Generation of hyperentangled four-photon cluster state via cross-Kerr nonlinearity*

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We propose a scheme for generating a hyperentangled four-photon cluster state that is simultaneously entangled in polarization modes and spatial modes. This scheme is based on linear optical elements, weak cross-Kerr nonlinearity, and homodyne detection. Therefore, it is feasible with current experimental technology.

Keywords: hyperentangled cluster state, cross-Kerr nonlinearity, homodyne detection, controlled-Z gate

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1. Introduction

Quantum entanglement is an important concept in quantum mechanics. It is a key ingredient in the field of quantum information processing,[1–7] such as quantum teleportation,[8,9] quantum key distribution (QKD),[10] and quantum secure direct communication (QSDC).[11] Recently, growing attention has been focused on the so-called hyperentanglement.[12–14] Hyperentanglement is a new form of entanglement, which involves simultaneous entanglements of more than one degree of freedom; for example, simultaneous polarization entanglement and spatial-mode entanglement. It can be applied to increase the level of security and simplifying quantum logic, which are offering significant advantages in quantum communication and quantum information processing.[15–17] Hyperentanglement plays a very important role in assisting the complete Bell-state analysis,[18,19] which is the completion of the most fundamental quantum communication processes involving bipartite entanglement. There has been much progress in terms of hyperentangled state. In 2010, Sheng *et al*. proposed a complete hyperentangled-Bell-state analysis scheme for quantum communication.[18] In 2012, Wang *et al*. generated and completely analyzed the hyperentangled Bell state for photons, assisted by quantum-dot spins in optical microcavities.[19] In 2013, Deng *et al*. presented a hyperentanglement concentration for two-photon four-qubit systems with linear optics.[20]

Currently, cross-Kerr nonlinearity has been well studied, both experimentally and theoretically. Nemoto *et al*. presented a pioneering cross-Kerr nonlinearity in quantum information processing.[21] In their paper, they constructed a quantum nondemolition detector (QND) based on the cross-Kerr nonlinearity. After their work, a series of studies have been done on the cross-Kerr nonlinearity, for example, Bell state analysis, entanglement purification, entanglement concentration, controlled-NOT (CNOT) gate construction,[22–25] and so on.[26–30]

The cluster state[31] is a kind of entangled state that is harder to be destroyed by local operations than the Greenberger–Horne–Zeilinger (GHZ) state and the W state.[32] Several physical systems have been proposed to prepare cluster states, such as cavity quantum electrodynamics (CQED) system and ion-trap system.[33–35] However, many schemes have focused on cluster states in a single degree of freedom.[36,37] Few schemes are proposed to generate hyperentangled cluster states. In this paper we present a scheme for generating a hyperentangled four-photon cluster state, which is based on weak cross-Kerr nonlinearity, linear optical elements, and homodyne detection.

This paper is organized as follows. In Section 2, we introduce the cross-Kerr nonlinear interaction between two light modes. In Section 3, we introduce the CZ gate and the parity analyzer with weak cross-Kerr nonlinearity. In Section 4, we present a scheme for preparation of the hyperentangled four-photon cluster state. A brief discussion and conclusion are given in Section 5.

2. Cross-Kerr nonlinearity

The cross-Kerr nonlinear interaction between a signal beam S and a probe beam P is shown in Fig. 1. The Hamiltonian for the interaction is

$$\hat{H} = -\hbar\chi\hat{n}_{\rm s}\hat{n}_{\rm p}, \tag{1}$$

where χ is the coupling strength, $\hat{n}_{\rm s}$ and $\hat{n}_{\rm p}$ are the photon number operators of the signal mode and the probe mode, re-

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spectively. If the signal mode is in a Fock state $|n\rangle_s$ and the probe mode is in a coherent state $|\alpha\rangle_p$, then the combined system (signal mode + probe mode) evolves as follows:

$$e^{i\chi\hat{n}_s\hat{n}_p t}|n\rangle_s|\alpha\rangle_p \rightarrow |n\rangle_s|\alpha e^{in\theta}\rangle_p, \quad (2)$$

where $\theta = \chi t$ and t is the interaction time. From expression (2) we can see that the state of the signal mode is unaffected, while the state of the probe mode picks up a phase shift that is proportional to the number of photons in the signal mode.[38] The phase shift of the probe mode can be measured by homodyne detection.[39] In this way, we can determine the state of the signal mode without destroying it by measuring the probe mode. This is a kind of non-destructive measurement.



