Nonautonomous dark soliton solutions in two-component Bose–Einstein condensates with a linear time-dependent potential*

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(Received 20 September 2013; revised manuscript received 21 November 2013; published online 20 April 2014)

We report the analytical nonautonomous soliton solutions (NSSs) for two-component Bose–Einstein condensates with the presence of a time-dependent potential. These solutions show that the time-dependent potential can affect the velocity of NSS. The velocity shows the characteristic of both increasing and oscillation with time. A detailed analysis for the asymptotic behavior of NSSs demonstrates that the collision of two NSSs of each component is elastic.

Keywords: nonautonomous soliton solution, interaction, Bose–Einstein condensates

PACS: 03.75.Lm, 05.30.Jp **DOI:** 10.1088/1674-1056/23/6/060310

1. Introduction

The concept of soliton was introduced firstly by Zabusky and Kruskal[1] to explain the observed phenomena that nonlinear solitary waves do not disperse and completely preserve their localized form and speeds, both during propagation and after a collision. Since the first experiment realization of Bose–Einstein condensates (BECs) in dilute atomic gases, great interest has been attributed to this new form of matter. A bright soliton[2–5] in a BECs denotes the density lumps and have attractive atomic interaction,[6] while a dark soliton[7,8] is a macroscopic excitation characterized by a local density minimum and a phase gradient of the wave function at the position of the minimum. The properties of autonomous solitons[7–14] in a BECs have been studied extensively by using different relevant techniques.[15–21]

When a physical system is subjected to various external time-dependent forces, nonautonomous nonlinear evolution models typically arise and the term of nonautonomous solitons[17] were first introduced. Different aspects of the dynamics in nonautonomous models[22–25] and the controllable soliton solutions[26–28] in optical fibers have been theoretically investigated. For BECs, this nonautonomous model can be realized by magnetically tuning the interatomic interaction near a Feshbach resonance[29–33] and temporal dependence external potential.[25,34] Controlling soliton trains[35] were also created starting from periodic waves. Therefore, the BEC forms a typical example of such nonautonomous solitons in external potentials and it offers a good opportunity for the exploration of nonlinear excitations.[36]

Recently, the development of trapping techniques has allowed the creation of multi-component condensates trapped in different internal states.[37,38] Furthermore, two-species of BECs with tunable interaction[39,40] were successfully produced. This experimental progress stimulates further explorations of two-species BECs.[41–44] The effective mean-field model[45,46] of a quasi-one-dimensional two-component BECs in a linear potential with an arbitrary time-dependence is given by

$$\mathrm{i}\hbar\frac{\partial\Psi_j}{\partial T}=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial X^2}+Xf(T)+\sum_{k=1}^{2}U_{jk}|\Psi_k|^2\right)\Psi_j, \quad (1)$$

where $\int|\Psi_j|^2\,\mathrm{d}X=N_j$ is the number of atoms of each component, with the subscript $j=1,2$, which is always held on in the following section. The interacting constant of two atoms is $U_{jk}=2\hbar^2a_{jk}/ml_0^2$,[47] where m is the mass of the atom, a_{jk} represents s-wave scattering lengths of intra- and inter-species atomic collisions ($a_{jk}<0$ for attractive interaction, while $a_{jk}>0$ for repulsive interaction), and $l_0\equiv\sqrt{\hbar/m\omega_0}$ is the characteristic extension length of the ground state wave function of an harmonic oscillator.

Equation (1) can be reduced to the dimensionless form of

$$\mathrm{i}\partial_t\psi_j=-\frac{1}{2}\partial_{xx}\psi_j-xf(t)\psi_j-\left(\sum_{k=1}^{2}\mu_{jk}|\psi_k|^2\right)\psi_j, \quad (2)$$

where $\partial_x\equiv\partial/\partial_x$, $\partial_t\equiv\partial/\partial_t$, $\mu_{jk}=2Na_{jk}/l_0$, $f(t)=-ml_0^3/\hbar^2f(T/(ml_0^2/\hbar))$. The time t and coordinate x have been measured in units $ml_0^2/\hbar$ and l_0, respectively. Generally, the coupled equations (2) are not integrable. When $f(t)=0$ and $\mu_{jk}=\mu$, equation (2) amounts to an integrable Manakov system.[48–51] However, the dynamic property of nonautonomous soliton solutions of Eq. (2) has not been well explored, and the Hirota method[52,53] developed for Eq. (2) with the repulsive interaction is also very interesting. In the present paper, we demonstrate how to construct an exact dark–dark

*Project supported by the Key Project of the Chinese Ministry of Education (Grant No. 2011015) and the Natural Science Foundation of Hebei Province of China (Grant No. A2012202023).

