# Efficient scheme for realizing quantum dense coding with GHZ state in separated low-Q cavities*

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We propose an efficient scheme for realizing quantum dense coding with three-particle GHZ state in separated low-Q cavities. In this paper, the GHZ state is first prepared with three atoms trapped, respectively, in three spatial separated cavities. Meanwhile, with the assistance of a coherent optical pulse and X-quadrature homodyne measurement, we can implement quantum dense coding with three-particle GHZ state with a higher probability. Our scheme can also be generalized to realize $N$-particle quantum dense coding.

**Keywords:** GHZ state, quantum dense coding, low-Q cavity, X-quadrature homodyne measurement

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## 1. Introduction

Quantum entanglement is one of the most fascinating features of quantum information theory and it plays a vital role in quantum mechanics. With the wide application of entanglement in quantum information processing, such as quantum teleportation,[1] quantum dense coding,[2] quantum cryptography,[3] etc., a lot of schemes for preparing various entangled states have been proposed.[4–15] Quantum dense coding,[16] as one of the most important applications of quantum entanglement in quantum communication, can transmit more bits of classical information with the help of entanglement. Utilizing the characteristic of quantum entanglement, many schemes of quantum dense coding have been proposed; for example, by using an optical system and nuclear magnetic resonance (NMR).[17–19] A cavity QED system has been considered as one of the most promising candidate systems for experimentally realizing quantum communication. Consequently, many kinds of protocols are proposed that carry out dense coding by using bipartite or tripartite entangled states.[20–27] There are also some examples that use an extended GHZ-type state.[28–30] Recently, Nemoto and Munro,[31] and Wang *et al.*[32] have put forward a protocol to simplify the original photon parity meter[31] by removing two polarization beam splitters (PBS) and the cross-Kerr nonlinearity.[32] Motivated by their ideas, we propose a new protocol to generate $N$-particle GHZ state with the help of the low-Q cavity ($\kappa \gg \gamma$) and X-quadrature homodyne measurement. Unlike previous schemes,[33,34] the present scheme has a low requirement for the quality factor of the cavity. The probability of success to achieve GHZ state is 100%, which is higher than that in Ref. [35], whose successful probability is 25%. Moreover, in this scheme we adopt the detection method of X-quadrature homodyne measurement, whose nature is to detect the phase of the coherent state. Since the photon detector is probabilistic while phase detection is deterministic, and this detection method can overcome the quantum noise caused by repeated operations, in principle the X-quadrature homodyne measurement can be accurate to an arbitrarily weak signal. In summary, this plan is much more efficient than single-photon detection and it is able to cater to the demands of modern experimental technology. That is to say, this scheme has a more highly successful probability than that in Ref. [36]. The most important thing is that our plan conforms to the features of a computer, which likes to operate the same and simple operation. From the perspective of dense coding, this scheme has some other significant advantages. Firstly, this scheme can carry more classical information than that of Refs. [25] and [37]. This advantage will be able to be put to good use in the design of superquantum computer in the near future because, properly speaking, it can reduce the number of nodes of the quantum computer but still maintain the stability of the superquantum computer. In our scheme, Alice encodes her two particles of three-bit GHZ state while three bits of classical information can be carried. However in Refs. [25] and [37] Alice encoded only one particle of the three-bit entangled state, and only two bits of classical information can be transmitted. So, our scheme carries more classical information. Secondly, our scheme is more feasible than those proposed in Refs. [37]–[42] due to the fact that it overcomes the defects of cross-Kerr nonlinearity medium, which can only achieve a small angle.

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The rest of this paper is organized as follows. In Section 2, we prepare the GHZ state depending on the interaction between coherent state and atomic state. In Section 3, we will show how to implement quantum dense coding with three particles and generalize it to $N$-particle dense coding. We will then give a conclusion in Section 4.

## 2. Generation of GHZ state

Firstly, we will explain the working mechanism of single-photon input–output process with respect to low-Q cavities. We consider the case that the confined atom has a three-level configuration. As shown in Fig. 1(b), in which there are shown an excited level and two ground levels, the atomic state transforms from $|1\rangle$ to $|e\rangle$, which can be driven by the input field. For the large hyperfine splitting result, the state $|0\rangle$ is decoupled from the cavity mode.

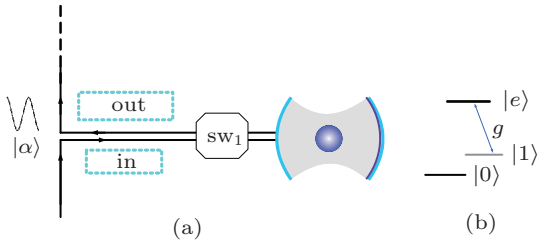



**Fig. 1.** (color online) (a) Schematic setup to implement a phase change in coherent beam induced by input–output process, and (b) the level structure of the atom, with g being the coupling induced by a cavity field.

The $\mathrm{sw}_1$ is a classical feedback control operation according to Ref. [43], which acts as a quantum switch to allow the coherent optical pulse to selectively pass through the in-path road or out-path road. The coherent light pulse enters into the cavity from the in-path road as a precondition to drive the atom interacting with its cavity and the coherent state. After their interaction, the coherent state obtains a phase but different atomic states (the ground state, the excited state) gives rise to different phases in the coherent state. Then, owing to the reflective coating on the inner surface of the cavity, the coherent optical pulse will leave out of this cavity and enter into the next cavity through the out-path road to participate in the interaction with the next set of atom and cavity, while this atom and its cavity have been out of the driving field, they should be stable in the present state after the first interaction. After repeating the above operation, the coherent light will eventually obtain a total phase. There is a corresponding relationship between the coherent light's phase and atomic state. In the following, we will recount how the systematic state evolves under the mechanism described in Fig. 1(a). In the interaction picture, the system Hamiltonian is[44]

