Tracking problem under a time-varying topology*

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This paper studies the multi-agent tracking problem of a third-order maneuvering target under uncertain communication environments. Each tracking agent is assumed to be a third-order system and can only use its own and neighbors' position, velocity, and acceleration information to design its control input. In this work, the uncertain communication environments are modelled by a finite number of constant Laplacian matrices together with their corresponding scheduling functions. Sufficient conditions for the existence of a tracking strategy have been expressed in terms of the solvability of linear matrix inequalities. Finally, a numerical example is employed to demonstrate the effectiveness of the proposed tracking strategy.

Keywords: third-order multi-agent systems, tracking strategy, time-varying topology

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1. Introduction

Multi-agent systems and their collective behavior have a variety of applications such as flocking, cooperative control of unmanned aerial vehicles, and formation control of mobile robots. Therefore, over the past decade multi-agent systems have attracted the increasing attention of researchers from various disciplines, including engineering, biology, and science. The consensus problem for multi-agent systems with first-order kinematics and second-order kinematics has been intensively investigated.[1–12] In Ref. [1], the consensus problem of first-order multi-agent systems with multiple time delays has been studied and in Refs. [2]–[7], the second-order consensus or formation algorithm for multi-agent systems under fixed topologies has been studied.

Although there are many results on the tracking problem of multi-agent systems with the first-order or second-order kinematics, there are only a few results on high-order multi-agent systems. Considering the behavior of a group of agents with different positions and speeds, the accelerations may have an impact on their final state. As mentioned in Ref. [13], when one bird in a flock of birds perceives danger, the bird flock may change direction abruptly. It is obvious that the other birds in this flock need to track the leading bird, not only in terms of their position and velocity but also in terms of their acceleration. Therefore, it is essential to investigate high-order tracking problems of multi-agent systems. Consequently, many researchers have recently become interested in high-order differential equations.[14–19] In particular, in Ref. [14], the authors investigated the third-order leader-following consensus problem of nonlinear multi-agent systems in undirected network topologies, and subsequently the authors in Refs. [17] and [19] considered consensus problems of high-order multi-agent systems with switching topologies.

Most of the works in this area have only considered multi-agent systems under fixed topologies or switching topologies. Such as, in Refs. [8]–[12], [17], and [19], consensus problems for multi-agent systems under switching topologies were discussed. In these papers, the communication network topology is modelled by switching between several fixed topologies. While, in practice, the velocity of each manoeuvring agent is time-varying and the communication radius of each agent is finite. Consequently, the communication topology between the target agent and the tracking agents may change from time to time rather than switching among several fixed topologies, thus the tracking problem of a manoeuvring target under time-varying topologies is of vital necessity.

Only a few researchers have paid attention to the consensus problem of multi-agent systems under time-varying topologies. In Ref. [20], the authors addressed the consensus problem of multi-agent systems, consisting of a set of identical multiple-input multiple-output (MIMO), linear time-invariant (LTI) systems, under a time-varying network that has a well-defined average. The author has investigated consensus problem of the first-order, the second-order, and the third-order leader-following multi-agent systems under a time-varying topology in Refs. [21], [22], and [23], respectively. Sufficient and necessary conditions of consensus by employing some knowledge of complex dynamical network are given in Ref. [21]. Based on the eigenvalues of the Laplacian matrix of the time-varying topology, some sufficient conditions

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and controller design principles that ensure the consensus of leader-following multi-agent system are explored in Refs. [22] and [23]. The authors in these papers have discussed both of the nonlinear and integral cases.

In this paper we will discuss the third-order multi-agent consensus problem under a time-varying topology. At the same time, it is worth mentioning that in this work we employ the same model as that in Ref. [23]. Sufficient conditions in Ref. [23] are based on the assumptions that Laplacian matrices are non-negative and their time derivatives are non-positive. However, these assumptions are not easy to be satisfied, which limits the range of application of the proposed theorem in Ref. [23]. Due to this fact, we develop another algorithm to solve the third-order nonlinear tracking problem under a time-varying topology. In this work, the time-varying topology is modelled by a finite Laplacian matrices and their corresponding scheduling functions.

Throughout this paper, the following notations will be used: $\mathbb{R}^{p\times q}$ is the set of $p\times q$ real matrix. $\boldsymbol{I}_n \in \mathbb{R}^{n\times n}$ denotes an $n\times n$ identity matrix. The Kronecker product is denoted by $\otimes$. The Euclidean norm of X is denoted by $\|X\|$.

2. Preliminaries

A weighted graph $G := \{\mathcal{N}, \mathcal{E}, \mathcal{A}\}$ is denoted by a node set $\mathcal{N} = \{1, 2, \ldots, n\}$, an edge set $\mathcal{E} \subseteq \mathcal{N}\times\mathcal{N}$ and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with non-negative elements. The set of neighbors of node i are denoted by $\mathcal{N}_i = \{j | j \in \mathcal{N}, (j,i) \in \mathcal{E}\}$. When $j \notin \mathcal{N}_i$, which means that node j cannot access node i, $a_{ij} = 0$, otherwise $a_{ij} > 0$.