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NSSs for two-component Bose–Einstein condensates with a time-dependent linear potential. The corresponding properties of such solutions are studied in detail.

2. Developed Hirota method and nonautonomous soliton solutions

The NSSs of Eq. (2) can be constructed by an effective straightforward Hirota method.[52] In the presence of a time-dependent potential the application of Hirota method should be careful to get NSSs of Eq. (2). Performing the normal procedure of Hirota method, the solution of Eq. (2) takes the form

$$\psi_j = \frac{G_j(x,t)}{F(x,t)}, \tag{3}$$

where the function $G_j(x,t)$ is complex and $F(x,t)$ is real. Substituting Eq. (3) into Eq. (2), we have

$$F\hat{\mathcal{A}}_1(G_j\cdot F) - G_j\left[\hat{\mathcal{A}}_2 F\cdot F - \mu\sum_{k=1}^{2}|G_k|^2\right] = 0, \tag{4}$$

where $\hat{\mathcal{A}}_1 = \mathrm{i}D_t + D_x^2/2 + xf(t) - \lambda$ and $\hat{\mathcal{A}}_2 = D_x^2/2 - \lambda$, here D_t and D_x^2 are called Hirota bilinear operators and are defined by $D_x^m D_t^n G(x,t)\cdot F(x,t) = (\partial_x - \partial_{x'})^m(\partial t - \partial t')^n G(x,t)F(x',t')|_{x'=x,t'=t}$. Equation (4) implies that the dark NSSs of Eq. (2) can be obtained with the following assumption

$$\hat{\mathcal{A}}_1 G_j\cdot F = 0, \quad \hat{\mathcal{A}}_2 F\cdot F = \mu\sum_{k=1}^{2}|G_k|^2. \tag{5}$$

The derivation of Eq. (5) has made Eq. (2) into the normal procedure of Hirota method. Next, the main task is how to get the expressions of $G_j(x,t)$ and $F(x,t)$. To this purpose, one can assume that $G_j(x,t)$ and $F(x,t)$ are, respectively, formed by a linear superposition of different powers for an arbitrary auxiliary parameter χ,

$$G_j = g_{0j}(1+\sum_{l_1=1}\chi^{l_1}g_{l_1 j}), \quad F = 1+\sum_{l_1=1}\chi^{l_1}F_{l_1}. \tag{6}$$

It should be noted that the arbitrary auxiliary parameter χ will be absorbed in the expression of NSSs. By substituting Eq. (6) into Eq. (5) and collecting the coefficients with the same power of χ, one can get a series of equations. With these equations, the nonautonomous multi-soliton solutions of Eq. (2) can be obtained analytically. In the following section, we focus on the nonautonomous one and two dark–dark soliton solutions, which demands $l_1 = 1,2$. Collecting the different powers of χ, equation (5) reduces to a series of equations, as listed below.

(i) For the coefficient of χ^0, we obtain

$$\hat{\mathcal{A}}_1 g_{0j}\cdot 1 = 0, \tag{7}$$

$$\hat{\mathcal{A}}_2(1\cdot 1) = \mu\sum_{k=1}^{2}|g_{0k}|^2; \tag{8}$$

(ii) For the coefficient of χ, we have

$$\hat{\mathcal{A}}_1\left(g_{0j}\cdot F_1 + g_{0j}g_{1j}\cdot 1\right) = 0, \tag{9}$$

$$\hat{\mathcal{A}}_2(1\cdot F_1 + F_1\cdot 1) = \mu\sum_{k=1}^{2}[|g_{0k}|^2(g_{1k}+\bar{g}_{1k})]; \tag{10}$$

(iii) For the coefficient of χ^2, we gain

$$\hat{\mathcal{A}}_1\left(g_{0j}\cdot F_2 + g_{0j}g_{1j}\cdot F_1 + g_{0j}g_{2j}\cdot 1\right) = 0, \tag{11}$$

$$\hat{\mathcal{A}}_2(1\cdot F_2 + F_2\cdot 1 + F_1\cdot F_1) = \mu\sum_{k=1}^{2}[|g_{0k}|^2(g_{2k}+\bar{g}_{2k}+|g_{1k}|^2)]; \tag{12}$$

