Icosahedral quasicrystals solids with an elliptic hole under uniform heat flow*

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The complex variable method for solving the two-dimensional thermal stress problem of icosahedral quasicrystals is stated. The closed-form solutions for icosahedral quasicrystals containing an elliptical hole subjected to a remote uniform heat flow are obtained. When the hole degenerates into a crack, the explicit solutions for the stress intensity factors is presented.

Keywords: icosahedral quasicrystals, thermal stress, elliptic hole, stress intensity factors

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1. Introduction

Icosahedral quasicrystals are fascinating substances that form a family of specific structures with strange physical and mechanical properties when compared to those of metallic alloys.[1] These properties open the way to technological applications, demonstrated or potential, mostly regarding energy savings. Icosahedral quasicrystals have potential applications in protective alloys and coatings and are also being studied for use in new materials that convert heat to electricity.[2]

In recent years, the elastic theory and fracture mechanics of quasicrystals have made great progress.[3–7] Some methods were developed to solve the defect problems in quasicrystallines.[8–14] However, relatively little work involving the thermal stress problems of quasicrystals has been done due to the mathematical complexity.

In the present work, we treat the generalized 2D problem of an elliptic hole in an infinite icosahedral quasicrystal subjected to uniform heat flow at infinity. The Lekhnitskii formulation of icosahedral quasicrystals including the thermal effect is developed. Then an explicit analytical solution for an elliptic hole in icosahedral quasicrystals is obtained by means of conformal mapping. The solution of the Griffith crack problem as a special case of the results is obtained. As a consequence, the stress intensity factor is derived analytically.

2. General solution

Consider a generalized two-dimensional problem of icosahedral quasicrystals in which all the field variables are independent of variable z (the z-axis is parallel to the fivefold direction).[15] According to the elastic theory of icosahedral quasicrystals,[5] we have the geometric equations ($x = x_1, y = x_2$)

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad w_{ij} = \frac{\partial w_i}{\partial x_j} \tag{1}$$

the generalized equilibrium equations (if the body force is neglected, $x = x_1, y = x_2$)

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \quad \frac{\partial H_{ij}}{\partial x_j} = 0, \tag{2}$$

and the constitutive equations

$$\begin{aligned}
\sigma_{xx} &= \lambda(\varepsilon_{xx} + \varepsilon_{yy}) + 2\mu\varepsilon_{xx} + R(w_{xx} + w_{yy}) - \beta T,\\
\sigma_{yy} &= \lambda(\varepsilon_{xx} + \varepsilon_{yy}) + 2\mu\varepsilon_{yy} - R(w_{xx} + w_{yy}) - \beta T,\\
\sigma_{zz} &= \lambda(\varepsilon_{xx} + \varepsilon_{yy}) - \beta T,\\
\sigma_{yz} &= \sigma_{zy} = 2\mu\varepsilon_{yz} + R(w_{zy} - w_{xy} - w_{yx}),\\
\sigma_{zx} &= \sigma_{xz} = 2\mu\varepsilon_{zx} + R(w_{xx} - w_{yy} - w_{zx}),\\
\sigma_{xy} &= \sigma_{yx} = 2\mu\varepsilon_{xy} + R(w_{yx} - w_{xy}),\\
H_{xx} &= R(\varepsilon_{xx} - \varepsilon_{yy} + 2\varepsilon_{zx}) + K_1 w_{xx} + K_2 w_{zx},\\
H_{yy} &= R(\varepsilon_{xx} - \varepsilon_{yy} - 2\varepsilon_{zx}) + K_1 w_{yy} - K_2 w_{zx},\\
H_{zz} &= R(\varepsilon_{xx} + \varepsilon_{yy}),\\
H_{yz} &= -2R\varepsilon_{xy} + K_2(w_{xy} - w_{yx}),\\
H_{zx} &= 2R\varepsilon_{zx} + (K_1 - K_2)w_{zx} + K_2(w_{xx} - w_{yy}),\\
H_{xy} &= -2R(\varepsilon_{yz} + \varepsilon_{xy}) + K_1 w_{xy} - K_2 w_{zy},\\
H_{zy} &= 2R\varepsilon_{yz} + (K_1 - K_2)w_{zy} - K_2(w_{xy} + w_{yx},)\\
H_{xz} &= R(\varepsilon_{xx} - \varepsilon_{yy}) + K_2(w_{xx} + w_{yy}),
\end{aligned}$$

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$$H_{yx} = 2R(\varepsilon_{xy} - \varepsilon_{yx}) + K_1 w_{yx} - K_2 w_{zy}, \tag{3}$$

where $\varepsilon_{zz} = w_{zz} = w_{xz} = w_{yz} = 0$ have been considered. σ_{ij}, ε_{ij}, and u_i are the stress, strain and displacement of phonon fields, respectively. λ and μ are the phonon field elastic constants. H_{ij}, w_{ij}, w_i, and K_i are the stress, strain, displacement, and elastic constants of phason fields. R and β are the phonon-phason coupling elastic constant and thermal-stress constant, respectively. T represents temperature change.

The heat conduction equation and the conduction law can be written as

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0, \tag{4}$$

$$q_x = -k_{11}\frac{\partial T}{\partial x}, q_y = -k_{11}\frac{\partial T}{\partial y}, \tag{5}$$

where k_{11} is the heat conduction coefficient. q_x and q_y denote the components of heat flux in the x and y directions, respectively.