The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ of graph G is defined as $\mathcal{L} := \mathcal{D} - \mathcal{A}$, where

$$\mathcal{D} := \mathrm{diag}\left\{\sum_{j\in\mathcal{N}_1} a_{1j}, \sum_{j\in\mathcal{N}_2} a_{2j}, \ldots, \sum_{j\in\mathcal{N}_n} a_{nj}\right\}.$$

In the view of graph theory, every agent can be treated as a node. Then, the topology of tracking agents and the target can be treated as a dynamic graph. To analyze the tracking problem, the following lemmas are essential.

Lemma 1 A nonlinear system is globally uniformly stable if there is a function $\boldsymbol{V}$ such that $\boldsymbol{V}$ is positive-definite and $\dot{\boldsymbol{V}} \leq 0$.

Lemma 2 For any vectors x, y of appropriate dimensions and any symmetric positive-definite matrix Z of appropriate dimension, the following inequality holds:

$$\pm 2x^{\mathrm{T}}y \leq x^{\mathrm{T}}Zx + y^{\mathrm{T}}Z^{-1}y.$$

Lemma 3[24] For a given symmetric matrix $S = \begin{pmatrix} S_{11} & S_{12} \\ * & S_{22} \end{pmatrix}$, the following inequalities are equivalent

(i) $S < 0$; (ii) $S_{11} < 0, S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$; (iii) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0$.

3. Problem description

Considering a team of n identical tracking agents which are distributed at n distinct positions in the three-dimensional space and denoted by $\boldsymbol{P} := \{x_{\boldsymbol{p}i} \in \mathbb{R}^3, i \in \mathcal{N}\}$, where $x_{\boldsymbol{p}i} = \{x_{\boldsymbol{p}i}, y_{\boldsymbol{p}i}, z_{\boldsymbol{p}i}\}$. The kinematic equation of the i-th tracking agent is described by

$$\begin{aligned} \dot{x}_{\boldsymbol{p}i}^{(0)}(t) &= x_{\boldsymbol{p}i}^{(1)}(t), \\ \dot{x}_{\boldsymbol{p}i}^{(1)}(t) &= x_{\boldsymbol{p}i}^{(2)}(t), \\ \dot{x}_{\boldsymbol{p}i}^{(2)}(t) &= f(t, x_{\boldsymbol{p}i}^{(0)}(t), x_{\boldsymbol{p}i}^{(1)}(t), x_{\boldsymbol{p}i}^{(2)}(t)) + u_{\boldsymbol{p}i}(t), \end{aligned} \tag{1}$$

where $x_{\boldsymbol{p}i}^{(d)} \in \mathbb{R}^3$, $d = \{0,1,2\}$ are position, velocity, and acceleration states of the i-th tracking agent, respectively. $f(t, x_{\boldsymbol{p}i}^{(0)}(t), x_{\boldsymbol{p}i}^{(1)}(t), x_{\boldsymbol{p}i}^{(2)}(t))$ is a nonlinear function which denotes the dynamics of the tracking agents. $u_{\boldsymbol{p}i} \in \mathbb{R}^3$ is the control input of the i-th tracking agent.

In this paper, a_{ij}, the weighted value of agent j to agent i, is time-varying, and it is denoted as $a_{ij}(t)$. The communication topology for the tracking agents is assumed to be modeled by a time-varying graph $G(t) := \{\mathcal{N}, \mathcal{E}(t), \mathcal{A}(t)\}$ with node set $\mathcal{N}$, time-varying edge $\mathcal{E}(t) \subseteq \mathcal{N}\times\mathcal{N}$ and continuous adjacency matrix $\mathcal{A}(t) = [a_{ij}(t)] \in \mathbb{R}^{n\times n}$. The topology is symmetric, that is, the corresponding Laplacian matrix $\mathcal{L}(t)$ satisfying $\mathcal{L}(t) = \mathcal{L}^{\mathrm{T}}(t)$.

The goal of each tracking agent, which is regarded as a point of the dynamic graph, is to track a manoeuvring target. The manoeuvring target is assumed to possess the following third-order kinematics

$$\begin{aligned} \dot{x}_{\boldsymbol{t}}^{(0)}(t) &= x_{\boldsymbol{t}}^{(1)}(t), \\ \dot{x}_{\boldsymbol{t}}^{(1)}(t) &= x_{\boldsymbol{t}}^{(2)}(t), \\ \dot{x}_{\boldsymbol{t}}^{(2)}(t) &= f(t, x_{\boldsymbol{t}}^{(0)}(t), x_{\boldsymbol{t}}^{(1)}(t), x_{\boldsymbol{t}}^{(2)}(t)), \end{aligned} \tag{2}$$

where $x_{\boldsymbol{t}}^{(d)} \in \mathbb{R}^3$, $d = \{0,1,2\}$ are position, velocity, and acceleration states of the target agent, respectively. $f(t, x_{\boldsymbol{t}}^{(0)}(t), x_{\boldsymbol{t}}^{(1)}(t), x_{\boldsymbol{t}}^{(2)}(t))$ denotes the change of the force which is imposed on the target.