(iv) For the coefficient of χ^3, we get

$$\hat{\mathcal{A}}_1\left(g_{0j}g_{1j}\cdot F_2 + g_{0j}g_{2j}\cdot F_1\right) = 0, \tag{13}$$

$$\hat{\mathcal{A}}_2(F_1\cdot F_2 + F_2\cdot F_1) = \mu\sum_{k=1}^{2}[|g_{0k}|^2(g_{1k}\bar{g}_{2k}+\bar{g}_{1k}g_{2k})]; \tag{14}$$

(v) For the coefficient of χ^4, we have

$$\hat{\mathcal{A}}_1 g_{0j}g_{2j}\cdot F_2 = 0, \tag{15}$$

$$\hat{\mathcal{A}}_2 F_2\cdot F_2 = \mu\sum_{k=1}^{2}|g_{0k}g_{2k}|^2, \tag{16}$$

where the overbar indicates the complex conjugate. It is obvious to see that the equations (7)–(16) can be solved in turn with the reasonable expression.

2.1. Dark–dark soliton solution

The nonautonomous one dark–dark soliton solution can be obtained by solving Eqs. (7)–(16) in turn, while with the assumption $g_{2j} = F_2 = 0$. Equation (7) admits the solution

$$g_{0j} = \tau_{0j}\mathrm{e}^{\mathrm{i}\eta_{0j}}, \tag{17}$$

where $\eta_{0j} = P_{0j}(t)x + \Omega_{0j}(t)$. The parameters $P_{0j}(t)$ and $\Omega_{0j}(t)$ are defined by

$$P_{0j} = \int_0^t f(\tau)\,\mathrm{d}\tau + \xi_{0j},$$
$$\Omega_{0j} = -\lambda t - \frac{1}{2}\int_0^t P_{0j}^2\,\mathrm{d}\tau + \zeta_{0j}, \tag{18}$$

here ξ_{0j} and ζ_{0j} are arbitrary real constants, respectively. Then, from Eq. (8) we get $\sum_{k=1}^{2}|\tau_{0k}|^2 = -\lambda/\mu$ which shows that the existence of dark–dark NSSs demands $\lambda > 0$. By substituting Eq. (17) into Eq. (9), we find that g_{1j} and F_1 admit the form

$$g_{1j} = Z_{gj}\mathrm{e}^{\eta_1}, \quad F_1 = \mathrm{e}^{\eta_1}, \tag{19}$$

where the parameter Z_{gj} is complex, and $\eta_1 = P_1(t)x + \Omega_1(t) + \eta_1^0$ with η_1^0 being a real constant. By substituting Eqs. (17) and (19) into Eq. (10) we have

$$\frac{1}{2}P_1^2(Z_{gj}+1) = -\mathrm{i}(Z_{gj}-1)\left(x\partial_t P_1 + P_1 P_{0j} + \partial_t\Omega_1\right). \tag{20}$$

A detailed analysis demonstrates that the above equation implies the solution that $P_{01} = P_{02} \equiv P_0$, $Z_{g1} = Z_{g2} \equiv Z_g$, $|Z_g|^2 = 1$, and $\partial_t P_1 = 0$ for the case of $Z_{gj} \neq \pm 1$ (the case of $Z_{gj} = \pm 1$ will be discussed below). With Eqs. (10), (19), and (20) we obtain

$$Z_g = \mathrm{e}^{\mathrm{i}\phi_1}, \quad \phi_1 = \frac{1}{2}\arctan\frac{P_1}{\sqrt{4\lambda - P_1^2}},$$
$$\Omega_1 = -P_1\int_0^t P_0(\tau)\,\mathrm{d}\tau + \sqrt{\lambda - P_1^2/4}P_1 t + \zeta_1, \quad (21)$$

which ensures that equations (11) and (12) are satisfied to the moment. In Eq. (21) the parameter P_1 is a real parameter, and $\lambda \geq P_1^2/4$.