$$H = \hbar\left(\omega_0-\omega_\mathrm{p}\right)|e\rangle\langle e| + \hbar\left(\omega_\mathrm{c}-\omega_\mathrm{p}\right)a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\sigma_-\right), \tag{1}$$

where $\sigma_+ = |e\rangle\langle 1|$, $\sigma_- = |1\rangle\langle e|$, $\omega_0$ denotes the resonant frequency between the excited state $|e\rangle$ and the ground state $|1\rangle$, $\omega_\mathrm{c}$ and $\omega_\mathrm{p}$ refer to the frequencies of the cavity and the input pulse, respectively. We use $\delta_1$ and $\delta_2$ to denote $\omega_0-\omega_\mathrm{p}$ and $\omega_\mathrm{c}-\omega_\mathrm{p}$, respectively, for simplicity. The cavity mode satisfying the commutation relation $\left[a_\mathrm{in}(t), a_\mathrm{in}^\dagger(t')\right] = \delta(t-t')$ is driven by the input field $a_\mathrm{in}$, and the quantum Langevin equation of the cavity is[45]

$$\dot{a} = -\left(\mathrm{i}\delta_2 + \frac{\kappa}{2}\right)a - \mathrm{i}g\sigma_- - \sqrt{\kappa}a_\mathrm{in}, \tag{2}$$

where $\kappa$ is the cavity damping rate. Moreover, the atomic lowing operator also obeys a similar equation

$$\dot{\sigma}_- = -\left(\mathrm{i}\delta_1 + \frac{\gamma}{2}\right)\sigma_- + \mathrm{i}g\sigma_z a + \sqrt{\gamma}\sigma_z b_\mathrm{in}, \tag{3}$$

where $\gamma$ is the decay rate of the two-level atom, $\sigma_z = |e\rangle\langle e| - |1\rangle\langle 1|$, and $b_\mathrm{in}$ is the vacuum input field with the commutation relation $\left[b_\mathrm{in}(t), b_\mathrm{in}^\dagger(t')\right] = \delta(t-t')$. The contribution of the vacuum field is much less than that of the photon pulse[44], so we regard it as being negligible. Thus, we will not consider $b_\mathrm{in}$ in the following. The relation between the input and output fields of the cavity is

$$a_\mathrm{out} = a_\mathrm{in} + \sqrt{\kappa}a. \tag{4}$$

Assume that $\kappa$ is large enough to determine that we have a weak excitation by the single-photon pulse on the atom, which is initially prepared in the ground state. In accordance with Eqs. (1), (2), (3), and (4), we can obtain the input–output relation that can be expressed as

$$a_\mathrm{out} = r_1 a_\mathrm{in},$$
$$r_1 = \frac{(\mathrm{i}\delta_2-\kappa/2)(\mathrm{i}\delta_1+\gamma/2)+g^2}{(\mathrm{i}\delta_2+\kappa/2)(\mathrm{i}\delta_1+\gamma/2)+g^2} \quad (\text{atom in state } |1\rangle), \tag{5}$$
$$a_\mathrm{out} = r_0 a_\mathrm{in}, r_0 = \frac{\mathrm{i}\delta_2-\kappa/2}{\mathrm{i}\delta_2+\kappa/2} \quad (\text{atom in state } |0\rangle). \tag{6}$$

When the initial input pulse is prepared in the coherent state $|\alpha\rangle$, and the atom is prepared in state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, then the initial state of the system in Fig. 1(a) will be

$$|\zeta\rangle_1 = \frac{|0\rangle+|1\rangle}{\sqrt{2}}\mathrm{e}^{\alpha a_\mathrm{in}^\dagger - \alpha^* a_\mathrm{in}}|0\rangle_\mathrm{in}. \tag{7}$$

When $\delta_1 \gg \gamma/2$, after applying the cavity-assisted transformation, state $|\zeta\rangle_1$ is evolved into

$$|\zeta\rangle_2 = \frac{1}{\sqrt{2}}(|0\rangle|\alpha\mathrm{e}^{\mathrm{i}\theta_0}\rangle_\mathrm{out} + |1\rangle|\alpha\mathrm{e}^{\mathrm{i}\theta_1}\rangle_\mathrm{out}), \tag{8}$$

where $\theta_n = \arg(r_n)$ $(n=0,1)$ are controlled by $\delta_1$, $\delta_2$, and $g$ in the case of low-Q cavity $(\kappa \gg \gamma)$.

Now, we will show the generation of three-particle GHZ state. According to Ref. [36] when $\delta_1 = \delta_2 = \kappa/2$ and $g = \kappa/\sqrt{2}$ the input coherent optical pulse will pick up $\pi/2(-\pi/2)$ phase shift conditioned the atomic state $|0\rangle(|1\rangle)$.

Now, we make $\theta_0 = \pi/2$; that is to say, as shown in the left dotted box of Fig. 2. Firstly, we input a sustainable and strong coherent light pulse $|\alpha\rangle$ and only turn on $sw_1$ and $sw_2$ to make atoms $A$ and $B$ interact with each cavity and the coherent state $|\alpha\rangle$, the initial state of the whole system

$$|\varphi\rangle_0 = \frac{1}{2\sqrt{2}}(|0\rangle + |1\rangle)_A \otimes (|0\rangle + |1\rangle)_B \otimes (|0\rangle + |1\rangle)_C |\alpha\rangle \quad (9)$$

will be evolved into the state

$$\begin{aligned}|\varphi\rangle_1 = & \frac{1}{2\sqrt{2}}\big[(|0\rangle_A|0\rangle_B|\alpha e^{i2\theta_0}\rangle + |1\rangle_A|1\rangle_B|\alpha e^{-i2\theta_0}\rangle) \\ & + (|0\rangle|1\rangle + |1\rangle|0\rangle)_{AB}|\alpha\rangle\big] \otimes (|0\rangle + |1\rangle)_C. \end{aligned} \quad (10)$$