2.1. The temperature field

Substituting Eq. (5) into Eq. (4), we have

$$\nabla^2 T = 0, \tag{6}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator. The general solution of Eq. (6) can be written as

$$T = 2\mathrm{Re}\{g'(z)\}, \tag{7}$$

where $g(z)$ is an analytic function of the complex variable $z = x + iy$, *Re* stands for the real part, the prime denotes the derivative with respect to the arguments. By Eq. (7), the resultant heat flux along an arc can be obtained as

$$\Theta = \int q_x \mathrm{d}y - \int q_y \mathrm{d}x = 2k_{11}\,\mathrm{Re}\{ig'(z)\}. \tag{8}$$

2.2. The elastic fields

Based on the constitutive Eqs. (3), the strain components of the phonon and phason fields can be expressed by the stresses and the temperature T, see Appendix A for detail. By the geometric Eqs. (1), the deformation compatibility equations are as follows:

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}, \quad \frac{\partial^2 \varepsilon_{yz}}{\partial x^2} = \frac{\partial^2 \varepsilon_{zx}}{\partial x \partial y},$$
$$\frac{\partial w_{xy}}{\partial x} = \frac{\partial w_{xx}}{\partial y}, \quad \frac{\partial w_{yx}}{\partial y} = \frac{\partial w_{yy}}{\partial x}, \quad \frac{\partial w_{zx}}{\partial y} = \frac{\partial w_{zy}}{\partial x}. \tag{9}$$

If we introduce the functions $\varphi_1(x,y)$, $\varphi_2(x,y)$, $\psi_1(x,y)$, $\psi_2(x,y)$, and $\psi_3(x,y)$ as follows

$$\sigma_{xx} = \frac{\partial^2 \varphi_1}{\partial y^2}, \quad \sigma_{xy} = -\frac{\partial^2 \varphi_1}{\partial x \partial y}, \quad \sigma_{yy} = \frac{\partial^2 \varphi_1}{\partial x^2},$$
$$\sigma_{zx} = \frac{\partial \varphi_2}{\partial y}, \quad \sigma_{zy} = -\frac{\partial \varphi_2}{\partial x},$$
$$H_{xx} = \frac{\partial \psi_1}{\partial y}, \quad H_{xy} = -\frac{\partial \psi_1}{\partial x}, \quad H_{yx} = \frac{\partial \psi_2}{\partial y},$$
$$H_{yy} = -\frac{\partial \psi_2}{\partial x}, \quad H_{zx} = \frac{\partial \psi_3}{\partial y}, \quad H_{zy} = -\frac{\partial \psi_3}{\partial x}, \tag{10}$$

then the generalized equilibrium Eqs. (2) involving stresses will be automatically satisfied.

Substituting the expressions of strain (Appendix A) into deformation compatibility equations (9), then by employing (10), we have

$$\begin{aligned}
&c_1 \nabla^2 \nabla^2 \varphi_1 + c_2\left(\frac{\partial}{\partial y}\Pi_1 \psi_1 + \frac{\partial}{\partial x}\Pi_2 \psi_2\right) = 0,\\
&(K_1 + K_2)\nabla^2 \varphi_2 + R\left(\Lambda^2 \psi_1 - 2\frac{\partial^2}{\partial x \partial y}\psi_2 - \nabla^2 \psi_3\right) = 0,\\
&-c_3 \frac{\partial}{\partial y}\Pi_1 \varphi_1 - R(2K_2 - K_1)\Lambda^2 \varphi_2\\
&+c_4 \nabla^2 \psi_1 + (\mu K_2 - R^2)\Lambda^2 \psi_3 = 0,\\
&c_3 \frac{\partial}{\partial x}\Pi_2 \varphi_1 - 2R(2K_2 - K_1)\frac{\partial^2}{\partial x \partial y}\varphi_2\\
&-c_4 \nabla^2 \psi_2 + 2(\mu K_2 - R^2)\frac{\partial^2}{\partial x \partial y}\psi_3 = 0,\\
&-R(2K_2 - K_1)\nabla^2 \varphi_2 - (\mu K_2 - R^2)(\Lambda^2 \psi_1\\
&-2\frac{\partial^2}{\partial x \partial y}\psi_2 + \nabla^2 \psi_3) = 0,
\end{aligned} \tag{11}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \Lambda^2 = \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2},$$
$$\Pi_1 = 3\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}, \quad \Pi_2 = 3\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2},$$

and

$$c_1 = \frac{1}{2}\left(\frac{1}{\lambda + \mu} + \frac{K_1}{\mu K_1 - 2R^2}\right), \quad c_2 = \frac{R}{\mu K_1 - 2R^2},$$
$$c_3 = \frac{R(2K_2 - K_1)(\mu K_1 + \mu K_2 - 3R^2)}{2(\mu K_1 - 2R^2)},$$
$$c_4 = \mu(K_1 - K_2) - R^2 - \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}.$$

Equations (11) can be solved using the method proposed by Yang.[16] It can be rewritten as

$$\boldsymbol{D}\begin{Bmatrix} \varphi_1(x,y) \\ \varphi_2(x,y) \\ \psi_1(x,y) \\ \psi_2(x,y) \\ \psi_3(x,y) \end{Bmatrix} = 0, \tag{12}$$

where the operator matrix $\boldsymbol{D}$ is

$$\boldsymbol{D}=\begin{bmatrix} c_1\nabla^2\nabla^2 & 0 & c_2\frac{\partial}{\partial y}\Pi_1 & c_2\frac{\partial}{\partial x}\Pi_2 & 0 \\ 0 & (K_1+K_2)\nabla^2 & R\Lambda^2 & -2R\frac{\partial^2}{\partial x\partial y} & -R\nabla^2 \\ -c_3\frac{\partial}{\partial y}\Pi_1 & -R(2K_2-K_1)\Lambda^2 & c_4\nabla^2 & 0 & (\mu K_2-R^2)\Lambda^2 \\ c_3\frac{\partial}{\partial x}\Pi_2 & -2R(2K_2-K_1)\frac{\partial^2}{\partial x\partial y} & 0 & -c_4\nabla^2 & 2(\mu K_2-R^2)\frac{\partial^2}{\partial x\partial y} \\ 0 & -R(2K_2-K_1)\nabla^2 & -(\mu K_2-R^2)\Lambda^2 & 2(\mu K_2-R^2)\frac{\partial^2}{\partial x\partial y} & -(\mu K_2-R^2)\nabla^2 \end{bmatrix}.$$

The determinant of matrix $\boldsymbol{D}$ is

$$\begin{aligned}\det\boldsymbol{D} &= d_1\frac{\partial^{12}}{\partial x^{12}}+d_2\frac{\partial^{12}}{\partial x^{10}\partial y^2}+d_3\frac{\partial^{12}}{\partial x^8\partial y^4}+d_4\frac{\partial^{12}}{\partial x^6\partial y^6}\\ &\quad+d_5\frac{\partial^{12}}{\partial x^4\partial y^8}+d_6\frac{\partial^{12}}{\partial x^2\partial y^{10}}+d_7\frac{\partial^{12}}{\partial y^{12}},\end{aligned}$$

where d_i $(i=1,2,\ldots,7)$ are given in Appendix B.

