Circular geodesics and accretion disk in the spacetime of a black hole including global monopole*

Sun Xu-Dong(孙旭东), Chen Ju-Hua(陈菊华), and Wang Yong-Jiu(王永久)†

College of Physics and Information Science, Hunan Normal University, Changsha 410081, *China*

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We study circular time-like geodesics in the spacetime of a black hole including global monopole. We show that when the range of parameter changed the properties of the circular geodesics and the radiation of accretion disks are different. It follows that the properties of the accretion disk around black hole including global monopole can be different from that of a disk around Schwarzschild black hole.

Keywords: global monopole, geodesics, accretion disk

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1. Introduction

Black hole including global monopole plays an important role in the early universe. The global monopole is producted when global symmetry is broken down. There exists goldstone field around global monopole which has tremendous energy. Analyses of the black hole including the global monopole have been studied as we try to understand the early universe. Wang,[1,2] Jing,[3] Chen,[4] Peng,[5] and Shao[6] have studied the black hole involving the global monopole. Analyses of circular geodesics involving accretion disks have also drawn much attention in recent years. Kovacs and Harko[7] studied a modified Kerr metric with scalar field. A similar analysis has been carried out for the Kerr and Reissner–Nordstrom metrics by Pugliese *et al.*[8–10] and Pradhan.[11] Anirban[12] studied the circular geodesics and accretion disks in the Janis–Newman–Winicour and gamma metric spacetimes. Chen Ju-Hua[13] and Lü Jian-Bo[14] studied time-like geodesics in the spherically symmetric black hole and the nut-taub-like spacetime respectively. In addition, the general relativistic setting of steady state accretion disks onto a rotating black hole on the equatorial plane was developed by Page, Thorne,[15] Novikov, Thorne,[16] and Shakura.[17]

2. Circular equatorial geodesics in the spacetime of a black hole including global monopole

A black hole including global monopole can be described by the following metric:[1]

$$\begin{aligned} \mathrm{d}s^2 = &-\left(1-8\pi G{\eta_0}^2-\frac{2Gm}{\overline{R}}\right)\mathrm{d}\tau^2 \\ &+\left(1-8\pi G{\eta_0}^2-\frac{2Gm}{\overline{R}}\right)^{-1}\mathrm{d}\overline{R}^2 \\ &+\overline{R}^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2), \end{aligned} \tag{1}$$

where m is the mass of the black hole including the global monopole, η_0 is symmetry breakdown scale factor during the formation of the global monopole. The event horizon of a black hole including the global monopole metric spacetime locates in

$$\overline{R_{\mathrm{eh}}}=\frac{2Gm}{1-8\pi G{\eta_0}^2}, \tag{2}$$

denoting that

$$t=(1-8\pi G{\eta_0}^2)^{1/2}\tau,\ r=(1-8\pi G{\eta_0}^2)^{-1/2}\overline{R}, \tag{3}$$

and rescaling the mass of the black hole and symmetry breakdown scale factor

$$M=(1-8\pi G{\eta_0}^2)^{-3/2}m,\ b=(1-8\pi G{\eta_0}^2), \tag{4}$$

using Eqs. (1), (3), and (4), the metric becomes

$$\begin{aligned} \mathrm{d}s^2 = &-\left(1-\frac{2GM}{r}\right)\mathrm{d}t^2+\left(1-\frac{2GM}{r}\right)^{-1}\mathrm{d}r^2 \\ &+br^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2), \end{aligned} \tag{5}$$

the Schwarzschild solution can be recovered from the metric (1) when $\eta_0\to 0$. The metric of a black hole including the global monopole endowed with two Killing vectors $\xi_t=(\partial/\partial t)^\mu$ and $\xi_\varphi=(\partial/\partial\varphi)^\mu$ for the spherically symmetric and static property. As a consequence, we have the following conserved quantities along geodesic motion

$$E=\left(1-\frac{2GM}{r}\right)U^t=-U_t, \tag{6}$$

$$\Phi=br^2U^\varphi, \tag{7}$$

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†Corresponding author. E-mail: wyj@hunnu.edu.cn

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where $U^\alpha = (U^t, U^r, U^\theta, U^\varphi)$ is the four-velocity of the test particle.

