 1 The proof of the above lemma is followed by the proof of Lemma 2 in Ref. [41], who considered that time-delay is constant. However, in this paper interval time-varying delay is considered, so we need to slightly modify the above-mentioned lemma for our convenience, which is given in Lemma 3.

3. Main results

Theorem 1 Given scalars $\alpha > 0$, ϵ, and γ, the master system $\mathcal{M}$ and slave system $\mathcal{S}$ in system (1) are exponentially synchronous if there exist matrices

$$\begin{aligned}
&P > 0,\ Q_1 > 0,\ Q_2 > 0,\ Q_3 > 0,\ Q_4 > 0,\ Q_5 > 0, \\
&\begin{bmatrix} R_1 & R_2 \\ \star & R_3 \end{bmatrix} > 0,\ \begin{bmatrix} S_1 & S_2 \\ \star & S_3 \end{bmatrix} > 0,\ T_1 > 0,\ T_2 > 0,\ T_3 > 0,\ U > 0, \\
&\begin{bmatrix} Z_1 & Z_2 \\ \star & Z_3 \end{bmatrix} > 0,\ X_1,\ X_2,\ X_3,\ X_4,\ X_5,\ G,\ L,
\end{aligned}$$

$H=[H_1\ H_2\ H_3\ H_4], X, Y, V, Z, Y_1, Y_2,$

and the diagonal matrices $V_1>0$ and $V_2>0$, such that the following expressions (15)–(17) holds

$$\begin{bmatrix} T_2 & Y_1 \\ \star & T_2 \end{bmatrix}>0,\ \begin{bmatrix} T_3 & Y_2 \\ \star & T_3 \end{bmatrix}>0,\ \begin{bmatrix} S_1 & S_2 & X & Y \\ \star & S_3 & V & Z \\ \star & \star & S_1 & S_2 \\ \star & \star & \star & S_3 \end{bmatrix}>0, \quad (15)$$

$$\Psi_{13\times 13}(\bar{h})<0, \quad (16)$$

$$\Omega_{14\times 14}(\bar{h})<0,\ \bar{h}=0,h, \quad (17)$$

where

$$\begin{aligned}
&\Psi_{11}=\Xi_{11}+\bar{h}Z_1+\alpha\bar{h}(X_1+x^{\mathrm{T}})+\bar{h}X_3+\bar{h}X_3^{\mathrm{T}},\\
&\Psi_{12}=\Xi_{12},\ \Psi_{13}=\Xi_{13},\ \Psi_{15}=\Xi_{15},\\
&\Psi_{16}=\Xi_{16},\ \Psi_{17}=\Xi_{17},\ \Psi_{18}=\Xi_{18}+\bar{h}\frac{X_1+X_1^{\mathrm{T}}}{2},\\
&\Psi_{111}=\Xi_{111}+\bar{h}Z_2+2\alpha\bar{h}(-X_1+X_2)+\bar{h}X_4^{\mathrm{T}},\\
&\Psi_{112}=\Xi_{112}+2\alpha\bar{h}X_3+\bar{h}\frac{X_5+X_5^{\mathrm{T}}}{2},\ \Psi_{113}=\Xi_{113}+D^{\mathrm{T}}SV_1,\\
&\Psi_{22}=\Xi_{22},\ \Psi_{23}=\Xi_{23},\ \Psi_{24}=\Xi_{24},\ \Psi_{25}=\Xi_{25},\\
&\Psi_{26}=\Xi_{26},\ \Psi_{27}=\Xi_{27},\ \Psi_{33}=\Xi_{33},\ \Psi_{34}=\Xi_{34},\\
&\Psi_{36}=\Xi_{36},\ \Psi_{37}=\Xi_{37},\ \Psi_{38}=\Xi_{38},\ \Psi_{311}=\Xi_{311},\\
&\Psi_{44}=\Xi_{44},\ \Psi_{46}=\Xi_{46},\ \Psi_{47}=\Xi_{47},\ \Psi_{55}=\Xi_{55},\\
&\Psi_{66}=\Xi_{66},\ \Psi_{67}=\Xi_{67},\ \Psi_{77}=\Xi_{77},\ \Psi_{88}=\Xi_{88}+\bar{h}U,\\
&\Psi_{811}=\Xi_{811}+\bar{h}(-X_1+X_2),\ \Psi_{812}=\bar{h}X_3,\ \Psi_{813}=\Xi_{813},\\
&\Psi_{99}=\Xi_{99},\ \Psi_{1010}=\Xi_{1010},\\
&\Psi_{1111}=\Xi_{1111}+\bar{h}Z_3+2\alpha\bar{h}\left[-X_2-X_2^{\mathrm{T}}+\frac{X_1+X_1^{\mathrm{T}}}{2}\right],\\
&\Psi_{1112}=\Xi_{1112}+2\alpha\bar{h}X_4,\ \Psi_{1113}=\Xi_{1113},\\
&\Psi_{1212}=\Xi_{1212}+\alpha\bar{h}(X_5+X_5^{\mathrm{T}}),\ \Psi_{1313}=-2V_1,\\
&\Omega_{11}=\Xi_{11},\ \Omega_{12}=\Xi_{12},\ \Omega_{13}=\Xi_{13},\ \Omega_{15}=\Xi_{15},\\
&\Omega_{16}=\Xi_{16},\ \Omega_{17}=\Xi_{17},\ \Omega_{18}=\Xi_{18},\ \Omega_{111}=\Xi_{111},\\
&\Omega_{112}=\Xi_{112},\ \Omega_{113}=\Xi_{113}+D^{\mathrm{T}}SV_2,\ \Omega_{114}=\sqrt{\bar{h}}H_1^{\mathrm{T}},\\
&\Omega_{22}=\Xi_{22},\ \Omega_{23}=\Xi_{23},\ \Omega_{24}=\Xi_{24},\ \Omega_{25}=\Xi_{25},\\
&\Omega_{26}=\Xi_{26},\ \Omega_{27}=\Xi_{27},\ \Omega_{33}=\Xi_{33},\ \Omega_{34}=\Xi_{34},\\
&\Omega_{36}=\Xi_{36},\ \Omega_{37}=\Xi_{37},\ \Omega_{38}=\Xi_{38},\ \Omega_{311}=\Xi_{311},\\
&\Omega_{44}=\Xi_{44},\ \Omega_{46}=\Xi_{46},\ \Omega_{47}=\Xi_{47},\ \Omega_{55}=\Xi_{55},\\
&\Omega_{66}=\Xi_{66},\ \Omega_{67}=\Xi_{67},\ \Omega_{77}=\Xi_{77},\ \Omega_{88}=\Xi_{88},\\
&\Omega_{811}=\Xi_{811},\ \Omega_{813}=\Xi_{813},\ \Omega_{814}=\sqrt{\bar{h}}H_2^{\mathrm{T}},\ \Omega_{99}=\Xi_{99},\\
&\Omega_{1010}=\Xi_{1010},\ \Omega_{1111}=\Xi_{1111}-\bar{h}e^{-2\alpha h}Z_3,\\
&\Omega_{1112}=\Xi_{1112},\ \Omega_{1113}=\Xi_{1113},\ \Omega_{1114}=\sqrt{\bar{h}}H_3^{\mathrm{T}},\\
&\Omega_{1212}=\Xi_{1212},\ \Omega_{1214}=\sqrt{\bar{h}}H_4^{\mathrm{T}},\ \Omega_{1313}=-2V_2,\\
&\Omega_{1414}=-\mathrm{e}^{-2\alpha h}U,
\end{aligned}$$

with

$$\begin{aligned}
\Xi_{11}&=2\alpha P+Q_1+d_1^2R_1+2(d_2-d_1)^2S_1-\mathrm{e}^{-2\alpha d_1}R_3\\
&\quad-d_1^2\mathrm{e}^{-2\alpha d_1}T_1-\beta_1^2\mathrm{e}^{-2\alpha d_2}T_2-\beta_1\beta_2Y_1\\
&\quad-\beta_2^2\mathrm{e}^{-2\alpha d_2}T_2-\frac{X_1+X_1^{\mathrm{T}}}{2}+\epsilon GA+\epsilon A^{\mathrm{T}}G^{\mathrm{T}}+H_1+H_1^{\mathrm{T}},\\
\Xi_{12}&=\mathrm{e}^{-2\alpha d_1}R_3,\ \Xi_{13}=\epsilon GB,\\
\Xi_{15}&=-\mathrm{e}^{-2\alpha d_1}R_2^{\mathrm{T}}+d_1\mathrm{e}^{-2\alpha d_1}T_1,\\
\Xi_{16}&=\beta_1\mathrm{e}^{-2\alpha d_2}T_2+\beta_2\mathrm{e}^{-2\alpha d_2}Y_1^{\mathrm{T}},\\
\Xi_{17}&=\beta_1\mathrm{e}^{-2\alpha d_2}Y_1+\beta_2\mathrm{e}^{-2\alpha d_2}T_2,\\
\Xi_{18}&=H_2+A^{\mathrm{T}}G^{\mathrm{T}}-\epsilon G+P+d_1^2R_2+2(d_2-d_1)^2S_2,\\
\Xi_{111}&=-(-X_1+X_2)-\epsilon LC+\gamma A^{\mathrm{T}}G^{\mathrm{T}}+H_3-H_1^{\mathrm{T}},\\
\Xi_{22}&=-\mathrm{e}^{-2\alpha d_1}Q_1+\mathrm{e}^{-2\alpha d_1}Q_2-\mathrm{e}^{-2\alpha d_2}R_3-\mathrm{e}^{-2\alpha d_2}S_3\\
&\quad-\beta_1^2\mathrm{e}^{-2\alpha d_2}T_3-\beta_1\beta_2\mathrm{e}^{-2\alpha d_2}Y_2-\beta_1\beta_2\mathrm{e}^{-2\alpha d_2}Y_2^{\mathrm{T}}\\
&\quad-\beta_2^2\mathrm{e}^{-2\alpha d_2}T_3,\\
\Xi_{23}&=\mathrm{e}^{-2\alpha d_2}S_3-\mathrm{e}^{-2\alpha d_2}Z,\ \Xi_{24}=\mathrm{e}^{-2\alpha d_2}Z,\\
\Xi_{25}&=\mathrm{e}^{-2\alpha d_1}R_2^{\mathrm{T}},\\
\Xi_{26}&=-\mathrm{e}^{-2\alpha d_2}S_2^{\mathrm{T}}+\beta_1\mathrm{e}^{-2\alpha d_2}T_3+\beta_2\mathrm{e}^{-2\alpha d_2}Y_2^{\mathrm{T}},\\
\Xi_{27}&=-\mathrm{e}^{-2\alpha d_2}V+\beta_1\mathrm{e}^{-2\alpha d_2}Y_2+\beta_2\mathrm{e}^{-2\alpha d_2}T_3,\\
\Xi_{33}&=-(1-d_\mu)\mathrm{e}^{-2\alpha d_2}Q_2+(1-d_\mu)\mathrm{e}^{-2\alpha d_1}Q_3\\
&\quad-2\mathrm{e}^{-2\alpha d_2}S_3+\mathrm{e}^{-2\alpha d_2}Z+\mathrm{e}^{-2\alpha d_2}Z^{\mathrm{T}},\\
\Xi_{34}&=\mathrm{e}^{-2\alpha d_2}S_3-\mathrm{e}^{-2\alpha d_2}Z,\ \Xi_{36}=-\mathrm{e}^{-2\alpha d_2}Y^{\mathrm{T}}+\mathrm{e}^{-2\alpha d_2}S_2^{\mathrm{T}},\\
\Xi_{37}&=\mathrm{e}^{-2\alpha d_2}V-\mathrm{e}^{-2\alpha d_2}S_2^{\mathrm{T}},\ \Xi_{38}=B^{\mathrm{T}}G^{\mathrm{T}},\\
\Xi_{311}&=\gamma B^{\mathrm{T}}G^{\mathrm{T}},\ \Xi_{44}=-\mathrm{e}^{-2\alpha d_2}Q_3-\mathrm{e}^{-2\alpha d_2}S_3,\\
\Xi_{46}&=\mathrm{e}^{-2\alpha d_2}Y^{\mathrm{T}},\ \Xi_{47}=\mathrm{e}^{-2\alpha d_2}S_2^{\mathrm{T}},\\
\Xi_{55}&=-\mathrm{e}^{-2\alpha d_2}R_1-\mathrm{e}^{-2\alpha d_2}T_1,\\
\Xi_{66}&=-\mathrm{e}^{-2\alpha d_2}S_1-\mathrm{e}^{-2\alpha d_2}T_2-\mathrm{e}^{-2\alpha d_2}T_3,\\
\Xi_{67}&=-\mathrm{e}^{-2\alpha d_2}X-\mathrm{e}^{-2\alpha d_2}Y_1^{\mathrm{T}}-\mathrm{e}^{-2\alpha d_2}Y_2^{\mathrm{T}},\\
\Xi_{77}&=-\mathrm{e}^{-2\alpha d_2}S_1-\mathrm{e}^{-2\alpha d_2}T_2-\mathrm{e}^{-2\alpha d_2}T_3,\\
\Xi_{88}&=Q_4+d_1^2R_3+2(d_2-d_1)^2S_3-G-G^{\mathrm{T}}+\frac{d_1^4}{4}T_1\\
&\quad+\frac{(d_2^2-d_1^2)^2}{4}T_2,\\
\Xi_{811}&=-\gamma G^{\mathrm{T}}-H_2^{\mathrm{T}}-LC,\ \Xi_{813}=GW,\\
\Xi_{99}&=-\mathrm{e}^{-2\alpha d_2}Q_4+\mathrm{e}^{-2\alpha d_2}Q_5+\frac{(d_2-d_1)^4}{4}T_3,\\
\Xi_{1010}&=-\mathrm{e}^{-2\alpha d_2}Q_5,\\
\Xi_{1111}&=X_2+X_2^{\mathrm{T}}-\frac{X_1+X_1^{\mathrm{T}}}{2}-\gamma LC-\gamma C^{\mathrm{T}}L^{\mathrm{T}}-H_3^{\mathrm{T}}-H_3,\\
\Xi_{1112}&=-\mathrm{e}^{-2\alpha h}Z_2^{\mathrm{T}}-X_4-H_4,\ \Xi_{1113}=\gamma GW,\\
\Xi_{1212}&=-\frac{\mathrm{e}^{-2\alpha h}}{h}Z_1-\frac{X_5+X_5^{\mathrm{T}}}{2}.
\end{aligned}$$