Based on the cofactors Δ_{ik} of matrix $\boldsymbol{D}$ $(i,k=1,2,3,4,5)$, the general solutions of Eq. (12) are

$$\begin{aligned}&[\varphi_1(x,y),\varphi_2(x,y),\psi_1(x,y),\psi_2(x,y),\psi_3(x,y)]^{\mathrm{T}}\\ &=[\Delta_{i1},\Delta_{i2},\Delta_{i3},\Delta_{i4},\Delta_{i5}]^{\mathrm{T}}G(x,y),\end{aligned}\tag{13}$$

with $G(x,y)$ satisfying the following equation:

$$\det\boldsymbol{D}\,G(x,y)=0.\tag{14}$$

In the following analysis, we use only $(\Delta_{11},\Delta_{12},\Delta_{13},\Delta_{14})$ for the present problem, which can be expressed as follows

$$\begin{aligned}\Delta_{11} &= e_{11}\nabla^2\nabla^2\nabla^2\nabla^2,\\ \Delta_{12} &= 2Rc_3c_4(\mu K_2-R^2)\Big(5\frac{\partial^9}{\partial x^8\partial y}-14\frac{\partial^9}{\partial x^4\partial y^5}\\ &\quad-8\frac{\partial^9}{\partial x^2\partial y^7}+\frac{\partial^9}{\partial y^9}\Big),\\ \Delta_{13} &= c_3\Big(e_{31}\frac{\partial^9}{\partial x^8\partial y}+e_{32}\frac{\partial^9}{\partial x^6\partial y^3}\\ &\quad+e_{33}\frac{\partial^9}{\partial x^4\partial y^5}+e_{34}\frac{\partial^9}{\partial x^2\partial y^7}+e_{35}\frac{\partial^9}{\partial y^9}\Big),\\ \Delta_{14} &= c_3\Big(e_{41}\frac{\partial^9}{\partial x^9}+e_{42}\frac{\partial^9}{\partial x^7\partial y^2}\\ &\quad+e_{43}\frac{\partial^9}{\partial x^5\partial y^4}+e_{44}\frac{\partial^9}{\partial x^3\partial y^6}+e_{45}\frac{\partial^9}{\partial x\partial y^8}\Big),\\ \Delta_{15} &= -c_3c_4K_2(\mu(K_1+K_2)-3R^2)\\ &\quad\times\Big(5\frac{\partial^9}{\partial x^8\partial y}-14\frac{\partial^9}{\partial x^4\partial y^5}-8\frac{\partial^9}{\partial x^2\partial y^7}+\frac{\partial^9}{\partial y^9}\Big),\end{aligned}$$

where the constants e_{ij} can be found in Appendix C. When $R=0$, Eq. (14) can be simplified as

$$\nabla^2\nabla^2\nabla^2\nabla^2\nabla^2\nabla^2G(x,y)=0.$$

Following Lekhnitskii,[17] the general solution of Eq. (14) can be expressed by six analytic functions $G_k(z_k)$ $(k=1,2,\ldots,6)$ in the form

$$G=2\,\mathrm{Re}\sum_{k=1}^{6}G_k(z_k),\quad z_k=x+\mu_k y,\tag{15}$$

where $\mu_k=\alpha_k+\mathrm{i}\beta_k$ $(\mathrm{i}=\sqrt{-1},\ k=1,2,\cdots,6)$ are distinct complex parameters to be determined by the characteristic equation

$$d_7\mu^{12}+d_6\mu^{10}+d_5\mu^8+d_4\mu^6+d_3\mu^4+d_2\mu^2+d_1=0.\tag{16}$$

Substituting the general solution (15) into Eqs. (13) and (10) yields

$$\begin{aligned}\sigma_{xx} &= 2e_{11}\,\mathrm{Re}\sum_{k=1}^{6}\mu_k^2(1+4\mu_k^2+6\mu_k^4+4\mu_k^6+\mu_k^{10})g_k'(z_k),\\ \sigma_{yy} &= 2e_{11}\,\mathrm{Re}\sum_{k=1}^{6}(1+4\mu_k^2+6\mu_k^4+4\mu_k^6+\mu_k^{10})g_k'(z_k),\\ \sigma_{xy} &= \sigma_{yx}=-2e_{11}\\ &\quad\times\mathrm{Re}\sum_{k=1}^{6}\mu_k(1+4\mu_k^2+6\mu_k^4+4\mu_k^6+\mu_k^{10})g_k'(z_k),\\ \sigma_{zx} &= 4Rc_3c_4(\mu K_2-R^2)\\ &\quad\times\mathrm{Re}\sum_{k=1}^{6}\mu_k(5\mu_k-14\mu_k^5-8\mu_k^7+\mu_k^9)g_k'(z_k),\\ \sigma_{zy} &= -4Rc_3c_4(\mu K_2-R^2)\\ &\quad\times\mathrm{Re}\sum_{k=1}^{6}(5\mu_k-14\mu_k^5-8\mu_k^7+\mu_k^9)g_k'(z_k),\\ H_{xx} &= 2c_3\,\mathrm{Re}\sum_{k=1}^{6}\mu_k(e_{31}\mu_k+e_{32}\mu_k^3\\ &\quad+e_{33}\mu_k^5+e_{34}\mu_k^7+e_{35}\mu_k^9)g_k'(z_k),\\ H_{xy} &= -2c_3\,\mathrm{Re}\sum_{k=1}^{6}(e_{31}\mu_k+e_{32}\mu_k^3\\ &\quad+e_{33}\mu_k^5+e_{34}\mu_k^7+e_{35}\mu_k^9)g_k'(z_k),\\ H_{yx} &= 2c_3\,\mathrm{Re}\sum_{k=1}^{6}\mu_k(e_{41}+e_{42}\mu_k^2\\ &\quad+e_{43}\mu_k^4+e_{44}\mu_k^6+e_{45}\mu_k^8)g_k'(z_k),\\ H_{yy} &= -2c_3\,\mathrm{Re}\sum_{k=1}^{6}(e_{41}+e_{42}\mu_k^2\\ &\quad+e_{43}\mu_k^4+e_{44}\mu_k^6+e_{45}\mu_k^8)g_k'(z_k),\\ H_{zx} &= -2c_3c_4K_2(\mu(K_1+K_2)-3R^2)\\ &\quad\times\mathrm{Re}\sum_{k=1}^{6}\mu_k(5\mu_k-14\mu_k^5-8\mu_k^7+\mu_k^9)g_k'(z_k),\\ H_{zy} &= 2c_3c_4K_2(\mu(K_1+K_2)-3R^2)\end{aligned}$$