Fig. 1. The cross-Kerr nonlinear interaction between a signal beam and a probe beam.

3. The controlled-Z gate and the parity analyzer

In our scheme for generating a hyperentangled four-photon cluster state, besides the cross-Kerr nonlinear interaction, there are also some other ingredients, for example: the controlled-Z (CZ) gate and the parity analyzer (PA). Let us first introduce the CZ gate and the parity analyzer.

The CZ gate with weak cross-Kerr nonlinearity is shown in Fig. 2. We suppose that the input photons are prepared in the state

$$|\varphi_1\rangle_{bc} = a|HH\rangle_{bc} + b|HV\rangle_{bc} + c|VH\rangle_{bc} + d|VV\rangle_{bc}, \quad (3)$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$, H and V represent horizontal and vertical polarizations, respectively. The controlling photon b passes through a balanced Mach–Zehnder (MZ) interferometer made up of two PBSs, while the target photon c passes through a balanced MZ interferometer made up of two BSs. The Pauli operator σ_z acts on one arm. The input photons b and c, respectively, interact with the probe mode (in a coherent state $|\alpha\rangle$) via the cross-Kerr nonlinearities, the state then evolves as follows:

$$\begin{aligned}|\varphi_1\rangle_{bc}\otimes|\alpha\rangle &= (a|HH\rangle_{bc} + b|HV\rangle_{bc} \\ &\quad + c|VH\rangle_{bc} + d|VV\rangle_{bc})\otimes|\alpha\rangle \rightarrow |\phi\rangle \\ &= \sqrt{\frac{R_1T_2}{2}}(a|HH\rangle_{bc} + b|HV\rangle_{bc})|\alpha e^{-i\theta}\rangle \\ &\quad + \sqrt{\frac{T_1R_2}{2}}(c|VH\rangle_{bc} - d|VV\rangle_{bc})|\alpha e^{i\theta}\rangle \\ &\quad + [\sqrt{\frac{T_1R_2}{2}}(a|HH\rangle_{bc} - b|HV\rangle_{bc}) \\ &\quad + \sqrt{\frac{R_1T_2}{2}}(c|VH\rangle_{bc} + d|VV\rangle_{bc})]\otimes|\alpha\rangle, \quad (4)\end{aligned}$$

where R and T represent reflection and transmission coefficients, respectively. We perform the homodyne detection on the quadrature X of the probe mode. If we find that the probe mode is in the coherent state $|\alpha e^{\pm i\theta}\rangle$, we do a feed-forward operation on photon c and obtain the following state

$$\begin{aligned}|\varphi_2\rangle_{bc} &= \sqrt{R_1T_2}(a|HH\rangle_{bc} + b|HV\rangle_{bc}) \\ &\quad + \sqrt{T_1R_2}(c|VH\rangle_{bc} - d|VV\rangle_{bc}). \quad (5)\end{aligned}$$

On the other hand, if we find that the probe mode is in the coherent state $|\alpha\rangle$, we carry out a single photon operation σ_z on photon c and obtain the the following state

$$\begin{aligned}|\varphi_3\rangle_{bc} &= \sqrt{T_1R_2}(a|HH\rangle_{bc} + b|HV\rangle_{bc}) \\ &\quad + \sqrt{R_1T_2}(c|VH\rangle_{bc} - d|VV\rangle_{bc}. \quad (6)\end{aligned}$$

When the conditions $\sqrt{T_1R_2} = \sqrt{R_1T_2}$ and $T_1 = R_2 = 1/2$ are satisfied, we obtain a nearly deterministic CZ gate; that is, the success probability is $p = 4T_1R_2 = 1$.