$b_i(t) > 0$ indicates a case where the i-th tracking agent can receive the position, velocity, and acceleration states of the target at time t, and $b_i(t) = 0$ indicates the case that information of the manoeuvring target is not accessible to the i-th tracking agent. Denote $\mathcal{B}(t) := \mathrm{diag}\{b_1(t), b_2(t), \ldots, b_n(t)\}$.

The objective of this paper is to design a tracking strategy that ensures the tracking agents effectively track the target agent in three-dimensional space under a time-varying topology. In this work, we assume $f(\cdot)$ satisfies the following assumption:

Assumption 1 There exists a finite constant $l \geq 0$ such that

$$|f(t, \hat{x}_{\boldsymbol{p}i}(t)) - f(t, \hat{x}_{\boldsymbol{t}}(t))| \leq l|\hat{x}_{\boldsymbol{p}i}(t) - \hat{x}_{\boldsymbol{t}}(t)|.$$

Definition 1 The third-order tracking agents (1) are said to track the target (2) successfully if

$$\lim_{t\to\infty}\sum_{d=0}^{2}\|x_{\boldsymbol{pi}}^{(d)}(t)-x_{\boldsymbol{t}}^{(d)}(t)\|=0, i\in\mathcal{N}. \tag{3}$$

4. Main section

This section presents a tracking strategy by which tracking agents can effectively track a third-order manoeuvring target under a time-varying communication topology.

The tracking control protocol consists of the disagreement vectors between the i-th tracking agent and its neighbor agents, and the disagreement vectors between the i-th tracking agent and the manoeuvring target are adopted in the following format for the i-th tracking agent:

$$u_i(t)=-\gamma\sum_{d=0}^{2}\left\{\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)e_{ij}^{(d)}(t)+b_i(t)e_{\boldsymbol{pi}}^{(d)}(t)\right\}, \tag{4}$$

where $\gamma>0$ is the control parameter to be designed. $e_{ij}^{(d)}(t)=x_{\boldsymbol{pi}}^{(d)}(t)-x_{\boldsymbol{pj}}^{(d)}(t), d=\{0,1,2\}$ are the position, velocity, and acceleration disagreement vectors between the i-th tracking agent and the j-th tracking agent, respectively. $e_{\boldsymbol{pi}}^{(d)}(t)=x_{\boldsymbol{pi}}^{(d)}(t)-x_{\boldsymbol{t}}^{(d)}(t), d=\{0,1,2\}$ are the position, velocity, and acceleration disagreement vectors between the i-th tracking agent and the target, respectively. Then, from Lemma 1 and Definition 1, the tracking problem can be interpreted as uniform stability problem of the following overall disagreement system:

$$\begin{aligned}\dot{\hat{e}}^{(0)}(t)&=\hat{e}^{(1)}(t),\\ \dot{\hat{e}}^{(1)}(t)&=\hat{e}^{(2)}(t),\\ \dot{\hat{e}}^{(2)}(t)&=F(t,\hat{e}^{(0)}(t),\hat{e}^{(1)}(t),\hat{e}^{(2)}(t))\\ &\quad-\gamma\tilde{\mathcal{L}}(t)\{\hat{e}^{(0)}(t)+\hat{e}^{(1)}(t)+\hat{e}^{(2)}(t)\},\end{aligned} \tag{5}$$

where $\hat{e}^{(d)}(t)=\mathrm{col}\{e_1^{(d)}(t),e_2^{(d)}(t),\ldots,e_n^{(d)}(t)\}\in\mathbb{R}^{n\times 3}$ are the collective position, velocity, and acceleration disagreement vectors between tracking agents and the target agent, at time t, where the corresponding $d=0,1,2$. $\tilde{\mathcal{L}}(t)=(\mathcal{L}(t)+\mathcal{B}(t))\otimes I_3$ is the associated Laplacian matrix of the dynamic graph which includes the target. $F(t,\hat{e}^{(0)}(t),\hat{e}^{(1)}(t),\hat{e}^{(2)}(t))=\mathrm{col}\{f_1(t),f_2(t),\ldots,f_n(t)\}$ satisfies Assumption 1, where $f_i(t)=f(t,x_{\boldsymbol{pi}}^{(0)}(t),x_{\boldsymbol{pi}}^{(1)}(t),x_{\boldsymbol{pi}}^{(2)}(t))-f(t,x_{\boldsymbol{t}}^{(0)}(t),x_{\boldsymbol{t}}^{(1)}(t),x_{\boldsymbol{t}}^{(2)}(t)), i\in\mathcal{N}$ is the collective nonlinear self-dynamic disagreement to the target at time t.