$$\times \mathrm{Re}\sum_{k=1}^{6}(5\mu_k - 14\mu_k^5 - 8\mu_k^7 + \mu_k^9)g_k'(z_k), \tag{17}$$

where

$$g_k(z_k) = \frac{d^9 G_k(z_k)}{dz_k^9}, \quad g_k'(z_k) = \frac{dg_k(z_k)}{dz_k}.$$

Introducing vectors $\boldsymbol{\Phi}(z_k)$, $\boldsymbol{t}_1$, $\boldsymbol{t}_2$ defined by

$$\begin{aligned}
&\boldsymbol{\Phi}(z_k) = [g_1(z_1), g_2(z_2), g_3(z_3), g_4(z_4), g_5(z_5), g_6(z_6)]^{\mathrm{T}},\\
&\boldsymbol{t}_1 = [\sigma_{xx}, \sigma_{yx}, \sigma_{zx}, H_{xx}, H_{yx}, H_{zx}]^{\mathrm{T}},\\
&\boldsymbol{t}_2 = [\sigma_{xy}, \sigma_{yy}, \sigma_{zy}, H_{xy}, H_{yy}, H_{zy}]^{\mathrm{T}}.
\end{aligned}$$

Equation (17) can be rewritten as

$$\boldsymbol{t}_1 = -2\mathrm{Re}\{\boldsymbol{B}\boldsymbol{\mu}\boldsymbol{\Phi}'\}, \quad \boldsymbol{t}_2 = 2\mathrm{Re}\{\boldsymbol{B}\boldsymbol{\Phi}'\}, \tag{18}$$

where the elements b_{ij} of the matrix $\boldsymbol{B}$ are given as

$$\begin{aligned}
b_{1k} &= -e_{11}\mu_k(1 + 4\mu_k^2 + 6\mu_k^4 + 4\mu_k^6 + \mu_k^{10}),\\
b_{2k} &= e_{11}(1 + 4\mu_k^2 + 6\mu_k^4 + 4\mu_k^6 + \mu_k^{10}),\\
b_{3k} &= -2Rc_3c_4(\mu K_2 - R^2)(5\mu_k - 14\mu_k^5 - 8\mu_k^7 + \mu_k^9),\\
b_{4k} &= -c_3(e_{31}\mu_k + e_{32}\mu_k^3 + e_{33}\mu_k^5 + e_{34}\mu_k^7 + e_{35}\mu_k^9),\\
b_{5k} &= -c_3(e_{41} + e_{42}\mu_k^2 + e_{43}\mu_k^4 + e_{44}\mu_k^6 + e_{45}\mu_k^8),\\
b_{6k} &= c_3c_4K_2(\mu(K_1 + K_2) - 3R^2)(5\mu_k - 14\mu_k^5 - 8\mu_k^7 + \mu_k^9),
\end{aligned}$$

where $k = 1, 2, \ldots, 6$ and $\boldsymbol{\mu}$ is the diagonal matrix, i.e. $\boldsymbol{\mu} = \mathrm{diag}[\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6]$. With Eqs. (10) and (18), the resultant forces along the arc can be expressed as

$$\boldsymbol{t} = -2\mathrm{Re}\{\boldsymbol{B}\boldsymbol{\Phi}\}, \tag{19}$$

in which $\boldsymbol{t}'_y = \boldsymbol{t}_1, \boldsymbol{t}'_x = -\boldsymbol{t}_2$, a comma stands for differentiation. By Eq. (17) and the strains expressed by stresses (Appendix A), the complex representation of displacements can also be given as