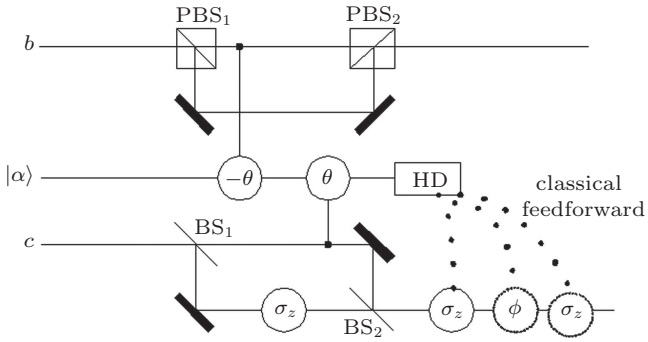


Fig. 2. The controlled-Z gate. PBS: polarization beam splitter; BS: beam splitter; HD: homodyne detection.

The schematic diagram of the PA is shown in Fig. 3. The input photons are prepared in the spatial mode

$$\frac{1}{2}(|b_1\rangle_b + |b_2\rangle_b)\otimes(|c_1'\rangle_c + |c_2'\rangle_c). \quad (7)$$

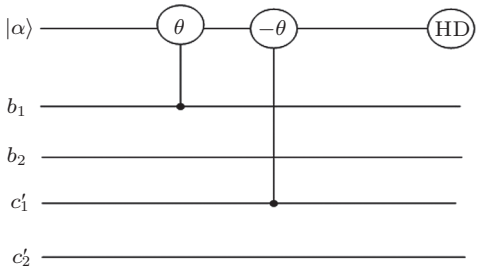


Fig. 3. The parity analyzer (PA). θ, $-\theta$: cross-Kerr nonlinearity. b_1 (b_2), c_1' (c_2'): the spatial modes of photons.

The function of the PA can be described as

$$\begin{aligned}&\frac{1}{2}(|b_1\rangle_b + |b_2\rangle_b)\otimes(|c_1'\rangle_c + |c_2'\rangle_c)\otimes|\alpha\rangle \rightarrow \\ &\frac{1}{2}(|b_1\rangle_b|c_1'\rangle_c + |b_2\rangle_b|c_2'\rangle_c)|\alpha\rangle \\ &+ \frac{1}{2}(|b_1\rangle_b|c_2'\rangle_c|\alpha e^{i\theta}\rangle + |b_2\rangle_b|c_1'\rangle_c|\alpha e^{-i\theta}\rangle). \quad (8)\end{aligned}$$

If the probe mode picks up no phase shift, we get the even parity state

$$\frac{1}{\sqrt{2}}(|b_1\rangle_b|c_1'\rangle_c+|b_2\rangle_b|c_2'\rangle_c). \tag{9}$$

If the probe mode picks up a $\pm\theta$ phase shift, we get the odd parity state

$$\frac{1}{\sqrt{2}}(|b_1\rangle_b|c_2'\rangle_c+|b_2\rangle_b|c_1'\rangle_c). \tag{10}$$

4. Preparation of the hyperentangled four-photon cluster state

In the following, we present the scheme for generating the hyperentangled four-photon cluster state in detail. The schematic diagram is shown in Fig. 4.

We suppose that the input photons are prepared in the state

$$|\varphi_1^{(\mathrm{p})}\rangle = \frac{1}{\sqrt{2}}(|HH\rangle_{ab}+|VV\rangle_{ab}) \otimes\frac{1}{\sqrt{2}}(|HH\rangle_{cd}+|VV\rangle_{cd}), \tag{11}$$

where H and V denote horizontal and vertical linear polarizations, respectively. Let us first apply a CZ gate on photons b and c. The unitary transformation of the CZ gate can be described as

$$\begin{aligned}&|HH\rangle_{bc}\to|HH\rangle_{bc},\quad |HV\rangle_{bc}\to|HV\rangle_{bc},\\ &|VH\rangle_{bc}\to|VH\rangle_{bc},\quad |VV\rangle_{bc}\to-|VV\rangle_{bc}.\end{aligned} \tag{12}$$