Now we carry out the first X-quadrature homodyne measurement (we call it $XQHM_1$) on particles $A$ and $B$, on the basis of Refs. [46] and [47], X-quadrature homodyne measurement can reveal the coherent states that have different size phases but it does not distinguish the phase signs (plus or minus) of the coherent states. So, according to the measurement result of the coherent state's phase, the corresponding states of atom $A$ and atom $B$ will collapse into $\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$. Next, we keep the $sw_2$ on, but turn off $sw_1$ and turn on $sw_3$ to just allow atom $B$ and atom $C$ to participate in the reaction with their respective cavity and pulse, the state of the system can be expressed as

$$\begin{aligned}|\varphi\rangle_2 = & \frac{1}{2\sqrt{2}}[(|0\rangle_A|0\rangle_B|0\rangle_C|\alpha e^{i3\theta_0}\rangle \\ & + |1\rangle_A|1\rangle_B|1\rangle_C|\alpha e^{-i3\theta_0}\rangle) \\ & + (|0\rangle_A|0\rangle_B|1\rangle_C|\alpha e^{i\theta_0}\rangle \\ & + |1\rangle_A|1\rangle_B|0\rangle_C|\alpha e^{-i\theta_0}\rangle)], \end{aligned} \quad (11)$$

this time we perform the second X-quadrature homodyne measurement, which is called $XQHM_2$, on particles $B$ and $C$. Then according to the results of detecting the coherent state's phase, we will have the corresponding atomic states

$$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C),$$
$$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|1\rangle_C + |1\rangle_A|1\rangle_B|0\rangle_C),$$

respectively. From above analyses we see that the scheme of preparation three-particle GHZ state is deterministic.

The generation of $N$-particle GHZ state is similar to the above operation process, the only things needed are more control switches, an X-quadrature homodyne measurement, and atoms dropped in low-Q cavity ($\kappa \gg \gamma$) with the original state of atom 1, 2, 3, ...$N$, and should be set to be

$$|\xi\rangle_{1,2,3,\ldots N} = \bigotimes_{i=1}^{N} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \quad (12)$$

After a series of similar operations, finally, according to the result of $(N-1)$-th X-quadrature homodyne measurement, the $N$-particle GHZ state

$$\begin{aligned}& |\mathrm{GHZ}\rangle_{1,2,3,\ldots N} \\ = & \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3\ldots|0\rangle_N - |1\rangle_1|1\rangle_2|1\rangle_3\ldots|1\rangle_N), \\ & |\mathrm{GHZ}\rangle_{1,2,3,\ldots N} \\ = & \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|1\rangle_3\ldots|0\rangle_N - |1\rangle_1|1\rangle_2|1\rangle_3\ldots|0\rangle_N) \end{aligned}$$

will, respectively, be prepared.

## 3. Dense coding with three-particle GHZ state

Now, we implement the quantum dense coding with the three-particle GHZ state.

The operations on the left dotted box are manipulated by Alice and are composed of two parts: the operations to prepare 3-particle GHZ state are designed in part I and Alice's encoding operations on her particles are presented in part II. The operations in the right dotted line box are performed by Bob. Parts 1, 2, and 3 separately represent the first, the second, and the third step, just taking advantage of the three steps Bob can successfully distinguish Alice's encoding operations on her particles.

After the operations contained in part I, we succeed in the preparation of the three-bit GHZ state. Suppose that Alice and Bob share the entangled state

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|000\rangle_{ABC} + |111\rangle_{ABC}). \quad (13)$$

Alice owns particles $A$ and $B$, while Bob has particle $C$. To realize the quantum dense coding, Alice simultaneously performs each of the four local operations $\{I, \sigma_x, i\sigma_y, \sigma_z\}$ ($I$ is the identity operator and $\sigma_i$ are three Pauli operators), respectively, on her atoms (atom $A$ and atom $B$) as shown in part II. The operations on atoms $A$ and $B$ evolve the state $|\psi\rangle_0$ into the states which are given in the following forms:

$$|\psi\rangle_{1,4} = \frac{1}{\sqrt{2}}(|000\rangle_{ABC} \pm |111\rangle_{ABC}), \quad (14)$$
$$|\psi\rangle_{2,3} = \frac{1}{\sqrt{2}}(|101\rangle_{ABC} \pm |010\rangle_{ABC}), \quad (15)$$
$$|\psi\rangle_{5,8} = \frac{1}{\sqrt{2}}(|100\rangle_{ABC} \pm |011\rangle_{ABC}), \quad (16)$$
$$|\psi\rangle_{6,7} = \frac{1}{\sqrt{2}}(|001\rangle_{ABC} \pm |110\rangle_{ABC}). \quad (17)$$

Then, Alice wants to send her atoms $A$ and $B$ to Bob. According to Refs. [48] and [49] she can adjust the powers of the laser beams used to trap atoms $A$ and $B$ to move out of her cavities, and load atom $A$ and atom $B$ to pass through other two cavities of Bob's laboratory. Here, Bob must ensure that the conditions of the two cavities are exactly the same as the previous ones to prevent the entanglement characteristics among the particles from being destroyed. The more important point is how Bob distinguishes the eight states.

