Cellular automata model for traffic flow with safe driving conditions*

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In this paper, a recently introduced cellular automata (CA) model is used for a statistical analysis of the inner microscopic structure of synchronized traffic flow. The analysis focuses on the formation and dissolution of clusters or platoons of vehicles, as the mechanism that causes the presence of this synchronized traffic state with a high flow. This platoon formation is one of the most interesting phenomena observed in traffic flows and plays an important role both in manual and automated highway systems (AHS). Simulation results, obtained from a single-lane system under periodic boundary conditions indicate that in the density region where the synchronized state is observed, most vehicles travel together in platoons with approximately the same speed and small spatial distances. The examination of velocity variations and individual vehicle gaps shows that the flow corresponding to the synchronized state is stable, safe and highly correlated. Moreover, results indicate that the observed platoon formation in real traffic is reproduced in simulations by the relation between vehicle headway and velocity that is embedded in the dynamics definition of the CA model.

Keywords: traffic flow models, synchronized traffic, cellular automata, platoons formation

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1. Introduction

Effective transportation systems for people and goods significantly contribute to the quality of life in modern societies. Because of increasing demands, the number of vehicles has seen a significant rise. However, in recent years, most industrialized societies have begun to see the limits of the growth of traffic infrastructure, leading to recurrent and systematic traffic congestion. Solutions are now oriented to a better management and control of existing transportation systems by designing techniques for dynamic management of traffic flows. Nevertheless, the design and implementation of such techniques requires suitable models for a proper understanding of traffic dynamics on a local and global system level, and on the effect of the associated phenomena. From this perspective, although a variety of models have been proposed in the open literature for this purpose, there is still room for more accurate models to analyze the chaotic behavior involved in today's transportation problem situations.

Computer simulations as a means for evaluating control and management strategies in traffic systems have gained considerable importance because of the possibility of taking into account the dynamical aspects of traffic (see for example, Refs. [1] and [2]). For a long time, the preferred method for traffic flow simulation was to derive macroscopic models that aggregate traffic variables in spatially discretized versions of highways and therefore to pay attention to the average value of the significant variables.[3–5] Recently, there has been a trend to perform microscopic traffic simulations that focus on individual vehicles due to the rapid development of computer capacity. Microscopic models have become popular as they can reproduce a large variety of phenomena observed in traffic that cannot be captured by most macroscopic models. Among the microscopic models, cellular automata (CA) have been increasingly used in simulations of traffic flow on account of their simplicity and flexibility.

In contrast to continuous models, which are usually discretized for their numerical simulations, in CA models, time and spacial state variables are discrete, and accordingly, velocity, acceleration, and deceleration have also discrete values. This discreteness is also taken into account in the definition of the model and its dynamics. These characteristics make them ideally suited for high-speed computer simulations, allowing to accurately obtain the desired behavior in a much simpler way. A relevant point is the fact that CA models are rule-based. Drivers' decisions are based on their current and neighbors' situation, as well as in their goals, etc., can be easily coded into rules. In addition, rule based models also allow one to take into account, for example, psychological aspects in a natural and efficient way. In contrast, force-inspired models specify interactions directly on the level of equations of motion, similar to classical mechanics. Daganzo[6] described how under some specific assumptions, CA models are equivalent to the kinematic traffic flow model and the car-following model. However, there are specific traffic behaviors that CA can model which cannot be reproduced by continuous time macroscopic models. Therefore, cellular automata have been

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used successfully in modeling the real behavior of traffic flow, and have become a popular tool for the management of traffic.

In 1992, Nagel and Schreckenberg[7] proposed the well-known NaSch model. Although the model is fairly simple, it can reproduce some complex phenomena appearing in real traffic, such as the occurrence of phantom traffic jams and start-stop waves. From then on, a considerable number of modified NaSch models have been developed or extended (e.g., Refs. [8]–[20]). In addition, several studies have been introduced to study more complex traffic situations using cellular automata.[21–26] Although some of these models have shown their success in capturing the essential features of traffic flow, which have not been reproduced by the NaSch model (e.g., metastability, hysteresis effect, etc.), the idea of a vehicle's limited deceleration has seldom been considered. In most CA models evolved from the NaSch model, arbitrarily large deceleration rates have been imposed, which can be far beyond the practical braking capability and realistic driver behavior. Consequently, vehicles tend to change speed abruptly; a vehicle can decelerate to zero from any velocity within one time step. This deceleration process exceeds the real vehicular acceleration/deceleration capabilities and produces erratic behavior. Few studies based on CA have emphasized the effect of the deceleration limitation before.[8,13,14,19,27]

