# Existence of a Hartmann layer in the peristalsis of Sisko fluid

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Analytical solutions for the peristaltic flow of a magneto hydrodynamic (MHD) Sisko fluid in a channel, under the effects of strong and weak magnetic fields, are presented. The governing nonlinear problem, for the strong magnetic field, is solved using the matched asymptotic expansion. The solution for the weak magnetic field is obtained using a regular perturbation method. The main observation is the existence of a Hartman boundary layer for the strong magnetic field at the location of the two plates of the channel. The thickness of the Hartmann boundary layer is determined analytically. The effects of a strong magnetic field and the shear thinning parameter of the Sisko fluid on the velocity profile are presented graphically.

**Keywords:** magneto hydrodynamic Sisko fluid, asymmetric channel, Peristaltic flow, non-Newtonian fluid

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## 1. Introduction

Recently, the peristaltic mechanism has become quite a popular research field because of its extensive applications in engineering and physiological problems. Specifically, such a mechanism appears in urine transport from the kidney to the bladder, chyme motion in the gastrointestinal tract, sperm transport, roller and finger pumps, a worm's locomotion, swallowing of food through the esophagus, sanitary fluid transport, and so on. Several articles of practical importance are presented since the pioneering work of Latham.[1] The relevant extended background can be found in Refs. [2]–[19]. The most significant paper from our point of view was contributed by Wang *et al.*[20] They presented numerical results for the MHD peristaltic flow of Sisko fluid. The present study provides deeper understanding and is different from Ref. [12], in the sense that it provides an analytical solution that may reveal the physics of the peristaltic flow. This is always the advantage of the analytical results over the numerical results. This motivation leads us to construct an analytical solution for nonlinear differential equation of the peristaltic motion of the Sisko fluid with a particular interest in the Hartmann boundary layer. For that we choose the problem considered by Wang *et al.*[20] and solve it analytically for small and large magnetic fields. The regular and singular perturbation methods are employed to determine the velocity field. The choice of Sisko fluid provides us with an opportunity to understand the boundary layer phenomenon for viscous and power law fluids as special cases. Our main objective has been to discuss various possibilities under which an analytical solution, under the influence of strong and weak magnetic fields, can be realized. These possibilities turn out to be the Newtonian $(n = 1)$ and the shear thinning $(n < 1)$ fluids. The analytical solutions for the Newtonian and the shear thinning fluids have been presented; whereas the solution for the shear thickening fluid is not tractable. The main focus of this study is thus to find the analytical solutions, under the influence of weak and strong magnetic fields, which to our knowledge, has not been realized so far. Equally important is the analysis of the Hartman boundary layer, which could be possible if we resort to the analytic solution. This is essentially the reason why this study is different from that of Wang *et al.*,[20] in terms of its usefulness. The phenomenal observation is the existence of a Hartmann layer in the presence of a strong magnetic field which combines the important concept of boundary layer with peristalsis in addition to other applications. It is expected that the existence and understanding of the boundary layer concept in physiological flows will help the engineers and biologists as well.

## 2. Problem formulation

We consider the flow of an electrically conducting incompressible Sisko fluid in a two-dimensional asymmetric channel of width $2\overline{h_0}$. The traveling waves of a small amplitude are propagating along the channel walls and a uniform magnetic field $\overline{B_0}$ is applied in the transverse direction to the flow.

The geometry of the problem is given in Fig. 1. Here $\overline{d_1}$ and $\overline{d_2}$ are the amplitudes of the peristaltic waves, $\lambda$ is the wave length, and $\phi$ is the phase difference between peristaltic waves at the two walls of the channel. The governing momentum equation and the associated boundary conditions for peristaltic flow in Sisko fluid are adopted directly from Ref. [12],