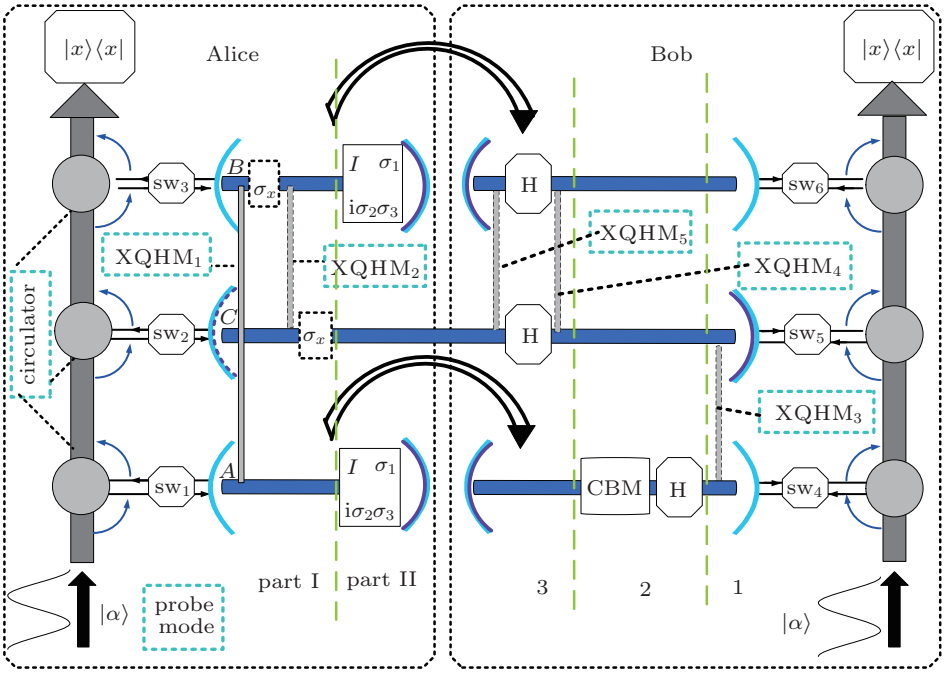



**Fig. 2.** (color online) Diagram for preparing GHZ state and achieving quantum dense coding, where H denotes the Hadamard gate operation, CBM the computation basis measurement, $|x\rangle\langle x|$ the X-quadrature homodyne measurement, $\mathrm{XQHM}_i$ $(i=0,1,2,3)$ the $i$-th X-quadrature homodyne measurement, and $\mathrm{sw}_i$ the $i$-th control switch.

**Step 1** Bob simultaneously turns on $\mathrm{sw}_4$ and $\mathrm{sw}_5$ but turns off $\mathrm{sw}_6$ to allow atoms $A$ and $C$ to participate in the interaction with their own cavity and the coherent state, then utilizes the third X-quadrature homodyne measurement ($\mathrm{XQHM}_3$) on atom $A$ and atom $C$, the coherent state gains a phase $\theta_1$, the phase change of Eqs. (14)–(17) similar to Eq. (10). By judging whether $\theta_1$ is equal to zero Bob can distinguish

$$|\psi\rangle_{1,4}=\frac{1}{\sqrt{2}}(|000\rangle\pm|111\rangle)_{ABC},\quad(\theta_1\neq 0),\tag{18a}$$

$$|\psi\rangle_{2,3}=\frac{1}{\sqrt{2}}(|101\rangle\pm|010\rangle)_{ABC},\quad(\theta_1\neq 0),\tag{18b}$$

and

$$|\psi\rangle_{5,8}=\frac{1}{\sqrt{2}}(|100\rangle\pm|011\rangle)_{ABC},\quad(\theta_1=0),\tag{19a}$$

$$|\psi\rangle_{6,7}=\frac{1}{\sqrt{2}}(|001\rangle\pm|110\rangle)_{ABC},\quad(\theta_1=0).\tag{19b}$$

**Step 2** Still keeping $\mathrm{sw}_4$ and $\mathrm{sw}_5$ on but $\mathrm{sw}_6$ turns off, Bob performs a Hadamard gate operation on atom $A$, which will cause the change

$$|0\rangle_A\xrightarrow{\mathrm{H}}\frac{1}{\sqrt{2}}(|0\rangle_A+|1\rangle_A)$$

and

$$|1\rangle_A\xrightarrow{\mathrm{H}}\frac{1}{\sqrt{2}}(|0\rangle_A-|1\rangle_A)$$

according to Ref. [50], equations (18) and (19) then evolve into

$$|\psi\rangle_1\Rightarrow\frac{1}{2}[|0\rangle_A(|00\rangle+|11\rangle)_{BC}+|1\rangle_A(|00\rangle-|11\rangle)_{BC}],\quad(\theta_1\neq 0),\tag{20a}$$

$$|\psi\rangle_4\Rightarrow\frac{1}{2}[|0\rangle_A(|00\rangle-|11\rangle)_{BC}+|1\rangle_A(|00\rangle+|11\rangle)_{BC}],\quad(\theta_1\neq 0),\tag{20b}$$

$$|\psi\rangle_2\Rightarrow\frac{1}{2}[|0\rangle_A(|01\rangle+|10\rangle)_{BC}+|1\rangle_A(-|01\rangle+|10\rangle)_{BC}],\quad(\theta_1\neq 0),\tag{20c}$$

$$|\psi\rangle_3\Rightarrow\frac{1}{2}[|0\rangle_A(|01\rangle-|10\rangle)_{BC}+|1\rangle_A(-|01\rangle-|10\rangle)_{BC}],\quad(\theta_1\neq 0),\tag{20d}$$

and

$$|\psi\rangle_5\Rightarrow\frac{1}{2}[|0\rangle_A(|00\rangle+|11\rangle)_{BC}+|1\rangle_A(-|00\rangle+|11\rangle)_{BC}],\quad(\theta_1=0),\tag{21a}$$

$$|\psi\rangle_8\Rightarrow\frac{1}{2}[|0\rangle_A(|00\rangle-|11\rangle)_{BC}+|1\rangle_A(-|00\rangle-|11\rangle)_{BC}],\quad(\theta_1=0),\tag{21b}$$

$$|\psi\rangle_6\Rightarrow\frac{1}{2}[|0\rangle_A(|01\rangle+|10\rangle)_{BC}+|1\rangle_A(|01\rangle-|10\rangle)_{BC}],\quad(\theta_1=0),\tag{21c}$$

$$|\psi\rangle_7\Rightarrow\frac{1}{2}[|0\rangle_A(|01\rangle-|10\rangle)_{BC}+|1\rangle_A(|01\rangle+|10\rangle)_{BC}],\quad(\theta_1=0).\tag{21d,21e}$$