Furthermore, the sampled-data controller gain matrix is given by

$$K=G^{-1}L. \quad (18)$$

Proof Consider the following Lyapunov–Krasovskii functional candidate as

$$V(t)=\sum_{i=1}^{7}V_i(t),\quad t\in[t_k,t_{k+1}), \quad (19)$$

where

$$
\begin{aligned}
V_1(t) &= \mathrm{e}^{2\alpha t} e^{\mathrm{T}}(t) P e(t),\\
V_2(t) &= \int_{t-d_1}^{t} \mathrm{e}^{2\alpha s} e^{\mathrm{T}}(s) Q_1 e(s)\,\mathrm{d}s + \int_{t-d(t)}^{t-d_1} \mathrm{e}^{2\alpha s} e^{\mathrm{T}}(s) Q_2 e(s)\,\mathrm{d}s\\
&\quad + \int_{t-d_2}^{t-d(t)} \mathrm{e}^{2\alpha s} e^{\mathrm{T}}(s) Q_3 e(s)\,\mathrm{d}s + \int_{t-d_1}^{t} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s) Q_4 \dot{e}(s)\,\mathrm{d}s\\
&\quad + \int_{t-d_1}^{t-d_2} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s) Q_5 \dot{e}(s)\,\mathrm{d}s,\\
V_3(t) &= d_1 \int_{-d_1}^{0}\int_{t+\theta}^{t} \mathrm{e}^{2\alpha s} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2\\ \star & R_3\end{bmatrix} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix} \mathrm{d}s\,\mathrm{d}\theta\\
&\quad + 2d_{12} \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \mathrm{e}^{2\alpha s} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2\\ \star & S_3\end{bmatrix} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix} \mathrm{d}s\,\mathrm{d}\theta,\\
V_4(t) &= \frac{d_1^2}{2} \int_{-d_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s) T_1 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta\\
&\quad + \frac{d_2^2 - d_1^2}{2} \int_{-d_2}^{-d_1}\int_{\theta}^{0}\int_{t+\lambda}^{t} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s) T_2 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta\\
&\quad + \frac{d_{12}^2}{2} \int_{t-d_2}^{t-d_1}\int_{\theta}^{t-d_1}\int_{\lambda}^{t-d_1} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s) T_3 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta,\\
V_5(t) &= (t_{k+1} - t) \int_{t_k}^{t} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s) U \dot{e}(s)\,\mathrm{d}s,\\
V_6(t) &= (t_{k+1} - t) \int_{t_k}^{t} \mathrm{e}^{2\alpha s} \begin{bmatrix} e(s)\\ e(t_k)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z_1 & Z_2\\ \star & Z_3\end{bmatrix} \begin{bmatrix} e(s)\\ e(t_k)\end{bmatrix} \mathrm{d}s,\\
V_7(t) &= (t_{k+1} - t)\,\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ e(t_k)\\ \int_{t_k}^{t} e(s)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}} \mathcal{H} \begin{bmatrix} e(t)\\ e(t_k)\\ \int_{t_k}^{t} e(s)\,\mathrm{d}s\end{bmatrix},
\end{aligned}
$$

and

$$
\mathcal{H} = \begin{bmatrix} \dfrac{X_1 + X_1^{\mathrm{T}}}{2} & -X_1 + X_2 & X_3\\ \star & -X_2 - X_2^{\mathrm{T}} + \dfrac{X_1 + X_1^{\mathrm{T}}}{2} & X_4\\ \star & \star & \dfrac{X_5 + X_5^{\mathrm{T}}}{2}\end{bmatrix}.
$$

Calculating the time derivative of $V(t)$ along the trajectories of expression (4), we then have

$$
\dot{V}(t) = \sum_{i=1}^{7} \dot{V}_i(t), \quad t \in [t_k, t_{k+1}), \tag{20}
$$

where

$$
\begin{aligned}
\dot{V}_1(t) &= 2\,\mathrm{e}^{2\alpha t} e^{\mathrm{T}}(t) P \dot{e}(t) + 2\alpha\,\mathrm{e}^{2\alpha t} e^{\mathrm{T}}(t) P e(t), \qquad (21)\\
\dot{V}_2(t) &= \mathrm{e}^{2\alpha t} e^{\mathrm{T}}(t) Q_1 e(t) - \mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_1} e^{\mathrm{T}}(t - d_1) Q_1 e(t - d_1)\\
&\quad + \mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_1} e^{\mathrm{T}}(t - d_1) Q_2 e(t - d_1)\\
&\quad - (1 - d_\mu)\,\mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d(t)} e^{\mathrm{T}}(t - d(t)) Q_2 e(t - d(t))\\
&\quad + (1 - d_\mu)\,\mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d(t)} e^{\mathrm{T}}(t - d(t)) Q_3 e(t - d(t))\\
&\quad - \mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_2} e^{\mathrm{T}}(t - d_2) Q_3 e(t - d_2)\\
&\quad + \mathrm{e}^{2\alpha t} \dot{e}^{\mathrm{T}}(t) Q_4 \dot{e}(t) - \mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_1} \dot{e}^{\mathrm{T}}(t - d_1) Q_4 \dot{e}(t - d_1)\\
&\quad + \mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_1} \dot{e}^{\mathrm{T}}(t - d_1) Q_5 \dot{e}(t - d_1)\\
&\quad - \mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_2} \dot{e}^{\mathrm{T}}(t - d_2) Q_5 \dot{e}(t - d_2), \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3(t) &= d_1^2\,\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2\\ \star & R_3\end{bmatrix} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&\quad + 2d_{12}^2\,\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2\\ \star & S_3\end{bmatrix} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&\quad - d_1 \int_{t-d_1}^{t} \mathrm{e}^{2\alpha s} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2\\ \star & R_3\end{bmatrix} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix} \mathrm{d}s\\
&\quad - 2d_{12} \int_{t-d_2}^{t-d_1} \mathrm{e}^{2\alpha s} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2\\ \star & S_3\end{bmatrix} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix} \mathrm{d}s\\
&\le d_1^2\,\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2\\ \star & R_3\end{bmatrix} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&\quad + 2d_{12}^2 \mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2\\ \star & S_3\end{bmatrix} \begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&\quad - d_1\,\mathrm{e}^{2\alpha t} \int_{t-d_1}^{t} \mathrm{e}^{-2\alpha d_1} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2\\ \star & R_3\end{bmatrix} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix} \mathrm{d}s\\
&\quad - 2d_{12} \mathrm{e}^{2\alpha t} \int_{t-d_2}^{t-d_1} \mathrm{e}^{-2\alpha d_2} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_2\\ \star & S_3\end{bmatrix} \begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix} \mathrm{d}s, \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_4(t) &\le \mathrm{e}^{2\alpha t} \dot{e}^{\mathrm{T}}(t) \left[\frac{d_1^4}{4} T_1 + \frac{(d_2^2 - d_1^2)^2}{4} T_2\right] \dot{e}(t)\\
&\quad + \mathrm{e}^{2\alpha t} \dot{e}^{\mathrm{T}}(t - d_1) \frac{d_{12}^4}{4} T_3 \dot{e}(t - d_1)\\
&\quad - \frac{d_1^2}{2}\,\mathrm{e}^{2\alpha(t - d_1)} \int_{-d_1}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s) T_1 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\quad - \frac{d_2^2 - d_1^2}{2}\,\mathrm{e}^{2\alpha(t - d_2)} \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s) T_2 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\quad + \frac{d_{12}^2}{2}\,\mathrm{e}^{2\alpha(t - d_1)} \int_{t-d_2}^{t-d_1}\int_{\theta}^{t-d_1} \dot{e}^{\mathrm{T}}(s) T_3 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta, \qquad (24)\\
\dot{V}_5(t) &= (t_{k+1} - t)\,\mathrm{e}^{2\alpha t} \dot{e}^{\mathrm{T}}(t) U \dot{e}(t) - \int_{t_k}^{t} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s) U \dot{e}(s)\,\mathrm{d}s\\
&\le (t_{k+1} - t)\,\mathrm{e}^{2\alpha t} \dot{e}^{\mathrm{T}}(t) U \dot{e}(t)\\
&\quad - \mathrm{e}^{2\alpha t} \int_{t_k}^{t} \mathrm{e}^{-2\alpha h} \dot{e}^{\mathrm{T}}(s) U \dot{e}(s)\,\mathrm{d}s, \qquad (25)\\
\dot{V}_6(t) &\le -\mathrm{e}^{2\alpha t} \int_{t_k}^{t} \mathrm{e}^{-2\alpha h} \begin{bmatrix} e(s)\\ e(t_k)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z_1 & Z_2\\ \star & Z_3\end{bmatrix} \begin{bmatrix} e(s)\\ e(t_k)\end{bmatrix} \mathrm{d}s\\
&\quad + (t_{k+1} - t)\,\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ e(t_k)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z_1 & Z_2\\ \star & Z_3\end{bmatrix} \begin{bmatrix} e(t)\\ e(t_k)\end{bmatrix}\\
&= -\mathrm{e}^{2\alpha t} \int_{t_k}^{t} \mathrm{e}^{-2\alpha h} e^{\mathrm{T}}(s) R_1 e(s)\,\mathrm{d}s\\
&\quad - 2\,\mathrm{e}^{2\alpha t} e^{\mathrm{T}}(t_k)\,\mathrm{e}^{-2\alpha h} R_2^{\mathrm{T}} \int_{t_k}^{t} e(s)\,\mathrm{d}s\\
&\quad - \mathrm{e}^{2\alpha t}(t - t_k) e^{\mathrm{T}}(t_k)\,\mathrm{e}^{-2\alpha h} R_3 e(t_k)\\
&\quad + (t_{k+1} - t)\,\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ e(t_k)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z_1 & Z_2\\ \star & Z_3\end{bmatrix} \begin{bmatrix} e(t)\\ e(t_k)\end{bmatrix}, \qquad (26)\\
\dot{V}_7(t) &\le -\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ e(t_k)\\ \int_{t_k}^{t} e(s)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}} \mathcal{H} \begin{bmatrix} e(t)\\ e(t_k)\\ \int_{t_k}^{t} e(s)\,\mathrm{d}s\end{bmatrix}\\
&\quad + 2\alpha(t_{k+1} - t)\,\mathrm{e}^{2\alpha t} \begin{bmatrix} e(t)\\ e(t_k)\\ \int_{t_k}^{t} e(s)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}} \mathcal{H} \begin{bmatrix} e(t)\\ e(t_k)\\ \int_{t_k}^{t} e(s)\,\mathrm{d}s\end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&+2(t_{k+1}-t)\,\mathrm{e}^{2\alpha t}e^{\mathrm{T}}(t)\frac{X_1+X_1^{\mathrm{T}}}{2}\dot{e}(t)\\
&+2(t_{k+1}-t)\,\mathrm{e}^{2\alpha t}e^{\mathrm{T}}(t_k)(-X_1+X_2)^{\mathrm{T}}\dot{e}(t)\\
&+2(t_{k+1}-t)\,\mathrm{e}^{2\alpha t}\int_{t_k}^{t}e^{\mathrm{T}}(s)\,\mathrm{d}sX_3^{\mathrm{T}}\dot{e}(t)\\
&+2(t_{k+1}-t)\,\mathrm{e}^{2\alpha t}e^{\mathrm{T}}(t)X_3e(t)\\
&+2(t_{k+1}-t)\,\mathrm{e}^{2\alpha t}e^{\mathrm{T}}(t_k)X_4e(t)\\
&+2(t_{k+1}-t)\,\mathrm{e}^{2\alpha t}\int_{t_k}^{t}e^{\mathrm{T}}(s)\,\mathrm{d}s\frac{X_5+X_5^{\mathrm{T}}}{2}e(t). \qquad (27)
\end{aligned}
$$