$$\begin{aligned}
u_x &= 2\mathrm{Re}\sum_{k=1}^{4}\{p_k\Phi_k(z_k) + \alpha g(z)\},\\
w_x &= 2\mathrm{Re}\sum_{k=1}^{4}\{s_k\Phi_k(z_k)\},\\
u_y &= 2\mathrm{Re}\sum_{k=1}^{4}\{q_k\Phi_k(z_k) - \alpha i g(z)\},\\
w_y &= 2\mathrm{Re}\sum_{k=1}^{4}\{m_k\Phi_k(z_k)\},\\
u_z &= 2\mathrm{Re}\sum_{k=1}^{4}\{r_k\Phi_k(z_k)\},\\
w_z &= 2\mathrm{Re}\sum_{k=1}^{4}\{n_k\Phi_k(z_k)\},
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
p_k &= -a_{11}b_{2k}\mu_k^2 - a_{12}b_{2k} - a_{13}(-\mu_k b_{4k} + b_{5k}),\\
q_k &= a_{12}b_{2k}\mu_k + a_{11}\frac{b_{2k}}{\mu_k} + a_{13}\left(-b_{4k} + \frac{b_{5k}}{\mu_k}\right),\\
r_k &= -\frac{1}{\Delta_1}[(K_1 + K_2)\mu_k b_{3k} + R(-\mu_k b_{4k} - \mu_k b_{6k} - b_{5k})],\\
s_k &= \frac{1}{\Delta_2}[a_{22}(-\mu_k^2 b_{2k} - b_{2k}) - a_{23}\mu_k b_{4k} - a_{24}b_{5k}\\
&\quad - a_{25}\mu_k b_{3k} + a_{26}\mu_k b_{6k}],\\
m_k &= \frac{1}{\Delta_2}[a_{22}(-\mu_k b_{2k} - \frac{b_{2k}}{\mu_k}) + a_{23}\frac{b_{5k}}{\mu_k}\\
&\quad + a_{24}b_{4k} + a_{25}b_{3k} - a_{26}b_{6k}],\\
n_k &= \frac{1}{\Delta_2}[(\mu K_2 - R^2)(b_{5k} + \mu_k b_{4k})\\
&\quad - (\mu K_1 - 2R^2)\mu_k b_{6k} - R(2K_2 - K_1)\mu_k b_{3k}],
\end{aligned}$$

and

$$\begin{aligned}
a_{11} &= \frac{K_1(\lambda + 2\mu) - 2R^2}{4(\lambda + \mu)(\mu K_1 - 2R^2)},\\
a_{12} &= \frac{\lambda K_1 + 2R^2}{4(\lambda + \mu)(\mu K_1 - 2R^2)}, \quad a_{13} = \frac{R}{2(\mu K_1 - 2R^2)},\\
a_{22} &= \frac{1}{2}\left[R(2K_2 - K_1) + \frac{R(2K_2 - K_1)(\mu K_2 - R^2)}{\mu K_1 - 2R^2}\right],\\
a_{23} &= \mu(K_1 - K_2) - R^2 - \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2},\\
a_{24} &= \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}, \quad a_{25} = R(2K_2 - K_1),\\
a_{26} &= \mu K_2 - R^2.
\end{aligned}$$

Introducing vectors $\boldsymbol{U}$, $\boldsymbol{c}$, and the matrix $\boldsymbol{A}$ as follows:

$$\begin{aligned}
&\boldsymbol{U} = [u_x,\ u_y,\ u_z,\ w_x,\ w_y,\ w_z]^{\mathrm{T}},\\
&\boldsymbol{c} = [\alpha,\ -\alpha\,\mathrm{i},\ 0,\ 0,\ 0,\ 0]^{\mathrm{T}},\\
&\boldsymbol{A} = \begin{bmatrix}
p_1 & p_2 & p_3 & p_4 & p_5 & p_6\\
q_1 & q_2 & q_3 & q_4 & q_5 & q_6\\
r_1 & r_2 & r_3 & r_4 & r_5 & r_6\\
s_1 & s_2 & s_3 & s_4 & s_5 & s_6\\
m_1 & m_2 & m_3 & m_4 & m_5 & m_6\\
n_1 & n_2 & n_3 & n_4 & n_5 & n_6
\end{bmatrix},
\end{aligned}$$

then equation (20) can be expressed by

$$\boldsymbol{U} = 2\mathrm{Re}\{\boldsymbol{A}\boldsymbol{\Phi} + \boldsymbol{c}g(z)\}. \tag{21}$$

3. Elliptic hole under uniform heat flow

Consider an infinite icosahedral quasicrystals containing an insulated elliptical hole and the heat h_0 is flowing steady and uniformly in the direction of the positive y-axis, as shown in Fig. 1.

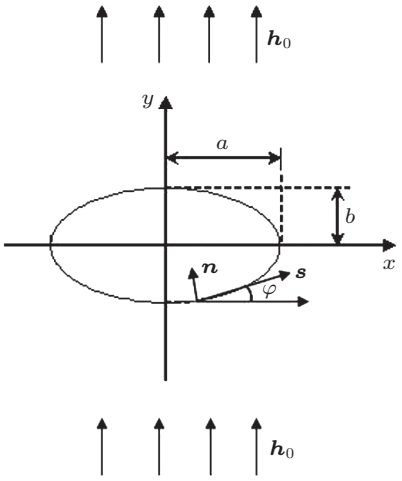


Fig. 1. An elliptical hole under uniform heat flux.

If the hole is assumed to be insulated and free of traction, the boundary conditions for this problem can be written as

$$q_x \to 0, \quad q_y \to h_0, \quad t_1 = 0, \quad t_2 = 0, \quad \text{at infinity}, \tag{22}$$

$$\Theta = 0, \quad t = 0, \quad \text{along the hole boundary}. \tag{23}$$

Similar to Ref. [17] by Lekhnitskii, we use the conformal mapping

$$z_k(\zeta) = R_k\left(\zeta_k + m_k\frac{1}{\zeta_k}\right), \quad k = 1,2,\ldots,6,t \tag{24}$$

with

$$R_k = \frac{a - \mathrm{i}\mu_k b}{2}, \quad m_k = \frac{a + \mathrm{i}\mu_k b}{a - \mathrm{i}\mu_k b}, \ \mu_t = \mathrm{i}, \tag{25}$$

which transform the interior of the ellipse at the z_k-plane onto the exterior of the unit circle at the ζ_k-plane. The boundary conditions of the heat flux shown in (22) are similar to those of the stresses in the isothermal condition. Similar to the classical elastic theory, the temperatures and resultant heat flux for the considered problem can be expressed as

$$\begin{aligned} T &= 2\,\mathrm{Re}\{g_h'(\zeta_t) + g_p'(\zeta_t)\}, \\ \Theta &= -2k_{11}\,\mathrm{Re}\{g_h'(\zeta_t) + g_p'(\zeta_t)\} \end{aligned} \tag{26}$$