We assume the global time-varying Laplacian matrix as $\mathcal{L}(t)+\mathcal{B}(t)=\sum_{i=1}^{N}\alpha_i(t)\mathcal{L}_i$, where $\mathcal{L}_i$ are constant matrices and N is a finite positive constant. $\alpha_i(t)>0$ are the corresponding scheduling functions and $\sum_{i=1}^{N}\alpha_i(t)=1$. We assume that the time derivative of $\alpha_i(t)$ satisfies the following assumption:

Assumption 2 $\dot{\alpha}_i(t)\le\alpha_{di}$, where α_{di} are known constants.

Let $E(t)=[\hat{e}^{(0)}(t),\hat{e}^{(1)}(t),\hat{e}^{(2)}(t)]^{\mathrm{T}}$, then the overall disagreement system (5) can be represented as

$$\dot{E}(t)=\sum_{i=1}^{N}\alpha_i(t)[(\hat{\mathcal{L}}_0-\gamma\hat{\mathcal{L}}_i)E(t)+BF(t,E(t))], \tag{6}$$

where

$$\begin{aligned}\hat{\mathcal{L}}_0&=\begin{pmatrix}\boldsymbol{0}&\boldsymbol{I}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{I}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\end{pmatrix}\otimes I_3,\hat{\mathcal{L}}_i=\begin{pmatrix}\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \mathcal{L}_i&\mathcal{L}_i&\mathcal{L}_i\end{pmatrix}\otimes I_3,B\\ &=\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{0}\\ \boldsymbol{I}\end{pmatrix}\otimes I_3,\end{aligned}$$

in which $\boldsymbol{0}$ and $\boldsymbol{I}$ are matrices with appropriate dimensions.

The following theorem provides a sufficient condition for successful tracking.

Theorem 1 Consider the multi-agent system (6) under a time-varying topology. Suppose that for given positive scalars γ, l, and $\alpha_{d1},\alpha_{d2},\ldots\alpha_{dN}$, there exist symmetric positive-definite matrices P_i and Z of appropriate dimensions such that

$$\begin{pmatrix}\alpha_{d1}\hat{P}_1+\alpha_{d2}\hat{P}_2+\cdots+\alpha_{dN}\hat{P}_N+(\hat{L}_0-\gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i+\hat{P}_i(\hat{L}_0-\gamma\hat{L}_j)+l^2B^{\mathrm{T}}\hat{Z}B & \hat{P}_i\\ * & -\hat{Z}\end{pmatrix}<0, \tag{7}$$

where $\hat{P}_i=P_i\otimes I_3,\hat{Z}=Z\otimes I_3$, for all $i=1,2,\ldots,N$ and $j=1,2,\ldots,N$. Then, the system (6) is asymptotically stable, which means the tracking agents can track the target.

Proof Let us select the Lyapunov function $V(t)$ as

$$V(t,E(t))=E^{\mathrm{T}}(t)\hat{P}(t)E(t)=\sum_{i=1}^{N}\alpha_i(t)E^{\mathrm{T}}(t)\hat{P}_iE(t). \tag{8}$$

Obviously, $V(t,E(t))>0$ and when $\|E(t)\|\to\infty$, $V(t)\to\infty$. The derivative of Lyapunov function (8) along the trajectory of the system (6) is

$$\dot{V}(t,E(t))|_{(6)}=V_1(t,E(t))+V_2(t,E(t)), \tag{9}$$

with

$$V_1(t,E(t))=\sum_{i=1}^{N}\dot{\alpha}_i(t)E^{\mathrm{T}}(t)\hat{P}_iE(t), \tag{10}$$

$$V_2(t,E(t))=\sum_{i=1}^{N}\alpha_i(t)[\dot{E}^{\mathrm{T}}(t)\hat{P}_iE(t)+E^{\mathrm{T}}(t)\hat{P}_i\dot{E}(t)]. \tag{11}$$

Taking Assumption 2 into consideration, we have

$$V_1(t,E(t)) \le \sum_{i=1}^{N} \alpha_{di} E^{\mathrm{T}}(t)\hat{P}_i E(t), \tag{12}$$

$$\begin{aligned} V_2(t,E(t)) &= \sum_{i=1}^{N} \alpha_i(t)\Bigg\{\Bigg[\sum_{j=1}^{N} \alpha_j(t)(\hat{\mathcal{L}}_0 - \gamma\hat{\mathcal{L}}_j)E(t) \\ &\quad + BF(t,E(t))\Bigg]^{\mathrm{T}} \hat{P}_i E(t) \\ &\quad + E^{\mathrm{T}}(t)\hat{P}_i\Bigg[\sum_{j=1}^{N} \alpha_j(t)(\hat{\mathcal{L}}_0 - \gamma\hat{\mathcal{L}}_j)E(t) \\ &\quad + BF(t,E(t))\Bigg]\Bigg\} \\ &= \sum_{i=1}^{N} \alpha_i(t) \sum_{j=1}^{N} \alpha_j(t)[E^{\mathrm{T}}(t)(\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i \\ &\quad + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j)E(t) \\ &\quad + 2(BF(t,E(t)))^{\mathrm{T}}\hat{P}_i E(t)]. \end{aligned} \tag{13}$$