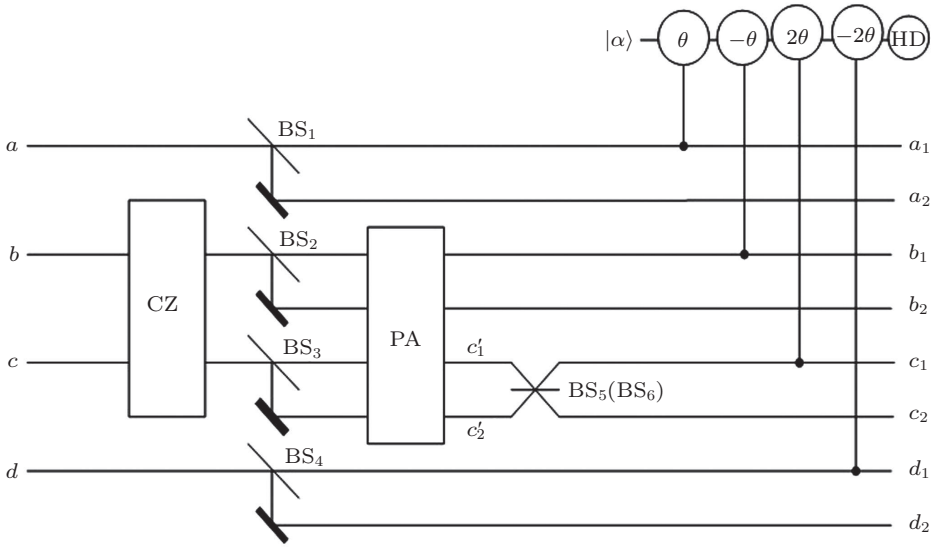


Fig. 4. The schematic diagram for generating a hyperentangled four-photon cluster state. BS: beam splitter; HD: homodyne detection; θ, $-\theta$, 2θ, -2θ: cross-Kerr nonlinearities; CZ: controlled-Z gate; PA: parity analyzer. The probe light is initially in a coherent state $|\alpha\rangle$, while photons a and b (c and d) are initially in the polarization entangled states.

After carrying out the CZ gate on photons b and c, we obtain the four-photon cluster state in the polarization mode

$$|\varphi_2^{(\mathrm{P})}\rangle = \frac{1}{2}(|HHHH\rangle_{abcd}+|HHVV\rangle_{abcd} +|VVHH\rangle_{abcd}-|VVVV\rangle_{abcd}). \tag{13}$$

Then, we let photons a, b, c, and d pass through the beam splitters BS$_1$, BS$_2$, BS$_3$, and BS$_4$, respectively. The functions of the four beam splitters can be described as

$$\begin{aligned}&a\to\frac{1}{\sqrt{2}}(|a_1\rangle_a+|a_2\rangle_a),\quad b\to\frac{1}{\sqrt{2}}(|b_1\rangle_b+|b_2\rangle_b),\\ &c\to\frac{1}{\sqrt{2}}(|c_1'\rangle_c+|c_2'\rangle_c),\quad d\to\frac{1}{\sqrt{2}}(|d_1\rangle_d+|d_2\rangle_d).\end{aligned} \tag{14}$$

The four-photon state in the spatial mode will evolve as

$$|\varphi_1^{(\mathrm{s})}\rangle = |abcd\rangle_{abcd}\to$$

$$|\varphi_2^{(\mathrm{s})}\rangle = \frac{1}{4}(|a_1\rangle_a+|a_2\rangle_a)\otimes(|b_1\rangle_b+|b_2\rangle_b) \otimes(|c_1'\rangle_c+|c_2'\rangle_c)\otimes(|d_1\rangle_d+|d_2\rangle_d). \tag{15}$$

The photons in the spatial mode b_1, b_2, c_1', and c_2' pass through a parity analyzer (PA). If the photons b and c are in the even parity state, the state of the four-photon spatial mode will evolve as

$$\begin{aligned}&|\varphi_2^{(\mathrm{s})}\rangle\to|\varphi_3^{(\mathrm{s}_1)}\rangle\\ &=\frac{1}{2\sqrt{2}}(|a_1\rangle_a+|a_2\rangle_a)\otimes(|b_1\rangle_b|c_1'\rangle_c+|b_2\rangle_b|c_2'\rangle_c)\\ &\otimes(|d_1\rangle_d+|d_2\rangle_d).\end{aligned} \tag{16}$$