with

$$\begin{aligned} &g_p(\zeta_t) = \gamma_1(\ln\zeta_t + m_0\zeta_t^{-2}/2), \\ &g_h(\zeta_t) = \gamma_2(\zeta_t^2 + m_0^2\zeta_t^{-2}/2), \\ &\gamma_1 = \frac{a(a+b)}{4ik_{11}}h_0, \quad \gamma_2 = -\frac{(a+b)^2}{16ik_{11}}h_0, \quad m_0 = \frac{a-b}{a+b}, \end{aligned}$$

where $g_h(\zeta_t)$ represents the solution for the homogeneous medium, and $g_p(\zeta_t)$ represents the perturbed solution due to the presence of the elliptical hole. According to the traction-free boundary condition and the requirement of single-valuedness of displacements,the complex function $\boldsymbol{\Phi}(\zeta_k)$ can be expressed as

$$\boldsymbol{\Phi}(\zeta_k) = \ll \ln\zeta_k \gg \boldsymbol{L}, \tag{27}$$

and by Eqs. (21), (23), and (27), we have

$$\begin{aligned} &\boldsymbol{A}\boldsymbol{L}\ln\sigma - \overline{\boldsymbol{A}}\,\overline{\boldsymbol{L}}\ln\sigma + \gamma_1\boldsymbol{c}\ln\sigma - \overline{\gamma}_1\overline{\boldsymbol{c}}\ln\sigma = 0, \\ &\boldsymbol{B}\boldsymbol{L}\ln\sigma - \overline{\boldsymbol{B}}\,\overline{\boldsymbol{L}}\ln\sigma = 0, \end{aligned} \tag{28}$$

where $\boldsymbol{L}$ is the complex constant to be determined. The angular bracket represents the diagonal matrix, i.e. $\ll \ln\zeta_k \gg = \mathrm{diag}[\ln\zeta_1,\ \ln\zeta_2,\ \ln\zeta_3,\ \ln\zeta_4,\ \ln\zeta_5,\ \ln\zeta_6]$.

By Eq. (28), the unknown coefficient $\boldsymbol{L}$ can be obtained as follows

$$\boldsymbol{L} = 2\boldsymbol{B}^{-1}(\boldsymbol{M} + \overline{\boldsymbol{M}})^{-1}\,\mathrm{Im}\{\gamma_1\boldsymbol{c}\}, \tag{29}$$

where

$$\boldsymbol{M} = \mathrm{i}\boldsymbol{A}\boldsymbol{B}^{-1}.$$

Then all the field variables can be determined by Eqs. (18), (21), and (27).

The stress of interest to us is that at the edge of the hole. To study the effect of the hole on the stress, we give the formula for the hoop phonon stress. Let $\boldsymbol{s}(\varphi)$ be the unit vector tangential to the elliptical boundary (see Fig. 1), one sees

$$\boldsymbol{s}(\varphi) = \{\cos\varphi, \sin\varphi\}^{\mathrm{T}}. \tag{30}$$

Then the phonon hoop stress σ_{ss} can be written as

$$\sigma_{\mathrm{ss}} = \boldsymbol{s}(\varphi)^{\mathrm{T}}\boldsymbol{t}_{\mathrm{s}}, \tag{31}$$

where $\boldsymbol{t}_{\mathrm{s}}$ denote the traction vectors on the surfaces with normals $\boldsymbol{s}$ and

$$\begin{aligned} &\boldsymbol{t}_{\mathrm{s}} = \Sigma_2\sin\varphi + \Sigma_1\cos\varphi, \\ &\Sigma_1 = \{\sigma_{xx}, \sigma_{yx}\}^{\mathrm{T}}, \Sigma_2 = \{\sigma_{xy}, \sigma_{yy}\}^{\mathrm{T}}. \end{aligned}$$

In order to have a clear and direct picture on how the ratio $e = b/a$ affects the hoop stress, the icosahedral Al–Pd–Mn is used as an example to numerically demonstrate the hoop stress of the phonon field. The material constants[18] are given by

$$\begin{aligned} &\lambda = 74.9\ \mathrm{GPa}, \quad \mu = 72.4\ \mathrm{GPa}, \\ &K_1 = 125\ \mathrm{MPa}, \quad K_2 = -50\ \mathrm{MPa}. \end{aligned}$$

It should be mentioned that exact values of the phonon–phason coupling constant R are not available in the literature. In this work, we adopt the coupling constant as $R = 0.004\mu$.

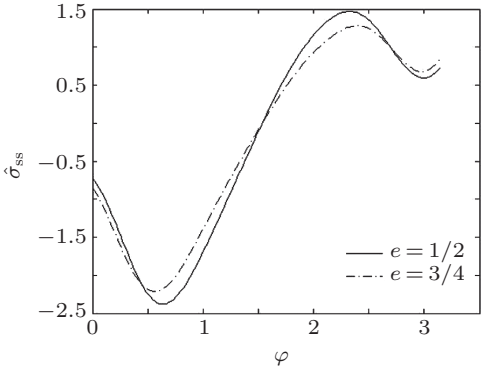


Fig. 2. The normalized phonon hoop stress $\hat{\sigma}_{\mathrm{ss}}$ versus angle φ.

Figure 2 show that the maximum values of the normalized phonon hoop stress $\hat{\sigma}_{ss} = (k_{11}\sigma_{ss})/(ah_0)$ decrease along with the increase of the ratio e.