From Lemma 2 and Assumption 1, the following inequality can be deduced

$$\begin{aligned} V_2(t,E(t)) &\le \sum_{i=1}^{N} \alpha_i(t) \sum_{j=1}^{N} \alpha_j(t)\{E^{\mathrm{T}}(t)[(\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i \\ &\quad + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j)]E(t) \\ &\quad + (BF(t,E(t)))^{\mathrm{T}}\hat{Z}(BF(t,E(t))) \\ &\quad + E^{\mathrm{T}}(t)\hat{P}_i\hat{Z}^{-1}\hat{P}_i E(t)\} \\ &\le \sum_{i=1}^{N} \alpha_i(t) \sum_{j=1}^{N} \alpha_j(t)\{E^{\mathrm{T}}(t)[(\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i \\ &\quad + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j)]E(t) \\ &\quad + l^2 E^{\mathrm{T}}(t)B^{\mathrm{T}}\hat{Z}BE(t) + E^{\mathrm{T}}(t)\hat{P}_i\hat{Z}^{-1}\hat{P}_i E(t)\} \\ &= \sum_{i=1}^{N} \alpha_i(t) \sum_{j=1}^{N} \alpha_j(t)\{E^{\mathrm{T}}(t)[(\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i \\ &\quad + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j) + l^2 B^{\mathrm{T}}\hat{Z}B + \hat{P}_i\hat{Z}^{-1}\hat{P}_i]E(t)\}. \end{aligned} \tag{14}$$

Putting expressions (12) and (14) into Eq. (9), we have

$$\begin{aligned} \dot{V}(t,E(t)) &= V_1(t,E(t)) + V_2(t,E(t)) \\ &\le \sum_{i=1}^{N} \alpha_i(t) \sum_{j=1}^{N} \alpha_j(t)\{E^{\mathrm{T}}(t)[\alpha_{d1}\hat{P}_1 + \alpha_{d2}\hat{P}_2 + \cdots \\ &\quad + \alpha_{dN}\hat{P}_N + (\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i \\ &\quad + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j) + l^2 B^{\mathrm{T}}\hat{Z}B + \hat{P}_i\hat{Z}^{-1}\hat{P}_i]E(t)\}. \end{aligned} \tag{15}$$

By making use of inequality (7) and Lemma 3, it is easy to see that $\dot{V}(t,E(t)) \le 0$ holds. From Lemma 1, we can obtain that Theorem 1 ensures the uniform stability of the disagreement system (6). The disagreement between the tracking agents and the target tends to 0 when $t \to \infty$; that is, tracking agents finally track the target according to Definition 1. This completes the proof. □

When the nonlinear function $f(\cdot)$ is ignored, the whole multi-agent system possess the third-order integral kinematics. Then the overall disagreement vector system can be derived as

$$\dot{E}(t) = \sum_{i=1}^{N} \alpha_i(t)(\hat{\mathcal{L}}_0 - \gamma\hat{\mathcal{L}}_i)E(t). \tag{16}$$

Then we establish the following Corollary.

Corollary 1 For the given positive scalars γ and $\alpha_{d1}, \alpha_{d2}, \ldots, \alpha_{dN}$, the system (16) is asymptotically stable, if there exist symmetric positive-definite matrices P_i of appropriate dimensions such that

$$\begin{aligned} &\alpha_{d1}\hat{P}_1 + \alpha_{d2}\hat{P}_2 + \cdots + \alpha_{dN}\hat{P}_N \\ &\quad + (\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j) < 0, \end{aligned} \tag{17}$$

where $\hat{P}_i = P_i \otimes I_3$, for all $i = 1,2,\ldots,N$ and $j = 1,2,\ldots,N$. According to Definition 1 and inequality (17), the tracking agents track the target with the third-order integral kinematics under a time-varying topology in a finite time.