Afterwards, Bob sets the computation basis measurement on atom $A$, when the result is $|0\rangle_A$, the above states will collapse into

$$|\psi\rangle_1\Rightarrow|\psi\rangle_1'=\frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle+|11\rangle)_{BC},\quad(\theta_1\neq 0),\tag{22a}$$

$$|\psi\rangle_4 \Rightarrow |\psi\rangle_4' = \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 \neq 0), \tag{22b}$$

$$|\psi\rangle_2 \Rightarrow |\psi\rangle_2' = \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 \neq 0), \tag{22c}$$

$$|\psi\rangle_3 \Rightarrow |\psi\rangle_3' = \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle - |10\rangle)_{BC}, \quad (\theta_1 \neq 0), \tag{22d}$$

and

$$|\psi\rangle_5 \Rightarrow |\psi\rangle_5' = \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle + |11\rangle)_{BC}, \quad (\theta_1 = 0), \tag{23a}$$

$$|\psi\rangle_8 \Rightarrow |\psi\rangle_8' = \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 = 0), \tag{23b}$$

$$|\psi\rangle_6 \Rightarrow |\psi\rangle_6' = \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 = 0), \tag{23c}$$

$$|\psi\rangle_7 \Rightarrow |\psi\rangle_7' = \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle - |10\rangle)_{BC}, \quad (\theta_1 = 0). \tag{23d}$$

Otherwise, if the result is $|1\rangle_A$, equations (20) and (21) will evolve into the following form

$$|\psi\rangle_1 \Rightarrow |\psi\rangle_1'' = \frac{1}{\sqrt{2}}|1\rangle_A(|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 \neq 0), \tag{24a}$$

$$|\psi\rangle_4 \Rightarrow |\psi\rangle_4'' = \frac{1}{\sqrt{2}}|1\rangle_A(|00\rangle + |11\rangle)_{BC}, \quad (\theta_1 \neq 0), \tag{24b}$$

$$|\psi\rangle_2 \Rightarrow |\psi\rangle_2'' = \frac{1}{\sqrt{2}}|1\rangle_A(-|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 \neq 0), \tag{24c}$$

$$|\psi\rangle_3 \Rightarrow |\psi\rangle_3'' = \frac{1}{\sqrt{2}}|1\rangle_A(-|01\rangle - |10\rangle)_{BC}, \quad (\theta_1 \neq 0), \tag{24d}$$

and

$$|\psi\rangle_5 \Rightarrow |\psi\rangle_5'' = \frac{1}{\sqrt{2}}|1\rangle_A(-|00\rangle + |11\rangle)_{BC}, \quad (\theta_1 = 0), \tag{25a}$$

$$|\psi\rangle_8 \Rightarrow |\psi\rangle_8'' = \frac{1}{\sqrt{2}}|1\rangle_A(-|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 = 0), \tag{25b}$$

$$|\psi\rangle_6 \Rightarrow |\psi\rangle_6'' = \frac{1}{\sqrt{2}}|1\rangle_A(|01\rangle - |10\rangle)_{BC}, \quad (\theta_1 = 0), \tag{25c}$$

$$|\psi\rangle_7 \Rightarrow |\psi\rangle_7'' = \frac{1}{\sqrt{2}}|1\rangle_A(|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 = 0). \tag{25d}$$

By comparing Eqs. (22) and (23) with Eqs. (24) and (25), Bob can see that the second case can be equal to the first case only if exchanging $|\psi\rangle_1'$ for $|\psi\rangle_4'$, $|\psi\rangle_2'$ for $|\psi\rangle_3'$, $|\psi\rangle_5'$ for $|\psi\rangle_8'$, and $|\psi\rangle_6'$ for $|\psi\rangle_7'$. So, the same operations can be implemented for these two cases, but the corresponding measurement results and output states will be different.

Bob takes the first situation for example in the following. The distinguishing result of the second case is given out later.

**Step 3** Bob keeps $sw_5$ working meanwhile turning off $sw_4$ and turning on $sw_6$ to take the fourth X-quadrature homodyne measurement on atom $B$ and atom $C$ ($XQHM_4$), and the coherent state obtains a phase $\theta_2$, depending on $\theta_2$, Bob can tell the above states (Eqs. (22) and (23)), as follows:

$$|\psi\rangle_1 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle + |11\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 \neq 0), \tag{26a}$$

$$|\psi\rangle_4 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 \neq 0), \tag{26b}$$

$$|\psi\rangle_2 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 = 0), \tag{27a}$$

$$|\psi\rangle_3 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle - |10\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 = 0), \tag{27b}$$

and

$$|\psi\rangle_5 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle + |11\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 \neq 0), \tag{28a}$$

$$|\psi\rangle_8 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 \neq 0), \tag{28b}$$

$$|\psi\rangle_6 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 = 0), \tag{29a}$$

$$|\psi\rangle_7 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle - |10\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 = 0). \tag{29b}$$