4. Cracks under uniform heat flow

The solutions for an icosahedral quasicrystal containing a crack can be obtained corresponding to the case $b = 0$ of the above solution. By Eqs. (24) and (26), we have

$$\zeta_k = \frac{z_k + \sqrt{z_k^2 - a^2}}{a}, \quad k = 1,2,\ldots,6,t,$$
$$\gamma_1 = \frac{a^2 h_0}{4\mathrm{i}k_{11}},$$
$$\boldsymbol{L} = -\frac{a^2 h_0}{2k_{11}} \boldsymbol{B}^{-1}(\boldsymbol{M} + \overline{\boldsymbol{M}})^{-1}\,\mathrm{Re}\{\boldsymbol{c}\}.$$

Differentiating functions $\boldsymbol{\Phi}(\zeta_k)$ with respect to z_k, by using Eq. (18) and setting $y = 0$, $x > a$, the stresses ahead of the crack tip along the x axis are obtained as

$$\boldsymbol{t}_2 = 2\,\mathrm{Re}\{\boldsymbol{BL}\}\frac{1}{\sqrt{x^2 - a^2}}. \tag{32}$$

Define

$$\boldsymbol{K} = \{K_{\mathrm{II}},\ K_{\mathrm{I}},\ K_{\mathrm{III}},\ K_{\mathrm{II}}^*,\ K_{\mathrm{I}}^*,\ K_{\mathrm{III}}^*\}^{\mathrm{T}}.$$

Then, at the right crack tip $(x = a)$, the field intensity factor vector $\boldsymbol{K}$ can be expressed as

$$\begin{aligned}\boldsymbol{K} &= \lim_{x\to a}\sqrt{2\pi(x-a)}\,\boldsymbol{t}_2(x,0)\\ &= 2\sqrt{\frac{\pi}{a}}\,\mathrm{Re}\{\boldsymbol{BL}\},\end{aligned} \tag{33}$$

K_{II}^*, K_{I}^*, K_{III}^* denote the phason field intensity factor. Equation (33) shows that the intensity factors are dependent on the material properties.

Similarly, by using Eq. (21) and setting $y = 0^{\pm}$ and $|x| < a$ where $\pm$ denotes the upper and lower surface of the crack, the crack opening displacements $\Delta\boldsymbol{U}$ are obtained as

$$\Delta\boldsymbol{U} = \boldsymbol{U}(x,0^+) - \boldsymbol{U}(x,0^-) = -\frac{2h_0}{k_{11}}\,\mathrm{Re}\{c\}x\sqrt{a^2 - x^2}. \tag{34}$$

By applying the virtual crack closure method, the strain energy release rate G can be calculated as

$$\begin{aligned}G &= \lim_{\Delta a\to 0}\frac{1}{2\Delta a}\int_0^{\Delta a}(u_i(x-\Delta a)\sigma_{i2}(x) + w_i(x-\Delta a)H_{i2}(x))\,\mathrm{d}x\\ &= \frac{\pi a^3 h_0^2}{2k_{11}^2}\{(\boldsymbol{M} + \overline{\boldsymbol{M}})^{-1}\,\mathrm{Re}\{\boldsymbol{c}\}\}^T\,\mathrm{Re}\{\boldsymbol{c}\}.\end{aligned} \tag{35}$$

5. Conclusion and discussion

The Lekhnitskii formulation including the thermal effect is extended to icosahedral quasicrystals. The two-dimensional problem of icosahedral quasicrystals containing an elliptic hole under heat loadings is studied. The hoop stress and the solution for crack are observed as an application of the solution. Based on the formulation, the stress intensity factor is derived analytically. The general solutions developed herein for heat sources can be used as fundamental solutions for more complicated thermal stress problems involving inclusions and cracks.

Appendix A

$$\begin{aligned}
\varepsilon_{xx} &= \frac{1}{4(\lambda+\mu)(\mu K_1 - 2R^2)}\\
&\quad\times[(K_1(\lambda+2\mu) - 2R^2)\sigma_{xx} - (\lambda K_1 + 2R^2)\sigma_{yy}\\
&\quad - 2R(\lambda+\mu)(H_{xx} + H_{yy})] + \alpha T,\\
\varepsilon_{yy} &= \frac{1}{4(\lambda+\mu)(\mu K_1 - 2R^2)}\\
&\quad\times[-(\lambda K_1 + 2R^2)\sigma_{xx} + (K_1(\lambda+2\mu) - 2R^2)\sigma_{yy}\\
&\quad + 2R(\lambda+\mu)(H_{xx} + H_{yy})] + \alpha T,\\
\varepsilon_{xy} &= \varepsilon_{yx} = \frac{K_1\sigma_{xy} + R(H_{xy} - H_{yx})}{2(\mu K_1 - 2R^2)},\\
\varepsilon_{xz} &= \varepsilon_{zx} = -\frac{1}{2\Delta_1}[-(K_1+K_2)\sigma_{xz} + R(H_{xx} + H_{zx} - H_{yy})],\\
\varepsilon_{yz} &= \varepsilon_{zy} = -\frac{1}{2\Delta_1}[-(K_1+K_2)\sigma_{yz} - R(H_{xy} - H_{zy} + H_{yx})],\\
w_{xx} &= \frac{1}{\Delta_2}\left\{\frac{1}{2}\left[R(2K_2 - K_1) + \frac{R(2K_2 - K_1)(\mu K_2 - R^2)}{\mu K_1 - 2R^2}\right]\right.\\
&\quad\times(\sigma_{xx} - \sigma_{yy}) + R(2K_2 - K_1)\sigma_{xz}\\
&\quad + \left[\mu(K_1 - K_2) - R^2 - \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}\right]H_{xx}\\
&\quad \left. - \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}H_{yy} - (\mu K_2 - R^2)H_{zx}\right\},\\
w_{yy} &= \frac{1}{\Delta_2}\left\{\frac{1}{2}\left[R(2K_2 - K_1) + \frac{R(2K_2 - K_1)(\mu K_2 - R^2)}{\mu K_1 - 2R^2}\right]\right.\\
&\quad\times(\sigma_{xx} - \sigma_{yy}) - R(2K_2 - K_1)\sigma_{xz} - \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}H_{xx}\\
&\quad + \left[\mu(K_1 - K_2) - R^2 -, \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}\right]H_{yy}\\
&\quad \left. + (\mu K_2 - R^2)H_{zx}\right\} - (\mu K_2 - R^2)H_{zx}\},\\
w_{xy} &= \frac{1}{\Delta_2}\left\{-\left[R(2K_2 - K_1) + \frac{R(2K_2 - K_1)(\mu K_2 - R^2)}{\mu K_1 - 2R^2}\right]\sigma_{xy}\right.\\
&\quad - R(2K_2 - K_1)\sigma_{yz}\\
&\quad + \left[\mu(K_1 - K_2) - R^2 - \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}\right]H_{xy}\\
&\quad \left. + \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}\right]H_{yx} + (\mu K_2 - R^2)H_{zy}\Big\},\\
w_{yx} &= \frac{1}{\Delta_2}\left\{\left[R(2K_2 - K_1) + \frac{R(2K_2 - K_1)(\mu K_2 - R^2)}{\mu K_1 - 2R^2}\right]\sigma_{xy}\right.\\
&\quad \left. - R(2K_2 - K_1)\sigma_{yz} + \frac{(\mu K_2 - R^2)^2}{\mu K_1 - 2R^2}\right]H_{xy}
\end{aligned}$$