Since the metric of the black hole including the global monopole is spherically symmetric, the motion of a test particle will be restricted to a plane which can be chosen by gauge freedom to be the equatorial plane. Thus we set $\theta = \pi/2$.

Using the normalization condition for the four-velocity $U^\alpha U_\alpha = -1$, from Eqs. (6) and (7) we obtain the equation for the motion of a particle

$$E^2 - \dot{r}^2 = \left(1 - \frac{2GM}{r}\right)(1 + b^{-1}(r\sin\theta)^{-2}\Phi^2 + br^2\dot{\theta}^2). \quad (8)$$

Denoting that

$$V^2 = \left(1 - \frac{2GM}{r}\right)(1 + b^{-1}(r\sin\theta)^{-2}\Phi^2 + br^2\dot{\theta}^2), \quad (9)$$

where V can be regarded as an effective potential for the test particle. The dot in Eqs. (8) and (9) denotes a derivative with respect to the affine parameter for geodesic, i.e., the proper time.

A particle orbit along a circular geodesic must obey the conditions $V = E, \partial V/\partial r = 0$ as r must stay constant. In this way we get the expression for the angular velocity, conserved energy, momentum of the particle orbit along the geodesic of radius r as

$$\omega = \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \sqrt{\frac{GM}{br^3}}, \quad (10)$$

$$E^2 = \frac{4G^2M^2 - 4GMr + r^2}{r^2 - 3GMr}, \quad (11)$$

$$\Phi^2 = \frac{bGMr^2}{r - 3GM}. \quad (12)$$

Further, we need to ensure that the circular orbit is stable against small perturbations in the radial direction, and this implies that for a stable orbit the effective potential must admit a minimum: $\partial^2 V/\partial r^2 > 0$. Using the condition for the stability of the circular geodesics against small perturbations along the radial direction, we get $GM(r - 6GM) > 0$. It follows that the radius of the stable orbit must satisfy the following condition

$$r > 6GM = r_{\mathrm{ms}},\ \overline{R_{\mathrm{ms}}} = 6(1 - 8\pi G\eta_0{}^2)^{-1}Gm. \quad (13)$$

The energy and angular momentum diverge at the radius

$$r_{\mathrm{ph}} = 3GM,\ \overline{R_{\mathrm{ph}}} = 3(1 - 8\pi G\eta_0{}^2)^{-1}Gm, \quad (14)$$

which represents the radius at which photons move on circular trajectories and it is known as photon sphere. The photon sphere plays a crucial role in the study of gravitational lensing since its presence delimits a boundary below which lensing effects cannot be observed, therefore implying that two spacetimes that possess a photon sphere at the same radius cannot be distinguished by the observation of lensing effects even though their compact sources might be different.

The condition $E^2 = 1$ gives the radius r_{mb} and it is the minimum radius of the bound circular orbit:

$$r_{\mathrm{mb}} = 4GM,\ \overline{R_{\mathrm{mb}}} = 4(1 - 8\pi G\eta_0{}^2)^{-1}Gm. \quad (15)$$

In Fig. 1 we plot $\overline{R}/m$ as a function of the symmetry breakdown scale factor η_0 in natural units. It is obvious that the larger the value of η_0 is, the higher the $\overline{R}$ is. As we can see that locations of the marginally stable orbits, marginally bound orbits, and photon orbits have a strong dependence on the parameter η_0, respectively.

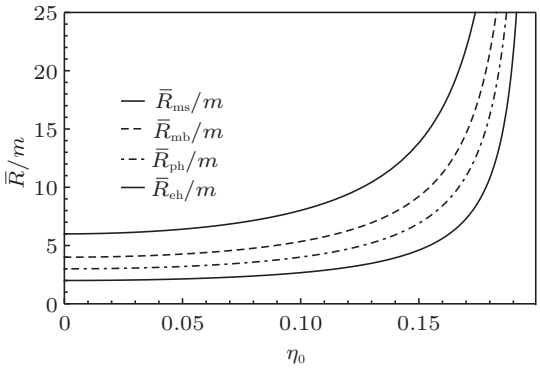


Fig. 1. The value of $\overline{R}/m$ in natural units for the event horizon ($\overline{R_{\mathrm{eh}}}$), photon sphere ($\overline{R_{\mathrm{ph}}}$), marginally bound orbits ($\overline{R_{\mathrm{mb}}}$), and marginally stable orbits ($\overline{R_{\mathrm{ms}}}$) as functions of the parameter η_0. It is clear that the radius of characteristic surface is monotonically increase with the symmetry breakdown scale factor η_0.