Proof Since the kinematics of tracking agents and the target are integral and hence the nonlinear function F is replaced by 0. The first part of the differential of Lyapunov function $V_1(t,E(t))$ is the same as that in Theorem 1. The second part $V_2(t,E(t))$ becomes

$$\begin{aligned} V_2(t,E(t)) &= \sum_{i=1}^{N} \alpha_i(t)\Bigg\{\Bigg[\sum_{j=1}^{N} \alpha_j(t)[(\hat{\mathcal{L}}_0 - \gamma\hat{\mathcal{L}}_j)E(t)]^{\mathrm{T}}\hat{P}_i E(t) \\ &\quad + E^{\mathrm{T}}(t)\hat{P}_i\Bigg[\sum_{j=1}^{N} \alpha_j(t)(\hat{\mathcal{L}}_0 - \gamma\hat{\mathcal{L}}_j)E(t)\Bigg]\Bigg\} \\ &= \sum_{i=1}^{N} \alpha_i(t) \sum_{j=1}^{N} \alpha_j(t)[E^{\mathrm{T}}(t)(\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i \\ &\quad + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j)E(t)]. \end{aligned} \tag{18}$$

Hence,

$$\begin{aligned} \dot{V}(t,E(t)) &= V_1(t,E(t)) + V_2(t,E(t)) \\ &\le \sum_{i=1}^{N} \alpha_i(t) \sum_{j=1}^{N} \alpha_j(t)\{E^{\mathrm{T}}(t) \\ &\quad \times [\alpha_{d1}\hat{P}_1 + \alpha_{d2}\hat{P}_2 + \cdots + \alpha_{dN}\hat{P}_N \\ &\quad + (\hat{L}_0 - \gamma\hat{L}_j)^{\mathrm{T}}\hat{P}_i + \hat{P}_i(\hat{L}_0 - \gamma\hat{L}_j)]E(t)\}. \end{aligned} \tag{19}$$

It is obvious that the inequality (17) leads to $\dot{V}(t,E(t))$, which means that the system (16) is asymptotically stable. Then, from Definition 1, the tracking agents track the target with the third-order integral kinematics under a time-varying topology in finite time. □

Note that Theorem 1 and Corollary 1 can be used to design the control parameter of the third-order tracking problem under a time-varying topology.

5. A numerical example

In this section we present a numerical example to illustrate the previous developments. Consider a scenario where

a group of three tracking agents track a manoeuvring target in three-dimensional space. The initial position, velocity, and acceleration of the target in three-dimensional space are $x_t^{(0)}(0) = [0,0,0], x_t^{(1)}(0) = [0.2,1,0.6], x_t^{(2)}(0) = [0.3,0.1,0.3]$, respectively. The initial positions of the three tracking agents in three-dimensional space are $x_{p1}^{(0)}(0) = [2,-3,1], x_{p2}^{(0)}(0) = [2.5,1,3], x_{p3}^{(0)}(0) = [3,-2.5,2]$, respectively. The initial velocities of the three tracking agents in three-dimensional space are $x_{p1}^{(1)}(0) = [-0.8,0.3,1], x_{p2}^{(1)}(0) = [0.2,-0.7,0.6], x_{p3}^{(1)}(0) = [-0.5,0.8,1]$, respectively. The initial accelerations of the three tracking agents are 0.

The nonlinear part is caused by the change of the force, which is imposed on the agents. The target can be regarded as a jerk system, which has a chaotic behavior. The nonlinear function is specified as[25]

$$f(\cdot) = -0.6x^{(2)}(t) - x^{(1)}(t) + |x^{(0)}(t)| - 1. \tag{20}$$

The function, $f(\cdot)$, satisfies Assumption 1 with $l = 1$.

We assume the overall time-varying communication topology of the tracking system as

$$\mathcal{L}(t) + \mathcal{B}(t) = \alpha_1(t)L_1 + \alpha_2(t)L_2, \tag{21}$$

where

$$\alpha_1(t) = \frac{1}{40}\sin^2(t), \alpha_2(t) = 1 - \alpha_1(t),$$

$$L_1 = \begin{pmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 2.5 & -1 \\ 0 & -1 & 1 \end{pmatrix},$$

$$L_2 = \begin{pmatrix} 1.2 & -0.2 & -0.5 \\ -0.2 & 0.2 & 0 \\ -0.5 & 0 & 2 \end{pmatrix}.$$

Figure 1 illustrates the considered time-varying topology.

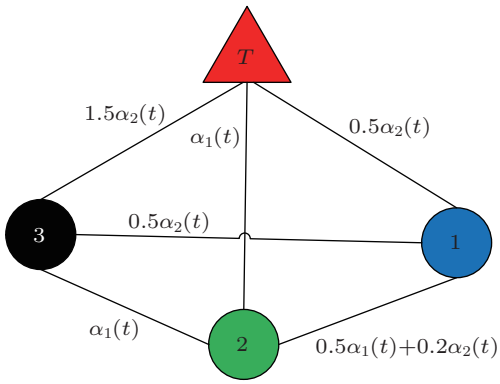


Fig. 1. (color online) The time-varying topology.

It should be noted that $\mathcal{L}(t) + \mathcal{B}(t)$ satisfies Assumption 2 in Ref. [23], but $\dot{\mathcal{L}}(t) + \dot{\mathcal{B}}(t)$ does not meet Assumption 3 in Ref. [23]. This means that the theorem in Ref. [23] cannot be employed to solve this example. Therefore, this numerical example vividly shows the advantage of the theorem in this work.