With the help of Eqs. (6), (17), and (19) the analytical dark–dark NSSs of Eq. (2) can be derived as

$$\psi_j = \frac{\tau_{0j}}{2}\mathrm{e}^{\mathrm{i}\eta_{0j}}\left[\left(1+\mathrm{e}^{\mathrm{i}\phi_1}\right) - \left(1-\mathrm{e}^{\mathrm{i}\phi_1}\right)\tanh\frac{\eta_1}{2}\right], \quad (22)$$

where the parameters η_{0j} and η_1 have been given before. From Eq. (22) we can clarify two special cases mentioned before; that is, $Z_g = \pm 1$. When $\phi_1 = 0$, the solution (22) reduces to the plane-wave solution $\psi_j = \tau_{0j}\mathrm{e}^{\mathrm{i}\eta_{0j}}$, which corresponds to the uniform distribution density of bosons. When $\phi = \pi$, the solution (22) becomes the form $\psi_j = -\tau_{0j}\mathrm{e}^{\mathrm{i}\eta_{0j}}\tanh(\eta_1/2)$, where

$$\eta_1 = 2\sqrt{\lambda}x - 2\sqrt{\lambda}\int_0^t P_0(\tau)\,\mathrm{d}\tau + \zeta_1.$$

This solution represents the black NSSs of a BEC in a linear potential with an arbitrary time-dependence, which results from the coupling effect of external magnetic field and gravity.

In the case of $f(t) = 0$, the solution in Eq. (22) reduces to one dark soliton solution of the normal nonlinear Schrödinger (NLS) equation, which is the same as that in Refs. [41] and [54]. When $f(t) =$ constant, the solution (22) represents the nonlinear dark wave propagation in a linearly inhomogeneous plasma[22] or optical fiber with the abnormal dispersion. As $f(t) = b_1 + l\cos(\omega t)$, the solution (22) denotes an NSS in BECs under the effect of the external magnetic field and gravity. The corresponding soliton dynamics is illustrated in Fig. 1. This shows that the soliton maintains its amplitude and shape.

We also find the effect of term $xf(t)$ from the NSSs in Eq. (22). The nonautonomous soliton width defined by $1/P_1$ is not affected by the time-dependent external potential. From the expression of η_1 and Eq. (21) we get the velocity

$$V_1 = -\frac{\partial}{\partial t}\frac{\Omega_1}{P_1} = \int_0^t f(\tau)\,\mathrm{d}\tau + \xi_0 - \frac{1}{2}\sqrt{4\lambda - P_1^2},$$

which shows that the time-dependent potential plays an important role for the velocity of dark NSSs. When $f(t) = b_1 + l\cos(\omega t)$, the velocity becomes

$$V_1 = b_1 t + l/\omega\sin(\omega t) + \xi_0 - \sqrt{\lambda - (P_1/2)^2},$$

which increases and oscillates with time. This result explains the soliton oscillation during propagation, as shown in Fig. 1. It is easy to find that the soliton will propagate without oscillation when $l = 0$ (see Fig. 2). These results show that the dark nonautonomous soliton slides down and oscillates with time under the coupling effect of gravity and the controlling external field.

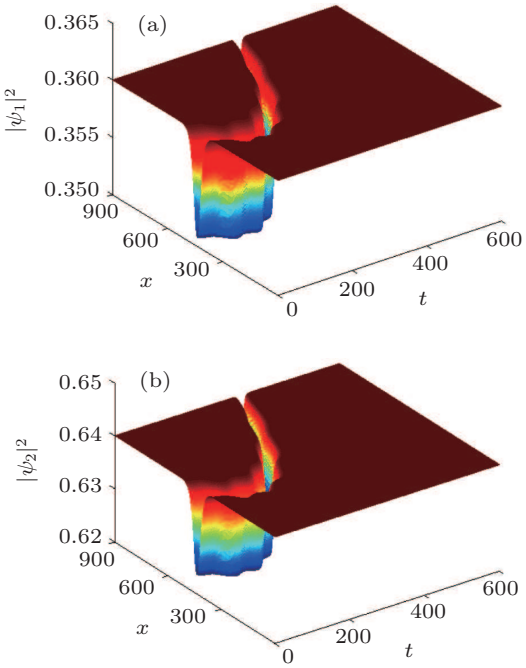


Fig. 1. (color online) Dynamics of the probability density profiles of (a) component 1 and (b) component 2. The parameter values are $b_1 = 0.5$, $l = 3$, $\omega = 2$, $\xi_{10} = \xi_{20} = 2$, $P_1 = 1.2$, $\tau_{10} = 0.6$, $\tau_{20} = 0.8$, $\zeta_{10} = 2$, $\zeta_{20} = -1.5$, $\zeta_1 = 0.9$, and $\mu = -1$.