On the other hand, if the photons b and c are in the odd parity state, we obtain the following four-photon state in the spatial mode

$$|\varphi_3^{(\mathrm{s}_2)}\rangle = \frac{1}{2\sqrt{2}}(|a_1\rangle_a+|a_2\rangle_a)\otimes(|b_1\rangle_b|c_2'\rangle_c$$

$$+|b_2\rangle_b|c_1'\rangle_c)\otimes(|d_1\rangle_d+|d_2\rangle_d). \quad (17)$$

When we have the four-photon state $|\varphi_3^{(s_1)}\rangle$, we let the photons in the spatial modes c_1' and c_2' pass through BS_5, which can realize the following transformation

$$\begin{aligned}|c_1'\rangle_c &\to \frac{1}{\sqrt{2}}(|c_1\rangle_c+|c_2\rangle_c),\\ |c_2'\rangle_c &\to \frac{1}{\sqrt{2}}(|c_1\rangle_c-|c_2\rangle_c).\end{aligned} \quad (18)$$

Otherwise, when we have the four-photon state $|\varphi_3^{(s_2)}\rangle$, we let the photons in the spatial modes c_1' and c_2' pass through BS_6. The beam splitter can accomplish the following transformation

$$|c_1'\rangle_c \to \frac{1}{\sqrt{2}}(|c_1\rangle-|c_2\rangle),\quad |c_2'\rangle_c \to \frac{1}{\sqrt{2}}(|c_1\rangle+|c_2\rangle). \quad (19)$$

After the transformation, both the states $|\varphi_3^{(s_1)}\rangle$ and $|\varphi_3^{(s_2)}\rangle$ will change into

$$\begin{aligned}&|\varphi_3^{(s_1)}\rangle(|\varphi_3^{(s_2)}\rangle)\to\\ &|\varphi_4^{(s)}\rangle=\frac{1}{4}(|a_1\rangle_a+|a_2\rangle_a)\otimes[|b_1\rangle_b(|c_1\rangle_c+|c_2\rangle_c)\\ &\quad+|b_2\rangle_b(|c_1\rangle_c-|c_2\rangle_c)]\otimes(|d_1\rangle_d+|d_2\rangle_d).\end{aligned} \quad (20)$$

After obtaining the state $|\varphi_4^{(s)}\rangle$, we let the photons in the spatial modes a_1, b_1, c_1, and d_1 interact with a coherent state via the cross-Kerr nonlinearities. Suppose the photons in the spatial modes a_1, b_1, c_1, d_1 induce, respectively, phase shifts θ, $-\theta$, 2θ, -2θ in the coherent state, that is

$$\begin{aligned}U|a_1\rangle_a|\alpha\rangle&=|a_1\rangle_a|\alpha\,\mathrm{e}^{\mathrm{i}\theta}\rangle,\\ U|b_1\rangle_b|\alpha\rangle&=|b_1\rangle_b|\alpha\,\mathrm{e}^{-\mathrm{i}\theta}\rangle,\\ U|c_1\rangle_c|\alpha\rangle&=|c_1\rangle_c|\alpha\,\mathrm{e}^{\mathrm{i}2\theta}\rangle,\\ U|d_1\rangle_d|\alpha\rangle&=|d_1\rangle_d|\alpha\,\mathrm{e}^{-\mathrm{i}2\theta}\rangle,\end{aligned} \quad (21)$$

then the state of the combined system will evolve as follows:

$$\begin{aligned}&|\varphi_4^{(s)}\rangle|\alpha\rangle\to|\phi_2\rangle\\ &=\frac{1}{4}(|\varphi_4^{(s_0)}\rangle|\alpha\rangle+|\varphi_4^{(s_1)}\rangle|\alpha\,\mathrm{e}^{\mathrm{i}\theta}\rangle+|\varphi_4^{(s_{-1})}\rangle|\alpha\,\mathrm{e}^{-\mathrm{i}\theta}\rangle\\ &\quad+|\varphi_4^{(s_2)}\rangle|\alpha\,\mathrm{e}^{\mathrm{i}2\theta}\rangle+|\varphi_4^{(s_{-2})}\rangle|\alpha\,\mathrm{e}^{-\mathrm{i}2\theta}\rangle\\ &\quad+|\varphi_4^{(s_3)}\rangle|\alpha\,\mathrm{e}^{\mathrm{i}3\theta}\rangle+|\varphi_4^{(s_{-3})}\rangle|\alpha\,\mathrm{e}^{-\mathrm{i}3\theta}\rangle),\end{aligned} \quad (22)$$