$\dot{\alpha}_1(t) = 0.05\sin(t)\cos(t)$ and $\dot{\alpha}_2(t) = -\dot{\alpha}_1(t)$. Hence, they satisfy Assumption 2 with $\alpha_{d1} = \alpha_{d2} = 0.05$. The tracking strategy (4) with the control parameter $\gamma = 45$ is selected. By applying Theorem 1, we obtain

$$P_1 = \begin{pmatrix} 0.0208 & 0.0151 & 0.0144 & 0.0096 & 0.0039 & 0.0032 & 0.0090 & 0.0033 & 0.0026 \\ 0.0151 & 0.0192 & 0.0137 & 0.0044 & 0.0085 & 0.0029 & 0.0033 & 0.0074 & 0.0019 \\ 0.0144 & 0.0137 & 0.0175 & 0.0041 & 0.0034 & 0.0072 & 0.0026 & 0.0019 & 0.0056 \\ 0.0096 & 0.0044 & 0.0041 & 0.0161 & 0.0109 & 0.0106 & 0.0074 & 0.0022 & 0.0020 \\ 0.0039 & 0.0085 & 0.0034 & 0.0109 & 0.0154 & 0.0103 & 0.0018 & 0.0063 & 0.0013 \\ 0.0032 & 0.0029 & 0.0072 & 0.0106 & 0.0103 & 0.0145 & 0.0010 & 0.0007 & 0.0050 \\ 0.0090 & 0.0033 & 0.0026 & 0.0074 & 0.0018 & 0.0010 & 0.0091 & 0.0035 & 0.0027 \\ 0.0033 & 0.0074 & 0.0019 & 0.0022 & 0.0063 & 0.0007 & 0.0035 & 0.0076 & 0.0020 \\ 0.0026 & 0.0019 & 0.0056 & 0.0020 & 0.0013 & 0.0050 & 0.0027 & 0.0020 & 0.0057 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 0.0390 & 0.0288 & 0.0279 & 0.0174 & 0.0073 & 0.0064 & 0.0116 & 0.0014 & 0.0005 \\ 0.0288 & 0.0594 & 0.0296 & -0.0014 & 0.0292 & -0.0005 & 0.0023 & 0.0329 & 0.0032 \\ 0.0279 & 0.0296 & 0.0303 & 0.0062 & 0.0080 & 0.0087 & 0.0006 & 0.0023 & 0.0030 \\ 0.0174 & -0.0014 & 0.0062 & 0.0402 & 0.0214 & 0.0290 & 0.0114 & -0.0074 & 0.0002 \\ 0.0073 & 0.0292 & 0.0080 & 0.0214 & 0.0434 & 0.0221 & 0.0021 & 0.0241 & 0.0028 \\ 0.0064 & -0.0005 & 0.0087 & 0.0290 & 0.0221 & 0.0313 & 0.0004 & -0.0065 & 0.0027 \\ 0.0116 & 0.0023 & 0.0006 & 0.0114 & 0.0021 & 0.0004 & 0.0116 & 0.0023 & 0.0006 \\ 0.0014 & 0.0329 & 0.0023 & -0.0074 & 0.0241 & -0.0065 & 0.0023 & 0.0338 & 0.0032 \\ 0.0005 & 0.0032 & 0.0030 & 0.0002 & 0.0028 & 0.0027 & 0.0006 & 0.0032 & 0.0031 \end{pmatrix},$$

$$Z = \begin{pmatrix} 0.7308 & 0.0292 & 0.0302 & 0.0165 & 0.0145 & 0.0155 & 0.0016 & -0.0004 & 0.0006 \\ 0.0292 & 0.7291 & 0.0285 & 0.0132 & 0.0135 & 0.0125 & 0.0013 & 0.0016 & 0.0006 \\ 0.0302 & 0.0285 & 0.7292 & 0.0141 & 0.0123 & 0.0135 & 0.0010 & -0.0007 & 0.0005 \\ 0.0165 & 0.0132 & 0.0141 & 0.7279 & 0.0251 & 0.0259 & 0.0041 & 0.0009 & 0.0017 \\ 0.0145 & 0.0135 & 0.0123 & 0.0251 & 0.7237 & 0.0230 & 0.0038 & 0.0029 & 0.0017 \\ 0.0155 & 0.0125 & 0.0135 & 0.0259 & 0.0230 & 0.7235 & 0.0035 & 0.0005 & 0.0015 \\ 0.0016 & 0.0013 & 0.0010 & 0.0041 & 0.0038 & 0.0035 & 0.4685 & -0.2313 & -0.2316 \\ -0.0004 & 0.0016 & -0.0007 & 0.0009 & 0.0029 & 0.0005 & -0.2313 & 0.4703 & -0.2316 \\ 0.0006 & 0.0006 & 0.0005 & 0.0017 & 0.0017 & 0.0015 & -0.2316 & -0.2316 & 0.4678 \end{pmatrix}.$$

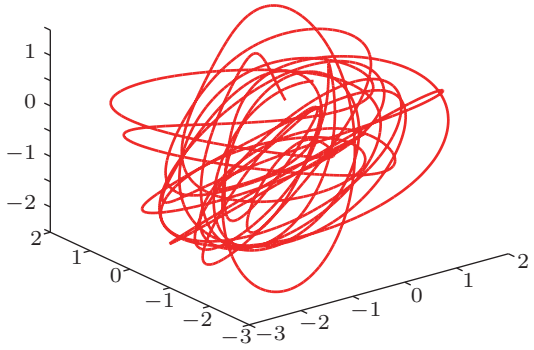


Fig. 2. (color online) Chaotic behavior of the target.