Using Lemmas 1 and 2, we have

$$
\begin{aligned}
\dot{V}_3(t) \le\ & \mathrm{e}^{2\alpha t}\begin{bmatrix}e(t)\\ \dot{e}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}d_1^2R_1 & d_1^2R_2\\ \star & d_1^2R_3\end{bmatrix}\begin{bmatrix}e(t)\\ \dot{e}(t)\end{bmatrix}\\
&+\mathrm{e}^{2\alpha t}\begin{bmatrix}e(t)\\ \dot{e}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}2d_{12}^2S_1 & 2d_{12}^2S_2\\ \star & 2d_{12}^2S_3\end{bmatrix}\begin{bmatrix}e(t)\\ \dot{e}(t)\end{bmatrix}\\
&-\mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_1}\begin{bmatrix}\int_{t-d_1}^{t}e(s)\,\mathrm{d}s\\ e(t)-e(t-d_1)\end{bmatrix}^{\mathrm{T}}\\
&\times\begin{bmatrix}R_1 & R_2\\ \star & R_3\end{bmatrix}\begin{bmatrix}\int_{t-d_1}^{t}e(s)\,\mathrm{d}s\\ e(t)-e(t-d_1)\end{bmatrix}\\
&-\mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_2}\begin{bmatrix}\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\\ e(t-d_1)-e(t-d(t))\end{bmatrix}^{\mathrm{T}}\\
&\times\begin{bmatrix}S_1 & S_2\\ \star & S_3\end{bmatrix}\begin{bmatrix}\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\\ e(t-d_1)-e(t-d(t))\end{bmatrix}\\
&-2\mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_2}\begin{bmatrix}\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\\ e(t-d_1)-e(t-d(t))\end{bmatrix}^{\mathrm{T}}\\
&\times\begin{bmatrix}X & Y\\ V & Z\end{bmatrix}\begin{bmatrix}\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\\ e(t-d(t))-e(t-d_2)\end{bmatrix}\\
&-\mathrm{e}^{2\alpha t}\,\mathrm{e}^{-2\alpha d_2}\begin{bmatrix}\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\\ e(t-d(t))-e(t-d_2)\end{bmatrix}^{\mathrm{T}}\\
&\times\begin{bmatrix}S_1 & S_2\\ \star & S_3\end{bmatrix}\begin{bmatrix}\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\\ e(t-d(t))-e(t-d_2)\end{bmatrix}. \qquad (28)
\end{aligned}
$$

On the other hand, let $\beta_1=d(t)-d_1$, $\beta_2=d_2-d(t)$, and using Lemmas 1 and 2, we can respectively estimate the three terms in inequality (24) to satisfy

$$
\begin{aligned}
&-\frac{d_1^2}{2}\mathrm{e}^{2\alpha(t-d_1)}\int_{-d_1}^{0}\int_{t+\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_1\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\le-\mathrm{e}^{2\alpha(t-d_1)}\left[d_1e(t)-\int_{t-d_1}^{t}e(s)\,\mathrm{d}s\right]^{\mathrm{T}}\\
&\quad\times T_1\left[d_1e(t)-\int_{t-d_1}^{t}e(s)\,\mathrm{d}s\right], \qquad (29)\\
&-\frac{d_2^2-d_1^2}{2}\mathrm{e}^{2\alpha(t-d_2)}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_2\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&=-\frac{d_2^2-d_1^2}{2}\left[\frac{d^2(t)-d_1^2}{d^2(t)-d_1^2}\right]\\
&\quad\times\mathrm{e}^{2\alpha(t-d_2)}\int_{-d(t)}^{-d_1}\int_{t+\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_2\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\quad-\frac{d_2^2-d_1^2}{2}\left[\frac{d_2^2-d^2(t)}{d_2^2-d^2(t)}\right]\\
&\quad\times\mathrm{e}^{2\alpha(t-d_2)}\int_{-d_2}^{-d(t)}\int_{t+\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_2\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\le-\mathrm{e}^{2\alpha(t-d_2)}\begin{bmatrix}\int_{-d(t)}^{-d_1}\int_{t+\theta}^{t}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ \int_{-d_2}^{-d(t)}\int_{t+\theta}^{t}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\end{bmatrix}^{\mathrm{T}}\\
&\quad\times\begin{bmatrix}T_2 & Y_1\\ \star & T_2\end{bmatrix}\begin{bmatrix}\int_{-d(t)}^{-d_1}\int_{t+\theta}^{t}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ \int_{-d_2}^{-d(t)}\int_{t+\theta}^{t}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\end{bmatrix}\\
&=-\mathrm{e}^{2\alpha t}\left[\beta_1e(t)-\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^{\mathrm{T}}\\
&\quad\times\mathrm{e}^{-2\alpha d_2}T_2\left[\beta_1e(t)-\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]\\
&\quad-2\mathrm{e}^{2\alpha t}\left[\beta_1e(t)-\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^{\mathrm{T}}\\
&\quad\times\mathrm{e}^{-2\alpha d_2}Y_1\left[\beta_2e(t)-\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]\\
&\quad-\mathrm{e}^{2\alpha t}\left[\beta_2e(t)-\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^{\mathrm{T}}\\
&\quad\times\mathrm{e}^{-2\alpha d_2}T_2\left[\beta_2e(t)-\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right], \qquad (30)\\
&-\frac{d_{12}^2}{2}\mathrm{e}^{2\alpha(t-d_1)}\int_{t-d_2}^{t-d_1}\int_{\theta}^{t-d_1}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&=-\frac{d_{12}^2}{2}\left[\frac{\beta_1^2}{\beta_1^2}\right]\int_{t-d(t)}^{t-d_1}\int_{\theta}^{t-d_1}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\quad-\frac{d_{12}^2}{2}\left[\frac{d_{12}^2-\beta_1^2}{d_{12}^2-\beta_1^2}\right]\int_{t-d_2}^{t-d(t)}\int_{\theta}^{t-d_1}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\le-\mathrm{e}^{2\alpha(t-d_2)}\begin{bmatrix}\int_{t-d(t)}^{t-d_1}\int_{\theta}^{t-d_1}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ \int_{t-d_2}^{t-d(t)}\int_{\theta}^{t-d_1}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\end{bmatrix}^{\mathrm{T}}\\
&\quad\times\begin{bmatrix}T_3 & Y_2\\ \star & T_3\end{bmatrix}\begin{bmatrix}\int_{t-d(t)}^{t-d_1}\int_{\theta}^{t-d_1}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ \int_{t-d_2}^{t-d(t)}\int_{\theta}^{t-d_1}\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta\end{bmatrix}\\
&=-\mathrm{e}^{2\alpha t}\left[\beta_1e(t-d_1)-\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^{\mathrm{T}}\\
&\quad\times\mathrm{e}^{-2\alpha d_2}T_2\left[\beta_1e(t-d_1)-\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]\\
&\quad-2\mathrm{e}^{2\alpha t}\left[\beta_1e(t-d_1)-\int_{t-d(t)}^{t-d_1}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^{\mathrm{T}}\\
&\quad\times\mathrm{e}^{-2\alpha d_2}Y_1\left[\beta_2e(t-d_1)-\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]\\
&\quad-\mathrm{e}^{2\alpha t}\left[\beta_2e(t-d_1)-\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]^{\mathrm{T}}\\
&\quad\times\mathrm{e}^{-2\alpha d_2}T_2\left[\beta_2e(t-d_1)-\int_{t-d_2}^{t-d(t)}e(s)\,\mathrm{d}s\,\mathrm{d}\theta\right]. \qquad (31)
\end{aligned}
$$

By using Lemma 1, inequality (26) can be rewritten in the following form,

$$
\dot{V}_6(t) = -\mathrm{e}^{2\alpha t}\int_{t_k}^{t}e^{\mathrm{T}}(s)\frac{\mathrm{e}^{-2\alpha h}}{h}Z_1e(s)\,\mathrm{d}s
$$

$$
\begin{aligned}
&-2\mathrm{e}^{2\alpha t}e^{\mathrm{T}}(t_k)\mathrm{e}^{-2\alpha h}Z_2^{\mathrm{T}}\int_{t_k}^{t}e(s)\mathrm{d}s\\
&-\mathrm{e}^{2\alpha t}(t-t_k)e^{\mathrm{T}}(t_k)\mathrm{e}^{-2\alpha h}Z_3e(t_k)\\
&+(t_{k+1}-t)\mathrm{e}^{2\alpha t}\begin{bmatrix}e(t)\\e(t_k)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_1 & Z_2\\ \star & Z_3\end{bmatrix}\begin{bmatrix}e(t)\\e(t_k)\end{bmatrix}. \quad (32)
\end{aligned}
$$

Moreover, based on Lemma 1, it can be found that for any approximately dimensioned matrix H,

$$
\begin{bmatrix}H^{\mathrm{T}}\mathrm{e}^{2\alpha h}U^{-1}H & H^{\mathrm{T}}\\ \star & \mathrm{e}^{-2\alpha h}U\end{bmatrix}\geq 0, \quad (33)
$$

which implies

$$
\int_{t_k}^{t}\begin{bmatrix}\phi(s)\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}H^{\mathrm{T}}\mathrm{e}^{2\alpha h}U^{-1}H & H^{\mathrm{T}}\\ \star & \mathrm{e}^{-2\alpha h}U\end{bmatrix}\begin{bmatrix}\phi(s)\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\geq 0, \quad (34)
$$

where $\phi(t)=\left[\,e^{\mathrm{T}}(t)\ \ \dot{e}^{\mathrm{T}}(t)\ \ e^{\mathrm{T}}(t_k)\ \ \int_{t_k}^{t}e^{\mathrm{T}}(s)\mathrm{d}s\,\right]$.