Then, Bob simultaneously performs Hadamard gate operations on both atom $B$ and atom $C$, and soon afterwards he puts into use the fifth X-quadrature homodyne measurement ($XQHM_5$) on these two particles, the above states will be transformed and differentiated into the following forms:

$$|\psi\rangle_1 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle + |11\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 \neq 0; \theta_3 \neq 0), \tag{30}$$

$$|\psi\rangle_4 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 \neq 0; \theta_3 = 0), \tag{31}$$

$$|\psi\rangle_2 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 = 0; \theta_3 \neq 0), \tag{32}$$

$$|\psi\rangle_3 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(-|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 \neq 0; \theta_2 = 0; \theta_3 = 0), \tag{33}$$

and

$$|\psi\rangle_5 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle + |11\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 \neq 0; \theta_3 \neq 0), \tag{34}$$

$$|\psi\rangle_8 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 \neq 0; \theta_3 = 0), \tag{35}$$

$$|\psi\rangle_6 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(|00\rangle - |11\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 = 0; \theta_3 \neq 0), \tag{36}$$

$$|\psi\rangle_7 \Rightarrow \frac{1}{\sqrt{2}}|0\rangle_A(-|01\rangle + |10\rangle)_{BC}, \quad (\theta_1 = 0; \theta_2 = 0; \theta_3 = 0). \tag{37}$$

So far, he has successfully distinguished eight states according to the different phases obtained by coherent state in different measurements. The discrimination results can be expressed as the following form (shown in Tables 1 and 2).

**Table 1.** Discrimination result of Bob when his computation basis measurement is on $|0\rangle_A$ state.

| Measurement results | | | | Final output states | Coding operations |
|---|---|---|---|---|---|
| $\theta_1 \neq 0$ | $\|0\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_1 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(\|00\rangle+\|11\rangle)_{BC}$ | $I, I(\sigma_3,\sigma_3)$ |
| $\theta_1 \neq 0$ | $\|0\rangle_A$ | $\theta_2 = 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_2 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(\|00\rangle-\|11\rangle)_{BC}$ | $I, \sigma_1(\sigma_3,\mathrm{i}\sigma_2)$ |
| $\theta_1 \neq 0$ | $\|0\rangle_A$ | $\theta_2 = 0$ | $\theta_3 = 0$ | $\|\psi\rangle_3 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(-\|01\rangle+\|10\rangle)_{BC}$ | $I, \mathrm{i}\sigma_2(\sigma_3,\sigma_1)$ |
| $\theta_1 \neq 0$ | $\|0\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 = 0$ | $\|\psi\rangle_4 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(\|01\rangle+\|10\rangle)_{BC}$ | $I, \sigma_3(\sigma_3,I)$ |
| $\theta_1 = 0$ | $\|0\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_5 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(\|00\rangle+\|11\rangle)_{BC}$ | $\sigma_1, I(\mathrm{i}\sigma_2,\sigma_3)$ |
| $\theta_1 = 0$ | $\|0\rangle_A$ | $\theta_2 = 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_6 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(\|00\rangle-\|11\rangle)_{BC}$ | $\sigma_1, \sigma_1(\mathrm{i}\sigma_2,\mathrm{i}\sigma_2)$ |
| $\theta_1 = 0$ | $\|0\rangle_A$ | $\theta_2 = 0$ | $\theta_3 = 0$ | $\|\psi\rangle_7 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(-\|01\rangle+\|10\rangle)_{BC}$ | $\sigma_1, \mathrm{i}\sigma_2(\mathrm{i}\sigma_2,\sigma_1)$ |
| $\theta_1 = 0$ | $\|0\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 = 0$ | $\|\psi\rangle_8 \Rightarrow \frac{1}{\sqrt{2}}\|0\rangle_A(\|01\rangle+\|10\rangle)_{BC}$ | $\sigma_1, \sigma_3(\mathrm{i}\sigma_2,I)$ |

**Table 2.** Discrimination result of Bob when his computation basis measurement is on $|1\rangle_A$ state.

| Measurement results | | | | Final output states | Coding operations |
|---|---|---|---|---|---|
| $\theta_1 \neq 0$ | $\|1\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 = 0$ | $\|\psi\rangle_1 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(\|01\rangle+\|10\rangle)_{BC}$ | $I, I(\sigma_3,\sigma_3)$ |
| $\theta_1 \neq 0$ | $\|1\rangle_A$ | $\theta_2 = 0$ | $\theta_3 = 0$ | $\|\psi\rangle_2 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(-\|01\rangle+\|10\rangle)_{BC}$ | $I, \sigma_1(\sigma_3,\mathrm{i}\sigma_2)$ |
| $\theta_1 \neq 0$ | $\|1\rangle_A$ | $\theta_2 = 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_3 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(\|00\rangle-\|11\rangle)_{BC}$ | $I, \mathrm{i}\sigma_2(\sigma_3,\sigma_1)$ |
| $\theta_1 \neq 0$ | $\|1\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_4 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(\|00\rangle+\|11\rangle)_{BC}$ | $I, \sigma_3(\sigma_3,I)$ |
| $\theta_1 = 0$ | $\|1\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 = 0$ | $\|\psi\rangle_5 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(\|01\rangle+\|10\rangle)_{BC}$ | $\sigma_1, I(\mathrm{i}\sigma_2,\sigma_3)$ |
| $\theta_1 = 0$ | $\|1\rangle_A$ | $\theta_2 = 0$ | $\theta_3 = 0$ | $\|\psi\rangle_6 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(-\|01\rangle+\|10\rangle)_{BC}$ | $\sigma_1, \sigma_1(\mathrm{i}\sigma_2,\mathrm{i}\sigma_2)$ |
| $\theta_1 = 0$ | $\|1\rangle_A$ | $\theta_2 = 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_7 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(\|00\rangle-\|11\rangle)_{BC}$ | $\sigma_1, \mathrm{i}\sigma_2(\mathrm{i}\sigma_2,\sigma_1)$ |
| $\theta_1 = 0$ | $\|1\rangle_A$ | $\theta_2 \neq 0$ | $\theta_3 \neq 0$ | $\|\psi\rangle_8 \Rightarrow \frac{1}{\sqrt{2}}\|1\rangle_A(\|00\rangle+\|11\rangle)_{BC}$ | $\sigma_1, \sigma_3(\mathrm{i}\sigma_2,I)$ |

That is to say, the target of dense coding is achieved.