Figure 2 illustrates the chaotic behavior of the target agent within 8000 steps. In order to clearly show the tracking performance, we only plot the trajectories of the group within 1500 steps. Figure 3 shows the group behavior of tracking agents and the target with nonlinear kinematics in the three-dimensional space. Figure 4 illustrates the nonlinear group traces in x–$\dot{x}$, y–$\dot{y}$, and z–$\dot{z}$ planes. From the figures, it can be clearly seen that the tracking agents track the manoeuvring target under the time-varying topology (21) with the proposed tracking strategy (4) in a finite time.

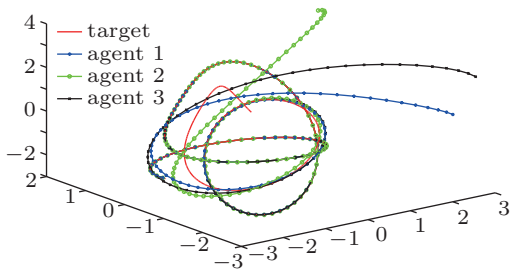


Fig. 3. (color online) Trajectories of the tracking agents and the target with nonlinear kinematics.

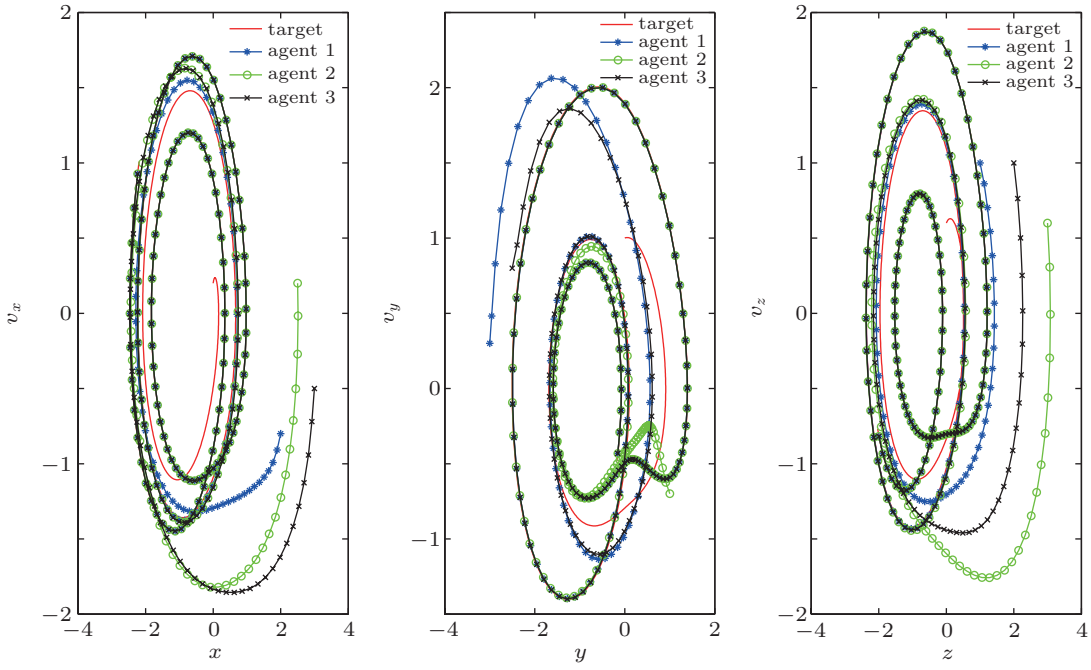


Fig. 4. (color online) The nonlinear group traces in x–$\dot{x}$, y–$\dot{y}$, and z–$\dot{z}$ planes.

6. Conclusions

We have proposed a tracking strategy for tracking a manoeuvring target by a group of the third-order tracking agents distributed in three-dimensional space. It is assumed that the tracking agents and the target communicate with each other according to an undirected time-varying topology. Actually, the proposed tracking strategy can also be applied to a class of tracking systems whose communication topologies contain a directed spanning tree, which will be explored in future work. The proposed tracking theorem is less conservative and offers a wider range of application comparing with that in Ref. [23]. With the proposed control strategy, the tracking agents are able to track the target with both nonlinear kinematics and integral kinematics under a time-varying topology in finite time.

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