From inequality (34), we can obtain

$$
\begin{aligned}
-\int_{t_k}^{t}\mathrm{e}^{-2\alpha h}\dot{e}^{\mathrm{T}}(s)U\dot{e}(s)\mathrm{d}s\leq{}&(t-t_k)\phi^{\mathrm{T}}(t)H^{\mathrm{T}}\mathrm{e}^{2\alpha h}U^{-1}H\phi(t)\\
&+2\phi^{\mathrm{T}}(t)H^{\mathrm{T}}e(t)-2\phi^{\mathrm{T}}(t)H^{\mathrm{T}}e(t_k).
\end{aligned}
$$

Therefore, inequality (25) can be rewritten as

$$
\begin{aligned}
\dot{V}_5(t)\leq{}&(t_{k+1}-t)\mathrm{e}^{2\alpha t}\dot{e}^{\mathrm{T}}(t)U\dot{e}(t)\\
&+(t-t_k)\phi^{\mathrm{T}}(t)H^{\mathrm{T}}\mathrm{e}^{2\alpha h}U^{-1}H\phi(t)\\
&+2\phi^{\mathrm{T}}(t)H^{\mathrm{T}}e(t)-2\phi^{\mathrm{T}}(t)H^{\mathrm{T}}e(t_k). \quad (35)
\end{aligned}
$$

According to the error system (4), for any appropriately dimensioned matrix G and scalar ϵ and γ, the following equation holds:

$$
\begin{aligned}
0={}&2\mathrm{e}^{2\alpha t}\left[\epsilon e^{\mathrm{T}}(t)G+\dot{e}^{\mathrm{T}}(t)G+\gamma e^{\mathrm{T}}(t_k)G\right]\\
&\times\left[-\dot{e}(t)+Ae(t)+Be(t-d(t))\right.\\
&\left.+W\eta(De(t),z(t))-KCe(t_k)\right]. \quad (36)
\end{aligned}
$$

Based on expression (8), the following inequality holds,

$$
\begin{aligned}
&2\mathrm{e}^{2\alpha t}\frac{t_{k+1}-t}{h_k}\left[e^{\mathrm{T}}(t)D^{\mathrm{T}}SV_1\eta(De(t),z(t))\right.\\
&\left.-\eta(De(t),z(t))^{\mathrm{T}}V_1\eta(De(t),z(t))\right]\\
&+2\mathrm{e}^{2\alpha t}\frac{t-t_k}{h_k}\left[e^{\mathrm{T}}(t)D^{\mathrm{T}}SV_2\eta(De(t),z(t))\right.\\
&\left.-\eta(De(t),z(t))^{\mathrm{T}}V_2\eta(De(t),z(t))\right]\geq 0. \quad (37)
\end{aligned}
$$

Applying expressions (21), (22), (26)–(32), (35) into $\dot{V}(t)$ and adding the right-hand side of Eq. (36) and the left-hand side of inequality (37) and letting $L=GK$, we arrive at

$$
\dot{V}(t)\leq\xi^{\mathrm{T}}(t)\left[\frac{t_{k+1}-t}{h_k}\Psi(h_k)+\frac{t-t_k}{h_k}\Omega(h_k)\right]\xi(t), \quad (38)
$$

where

$$
\xi(t)=\left[\xi_1^{\mathrm{T}}(t)\ \ \xi_2^{\mathrm{T}}(t)\right]^{\mathrm{T}}, \quad (39)
$$

$$
\begin{aligned}
\xi_1(t)={}&\left[e^{\mathrm{T}}(t)\ \ e^{\mathrm{T}}(t-d_1)\ \ e^{\mathrm{T}}(t-d(t))\ \ e^{\mathrm{T}}(t-d_2)\right.\\
&\int_{t-d_1}^{t}e^{\mathrm{T}}(s)\mathrm{d}s\ \ \int_{t-d(t)}^{t-d_1}e^{\mathrm{T}}(s)\mathrm{d}s\ \ \int_{t-d_2}^{t-d(t)}e^{\mathrm{T}}(s)\mathrm{d}s\\
&\left.\dot{e}^{\mathrm{T}}(t)\ \ \dot{e}^{\mathrm{T}}(t-d_1)\ \ \dot{e}^{\mathrm{T}}(t-d_2)\right], \quad (40)\\
\xi_2(t)={}&\left[e^{\mathrm{T}}(t_k)\ \ \int_{t_k}^{t}e(s)\mathrm{d}s\ \ \eta(De(t),z(t))\right].
\end{aligned}
$$

It is noted that

$$
\Psi(h_k)=\frac{h_k}{h}\Psi(h)+\frac{h-h_k}{h}\Psi(0) \quad (41)
$$

and

$$
\Omega(h_k)=\frac{h_k}{h}\Omega(h)+\frac{h-h_k}{h}\Omega(0). \quad (42)
$$

From expressions (16) and (17), we can obtain

$$
\Psi(h_k)<0,\quad \Omega(0)<0. \quad (43)
$$

By using Schur complement, we have from expression (17)

$$
\Omega(h)<0, \quad (44)
$$

which, in combination with Eqs. (42) and (43), implies

$$
\Omega(h_k)<0. \quad (45)
$$

It is clear from expressions (38), (43), and (45) that

$$
\dot{V}(t)<0,\quad t\in[t_k,t_{k+1}). \quad (46)
$$

From the above, the following inequality holds for $t\in[t_k,t_{k+1})$,

$$
V(t)\leq V(t_k)\leq V(t_{k-1})\leq\cdots\leq V(0). \quad (47)
$$

Based on Lemma 3 and inequality (47), we can conclude that for $t_k\leq t\leq t_{k+1}$,

$$
\begin{aligned}
\|e(t)\|^2\leq{}&\theta_1\|e(t_k)\|^2+\theta_2\int_{t_k-d(t_k)}^{t_k-d_1}\|e(\alpha)\|^2\mathrm{d}\alpha\\
={}&\frac{\theta_1}{\lambda_{\min}(P)\mathrm{e}^{2\alpha t_k}}\mathrm{e}^{2\alpha t_k}\lambda_{\min}(P)\|e(t_k)\|^2\\
&+\frac{\theta_2}{\lambda_{\min}(Q_2)\mathrm{e}^{2\alpha t_k}}\mathrm{e}^{2\alpha t_k}\lambda_{\min}(Q_2)\int_{t_k-d(t_k)}^{t_k-d_1}\|e(\alpha)\|^2\mathrm{d}\alpha\\
\leq{}&\frac{\theta_1}{\lambda_{\min}(P)\mathrm{e}^{2\alpha t_k}}\mathrm{e}^{2\alpha t_k}e^{\mathrm{T}}(t_k)Pe(t_k)\\
&+\frac{\theta_2}{\lambda_{\min}(Q_2)\mathrm{e}^{2\alpha t_k}}\mathrm{e}^{2\alpha t_k}\int_{t_k-d(t_k)}^{t_k-d_1}\mathrm{e}^{2\alpha s}e^{\mathrm{T}}(s)Q_2e(s)\mathrm{d}s\\
\leq{}&\frac{\max\left\{\frac{\theta_1}{\lambda_{\min}(P)},\frac{\theta_2\mathrm{e}^{2\alpha d_2}}{\lambda_{\min}(Q_2)}\right\}}{\mathrm{e}^{2\alpha t_k}}(V_1(t_k)+V_2(t_k))\\
\leq{}&\frac{\max\left\{\frac{\theta_1}{\lambda_{\min}(P)},\frac{\theta_2\mathrm{e}^{2\alpha d_2}}{\lambda_{\min}(Q_2)}\right\}}{\mathrm{e}^{2\alpha t_k}}V(t_k)\\
\leq{}&\frac{\max\left\{\frac{\theta_1}{\lambda_{\min}(P)},\frac{\theta_2\mathrm{e}^{2\alpha d_2}}{\lambda_{\min}(Q_2)}\right\}}{\mathrm{e}^{2\alpha t_k}}V(0)
\end{aligned}
$$

$$= \max\left\{\frac{\theta_1}{\lambda_{\min}(P)}, \frac{\theta_2 \mathrm{e}^{2\alpha d_2}}{\lambda_{\min}(Q_2)}\right\} \mathrm{e}^{-2\alpha t} \mathrm{e}^{2\alpha(t-t_k)} V(0)$$
$$\leq \mathrm{e}^{2\alpha h} \max\left\{\frac{\theta_1}{\lambda_{\min}(P)}, \frac{\theta_2 \mathrm{e}^{2\alpha d_2}}{\lambda_{\min}(Q_2)}\right\} \mathrm{e}^{-2\alpha t} V(0). \quad (48)$$

$V(0)$ can be calculated from expression (20), we obtain

$$V(0) \leq \theta_3 \left(\sup_{-d\leq\theta\leq 0}\{\|e(\theta)\|, \|\dot{e}(\theta)\|\}\right)^2, \quad (49)$$

where

$$\begin{aligned}\theta_3 =\ & \lambda_{\max}(P) + d_1\lambda_{\max}(Q_1) + d_{12}\lambda_{\max}(Q_2) + d_{12}\lambda_{\max}(Q_3) \\ & + d_1\lambda_{\max}(Q_4) + d_{12}\lambda_{\max}(Q_5) + d_1^3(\lambda_{\max}(R_1) \\ & + \lambda_{\max}(R_2^{\mathrm{T}}) + \lambda_{\max}(R_2) + \lambda_{\max}(R_3)) + 2d_{12}^3(\lambda_{\max}(S_1) \\ & + \lambda_{\max}(S_2^{\mathrm{T}}) + \lambda_{\max}(S_2) + \lambda_{\max}(S_3)) + d_1^5\lambda_{\max}(T_1) \\ & + (d_2^2 - d_1^2)^4\lambda_{\max}(T_2) + d_{12}^5\lambda_{\max}(T_3).\end{aligned}$$

Combining expressions (48) and (49), we have

$$\|e(t)\| \leq \mathrm{e}^{\alpha h}\sqrt{\max\left\{\frac{\theta_1}{\lambda_{\min}(P)}, \frac{\theta_2 \mathrm{e}^{2\alpha d_2}}{\lambda_{\min}(Q_2)}\right\}\theta_3}\, \mathrm{e}^{-\alpha t}\|e_0\|_c. \quad (50)$$

Then by definition of exponential stability, master system $\mathcal{M}$ and slave system $\mathcal{S}$ in system (1) are exponentially synchronized. This completes the proof of Theorem 1.