In a way similar to the above principle, we can achieve $N$-particle quantum dense coding by making use of $N$-particle GHZ state while encoding $N-1$ particles. We briefly describe the steps in theory.

Alice and Bob share the entangled state

$$|\phi\rangle_0 = \frac{1}{\sqrt{2}}(|000\cdots0\rangle_{1,2,3,\ldots,N} + |111\cdots1\rangle_{1,2,3,\ldots,N}). \quad (38)$$

Alice owns particles 1, 2, 3, ..., $(N-1)$, while Bob has particle $N$. After Alice simultaneously performs each of the four local operations $\{I, \sigma_x, \mathrm{i}\sigma_y, \sigma_z\}$ on her atoms 1, 2, 3, ..., and $(N-1)$, the state $|\phi\rangle_0$ will evolve into $2^N$ states, Alice then sends her atoms to Bob.

**Step 1** Bob performs the X-quadrature homodyne measurement on atoms 1 and $N$, and the $2^N$ states are divided into two parts.

**Step 2** Bob carries out a Hadamard gate operation and sets computation basis measurements on atom 1, afterwards he performs the second X-quadrature homodyne measurement on the atoms $(N-1)$ and $N$, the above states round into four parts.

**Step 3** Bob simultaneously performs Hadamard gate operations on both atoms $(N-1)$ and $N$ and carries out the third X-quadrature homodyne measurement on atom $(N-1)$ and atom $N$, $2^N$ states will then be divided into eight parts. The following operations are similar to the above: only requiring us to change the corresponding every step action objects. The action objects will be arranged as follows:

Action objects of the first step will become respectively 2 and $N$; 3 and $N$; 4 and $N$; ...; $(N-2)$, and $(N)$. Correspondingly, in the second step, the action objects will be 2, $(N-1)$ and $N$; 3, $(N-1)$ and $N$; 4, $(N-1)$ and $N$; ...; $(N-2)$, $(N-2)$, and $(N)$. In Step 3, the corresponding action objects are $N$ and $(N-1)$; $N$ and $(N-1)$; $N$ and $(N-1)$; ...; $N$ and $(N-1)$. Up to now, we have implemented the $N$-atom quantum dense coding with $N$-particle GHZ state.

We will briefly discuss the feasibility of our protocol under the condition of modern experimental technology and the success probability of our scheme. During this work, atoms composed of low-Q cavities ($\kappa \gg \gamma$) are working in the intermediate coupling region. In contrast to the previous schemes,[51,52] our scheme places fewer demands on the Purcell factor $g^2/\kappa\gamma$, and is better suited to current experimen-

tal technology. Moreover, according to recent reports,[53,54] researchers have demonstrated that the position of a single atom could be manipulated experimentally. In addition, X-quadrature homodyne measurement plays a practical role in current quantum information processing,[55,56] which is a standard tool of continuous variable experiments[57] and which is also highly efficient (more than 0.99). The present experiment has demonstrated that between the photon and the trapped atom,[58] the entanglement lifetime $T_{\rm pa}$ is about 150 μs; the value of radiative time of atoms, $T_{\rm R}$, is about $3\times10^{-2}$ s; the decay time of the cavity, $T_{\rm C}$, is approximately $1\times10^{-3}$ s. The time of single particle rotation is much less than $10^{-6}$, so we could neglect it. After the analysis, we find that the interaction time of this scheme is ideally about $T_{\rm total} < 5\times150\times10^{-6}$ s = $T_{\rm pa}$ s, so $T_{\rm total} < T_{\rm c} < T_{\rm r}$. Therefore, based on the present techniques, the proposed scheme may be realizable.

Next, we may simply assess the implementation of our schemes, where the failure rate due to atomic decay is about $\gamma/g = 2\%$.[59] Moreover, another imperfection rate, the photon loss which occurs due to the cavity mirror absorption and scattering, can be assumed to be 6%.[60] Besides, the error rate of the homodyne measurement is mainly caused by the overlap of adjacent curves $f(x,\alpha\cos(n\theta))$ and $f(x,\alpha\cos[(n+1)\theta])$, and it is given by $p_{\rm error} = \frac{1}{2}(1-{\rm erf}[\chi_d/2\sqrt{2}]) \approx 2.3\times10^{-4}$.[56] In summary, the success rate of our implementation is about $P = (1-2\%)^3\times(1-6\%)\times(1-2.3\times10^{-4})^5 \approx 0.8837$.

## 4. Conclusions

In this paper, we provide an efficient scheme for realizing quantum dense coding with three-particle GHZ state utilizing a single atom trapped in separated low-Q cavities by using coherent optical pulse as a quantum channel and X-quadrature homodyne measurement as the main method of measurement. Beyond that, this scheme does not need a strong coupling cavity nor single-photon source, so it is very well suited to the current experimental condition. According to the above analyses, our protocol has not only a high probability and easy operability but also feasibility for large-scale and nonlocal quantum computation and quantum communication.