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i.e.,

$$\frac{\partial^2}{\partial y^2}\left[\left(1+b^*\left(\frac{\partial^2\Psi}{\partial y^2}\right)^{n-1}\right)\frac{\partial^2\Psi}{\partial y^2}\right]-B^2\frac{\partial^2\Psi}{\partial y^2}=0, \quad (1)$$

$$\Psi=\frac{q}{2}, \quad \frac{\partial\Psi}{\partial y}=-1, \quad \text{at } y=h_1(x),$$

$$\Psi=-\frac{q}{2}, \quad \frac{\partial\Psi}{\partial y}=-1, \quad \text{at } y=h_2(x). \quad (2)$$

Here, $B$ is the magnetic field parameter (Hartman number), and $b^*$ is the dimensionless material parameter which indicates the ratio of the power law part to the viscous part in a Sisko fluid ($b^*\to\infty$ describes a hydrodynamic power law fluid). The case $n=1$ corresponds to the Newtonian fluid, $n<1$ to the shear thinning fluid, and $n>1$ to the shear thickening fluid. The variable $q$ is the dimensionless flow rate in the wave frame at any axial position of the channel. We are concerned with the analytical solution of Eq. (1) with the boundary conditions (2) for the cases of strong and weak magnetic fields.

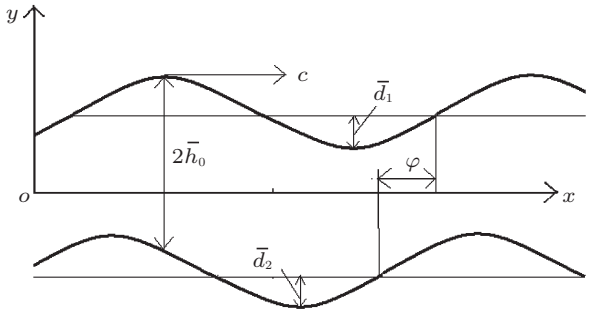



**Fig. 1.** Geometry of the problem.

## 3. Solution methodology

Regular and singular perturbation techniques are used to find the analytic solution of the proposed boundary value problem. A number of cases will be discussed to show the impact of strong and weak magnetic field parameters on the fluid flow.

### 3.1. Strong Magnetic field and Newtonian fluid ($n=1$)

For the Newtonian fluid ($n=1$) and a large (strong) magnetic field ($B$), equation (1) is conveniently written as

$$\varepsilon^2(1+b^*)\frac{\partial^4\Psi}{\partial y^4}-\frac{\partial^2\Psi}{\partial y^2}=0, \quad (3)$$

where $B=1/\varepsilon$, and $\varepsilon\ll 1$. Equation (3) is a singular perturbation problem giving rise to boundary layer behavior at the location of the two walls. The best way to obtain the solution is matched asymptotic analysis, which requires an inner solution at the location of two walls and an outer solution away from the walls while remaining inside the channel.

#### 3.1.1. Inner solutions

For the boundary layer equation at $y=h_1(x)$, let us introduce the stretched variable

$$\eta=\frac{h_1-y}{\varepsilon^p}. \quad (4)$$

In the new variable, equation (3) takes the form of

$$\varepsilon^{2-4p}(1+b^*)\frac{\partial^4\Psi^{\text{in}}}{\partial\eta^4}-\varepsilon^{-2p}\frac{\partial^2\Psi^{\text{in}}}{\partial\eta^2}=0, \quad (5)$$

where $\Psi^{\text{in}}(\eta)$ is the solution satisfying the inner boundary condition at $y=h_1(x)$, and $p$ is to be determined by the principle of the least degeneracy. In order to balance the terms in Eq. (5), we require $p=1$, and thus

$$\frac{\partial^4\Psi^{\text{in}}}{\partial\eta^4}-\frac{1}{k^2}\frac{\partial^2\Psi^{\text{in}}}{\partial\eta^2}=0, \quad (6)$$

where $1+b^*=k^2$. The boundary conditions satisfied by $\Psi^{\text{in}}$ at $y=h_1(x)$ are

$$\Psi^{\text{in}}(\eta)=\frac{q}{2}, \quad \Psi^{\text{in}}(\eta)=\varepsilon, \quad \text{at } \eta=0. \quad (7)$$