where

$$\begin{aligned}|\varphi_4^{(s_0)}\rangle &= \frac{1}{2}(|a_1b_1c_1d_1\rangle_{abcd}+|a_1b_1c_2d_2\rangle_{abcd}\\ &\quad+|a_2b_2c_1d_1\rangle_{abcd}-|a_2b_2c_2d_2\rangle_{abcd}),\\ |\varphi_4^{(s_1)}\rangle &= \frac{1}{\sqrt{3}}(|a_1b_2c_1d_1\rangle_{abcd}-|a_1b_2c_2d_2\rangle_{abcd}\\ &\quad+|a_2b_1c_1d_2\rangle_{abcd}),\\ |\varphi_4^{(s_{-1})}\rangle &= \frac{1}{\sqrt{3}}(|a_2b_1c_1d_1\rangle_{abcd}-|a_1b_2c_2d_1\rangle_{abcd}\\ &\quad+|a_2b_1c_2d_2\rangle_{abcd}),\\ |\varphi_4^{(s_2)}\rangle &= \frac{1}{\sqrt{2}}(|a_1b_1c_1d_2\rangle_{abcd}+|a_2b_2c_1d_2\rangle_{abcd}),\\ |\varphi_4^{(s_{-2})}\rangle &= \frac{1}{\sqrt{2}}(|a_1b_1c_2d_1\rangle_{abcd}-|a_2b_2c_2d_1\rangle_{abcd}),\\ |\varphi_4^{(s_3)}\rangle &= |a_1b_2c_1d_2\rangle_{abcd},\\ |\varphi_4^{(s_{-3})}\rangle &= |a_2b_1c_2d_1\rangle_{abcd}.\end{aligned} \quad (23)$$

With the help of the homodyne detection on the quadrature P, if the probe mode does not have a phase shift, the state $|\phi_2\rangle$ will collapse into

$$\begin{aligned}|\varphi_4^{(s_0)}\rangle &= \frac{1}{2}(|a_1b_1c_1d_1\rangle_{abcd}+|a_1b_1c_2d_2\rangle_{abcd}\\ &\quad+|a_2b_2c_1d_1\rangle_{abcd}-|a_2b_2c_2d_2\rangle_{abcd}).\end{aligned} \quad (24)$$

This is a four-photon cluster state in the spatial mode. Thus, by combing Eq. (13) with Eq. (24), we generate a hyperentangled four-photon cluster state in the following form

$$\begin{aligned}|\varphi_5^{(ps)}\rangle &= \frac{1}{4}(|HHHH\rangle_{abcd}+|HHVV\rangle_{abcd}\\ &\quad+|VVHH\rangle_{abcd}-|VVVV\rangle_{abcd})\\ &\quad\otimes(|a_1b_1c_1d_1\rangle_{abcd}+|a_1b_1c_2d_2\rangle_{abcd}\\ &\quad+|a_2b_2c_1d_1\rangle_{abcd}-|a_2b_2c_2d_2\rangle_{abcd}).\end{aligned} \quad (25)$$