$$+\left[\mu(K_1-K_2)-R^2-\frac{(\mu K_2-R^2)^2}{\mu K_1-2R^2}\right]H_{yx}$$
$$+(\mu K_2-R^2)H_{zy}\Big\},$$
$$w_{zx} = \frac{1}{\Delta_2}[(\mu K_2-R^2)(H_{yy}-H_{xx})$$
$$+(\mu K_1-2R^2)H_{zx}+R(2K_2-K_1)\sigma_{xz}],$$
$$w_{zy} = \frac{1}{\Delta_2}[(\mu K_2-R^2)(H_{xy}+H_{yx})$$
$$+(\mu K_1-2R^2)H_{zy}+R(2K_2-K_1)\sigma_{yz}],$$

in which

$$\Delta_1=\mu(K_1+K_2)-3R^2,\quad \Delta_2=(K_1-2K_2)\Delta_1,$$
$$\alpha=\frac{\beta}{2(\lambda+\mu)}.$$

Appendix B

$$d_1=e_1+e_2+6e_3,\quad d_2=6e_1-19e_2+6e_3,$$
$$d_3=15e_1+90e_2+15e_3,\quad d_4=20e_1+10e_2+20e_3,$$
$$d_5=15e_1-75e_2+15e_3,\quad d_6=6e_1+25e_2+6e_3,$$
$$d_7=e_1+e_3,$$

where

$$e_1 = c_2c_3c_4\left(K_1\left(\mu K_2-2R^2\right)+K_2\left(R^2+\mu K_2\right)\right),$$
$$e_2 = c_2c_3\left(R^2-\mu K_2\right)\left(K_2\left(\mu K_2-7R^2\right)+K_1\left(2R^2+\mu K_2\right)\right),$$
$$e_3 = c_1c_4(K_1(2R^4-\mu K_2R^2-\mu^2K_2^2+c_4(\mu K_2-2R^2))$$
$$+K_2(-7R^4+8\mu K_2R^2-\mu^2K_2^2+c_4(R^2+\mu K_2))).$$

Appendix C

$$e_{11} = c_4(K_1(2R^4-\mu K_2R^2-\mu^2K_2^2+c_4(\mu K_2-2R^2))$$
$$+K_2(-7R^4+8\mu K_2R^2-\mu^2K_2^2+c_4(R^2+\mu K_2))),$$
$$e_{31} = -3\alpha_1^{(3)}+3\alpha_2^{(3)}-2\alpha_3^{(3)},$$
$$e_{32} = -8\alpha_1^{(3)}+8\alpha_2^{(3)}+6\alpha_3^{(3)}+12\alpha_4^{(3)}-12\alpha_5^{(3)}$$
$$e_{33} = -6\alpha_1^{(3)}+6\alpha_2^{(3)}+2\alpha_3^{(3)}+8\alpha_4^{(3)}-8\alpha_5^{(3)},$$
$$e_{34} = -6\alpha_3^{(3)}-4\alpha_4^{(3)}+4\alpha_5^{(3)},\quad e_{35}=\alpha_1^{(3)}-\alpha_2^{(3)},$$
$$e_{41} = \alpha_1^{(4)}+2\alpha_2^{(4)}-\alpha_3^{(4)}-\alpha_4^{(4)},$$
$$e_{42} = -8\alpha_2^{(4)}+4\alpha_4^{(4)}-6\alpha_3^{(3)},$$
$$e_{43} = -6\alpha_1^{(4)}+4\alpha_2^{(4)}+6\alpha_3^{(4)}-2\alpha_4^{(4)}+2\alpha_3^{(3)},$$
$$e_{44} = -8\alpha_1^{(4)}+8\alpha_2^{(4)}+8\alpha_3^{(4)}-4\alpha_4^{(4)}+6\alpha_3^{(3)},$$
$$e_{45} = -3\alpha_1^{(4)}-6\alpha_2^{(4)}+3\alpha_3^{(4)}+3\alpha_4^{(4)}-2\alpha_3^{(3)},$$

where

$$\alpha_1^{(3)} = c_4K_1(2R^2-\mu K_2),\quad \alpha_2^{(3)}=c_4K_2(R^2+\mu K_2),$$
$$\alpha_3^{(3)} = (R^2-\mu K_2)(K_2(\mu K_2-7R^2)+K_1(2R^2+\mu K_2)),$$
$$\alpha_4^{(3)} = K_1(2R^4-\mu K_2R^2-\mu^2K_2^2),$$
$$\alpha_5^{(3)} = K_2(7R^4-8\mu K_2R^2+\mu^2K_2^2),$$
$$\alpha_1^{(4)} = R^2c_4(2K_2-K_1),$$
$$\alpha_2^{(4)} = R^2(K_1-2K_2)(R^2-\mu K_2),$$
$$\alpha_3^{(4)} = c_4(K_1+K_2)(R^2-\mu K_2),$$
$$\alpha_4^{(4)} = K_2(\mu K_2-R^2)(-3R^2+\mu K_1+\mu K_2).$$

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