The two terms perturbation solution of Eq. (6) can be expressed as

$$\Psi^{\text{in}}(\eta)=\Psi_0^{\text{in}}(\eta)+\varepsilon\Psi_1^{\text{in}}(\eta). \quad (8)$$

Substituting Eq. (7) into Eq. (8), we obtain the leading order system as

$$\frac{\partial^4\Psi_0^{\text{in}}}{\partial\eta^4}-\frac{1}{k^2}\frac{\partial^2\Psi_0^{\text{in}}}{\partial\eta^2}=0, \quad (9)$$

$$\Psi_0^{\text{in}}(0)=\frac{q}{2}, \quad \Psi_0'^{\text{in}}(0)=0. \quad (10)$$

The above solution can be constructed as

$$\Psi_0^{\text{in}}=\frac{q}{2}+C_0\left(\mathrm{e}^{\eta/K}-1-\frac{\eta}{k}\right)+D_0\left(\mathrm{e}^{-\eta/K}-1+\frac{\eta}{k}\right).$$

To avoid the exponential growth in the above equation, we choose $C_0=0$, yielding

$$\Psi_0^{\text{in}}=\frac{q}{2}+D_0\left(\mathrm{e}^{-\eta/k}-1+\frac{\eta}{k}\right). \quad (11)$$

The first-order system is written as

$$\frac{\partial^4\Psi_1^{\text{in}}}{\partial\eta^4}-\frac{1}{k^2}\frac{\partial^2\Psi_1^{\text{in}}}{\partial\eta^2}=0, \quad (12)$$

$$\Psi_1^{\text{in}}(0)=0, \quad \Psi_1'^{\text{in}}(0)=1. \quad (13)$$

The solution is expressed as

$$\Psi_1^{\text{in}}(\eta)=\eta+C_1\left(\mathrm{e}^{\eta/K}-1-\frac{\eta}{k}\right)+D_1\left(\mathrm{e}^{-\eta/K}-1+\frac{\eta}{k}\right).$$

The constant $C_1$ is still zero and the above equation reduces to

$$\Psi_1^{\text{in}}(\eta)=\eta+D_1\left(\mathrm{e}^{-\eta/K}-1+\frac{\eta}{k}\right). \quad (14)$$

The two terms of the inner solution are finally written as

$$\Psi^{\text{in}}_{\text{2terms}}(\eta) = \frac{q}{2} + D_0\left(\mathrm{e}^{-\eta/K} - 1 + \frac{\eta}{k}\right) + \varepsilon\left[\eta + D_1\left(\mathrm{e}^{-\eta/K} - 1 + \frac{\eta}{k}\right)\right], \quad (15)$$

where $D_0$ and $D_1$ are unknown constants to be determined by the matching technique.

For the inner solution $\Psi^{\text{IN}}(\xi)$ at $y = h_2(x)$, we define

$$\xi = \frac{y - h_2}{\varepsilon^r}. \quad (16)$$

Following the same procedure as above, we have

$$\Psi^{\text{IN}}_{\text{2terms}}(\xi) = -\frac{q}{2} + C_2\left(\mathrm{e}^{-\xi/K} - 1 + \frac{\xi}{K}\right) + \varepsilon\left[-\xi + D_2\left(\mathrm{e}^{-\xi/K} + \frac{\xi}{K} - 1\right)\right]. \quad (17)$$

In Eqs. (15) and (17), the constants $C_2$ and $D_2$ are determined by matching with the outer solution.

### 3.1.2. Outer solution

For the outer solution away from the two surfaces, in the channel, equation (3) reduces to

$$\frac{\partial^2 \Psi}{\partial y^2} = 0. \quad (18)$$

By virtue of

$$\Psi^{\text{out}}(y) = \Psi^{\text{out}}_0 + \varepsilon\Psi^{\text{out}}_1, \quad (19)$$

the two terms of the outer solution are given by

$$\Psi^{\text{out}}(y) = a_0 y + b_0 + \varepsilon(a_1 y + b_1), \quad (20)$$

where $a_0$, $b_0$, $a_1$, and $b_1$ are unknown constants to be determined by higher-order matching of two inner expansions and one outer expansion.