Remark 2 In Theorem 1, the convex combination technique is used for sampling delay, whereas the convex combination technique is applied for interval time-varying delay in the following theorem. In order to apply this technique, we slightly modify Lyapunov–Krasovskii functional (19), which is given in Eq. (55).

Theorem 2 $\alpha > 0, \epsilon$, and γ, the master system $\mathcal{M}$ and slave system $\mathcal{S}$ in system (1) are exponentially synchronous if there exist matrices $P > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, Q_4 > 0, Q_5 > 0, \begin{bmatrix} R_1 & R_2 \\ \star & R_3 \end{bmatrix} > 0, \begin{bmatrix} S_1 & S_2 \\ \star & S_3 \end{bmatrix} > 0, T_1 > 0, T_2 > 0, T_3 > 0, U > 0, \begin{bmatrix} Z_1 & Z_2 \\ \star & Z_3 \end{bmatrix} > 0, G, L, X, Y, V, Z, Y_1, Y_2$ and the diagonal matrices $V_1 > 0$ and $V_2 > 0$, such that the following formulas (51)–(53) hold

$$\begin{bmatrix} S_1 & S_2 & X & Y \\ \star & S_3 & V & Z \\ \star & \star & S_1 & S_2 \\ \star & \star & \star & S_3 \end{bmatrix} > 0, \quad \begin{bmatrix} Z_2 & S \\ * & Z_2 \end{bmatrix} > 0, \quad (51)$$

$$\bar{\Phi}_1 = \bar{\Phi} - \vartheta_1^{\mathrm{T}} U_1 \vartheta_1 - \vartheta_2^{\mathrm{T}} U_2 \vartheta_2 < 0, \quad (52)$$

$$\bar{\Phi}_2 = \bar{\Phi} - \vartheta_3^{\mathrm{T}} U_1 \vartheta_3 - \vartheta_4^{\mathrm{T}} U_2 \vartheta_4 < 0, \quad (53)$$

with

$$\begin{aligned}\bar{\Phi}_{11} =\ & 2\alpha P + Q_1 + d_1^2 R_1 + 2(d_2 - d_1)^2 S_1 \\ & - \mathrm{e}^{-2\alpha d_1} R_3 - d_1^2 \mathrm{e}^{-2\alpha d_1} T_1 - d_{12}^2 \mathrm{e}^{-2\alpha d_2} T_2 + Z_1 \\ & - \mathrm{e}^{-2\alpha h} Z_2 - h^2 \mathrm{e}^{-2\alpha h} Z_3 + \epsilon G A + \epsilon A^{\mathrm{T}} G^{\mathrm{T}}, \\ \bar{\Phi}_{12} =\ & \mathrm{e}^{-2\alpha d_1} R_3, \quad \bar{\Phi}_{13} = \epsilon G B, \\ \bar{\Phi}_{15} =\ & -\mathrm{e}^{-2\alpha d_1} R_2^{\mathrm{T}} + d_1 \mathrm{e}^{-2\alpha d_1} T_1, \quad \bar{\Phi}_{16} = d_{12} \mathrm{e}^{-2\alpha d_2} T_2, \\ \bar{\Phi}_{17} =\ & d_{12} \mathrm{e}^{-2\alpha d_2} T_2, \\ \bar{\Phi}_{18} =\ & P + d_1^2 R_2 + 2(d_2 - d_1)^2 S_2 + A^{\mathrm{T}} G^{\mathrm{T}} - \epsilon G, \\ \bar{\Phi}_{111} =\ & \mathrm{e}^{-2\alpha h} S, \quad \bar{\Phi}_{111} = \mathrm{e}^{-2\alpha h}(Z_2 - S) - \epsilon L C + \gamma A^{\mathrm{T}} G^{\mathrm{T}}, \\ \bar{\Phi}_{22} =\ & -\mathrm{e}^{-2\alpha d_1} Q_1 + \mathrm{e}^{-2\alpha d_1} Q_2 - \mathrm{e}^{-2\alpha d_2} R_3 - \mathrm{e}^{-2\alpha d_2} S_3 \\ & - d_{12}^2 \mathrm{e}^{-2\alpha d_1} U_1 - \mathrm{e}^{-2\alpha d_2} U_2 - d_{12}^2 \mathrm{e}^{-2\alpha d_2} T_3 \\ & - (1-\rho)\mathrm{e}^{-2\alpha d_2} U_2, \\ \bar{\Phi}_{23} =\ & \mathrm{e}^{-2\alpha d_2} S_3 - \mathrm{e}^{-2\alpha d_2} Z + \mathrm{e}^{-2\alpha d_2} U_2 + (1-\rho)\mathrm{e}^{-2\alpha d_2} U_2, \\ \bar{\Phi}_{24} =\ & \mathrm{e}^{-2\alpha d_2} Z, \quad \bar{\Phi}_{25} = \mathrm{e}^{-2\alpha d_1} R_2^{\mathrm{T}}, \\ \bar{\Phi}_{26} =\ & -\mathrm{e}^{-2\alpha d_2} S_2^{\mathrm{T}} + d_{12} \mathrm{e}^{-2\alpha d_2} T_3, \\ \bar{\Phi}_{33} =\ & -(1-d_\mu)\mathrm{e}^{-2\alpha d_2} Q_2 + (1-d_\mu)\mathrm{e}^{-2\alpha d_1} Q_3 \\ & - 2\mathrm{e}^{-2\alpha d_2} S_3 + \mathrm{e}^{-2\alpha d_2} Z + \mathrm{e}^{-2\alpha d_2} Z^{\mathrm{T}} - 2\mathrm{e}^{-2\alpha d_2} U_2 \\ & - \rho\,\mathrm{e}^{-2\alpha d_2} U_2 - (1-\rho)\mathrm{e}^{-2\alpha d_2} U_2, \\ \bar{\Phi}_{34} =\ & \mathrm{e}^{-2\alpha d_2} S_3 - \mathrm{e}^{-2\alpha d_2} Z + \mathrm{e}^{-2\alpha d_2} U_2 + \rho\,\mathrm{e}^{-2\alpha d_2} U_2, \\ \bar{\Phi}_{36} =\ & -\mathrm{e}^{-2\alpha d_2} Y^{\mathrm{T}} + \mathrm{e}^{-2\alpha d_2} S_2^{\mathrm{T}}, \\ \bar{\Phi}_{37} =\ & \mathrm{e}^{-2\alpha d_2} V - \mathrm{e}^{-2\alpha d_2} S_2^{\mathrm{T}}, \quad \bar{\Phi}_{38} = B^{\mathrm{T}} G^{\mathrm{T}}, \quad \bar{\Phi}_{312} = \gamma B^{\mathrm{T}} G^{\mathrm{T}}, \\ \bar{\Phi}_{44} =\ & -\mathrm{e}^{-2\alpha d_2} Q_3 - \mathrm{e}^{-2\alpha d_2} S_3 - \mathrm{e}^{-2\alpha d_2} U_2 - \rho\,\mathrm{e}^{-2\alpha d_2} U_2, \\ \bar{\Phi}_{46} =\ & \mathrm{e}^{-2\alpha d_2} Y^{\mathrm{T}}, \quad \bar{\Phi}_{47} = \mathrm{e}^{-2\alpha d_2} S_2^{\mathrm{T}}, \\ \bar{\Phi}_{55} =\ & -\mathrm{e}^{-2\alpha d_1} R_1 - \mathrm{e}^{-2\alpha d_1} T_1, \\ \bar{\Phi}_{66} =\ & -\mathrm{e}^{-2\alpha d_2} S_1 - \mathrm{e}^{-2\alpha d_2} T_2 - \mathrm{e}^{-2\alpha d_2} T_3 - \mathrm{e}^{-2\alpha d_2} U_1 \\ & - (1-\rho)\mathrm{e}^{-2\alpha d_2} U_1, \\ \bar{\Phi}_{67} =\ & -\mathrm{e}^{-2\alpha d_2} X - \mathrm{e}^{-2\alpha d_2} Y_1^{\mathrm{T}} - \mathrm{e}^{-2\alpha d_2} Y_2^{\mathrm{T}}, \\ \bar{\Phi}_{77} =\ & -\mathrm{e}^{-2\alpha d_2} S_1 - \mathrm{e}^{-2\alpha d_2} T_2 - \mathrm{e}^{-2\alpha d_2} T_3 - \mathrm{e}^{-2\alpha d_2} U_1 \\ & - \rho\,\mathrm{e}^{-2\alpha d_2} U_1, \\ \bar{\Phi}_{88} =\ & Q_4 + d_1^2 R_3 + 2(d_2 - d_1)^2 S_3 + \frac{d_1^4}{4} T_1 + \frac{(d_2^2 - d_1^2)^2}{4} T_2 \\ & + h^2 Z_2 + \frac{h^4}{4} Z_3 - G - G^{\mathrm{T}}, \\ \bar{\Phi}_{812} =\ & -LC - \gamma G^{\mathrm{T}}, \quad \bar{\Phi}_{814} = GW, \\ \bar{\Phi}_{99} =\ & -\mathrm{e}^{-2\alpha d_2} Q_4 + \mathrm{e}^{-2\alpha d_2} Q_5 + \frac{d_{12}^4}{4} \mathrm{e}^{-2\alpha d_1} T_3 \\ & + d_{12}^2 \mathrm{e}^{-2\alpha d_1} U_2, \\ \bar{\Phi}_{1010} =\ & -\mathrm{e}^{-2\alpha d_2} Q_5, \quad \bar{\Phi}_{1111} = -\mathrm{e}^{-2\alpha h} Z_1 - \mathrm{e}^{-2\alpha h} Z_2, \\ \bar{\Phi}_{1112} =\ & \mathrm{e}^{-2\alpha h}(-S^{\mathrm{T}} + Z_2^{\mathrm{T}}), \\ \bar{\Phi}_{1212} =\ & \mathrm{e}^{-2\alpha h}(-2Z_2 + S + S^{\mathrm{T}}) - \gamma L C - \gamma C^{\mathrm{T}} L^{\mathrm{T}}, \\ \bar{\Phi}_{1214} =\ & \gamma G W, \quad \bar{\Phi}_{1313} = -\mathrm{e}^{-2\alpha h} Z_3, \quad \bar{\Phi}_{1414} = -2V_1 - 2V_2, \\ \vartheta_1 =\ & [0\,0\,0\,0\,0\,0\,I\,0\,0\,0\,0\,0\,0\,0\,0\,], \\ \vartheta_2 =\ & [0\,0\,I\ -I\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,], \\ \vartheta_3 =\ & [0\,0\,0\,0\,0\,0\,I\,0\,0\,0\,0\,0\,0\,0\,0\,], \\ \vartheta_4 =\ & [0\,I\ -I\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,].\end{aligned}$$