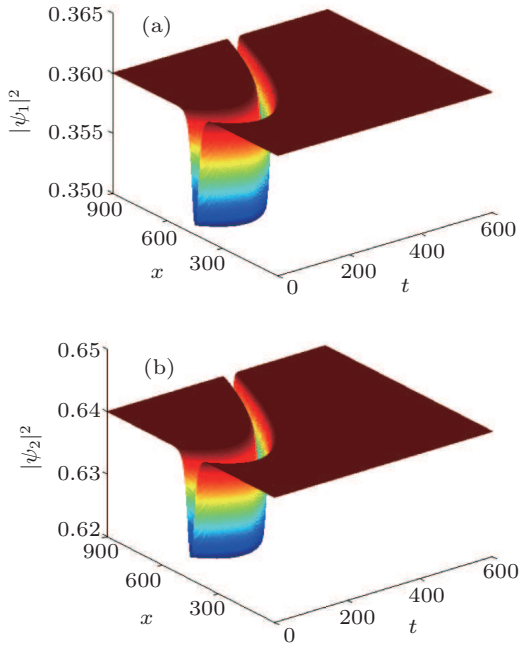


Fig. 2. (color online) Dynamics of the probability density profiles of (a) component 1 and (b) component 2. The parameters are the same as those in Fig. 1 except $l = 0$ and $\zeta_1 = 0$.

2.2. Collision of two dark–dark nonautonomous solitons

In this section, we will give the analytical expression of two dark–dark NSSs of Eq. (2) which requires solving Eqs. (7)–(16) in turn with reasonable expressions of g_{1j}, g_{2j}, and F_j, while g_0 has been obtained in Eq. (17). Employing a similar procedure, we assume that g_{1j} and F_1 take the following form

$$g_{1j} = \sum_{k=1}^{2} Z_k \mathrm{e}^{\eta_k}, \quad F_1 = \sum_{k=1}^{2} \mathrm{e}^{\eta_k}, \tag{23}$$

where $\eta_k = P_k(t)x + \Omega_k(t) + \eta_k^0$ and the parameter Z_k is complex.

By substituting Eq. (23) into Eq. (9) we have

$$\sum_{k=1}^{2} \left[\frac{P_k^2}{2}(Z_k+1) - \mathrm{i}(1-Z_k)(x\partial_t P_k + P_0 P_k + \partial_t \Omega_k) \right] \mathrm{e}^{\eta_k} = 0.$$

The above equation implies that $\partial_t P_k = 0$, $k = 1,2,$; that is, P_k is independent of t. So we get

$$Z_k = \frac{P_k P_0 + \mathrm{i}\partial_t \Omega_k - P_k^2/2}{P_k P_0 + \mathrm{i}\partial_t \Omega_k + P_k^2/2}, \tag{24}$$

which shows that $|Z_k|^2 = 1$. From Eq. (10) we have the restriction

$$P_k^2 = \lambda \left[2 - \left(Z_k + \overline{Z}_k \right) \right], k = 1,2. \tag{25}$$

From Eqs. (24) and (25) we have

$$Z_k = \mathrm{e}^{\mathrm{i}\phi_k}, \quad \phi_k = \frac{1}{2}\arctan\frac{P_k}{\sqrt{4\lambda - P_k^2}},$$
$$\Omega_k = P_k \int_0^t P_0(\tau)\,\mathrm{d}\tau + \sqrt{\lambda - P_k^2/4} P_k t + \zeta_k, \tag{26}$$

where ζ_k is a real constant, P_k is an arbitrary real parameter, and $\lambda \geq P_k^2/4$, $k = 1,2$.

Substituting Eq. (23) into Eqs. (11) and (12), after a tedious and expatiatory calculation we obtain the expressions of g_{2j}, and F_2 as

$$g_{2j} = A_{12} Z_1 Z_2 \mathrm{e}^{\eta_1+\eta_2}, \quad F_2 = A_{12}\mathrm{e}^{\eta_1+\eta_2}, \tag{27}$$

where the real parameter A_{12} is given by

$$A_{12} = \frac{4\lambda - P_1P_2 - \sqrt{(4\lambda - P_1^2)(4\lambda - P_2^2)}}{4\lambda + P_1P_2 - \sqrt{(4\lambda - P_1^2)(4\lambda - P_2^2)}}.$$

With the help of Eqs. (23)–(27) one can find equations (13) and (14) are satisfied to the moment.