For higher-order matching at $y = h_1(x)$, we define an intermediate region of $O(\varepsilon^\alpha)$ and an intermediate parameter $t$ by

$$t = \frac{h_1 - y}{\varepsilon^\alpha}. \quad (21)$$

In the following, we convert inner and outer solutions given by Eqs. (15) and (20) in the intermediate parameter $t$ and discard the terms higher than $O(\varepsilon^\alpha)$. This simply yields

$$\left(\Psi^{\text{out}}\right)^{\text{int}} = a_0 h_1 + b_0 - a_0 t(\varepsilon)^\alpha + \varepsilon(a_1 h_1 + b_1) + O(\varepsilon)^{\alpha+1}, \quad (22)$$

$$\left(\Psi^{\text{in}}\right)^{\text{int}} = \frac{q}{2} + D_0\left(-1 + \frac{t(\varepsilon)^{\alpha-1}}{k}\right) + t(\varepsilon)^\alpha\left(1 + \frac{D_1}{k}\right) - D_1\varepsilon. \quad (23)$$

Matching the two solutions given by Eqs. (22) and (23) yields

$$a_0 h_1 + b_0 = \frac{q}{2} - D_0, \ \ D_0 = 0, \quad \left(1 + \frac{D_1}{k}\right) = -a_0, \ \ (a_1 h_1 + b_1) = -D_1. \quad (24)$$

Similar analysis of matching outer and inner solutions at $y = h_2(x)$ leads to

$$a_0 h_2 + b_0 = -\frac{q}{2} - H_0, \ \ H_0 = 0, \quad a_0 = \frac{H_1}{k} - 1, \ \ -H_1 = a_1 h_2 + b_1. \quad (25)$$

The unknown constants are now determined by Eqs. (24) and (25) and given by

$$a_0 = \frac{q}{h_1 - h_2}, \ a_1 = \frac{2k}{(h_1 - h_2)}\left(1 + \frac{q}{h_1 - h_2}\right),$$
$$b_0 = \frac{-q}{2}\left(\frac{h_1 + h_2}{h_1 - h_2}\right), \ b_1 = \left(1 + \frac{q}{h_1 - h_2}\right)\left(\frac{-k(h_1 + h_1)}{(h_1 - h_2)}\right),$$
$$D_1 = k\left(-\frac{q}{h_1 - h_2} - 1\right), \ H_1 = k\left(1 + \frac{q}{h_1 - h_2}\right).$$

The two terms of the composite solution defined as

$$\Psi_{\text{composite}} = \Psi^{\text{out}} + \Psi^{\text{in}}(\eta) - (\Psi^{\text{in}})^{\text{int}} + \Psi^{\text{IN}}(\xi) - (\Psi^{\text{IN}})^{\text{int}}, \quad (26)$$

are finally written as

$$\Psi_{\text{composite}} = \frac{q\,y}{h_1 - h_2} - \frac{q}{2}\left(\frac{h_1 + h_2}{h_1 - h_2}\right) + \varepsilon k\left(1 + \frac{q}{h_1 - h_2}\right) \times \left[\frac{2y}{h_1 - h_2} - \frac{h_1 + h_1}{h_1 - h_2} - \mathrm{e}^{-\frac{h_1 - y}{k\varepsilon}} + \mathrm{e}^{-\frac{y - h_2}{k\varepsilon}}\right]. \quad (27)$$