3. Accretion disk properties

In astrophysics an accretion disk is a nearly flat disk of heated gas whose particles slowly spiral onto a central accreting compact object. Such accretion disks can be detected by their emitted radiation. We use the equatorial approximation ($(\theta - \pi/2)^2 \ll 1$). The gas is represented by test particles moving with angular velocity $\omega = \mathrm{d}\varphi/\mathrm{d}t$ on circular orbits around the black hole including the global monopole.

The radius of a test particle moving on the circular orbit could not be smaller than r_{ph}. The circular orbit of a test particle may exist when $r > r_{\mathrm{ph}}$. When $r_{\mathrm{ph}} < r < r_{\mathrm{mb}}$, particles move on the circular orbit are not stable against a small perturbation. They may fall into the black hole or flee away to a distance. When $r_{\mathrm{mb}} < r < r_{\mathrm{ms}}$, particles moving on the circular orbit are not stable either. They may fall into the black hole or flee away to a limited radius and then get back as they are bounded. When a particle moves on the circular orbit $r > r_{\mathrm{ms}}$, the orbit of the particle is stable. The radius r_{ms} represents the inner edge of the outer disk.

It is possible to construct the quantities relevant for observations,[15] using the structure equations describing the gas in the disk. The radiant energy flux can be written as

$$f(r) = -\frac{M_0}{4\pi\sqrt{-\det g}}\frac{\omega_{,r}}{(E - \omega\Phi)^2}\int_{r_{\mathrm{ms}}}^{r}(E - \omega\Phi)\Phi_{,r}\,\mathrm{d}r, \quad (16)$$

where M_0 assumed to be constant is the mass accretion rate.

Here we study the behavior of the radiant energy flux in the metric in natural units and assume that $M = 1$. Using Eq. (16), the function $f(r)$ becomes

$$f(r) = \frac{3M_0(2\sqrt{r}+\sqrt{3}\ln\frac{(\sqrt{r}+\sqrt{3})^2}{r-3}-2\sqrt{6}-2\sqrt{3}\ln(1+\sqrt{2}))}{16\pi b(r-3)\sqrt{r^7}}. \quad (17)$$

The feature of a black hole including the global monopole is that the radiant energy flux is lager than that of a Schwarzschild black hole (see Fig. 2).

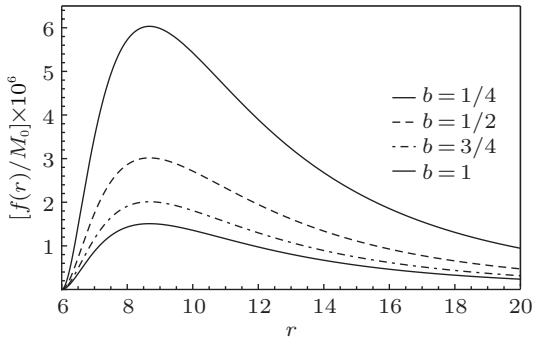


Fig. 2. Radiant energy flux ($\times10^6$) for unit mass accretion rate in natural units assuming that $M = 1$. The lowest curve represents the Schwarzschild black hole case. From the graph we can see that the radiant energy flux decreases with the parameter b for a fixed radius.

4. Conclusions

In this paper we have investigated in detail the structure of circular geodesics in the space-time of a black hole including the global monopole. The Schwarzschild solution could be acquired when the parameter $\eta_0 = 0$ is satisfied. The parameter η_0 affects locations of the marginally stable orbits, marginally bound orbits, and photon orbits respectively. There exists difference between the Schwarzschild black hole and the black hole including the global monopole as far as the structure of circular geodesics is concerned. In order to distinguish the black hole including the global monopole from classical Schwarzschild black hole observationally, we could analyze spectrum of the radiation from accretion disks.

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