Furthermore, the sampled-data controller gain matrix is given by

$$K = G^{-1} L. \quad (54)$$

Proof Let us consider the following Lyapunov–Krasovskii functional

$$V(t)=\sum_{i=1}^{6}V_i(t),\quad t\in[t_k,t_{k+1}), \tag{55}$$

where $V_1(t)$, $V_2(t)$, $V_3(t)$, and $V_4(t)$ follow the same definitions as those in expression (19), and

$$\begin{aligned}V_5(t) &= d_{12}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t-d_1}\mathrm{e}^{2\alpha s}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\mathrm{d}\theta\\ &\quad+d_{12}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t-d_1}\mathrm{e}^{2\alpha s}\dot{e}^{\mathrm{T}}(s)U_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta,\\ V_6(t) &= \int_{t-h}^{t}\mathrm{e}^{2\alpha s}e^{\mathrm{T}}(S)Z_1e(s)\mathrm{d}s\\ &\quad+h\int_{-h}^{0}\int_{t+\theta}^{t}\mathrm{e}^{2\alpha s}\dot{e}^{\mathrm{T}}(S)Z_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\ &\quad+\frac{h^2}{2}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\mathrm{e}^{2\alpha s}\dot{e}^{\mathrm{T}}(S)Z_2\dot{e}(s)\mathrm{d}s\mathrm{d}\lambda\,\mathrm{d}\theta.\end{aligned}$$

Calculating the derivatives of $V_5(t)$ and $V_6(t)$ along the trajectories of the system (4) yields

$$\begin{aligned}\dot{V}_5(t) &\le d_{12}^2\mathrm{e}^{2\alpha t}\mathrm{e}^{-2\alpha d_1}e^{\mathrm{T}}(t-d_1)U_1e(t-d_1)\\ &\quad-d_{12}\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\quad+d_{12}^2\mathrm{e}^{2\alpha s}\mathrm{e}^{-2\alpha d_1}\dot{e}^{\mathrm{T}}(t-d_1)U_2\dot{e}(t-d_1)\mathrm{d}s\\ &\quad-d_{12}\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d_1}\mathrm{e}^{-2\alpha d_2}\dot{e}^{\mathrm{T}}(s)U_2\dot{e}(s)\mathrm{d}s,\end{aligned} \tag{56}$$

$$\begin{aligned}\dot{V}_6(t) &\le \mathrm{e}^{2\alpha t}e^{\mathrm{T}}(t)Z_1e(t)-\mathrm{e}^{2\alpha t}\mathrm{e}^{-2\alpha h}e^{\mathrm{T}}(t-h)Z_1e(t-h)\\ &\quad+\mathrm{e}^{2\alpha t}\dot{e}^{\mathrm{T}}(t)\left[h^2Z_2+\frac{h^4}{4}Z_3\right]\dot{e}(t)\\ &\quad-h\mathrm{e}^{2\alpha t}\int_{t-h}^{t}\mathrm{e}^{-2\alpha h}\dot{e}^{\mathrm{T}}(S)Z_2\dot{e}(s)\mathrm{d}s\\ &\quad-\frac{h^2}{2}\mathrm{e}^{2\alpha t}\int_{-h}^{0}\int_{t+\theta}^{t}\mathrm{e}^{-2\alpha h}\dot{e}^{\mathrm{T}}(S)Z_3\dot{e}(s)\mathrm{d}s\mathrm{d}\theta.\end{aligned} \tag{57}$$

Note that,

$$\begin{aligned}&d_{12}\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &=d_{12}\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\quad+d_{12}\mathrm{e}^{2\alpha t}\int_{t-d(t)}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\le(d_2-d(t))\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\quad-(d(t)-d_1)\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\quad-(d(t)-d_1)\mathrm{e}^{2\alpha t}\int_{t-d(t)}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\quad-(d_2-d(t))\mathrm{e}^{2\alpha t}\int_{t-d(t)}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s.\end{aligned}$$

Let $\rho=(d(t)-d_1)/d_{12}$, we have

$$\begin{aligned}&-(d(t)-d_1)\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &=\rho d_{12}\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\le\rho(d_2-d(t))\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s,\\ &\quad-(d_2-d(t))\mathrm{e}^{2\alpha t}\int_{t-d(t)}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\le(1-\rho)(d(t)-d_1)\mathrm{e}^{2\alpha t}\int_{t-d(t)}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s.\end{aligned}$$

Therefore, Lemma 1 gives that

$$\begin{aligned}&-d_{12}\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d_1}\mathrm{e}^{-2\alpha d_2}e^{\mathrm{T}}(s)U_1e(s)\mathrm{d}s\\ &\le-\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}e^{\mathrm{T}}(s)\mathrm{d}s\mathrm{e}^{-2\alpha d_2}U_1\int_{t-d_2}^{t-d(t)}e^{\mathrm{T}}(s)\mathrm{d}s\\ &\quad-\rho\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d(t)}e^{\mathrm{T}}(s)\mathrm{d}s\mathrm{e}^{-2\alpha d_2}U_1\int_{t-d_2}^{t-d(t)}e^{\mathrm{T}}(s)\mathrm{d}s\\ &\quad-\mathrm{e}^{2\alpha t}\int_{t-d(t)}^{t-d_1}e^{\mathrm{T}}(s)\mathrm{d}s\mathrm{e}^{-2\alpha d_2}U_1\int_{t-d(t)}^{t-d_1}e^{\mathrm{T}}(s)\mathrm{d}s\\ &\quad-(1-\rho)\mathrm{e}^{2\alpha t}\int_{t-d(t)}^{t-d_1}e^{\mathrm{T}}(s)\mathrm{d}s\mathrm{e}^{-2\alpha d_2}U_1\int_{t-d(t)}^{t-d_1}e^{\mathrm{T}}(s)\mathrm{d}s,\end{aligned} \tag{58}$$

similarly,

$$\begin{aligned}&-d_{12}\mathrm{e}^{2\alpha t}\int_{t-d_2}^{t-d_1}\mathrm{e}^{-2\alpha d_2}\dot{e}^{\mathrm{T}}(s)U_2\dot{e}(s)\mathrm{d}s\\ &\le-\mathrm{e}^{2\alpha t}[e(t-d(t))-e(t-d_2)]^{\mathrm{T}}\\ &\quad\times\mathrm{e}^{-2\alpha d_2}U_2[e(t-d(t))-e(t-d_2)]\\ &\quad-\rho\mathrm{e}^{2\alpha t}[e(t-d(t))-e(t-d_2)]^{\mathrm{T}}\\ &\quad\times\mathrm{e}^{-2\alpha d_2}U_2[e(t-d(t))-e(t-d_2)]\\ &\quad-\mathrm{e}^{2\alpha t}[e(t-d_1)-e(t-d(t))]^{\mathrm{T}}\\ &\quad\times\mathrm{e}^{-2\alpha d_2}U_2[e(t-d_1)-e(t-d(t))]\\ &\quad-(1-\rho)\mathrm{e}^{2\alpha t}[e(t-d_1)-e(t-d(t))]^{\mathrm{T}}\\ &\quad\times\mathrm{e}^{-2\alpha d_2}U_2[e(t-d_1)-e(t-d(t))].\end{aligned} \tag{59}$$

Here, we take

$$\delta_1(t)=\int_{t-h(t)}^{t}\dot{e}(s)\mathrm{d}s,\quad\delta_2(t)=\int_{t-h}^{t-h(t)}\dot{e}(s)\mathrm{d}s,$$

where $0<h(t)<h$, according to Lemma 2, we have

$$\begin{aligned}&\mathrm{e}^{\alpha t}h\int_{t-h}^{t}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\mathrm{d}s\\ &=\mathrm{e}^{\alpha t}h\int_{t-h(t)}^{t}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\mathrm{d}s+\mathrm{e}^{\alpha t}h\int_{t-h}^{t-h(t)}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\mathrm{d}s\\ &\ge\mathrm{e}^{\alpha t}\frac{h}{h(t)}\delta_1^{\mathrm{T}}(t)Z_2\delta_1(t)+\mathrm{e}^{\alpha t}\frac{h}{h-h(t)}\delta_2^{\mathrm{T}}(t)Z_2\delta_2(t)\\ &=\mathrm{e}^{\alpha t}\left[\delta_1^{\mathrm{T}}(t)Z_2\delta_1(t)+\frac{h-h(t)}{h(t)}\delta_2^{\mathrm{T}}(t)Z_2\delta_2(t)+\delta_2^{\mathrm{T}}(t)Z_2\delta_2(t)\right.\\ &\quad\left.+\frac{h(t)}{h-h(t)}\delta_2^{\mathrm{T}}(t)Z_2\delta_2(t)\right].\end{aligned} \tag{60}$$

It is clear from expression (51) that

$$\mathrm{e}^{\alpha t}\begin{bmatrix}\sqrt{\frac{h-h(t)}{h(t)}}\delta_1(t)\\ \sqrt{\frac{h(t)}{h-h(t)}}\delta_2(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_2 & S\\ * & Z_2\end{bmatrix}\begin{bmatrix}\sqrt{\frac{h-h(t)}{h(t)}}\delta_1(t)\\ \sqrt{\frac{h(t)}{h-h(t)}}\delta_2(t)\end{bmatrix}\geq 0, \quad (61)$$

which implies that

$$\begin{aligned}&\mathrm{e}^{\alpha t}\frac{h-h(t)}{h(t)}\delta_1^{\mathrm{T}}(t)Z_2\delta_1(t)+\mathrm{e}^{\alpha t}\frac{h(t)}{h-h(t)}\delta_2^{\mathrm{T}}(t)Z_2\delta_2(t)\\ &\geq \mathrm{e}^{\alpha t}\delta_1^{\mathrm{T}}(t)S\delta_2(t)+\mathrm{e}^{\alpha t}\delta_2^{\mathrm{T}}(t)S\delta_1(t). \quad (62)\end{aligned}$$

From expressions (60) and (62), we get

$$\begin{aligned}&\mathrm{e}^{\alpha t}h\int_{t-h}^{t}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\,\mathrm{d}s\\ &\geq \mathrm{e}^{\alpha t}[\delta_1^{\mathrm{T}}(t)Z_2\delta_1(t)+\delta_2^{\mathrm{T}}(t)Z_2\delta_2(t)+\delta_1^{\mathrm{T}}(t)S\delta_2(t)\\ &\quad+\delta_2^{\mathrm{T}}(t)S\delta_1(t)]\\ &=\mathrm{e}^{\alpha t}\begin{bmatrix}\delta_1(t)\\ \delta_2(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_2 & S\\ * & Z_2\end{bmatrix}\begin{bmatrix}\delta_1(t)\\ \delta_2(t)\end{bmatrix}. \quad (63)\end{aligned}$$

It should be noted that when $h(t)=0$ or $h(t)=h$, we have $\delta_1(t)=0$ and $\delta_2(t)=0$, respectively, and thus inequality (62) still holds. It is clear that inequality (62) implies

$$-\mathrm{e}^{\alpha t}h\int_{t-h}^{t}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\,\mathrm{d}s\leq \mathrm{e}^{\alpha t}\varpi^{\mathrm{T}}(t)\Omega^{*}\varpi(t), \quad (64)$$

where

$$\begin{aligned}&\varpi(t)=[e^{\mathrm{T}}(t)\ \ e^{\mathrm{T}}(t-h(t))\ \ e^{\mathrm{T}}(t-h)]^{\mathrm{T}},\\ &\Omega^{*}=\begin{bmatrix}-Z_2 & Z_2-S & S\\ * & -2Z_2+S+S^{\mathrm{T}} & -S+Z_2\\ * & * & -Z_2\end{bmatrix}.\end{aligned}$$