We recall that $B = 1/\varepsilon$ and the appearance of exponential terms in Eq. (27) confirm the existence of boundary layers at the location of the two walls at $y = h_1(x)$ and $y = h_2(x)$. The velocity profile for different values of $B$ and $b^*$ are shown in Figs. 2 and 3 for $x = -\pi$. We clearly see that the value of the velocity increases for $-0.5 < y < 0$, reaching its maximum value at $y = 0$. For $0 < y < 0.5$, the velocity shows a decreasing trend. This is actually the same behavior as that shown by Fig. 1 in Ref. [20]. The boundary layer called the Hartmann boundary layer has a thickness of $O(\sqrt{1 + b^*}/B)$, thus depending inversely upon the magnetic field parameter $B$ and directly on $b$. This suggests that the boundary layer decreases with the increase in magnetic field and increases with the increase in $b^*$. The impact of $b^*$ however is not very significant. The existence, location, and thickness of the boundary layer have been confirmed by analytical results for the Newtonian case. Similar observations were reported by Wang *et al.*[20] for the Newtonian fluid (see Fig. 1), but numerically.

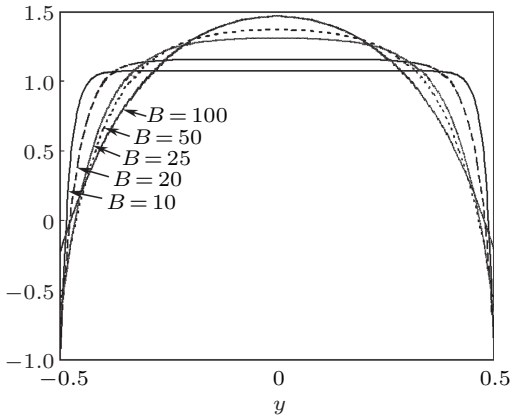



**Fig. 2.** Velocity profile for $x=-\pi$, $b^*=3$, and $d_1=d_2=0.5$.

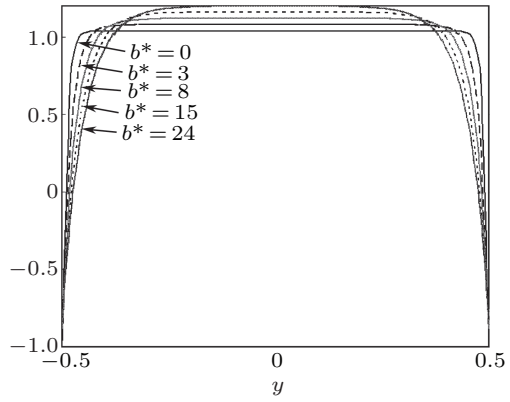



**Fig. 3.** Velocity profile for $x=-\pi$, $B=100$, and $d_1=d_2=0.5$.

### 3.2. Shear thinning fluid ($n<1$) and large (strong) magnetic field

In this section, the solutions for the shear thinning fluid ($n<1$) and the large magnetic field are presented. Taking $B=1/\varepsilon$ ($\varepsilon<<1$), equation (1) leads to

$$\varepsilon^2\left[\frac{\partial^4\Psi}{\partial y^4}+b^*\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\Psi}{\partial y^2}\right)^n\right]-\frac{\partial^2\Psi}{\partial y^2}=0. \tag{28}$$

It is important to note that the power law index $n$ plays an important role in the order of magnitude analysis. We observe that the solution of Eq. (28) is tractable for $n<1$ and as a particular choice, we take $n=1/2$. Following the method discussed in Section 3.1, the inner solutions at $y=h_1(x)$ and $y=h_2(x)$ are given by