## References


[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[2] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[3] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[4] Man Z X and Xia Y J 2008 *Chin. Phys. B* **17** 4375
[5] Chen M F 2006 *Chin. Phys.* **15** 2847
[6] Wang J, Yu L B and Ye L 2007 *Chin. Phys.* **16** 2211
[7] Guo G C and Zhang Y S 2002 *Phys. Rev. A* **65** 054302
[8] Zou X B and Mathis W 2005 *Phys. Rev. A* **72** 013809
[9] Deng Z J, Feng M and Gao K L 2006 *Phys. Rev. A* **73** 014302
[10] Hagley E, Maitre X, Nogues G, Wunderlich C, Brune M, Raimand J M and Haroche S 1997 *Phys. Rev. Lett.* **79** 03502
[11] Gorbachev V N, Trubilko A I and Rodichkina A A 2003 *Phys. Lett. A* **314** 267
[12] Jin G S, Li S S, Feng S L and Zheng H Z 2004 *Phys. Rev. A* **69** 034302
[13] Zha X W and Zhang C M 2008 *Acta Phys. Sin.* **57** 1339 (in Chiense)
[14] Zhang B L, Meng X G and Wang J S 2012 *Chin. Phys. B* **21** 030304
[15] Gao G L, Song F Q, Huang S S, Wang H, Yuan X Z, Wang M F and Jiang N Q 2012 *Chin. Phys. B* **21** 044209
[16] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[17] Mattle K, Weinfurter H, Kwiat P G and Zeilinger A 1996 *Phys. Rev. Lett.* **76** 4656
[18] Li X, Pan Q, Jing J, Zhang J, Xie C and Peng K 2002 *Phys. Rev. Lett.* **88** 047904
[19] Fang X, Zhu X, Feng M, Mao X and Du F 2000 *Phys. Rev. A* **61** 022307
[20] Hao J C, Li C F and Guo G C 2001 *Phys. Rev. A* **63** 054301
[21] Lee H J, Ahn D and Hwang S W 2002 *Phys. Rev. A* **66** 024304
[22] Hao J C, Li C F and Guo G C 2000 *Phys. Lett. A* **278** 113
[23] Lin X M, Zhou Z W, Xue P, Gu Y J and Guo G C 2003 *Phys. Lett. A* **313** 351
[24] Ye L and Guo G C 2005 *Phys. Rev. A* **71** 034304
[25] Ye L and Yu L B 2005 *Phys. Lett. A* **346** 330
[26] Karlsson A and Bourennane M 1998 *Phys. Rev. A* **58** 4394
[27] DÄur W, Vidal G and Cirac J I 2000 *Phys. Rev. A* **62** 062314
[28] Acín A, Bruß D, Lewenstein M and Sanpera A 2011 *Phys. Rev. Lett.* **87** 040401
[29] Sim S, Jin J and Kwon Y 2005 *Int. J. Theor. Phys.* **44** 1419
[30] Guo Q, Cheng L Y, Wang H F, Zhang S and Yeon K H 2012 *Opt. Commun.* **285** 1571
[31] Nemoto K and Munro W J 2004 *Phys. Rev. Lett.* **93** 250502
[32] Wang X W, Zhang D Y, Tang S Q, Xie L J, Wang Z Y and Kuang L M 2012 *Phys. Rev. A* **85** 052326
[33] Chen X D, Xiao S J, Gu Y J and Lin X M 2010 *Acta Phys. Sin.* **59** 5251 (in Chiense)
[34] Ye L and Guo G C 2002 *Chin. Phys.* **11** 996
[35] Yu C S, Yi X X, Song H S and Mei D 2007 *Phys. Rev. A* **75** 044301
[36] Su S L, Guo Q, Zhu L, Wang H F and Zhang S 2012 *J. Opt. Soc. Am. B* **29** 002827
[37] He J, Ye L and Ni Z X 2008 *Chin. Phys. B* **17** 1597
[38] Zhu M Z, Zhao C R and Ye L 2011 *Opt. Commun.* **284** 5394
[39] Zhao C R and Ye L 2011 *Opt. Commun.* **284** 541
[40] Zhao C R and Ye L 2011 *Phys. Lett. A* **375** 401
[41] Gao M, Hu W H and Li C Z 2007 *J. Phys. B: At. Mol. Opt. Phys.* **40** 3525
[42] Jin G S, Lin Y and Wu B 2007 *Phys. Rev. A* **75** 054302
[43] Zhu M Z and Yin X G 2013 *J. Opt. Soc. Am. B* **30** 355
[44] Mei F, Yu Y F, Feng X L, Zhu S L and Zhang Z M 2010 *Europhys. Lett.* **91** 10001
[45] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer-Verlag) p. 124
[46] Zhao C R, Zhu M Z and Ye L 2011 *J. Opt. Soc. Am. B* **28** 1740
[47] Barrett S D, Kok P, Nemoto K, Beausoleil R G, Munro W J and Spiller T P 2005 *Phys. Rev. A* **71** 060302
[48] Riebe M, Häffner H, Roos C F, Hänsel W, Benhelm J, Lancaster G P T, Körber T W, Becher C, Schmidt-Kaler F, James D F V and Blatt R 2004 *Nature* **429** 734
[49] Fortier K M, Kim S Y, Gibbons M J, Ahmadi P and Chapman M S 2007 *Phys. Rev. Lett.* **98** 233601
[50] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
[51] Wei H, Yang W L, Deng Z J and Feng M 2008 *Phys. Rev. A* **78** 014304
[52] Mei F, Feng M, Yu Y F and Zhang Z M 2009 *Phys. Rev. A* **80** 042319
[53] Fortier K M, Kim S Y, Gibbons M J, Ahmadi P and Chapman M S 2007 *Phys. Rev. Lett.* **98** 233601
[54] Numann S, Hijlkema M, Weber B, Rohde F, Rempe G and Kuhn A 2005 *Phys. Rev. Lett.* **95** 173602
[55] Su S L, Cheng L Y, Wang H F and Zhang S 2013 *Opt. Commun.* **293** 172
[56] Nemoto K and Munro W J 2004 *Phys. Rev. Lett.* **93** 250502
[57] Polzik E S, Carri J and Kimble H J 1992 *Phys. Rev. Lett.* **68** 3020
[58] Rosenfeld W, Hocke F, Henkel F, Krug M, Volz J, Weber M and Weinfurter H 2008 *Phys. Rev. Lett.* **101** 260403
[59] Chen Q and Feng M 2009 *Phys. Rev. A* **79** 064304
[60] An J H, Feng M and Oh C H 2009 *Phys. Rev. A* **79** 032303