On the other hand based on expression (8), the following inequality holds,

$$\begin{aligned}&2\mathrm{e}^{2\alpha t}\left[e^{\mathrm{T}}(t)D^{\mathrm{T}}SV_1\eta(De(t),z(t))\right.\\ &\left.-\eta(De(t),z(t))^{\mathrm{T}}V_1\eta(De(t),z(t))\right]\\ &+2\mathrm{e}^{2\alpha t}\left[e^{\mathrm{T}}(t)D^{\mathrm{T}}SV_2\eta(De(t),z(t))\right.\\ &\left.-\eta(De(t),z(t))^{\mathrm{T}}V_2\eta(De(t),z(t))\right]\geq 0. \quad (65)\end{aligned}$$

Applying formulas (21), (22), (28)–(31), (58), (59), (64) into $\dot{V}(t)$ and adding the right-hand side of expression (36) and the left-hand side of inequality (65) and letting $L=GK$, we arrive at

$$\dot{V}(t)\leq\bar{\xi}^{\mathrm{T}}(t)\Phi^{\star}\bar{\xi}(t), \quad (66)$$

where

$$\bar{\xi}_1(t)=\left[\xi_1^{\mathrm{T}}(t)\ \ \xi_3^{\mathrm{T}}(t)\right]^{\mathrm{T}}, \quad (67)$$

$$\xi_3(t)=\left[e^{\mathrm{T}}(t-h)\ e^{\mathrm{T}}(t-h(t))\int_{t-h}^{t}e^{\mathrm{T}}(s)\,\mathrm{d}s\ \eta(De(t),z(t))\right], \quad (68)$$

$$\Phi^{\star}=\rho\bar{\Phi}_1+(1-\rho)\bar{\Phi}_2. \quad (69)$$

Since $0\leq\rho\leq 1$, $\rho\bar{\Phi}_1+(1-\rho)\bar{\Phi}_2$ is a convex combination of $\bar{\Phi}_1<0$ and $\bar{\Phi}_2<0$. Therefore, $\Phi^{*}<0$ is equivalent to $\bar{\Phi}_1<0$ and $\bar{\Phi}_2<0$. Applying the Schur complement, which is equivalent to expressions (52) and (53).

By using Lemma 3 and Eq. (55), we have the following inequality

$$\|e(t)\|\leq\mathrm{e}^{\alpha h}\sqrt{\max\left\{\frac{\theta_1}{\lambda_{\min}(P)},\frac{\theta_2\mathrm{e}^{2\alpha d_2}}{\lambda_{\min}(Q_2)}\right\}\theta_3}\,\mathrm{e}^{-\alpha t}\|e_0\|_c, \quad (70)$$

where

$$\begin{aligned}\theta_3=&\lambda_{\max}(P)+d_1\lambda_{\max}(Q_1)+d_{12}\lambda_{\max}(Q_2)+d_{12}\lambda_{\max}(Q_3)\\ &+d_1\lambda_{\max}(Q_4)+d_{12}\lambda_{\max}(Q_5)+d_1^3(\lambda_{\max}(R_1)+\lambda_{\max}(R_2^{\mathrm{T}})\\ &+\lambda_{\max}(R_2)+\lambda_{\max}(R_3))+2d_{12}^3(\lambda_{\max}(S_1)+\lambda_{\max}(S_2^{\mathrm{T}})\\ &+\lambda_{\max}(S_2)+\lambda_{\max}(S_3))+d_1^5\lambda_{\max}(T_1)\\ &+(d_2^2-d_1^2)^4\lambda_{\max}(T_2)+d_{12}^5\lambda_{\max}(T_3)\\ &+d_{12}^3\lambda_{\max}(U_1)+d_{12}^3\lambda_{\max}(U_2).\end{aligned}$$

Therefore, based on definition of exponential stability, the master system $\mathcal{M}$ and slave system $\mathcal{S}$ in system (1) are exponentially synchronizes. This completes the proof of Theorem 2.

Remark 3 In the proof of Theorems 1 and 2, a less conservative results are presented by introducing new Lyapunov–Krasovskii functionals (19) and (55). In the most existing literature authors considered the integral terms

$$\int_{t}^{t-d_1}\mathrm{e}^{2\alpha s}e^{\mathrm{T}}(s)Q_2e(s)\,\mathrm{d}s$$

and

$$\int_{t-d_2}^{t-d_1}\int_{\theta}^{t}\int_{\lambda}^{t}\mathrm{e}^{2\alpha s}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta.$$

In contrast, in this paper new integral terms

$$\int_{t-d(t)}^{t-d_1}\mathrm{e}^{2\alpha s}e^{\mathrm{T}}(s)Q_2e(s)\,\mathrm{d}s$$

and

$$\int_{t-d_2}^{t-d_1}\int_{\theta}^{t-d_1}\int_{\lambda}^{t-d_1}\mathrm{e}^{2\alpha s}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta$$

are introduced, which has fully included information about the lower bound of the time varying delay. On the other hand, the terms in $V_5(t)$, $V_6(t)$, and $V_7(t)$ of Lyapunov–Krasovskii functional (19) are (t_k,t_{k+1})-dependent terms. It is clear that these terms make available information about the sampling period. To the best of the author's knowledge, Lyapunov–Krasovskii functional containing triple integral terms is a new way to drive a synchronization criterion for Lur'e system.

Remark 4 In order to reduce the conservativeness of Theorems 1 and 2, convex combination technique is used. By using Lemma 2 and adding zero equation, which may also lead to less conservative results. The main difference between Theorems 1 and 2 is how to apply the convex combination technique for time delays; that is, sampling delay or interval time

varying delay. In Theorem 1, the convex combination technique is used for a sampling period and it is less conservative than that in Ref. [41]. In order to apply the convex combination technique for time-varying delay in Theorem 2, the new double integral terms

$$d_{12}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t-d_1} \mathrm{e}^{2\alpha s} r^{\mathrm{T}}(s)U_1 e(s)\,\mathrm{d}s\,\mathrm{d}\theta$$

and

$$d_{12}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t-d_1} \mathrm{e}^{2\alpha s} \dot{e}^{\mathrm{T}}(s)U_2 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta$$

are introduced in Eq. (55).

4. Numerical examples

In this section, we shall present numerical examples to demonstrate the effectiveness of the proposed method.

Example 1: Chau's circuit

Consider the following Chau's circuit with time-varying delay

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - m_1 x_1(t) - g(x_1(t))) - cx_1(t-d(t)), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t) - cx_1(t-d(t)), \\ \dot{x}_3(t) = -bx_2(t) + c(2x_1(t-d(t)) - x_3(t-d(t))) \end{cases} \tag{71}$$

with the nonlinear characteristics

$$g(x_1(t)) = \frac{1}{2}(m_1 - m_0)(|x_1(t)+1| - |x_1(t)-1|)$$

and the parameters $m_0 = -1/7$, $m_1 = 2/7$, $a = 9$, $b = 14.28$, $c = 0.1$, and the time varying delay $d(t) = 1.0|\sin t|$. The above system can be expressed in the Lur'e form (1) with the following matrices

$$A = \begin{bmatrix} -am_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -c & 0 & 0 \\ -c & 0 & 0 \\ 2c & 0 & -c \end{bmatrix},$$

$$W = \begin{bmatrix} -a(m_1 - m_0) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and $\varphi(x_1(t)) = (|x_1(t)+1| - |x_1(t)-1|)/2$ belonging to the sector $[0,1]$ and $\varphi_2(x_2(t)) = \varphi_3(x_3(t)) = 0$. In this example we choose $C = [\,1\ 0\ 0\,]$, $\epsilon = 2$, and $\gamma = 0$. Using Matlab LMI toolbox and Theorems 1 and 2, we obtain maximum allowable upper bounds of h for different α and list the results in Table 1. It can be seen from Table 1 that the maximum allowable upper bounds h decrease as α increases. Table 2 shows that maximum allowable upper bounds of α for different h, from which we can find that α decreases as h increases. From Tables 1 and 2, it is clear that Theorems 1 and 2 in our paper are less conservative than Theorem 1 in Ref. [41].

Table 1. Maximum allowable upper bounds h for different α.

	α	0.1	0.2	0.3	0.4	0.5
Theorem[41]	h	0.3247	0.2941	0.2658	0.2396	0.2154
Theorem 1	h	0.3946	0.3656	0.3423	0.3221	0.3045
Theorem 2	h	0.4096	0.3907	0.3736	0.3578	0.3432

Table 2. Maximum allowable upper bounds α for different h.

	h	0.10	0.15	0.20	0.25	0.30
Theorem[41]	α	1.1650	0.8240	0.5683	0.3596	0.1802
Theorem 1	α	9.7096	4.2312	1.9506	1.0508	0.5571
Theorem 2	α	14.0693	6.6764	3.5689	1.9625	1.0959

Choosing $\alpha = 0.1$ and $h = 0.324$, and using Matlab LMI toolbox and Theorem 1, we can get the following gain matrix $K = \left[\,4.0304\ \ 0.7769\ \ -0.9536\,\right]^{\mathrm{T}}$. During the simulations, the initial conditions of master and slave systems are taken as $x(0) = (0.2, 0.2, 0.2)$, $z(0) = (0.2, -0.2, 0.33)$. The chaotic attractor of master and slave systems of system (71) is shown Fig. 1. Figure 2 displays the synchronization errors of Chau's circuit system (71).

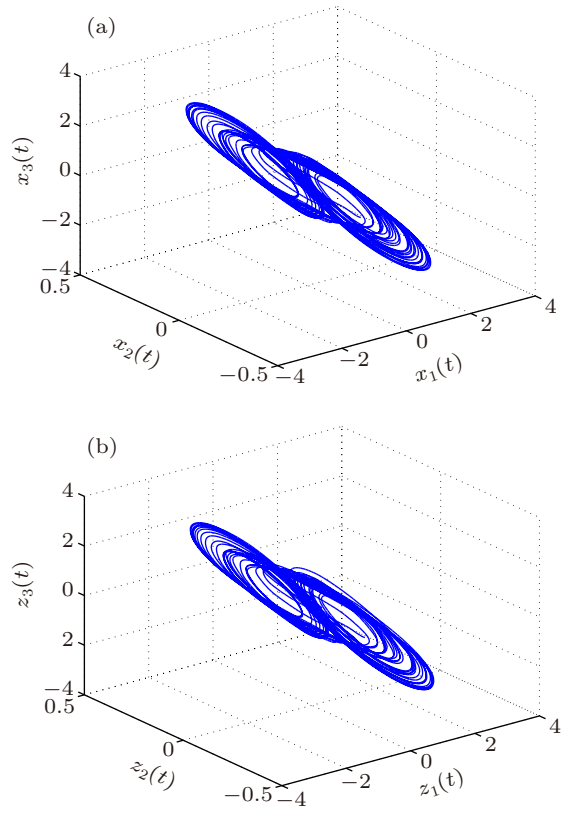


Fig. 1. (color online) Chaotic attractor of master and slave systems of system (71).

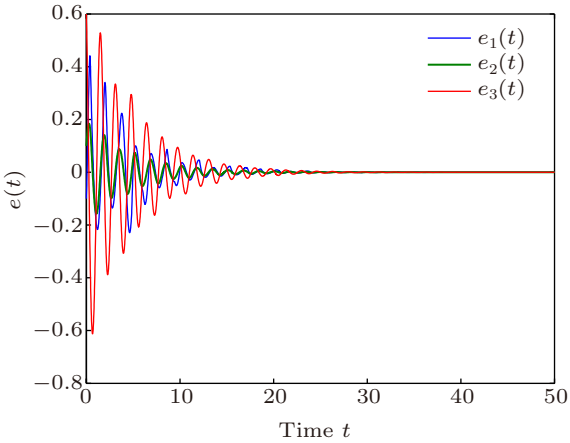


Fig. 2. (color online) Synchronization errors of Chau's circuit system (72).