5. Discussion and conclusion

Now, we discuss the probability of getting the four-photon cluster state in the spatial mode $|\varphi_4^{(s_0)}\rangle$ Eq. (24) from the state $|\phi_2\rangle$ Eq. (22) with the homodyne detection on the quadrature P. The state (22) are made of a spatial mode and a coherent beam mode, so with the homodyne detection on the quadrature P of the coherent beam, the state $|\phi_2\rangle$ will change into

$$|\phi_2(p)\rangle=\langle p|\phi_2\rangle=\sum_{m=-3}^{3}c_m|\varphi_4^{(s_m)}\rangle, \quad (26)$$

where

$$\begin{aligned}&c_0=\frac{1}{2}f_0\,\mathrm{e}^{\mathrm{i}\tau_0},\quad c_{\pm1}=\frac{\sqrt{3}}{4}f_{\pm1}\,\mathrm{e}^{\mathrm{i}\tau_{\pm1}},\\ &c_{\pm2}=\frac{\sqrt{2}}{4}f_{\pm2}\,\mathrm{e}^{\mathrm{i}\tau_{\pm2}},\quad c_{\pm3}=\frac{1}{4}f_{\pm3}\,\mathrm{e}^{\mathrm{i}\tau_{\pm3}},\end{aligned} \quad (27)$$

$$\begin{aligned}&f_m=\left(\frac{1}{\pi}\right)^{1/4}\exp\left[-\frac{1}{2}(p-\sqrt{2}\alpha\sin m\theta)^2\right],\\ &\tau_m=\sqrt{2}\alpha\left(p-\frac{1}{\sqrt{2}}\sin m\theta\right)\cos m\theta.\end{aligned} \quad (28)$$

The density operator of the combined system can be written as

$$\rho=|\phi_2\rangle\langle\phi_2|. \quad (29)$$

And the reduced density operator of the spatial mode will be

$$\rho_{\mathrm{r}}=\mathrm{Tr}_\alpha(\rho)=\int_{-\infty}^{+\infty}\langle p|\phi_2\rangle\langle\phi_2|p\rangle\,\mathrm{d}p$$

$$= \int_{-\infty}^{+\infty} \sum_{mn} c_m c_n^* |\varphi_4^{(s_m)}\rangle\langle\varphi_4^{(s_n)}| \, \mathrm{d}p. \tag{30}$$

Therefore, the probability of getting the four-photon cluster state in a spatial mode $|\varphi_4^{(s_0)}\rangle$ is

$$P_0 = \langle\varphi_4^{(s_0)}|\rho_r|\varphi_4^{(s_0)}\rangle = \int_{-\infty}^{+\infty} |c_0|^2 \, \mathrm{d}p = \frac{1}{4}. \tag{31}$$

So far, we have fully presented our scheme. Let us now discuss its experimental feasibility. Our scheme is based on the cross-Kerr nonlinearity. However, we know that natural cross-Kerr nonlinearities are extremely weak. If the signal mode is in a single photon state, then the phase shift of the coherent probe field is very small, and this creates a considerable challenge in experiments. Hofmann, Banacloche and Shapiro pointed out these problems in their work.[40,41] Fortunately, much effort has been made for the large phase shift with weak cross-Kerr nonlinearity. Hofmann *et al.* showed that a phase shift of π can be achieved with a single two-level atom in a one-sided cavity.[42] Feizpour *et al.* showed that weak measurement could be used to amplify optical nonlinearities at the single-photon level, such that the effect of one properly postselected photon on a classical beam may be as large as that of many unpostselected photons.[43] Hoi *et al.* showed that they could get giant cross-Kerr effect for propagating microwaves induced by an artificial atom.[44] That is to say, if we can choose appropriate Kerr media and effective measurement, the weak cross-Kerr nonlinearity can be efficiently enhanced. The non-destructive measurement is also a crucial ingredient in our scheme. With weak cross-Kerr nonlinearity, we can implement non-destructive measurements with a high precision. This scheme is also influenced by the quantum CZ gates, and we have introduced a nearly deterministic CZ gate with a simple structure.

In summary, we have presented a scheme for generating the hyperentangled four-photon cluster state, the advantage of this scheme is the achievement of the simultaneous entanglement in polarization modes and spatial modes. In our scheme, we first construct a controlled-Z (CZ) gate and a parity analyzer (PA) by using the cross-Kerr nonlinearity, the homodyne detection, and some linear optical elements (for example, BS and PBS). It is interesting that the constructed CZ gate is nearly deterministic. Then, by using the CZ gate and the PA, combining with the cross-Kerr nonlinearity, the homodyne detection, and some linear optical elements, we construct a scheme for generating the hyperentangled four-photon cluster state.

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