With Eqs. (3), (17), (23), and (27) we obtain two dark–dark NSSs of Eq. (2) as

$$\psi_j = \tau_{0j}\mathrm{e}^{\mathrm{i}\eta_{0j}} \frac{1 + Z_1\mathrm{e}^{\eta_1} + Z_2\mathrm{e}^{\eta_2} + A_{12}Z_1Z_2\mathrm{e}^{\eta_1+\eta_2}}{1 + \mathrm{e}^{\eta_1} + \mathrm{e}^{\eta_2} + A_{12}\mathrm{e}^{\eta_1+\eta_2}}. \tag{28}$$

When $f(t) = 0$, the solution (28) denotes two autonomous dark–dark soliton interaction of the normal NLS equation. When $f(t) =$ constant, the solution (28) represents the dynamics of two nonautonomous nonlinear waves in a linearly inhomogeneous plasma or optical fibre with abnormal dispersion.

The solution in Eq. (28) describes a general scattering process of two dark NSSs for each component in BECs with different center velocities V_{1j} and V_{2j}, respectively. From Eq. (26) we get each velocity

$$V_{kj} = \int_0^t f(\tau)\,\mathrm{d}\tau + \xi_k - \sqrt{\lambda - P_k^2/4}.$$

Under the proper parameters two NSS can move toward each other, one with the velocity V_{1j}, while the other with V_{2j}. In order to understand the nature of two NSSs interaction, we analyze the asymptotic behavior of the solution in Eq. (28). Asymptotically, the solution in Eq. (28) can be written as a combination of two NSSs in Eq. (22). The asymptotic form of the two NSSs in limits $t \to -\infty$ and $t \to \infty$ is similar to that of one of the NSSs in Eq. (22).

(I) Before collision (limit $t \to -\infty$)

a) Nonautonomous soliton 1 ($\eta_1 \approx 0$, $\eta_2 \to -\infty$)

$$\psi_{1,j} \to \frac{\tau_{0j}}{2}\mathrm{e}^{\mathrm{i}\eta_{0j}}\left[1 + Z_1 - (1-Z_1)\tanh\frac{\eta_1}{2}\right]; \tag{29}$$

b) Nonautonomous soliton 2 ($\eta_2 \approx 0$, $\eta_1 \to \infty$)

$$\psi_{2,j} \to \frac{\tau_{0j}Z_2\mathrm{e}^{\mathrm{i}\eta_{0j}}}{2}\left[1 + Z_2 - (1-Z_2)\tanh\frac{\eta_2+\delta_0}{2}\right]. \tag{30}$$

(II) After collision (limit $t \to \infty$)

a) Nonautonomous soliton 1 ($\eta_1 \approx 0$, $\eta_2 \to \infty$)

$$\psi_{2,j} \to \frac{\tau_{0j}Z_2\mathrm{e}^{\mathrm{i}\eta_{0j}}}{2}\left[1 + Z_1 - (1-Z_1)\tanh\frac{\eta_1+\delta_0}{2}\right]; \tag{31}$$

b) Nonautonomous soliton 2 ($\eta_2 \approx 0$, $\eta_1 \to -\infty$)

$$\psi_{2,j} \to \frac{\tau_{0j}}{2}\mathrm{e}^{\mathrm{i}\eta_{0j}}\left[1 + Z_2 - (1-Z_2)\tanh\frac{\eta_1}{2}\right], \tag{32}$$

where the center shift is given by $\delta_0 = \ln A_{12}$. The asymptotic behavior demonstrates that there is no change in each soliton amplitude during collision, while one should notice that the factor $|Z_k| = 1$, $k = 1,2$. However, from Eqs. (29)–(32) we find a phase exchange δ_0 for soliton 1 and soliton 2 during collisions. These results show that the collision of two NSSs in each component is elastic.

3. Conclusion

In summary, we have reported on the analytical dark NSSs of two-component BECs with a time-dependent potential. From these results, we find that the time-dependent potential can affect the velocity of NSSs. The velocity of NSSs in quasi-one-dimensional BECs increases and oscillates with time. We also investigate the asymptotic behavior of two NSSs, which denotes the elastic collision.

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