Remark 5 Comparison results are presented in Table 1. In Ref. [41] the constant delay $d = 1$ was taken, but our results are presented for an upper bound on the time-varying delay as 1 (i.e., $d_2 = 1$). From Tables 1 and 2, it is clear that Theorem 2 is less conservative than Theorems 1 and Ref. [41].

Example 2: Hyperchaotic system

Consider the following hyperchaotic system with time-varying delay

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - m_1 x_1(t) - g(x_1(t))) - cx_1(t - d(t)), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t) - cx_1(t - d(t)), \\ \dot{x}_3(t) = -bx_2(t) + c(2x_1(t - d(t)) - x_3(t - d(t))), \\ \dot{x}_4(t) = a(x_5(t) - m_1 x_4(t) - g(x_4(t))) - cx_4(t - d(t)) \\ \qquad + \rho(x_4(t) - x_1(t)), \\ \dot{x}_5(t) = x_4(t) - x_5(t) + x_6(t) - cx_4(t - d(t)), \\ \dot{x}_6(t) = -bx_5(t) + c(2x_4(t - d(t)) - x_6(t - d(t))) \end{cases} \quad (72)$$

with the nonlinear characteristics

$$g(x_i(t)) = \frac{1}{2}(m_1 - m_0)(|x_i(t) + 1| - |x_i(t) - 1|), \quad i = 1, 4,$$

and the parameters $m_0 = -1/7$, $m_1 = 2/7$, $a = 9$, $b = 14.28$, $c = 0.1$, $\rho = 0.01$, and the time varying delay $d(t) = 1.0|\sin t|$. The above system can be expressed in Lur'e form (1) with

$$A = \begin{bmatrix} -am_1 & a & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -am_1 + \rho & a & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -b & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -c & 0 & 0 & 0 & 0 & 0 \\ -c & 0 & 0 & 0 & 0 & 0 \\ 2c & 0 & -c & 0 & 0 & 0 \\ 0 & 0 & 0 & -c & 0 & 0 \\ 0 & 0 & 0 & -c & 0 & 0 \\ 0 & 0 & 0 & 2c & 0 & -c \end{bmatrix},$$

$$W = \begin{bmatrix} -a(m_1 - m_0) & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -a(m_1 - m_0) \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

and $\varphi(x_i(t)) = (|x_i(t) + 1| - |x_1(t) - i|)/2$, $i = 1, 4$ belonging to the sector $[0, 1]$, $\varphi(x_i(t)) = 0$, $i = 2, 3, 5, 6$. In this example we choose $C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$, $\epsilon = 2$, and $\gamma = 0$. The initial conditions are taken $x(0) = (0.3, 0.1, 0.4, 0.5, 0.1, -0.66)$ and $z(0) = (0.1, -0.1, 0.1, 0.0, 0.1, 0.33)$ for the master and slave systems of system (72), respectively, and $\epsilon = 2$ and $\gamma = 0$. Choosing $\alpha = 0.1$ and $h = 0.324$, and using Matlab LMI toolbox and Theorem 1, we can get the following gain matrix

$$K = \begin{bmatrix} 4.1696 & 1.1252 & -0.0877 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.8492 & 1.4800 & -0.2832 \end{bmatrix}^{\mathrm{T}}.$$

Figures 3 and 4 shows the chaotic attractors of master and slave systems of system (72) and the synchronization errors of system (72), respectively.

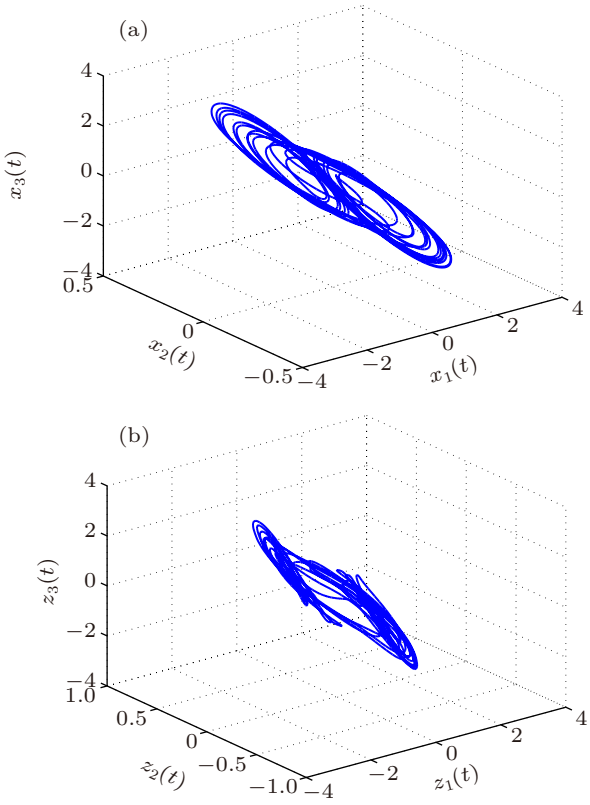


Fig. 3. (color online) Chaotic attractor of master and slave systems of system (72).

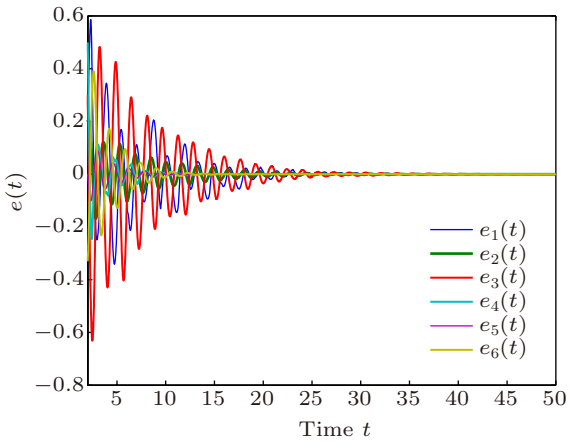


Fig. 4. (color online) Synchronization errors of system (72).

Example 3: Five-scroll attractors

Consider the following n-scroll circuit system with time-varying delay

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - m_{2q-1}x_1(t) - g(x_1(t))) - \delta x_1(t - d(t)), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t) - \delta x_1(t - d(t)), \\ \dot{x}_3(t) = -bx_2(t) + \delta(2x_1(t - d(t)) - x_3(t - d(t))), \end{cases} \quad (73)$$

with the nonlinear characteristics having additional break points

$$g(x_1(t)) = \frac{1}{2}\sum_{i=1}^{2q-1}(m_{i-1}-m_i)(|x_1(t)+c_i|-|x_1(t)-c_i|),$$

where q denotes a natural number. Here, we will consider the 5-scroll attractor, the following parameters are chosen $m = [0.9/7, -3/7, 3.5/7, -2.7/7, 4/7, -2.4/7]$, $c = [1, 2.15, 3.6, 6.2, 9]$, $\alpha = 9$, $\beta = 14.28$, $\delta = 0.1$, and the time varying delay $d(t) = 1.0|\sin t|$. The above system can be represented in Lur'e form by

$$A = \begin{bmatrix} -am_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -c & 0 & 0 \\ -c & 0 & 0 \\ 2c & 0 & -c \end{bmatrix},$$

$$W = \begin{bmatrix} -a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$\varphi(x_1(t)) = \frac{1}{2}\sum_{i=1}^{2q-1}(m_{i-1}-m_i)(|x_1(t)+c_i|-|x_1(t)-c_i|)$$

belonging to the sector $[0, 2.5]$ and $\varphi_2(x_2(t)) = \varphi_3(x_3(t)) = 0$. In this example we choose $C = [1\ 0\ 0]$, $\epsilon = 2$, and $\gamma = 0$. By choosing $\alpha = 0.1$ and $h = 0.324$, and using Matlab LMI toolbox and Theorem 1, we can get the following gain matrix $K = \begin{bmatrix} 6.3799 & 1.9354 & 0.1480 \end{bmatrix}^{\mathrm{T}}$. During the simulations, the initial conditions of master and slave systems are taken as $x(0) = (0.2, 0.2, 0.2)$, $z(0) = (0.2, -0.2, 0.33)$. Figures 5 and 6 displays the chaotic attractors of master and slave systems of system (73) and the synchronization errors of system (73), respectively.

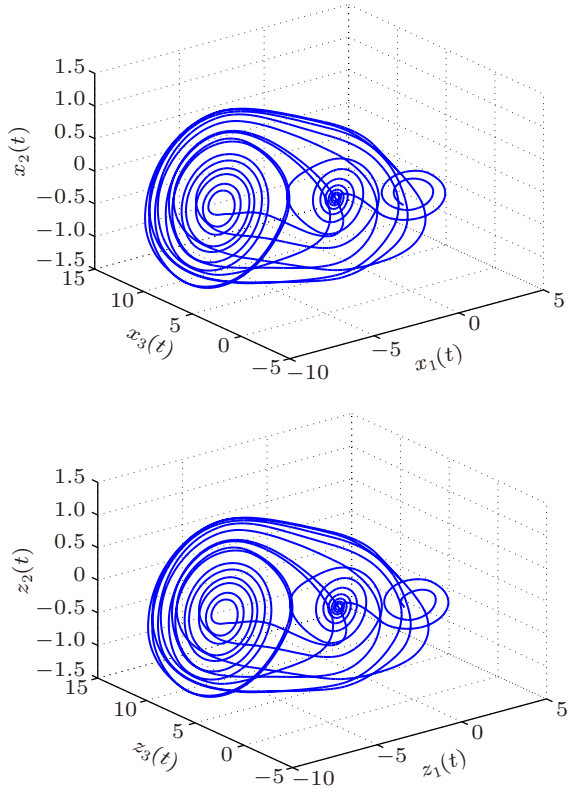


Fig. 5. (color online) Chaotic attractor of master and slave systems of system (73).

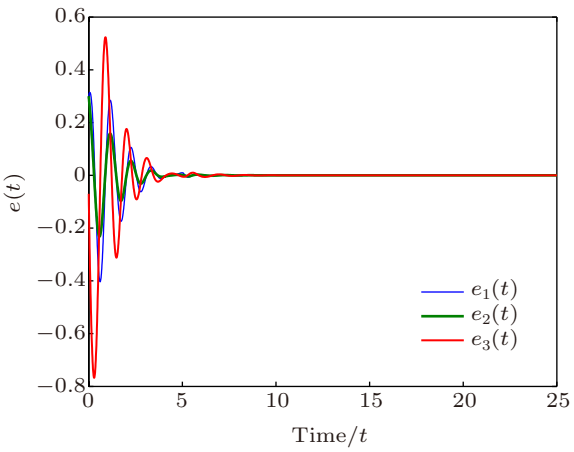


Fig. 6. (color online) Synchronization errors of system (73).

5. Conclusion

In this paper, the exponential synchronization of chaotic Lur'e systems with time-varying delays using sampled-data control has been discussed. Note that nonlinearties satisfies the sector bound constraints. By constructing a new Lyapunov–Krasovskii functional with triple integral and using convex combination technique, a new synchronization criterion is given in terms of LMIs. Finally, our three numerical examples have shown the effectiveness and improvements of this method over the previous results.

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