$$\begin{aligned}\Psi^{\rm in}_{\rm 2terms}(\eta) &= \frac{q}{2}+M_0\left(\mathrm{e}^{-\eta}-1+\eta\right)\\ &\quad+\varepsilon\left[\eta+M_1\left(\mathrm{e}^{-\eta}-1+\eta\right)\right.\\ &\quad\left.+\frac{4}{3}b^*\sqrt{M_0}\left(\mathrm{e}^{-\eta/2}-1+\frac{\eta}{2}\right)\right], \qquad (29)\\ \Psi^{\rm IN}_{\rm 2terms}(\xi) &= -\frac{q}{2}+V_0\left(\mathrm{e}^{-\xi}-1+\xi\right)\\ &\quad+\varepsilon\left[-\xi+V_1\left(\mathrm{e}^{-\xi}+\xi-1\right)\right.\\ &\quad\left.+\frac{4}{3}b^*\sqrt{V_0}\left(\mathrm{e}^{-\xi/2}-1+\frac{\xi}{2}\right)\right], \qquad (30)\end{aligned}$$

where $M_0$, $M_1$, $V_0$, and $V_1$ are constants to be determined by the matching principle.

The outer solution remains the same (Eq. (20)) and is rewritten as

$$\Psi^{\rm out}(y)=a_0y+b_0+\varepsilon(a_1y+b_1). \tag{31}$$

For higher-order matching, the outer solution and the inner solution at $y=h_1(x)$ when written in terms of intermediate parameter $t$, are expressed as

$$\begin{aligned}\left(\Psi^{\rm out}\right)^{\rm int} &= a_0h_1+b_0-a_0t(\varepsilon)^\alpha\\ &\quad+\varepsilon(a_1h_1+b_1)+O(\varepsilon)^{\alpha+1}, \qquad (32)\\ (\Psi^{\rm in})^{\rm int} &= \frac{q}{2}+U_0\left(-1+t(\varepsilon)^{\alpha-1}\right)\\ &\quad+t(\in)^\alpha(1+U_1)-O_1\varepsilon. \qquad (33)\end{aligned}$$

The matching between Eqs. (32) and (33) results in

$$\begin{aligned}&a_0h_1+b_0=\frac{q}{2}-U_0,\ \ U_0=0,\ \ (1+U_1)=-a_0,\\ &(a_1h_1+b_1)=-U_1. \qquad (34)\end{aligned}$$

Similarly, higher-order matching, in terms of intermediate parameter at $y=h_2(x)$, yields

$$\begin{aligned}&a_0h_2+b_0=-\frac{q}{2}-V_0,\ \ V_0=0,\\ &a_0=V_1-1,\ \ -V_1=a_1h_2+b_1. \qquad (35)\end{aligned}$$

Solving Eqs. (34) and (35) simultaneously determines the unknown constants as

$$\begin{aligned}&a_0=\frac{q}{h_1-h_2},\ \ a_1=\frac{2}{h_1-h_2}\left(1+\frac{q}{h_1-h_2}\right),\\ &b_0=\frac{q}{2}-\frac{q}{h_1-h_2}h_1,\ \ b_1=\left(1+\frac{q}{h_1-h_2}\right)\left(\frac{-(h_1+h_1)}{h_1-h_2}\right),\\ &O_1=\left(-\frac{q}{h_1-h_2}-1\right),\ \ V_1=1+\frac{q}{h_1-h_2}.\end{aligned}$$

The composite solution of the boundary value problem (28) defined by

$$\Psi_{\rm composite}=\Psi^{\rm out}+\Psi^{\rm in}(\eta)-(\Psi^{\rm in})^{\rm int}+\Psi^{\rm IN}(\xi)-(\Psi^{\rm IN})^{\rm int},$$

is finally written as

$$\begin{aligned}\Psi_{\rm uniform} &= \frac{q}{h_1-h_2}y-\frac{q}{2}\left(\frac{h_1+h_2}{h_1-h_2}\right)+\varepsilon\left(1+\frac{q}{h_1-h_2}\right)\\ &\quad\times\left[\frac{2y}{h_1-h_2}-\left(\frac{h_1+h_1}{h_1-h_2}\right)-\mathrm{e}^{-\frac{h_1-y}{\varepsilon}}+\mathrm{e}^{-\frac{y-h_2}{\varepsilon}}\right].\end{aligned} \tag{36}$$

The graphical representation of the results for the case of shear thinning fluid, for various values of magnetic field parameter $B$, is shown in Fig. 4. This graph clearly shows that the boundary layer thickness increases as the magnetic field parameter increases. In this case, the maximum value of the boundary layer thickness is at $y=0$, which decreases with increasing values of $B$.

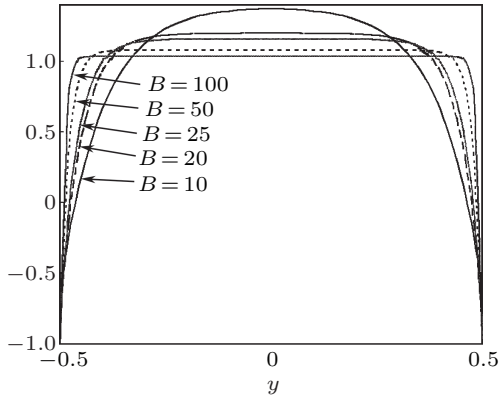



**Fig. 4.** Velocity profile for $x = -\pi$ and $d_1 = d_2 = 0.5$.

The analytical solution for the shear thickening case $n > 1$ is not tractable, in the sense that the corresponding nonlinear equation can be solved but its solution does not satisfy the required boundary conditions.

## 4. Conclusions

The analytical solution for the magneto hydrodynamic peristaltic flow of a Sisko fluid in an asymmetric channel is obtained. The study is carried out under the influence of weak and strong magnetic fields. Two special cases of Sisko fluid, Newtonian fluid and shear thinning fluid, are treated separately. The problem for a weak magnetic field can be treated using the regular perturbation method, whereas the singular perturbation technique is required for a strong magnetic field. The procedure for a weak magnetic field is pretty simple and the results are quoted directly below.

For small magnetic field $B$ and Newtonian fluid $n = 1$, the solution can be expressed as

$$\begin{aligned}\Psi = & \frac{3y^2(h_2+h_1)(q-h_2+h_1)}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)} \\ & - \frac{2y^3(q-h_2+h_1)}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)} \\ & - \frac{h_1^2(3h_2+h_1)-h_2^2(3h_1+h_2)+6qh_1h_2}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)}y \\ & - \frac{1}{2}\left(\frac{2h_1h_2\left(h_2^2-h_1^2\right)-3qh_1h_2(h_1+h_2)+q\left(h_1^3+h_2^3\right)}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)}\right).\end{aligned}$$

For small magnetic field $B$ and shear thinning fluid $n < 1$, the solution can be given by

$$\begin{aligned}\Psi = & -\frac{2y^3(q-h_2+h_1)}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)} \\ & - \frac{3y^2(q-h_2+h_1)(h_1+h_2)}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)} \\ & - \frac{h_1^2(3h_2+h_1)-h_2^2(3h_1+h_2)+6qh_1h_2}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)}y \\ & - \frac{1}{2}\left(\frac{2h_1h_2\left(h_2^2-h_1^2\right)-3qh_1h_2(h_1+h_2)+q\left(h_1^3+h_2^3\right)}{h_2^2(3h_1-h_2)-h_1^2(3h_2-h_1)}\right).\end{aligned}$$

We observe that these expressions do not have a boundary layer behavior. Some other phenomena of significant value are summarized below.

(i) A large magnetic field gives rise to the singular perturbation problem, whereas a small magnetic field allows the regular perturbation technique.

(ii) The boundary layer thickness is of $O\left(\sqrt{1+b^*}/B\right)$ for viscous fluid and $O(1/B)$ for shear thinning fluid.

(iii) The boundary layer thickness decreases as the magnetic parameter increases.

(iv) The results of the magnetic field for a peristaltic flow in Sisko fluid can be well utilized in understanding the transport of physiological fluids in humans. The role of a strong magnetic field is to control the boundary layer thickness, thus confining the effects of the non-Newtonian properties of the fluid close to the walls of the channel and allowing a better and convenient passage to the physiological fluids away from the boundary layer.

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