Krauss and Wagner[8] was perhaps the first effort to introduce limited deceleration capability into CA modeling. Later, Lee *et al.*[13] introduced a CA model that considers mechanical restrictions of vehicles with limited capabilities of acceleration and deceleration in their model. Both of the aforementioned modifications are established under the assumption that the following driver will always be aware of the speed of the lead vehicle and hence will continuously maintain an adequate distance to prevent a collision in case the preceding vehicle in the next time step decelerates to a complete stop. However, the argument that the following vehicle is always taking caution to maintain an adequate distance to the lead vehicle is over-conservative. Knospe *et al.* developed a brake-light CA model by incorporating anticipation effects, reduced acceleration capabilities and an enhanced intersection horizon for braking.[9] Jin *et al.* proposed a new CA model without the slow-to-start effect and showed that the slow-to-start effect might not be the only possible mechanism for a first-order transition to a jam.[22] These models took the deceleration into account and can flexibly reproduce different aspects of real traffic. However, they focused on the velocity rather than on the deceleration itself. In reality, a driver commands its vehicle to accelerate, decelerate or maintain speed. Thus new CA approaches for realistic driver behaviors have been recently developed based on the process followed in drivers' decisions.[14,19,27,28]

In the search for a realistic CA model for traffic flow to capture the drivers' reactions to the traffic in a better way while preserving safety on the road, recently a new CA was proposed by Lárraga and Alvarez-Icaza[28] (hereafter referred as the LAI model). A discussion of the differences between the LAI model and other CA can also be found in Ref. [28]. The term 'realistic' is used in the sense that macroscopic and microscopic empirical data are reproduced with a high degree of realism. The LAI model was inspired by the work presented in Ref. [29], for manual and automated highway systems. Thus, human behavior is modeled as the driver response to the local traffic conditions and it is based on a safety analysis that determines the most appropriate action for a vehicle to undertake. The velocity updating of vehicles is based on the inclusion of three safe distances required by the follower vehicle to accelerate, slow down or maintain its speed. Besides, the space gap, relative speed and limited acceleration/deceleration capabilities are introduced into simulations. Values to limit acceleration/deceleration are derived from safe driving principles for normal drivers, and in accordance with the practices of transportation engineering and human reactions. In this way, in the LAI model, vehicles cannot change speeds instantly. The comfortable deceleration rate under normal situations that an individual driver is willing to use is bounded by a limited value. Larger braking applies only in emergency situations with a limited deceleration similar to real vehicles ones. Thus, extreme decelerations are avoided. Moreover, the LAI model leads to a faithful representation of driver behavior without jeopardizing the computational efficiency. Simulation results of the LAI model already have revealed that depending on the vehicle density, the model reproduces the three known traffic phases of the flow: free flow, synchronized flow, and jammed flow.[28]

In this paper, a statistical analysis of the inner microscopic structure of traffic phases reproduced by the LAI model is presented. The emphasis of the analysis focuses on the formation of platoons or clusters, as the mechanism that induces the presence of synchronized traffic flow. Platoon formation plays an important role both in manual and automated highway systems (AHS), as it increases freeway capacity. Organization of traffic into vehicle platoons generates a small variation of vehicles velocities and therefore, a safer flow. Formation and dissolution of platoons have been previously observed in other existing CA models (see for example, Refs. [10], [11], [30]–[32]).

Simulation results of the LAI model for density values associated with the synchronized phase, exhibit the presence of several coherently moving platoons with velocity larger than 0. In these dense platoons two or more vehicles move with similar velocity v_p and have a reduced headway induced by the safe following distances that a vehicle requires to accelerate, decelerate and maintain its velocity in a safe way. In

addition, statistical analysis based on the velocities and gaps of individual vehicles indicates that in this coherent state resulting from the platoon formation (dissolution), the flow is high and stable.

Simulation results of the model show that the formation and dissolution of platoons, as a function of the density observed at intermediate densities, are the basic mechanisms to maintain a high and stable synchronized flow, which is not a mechanism induced by the discretization of the LAI model, but a more complex behavior resulting from the interaction of vehicles dynamics and safe driving policies. To our knowledge, the behavior of such a coherent structure has been seldom observed before in other CA models.

The rest of the paper is organized as follows. In Section 2, a brief description of the LAI model is presented. In Section 3, a statistical analysis of the simulation results obtained for the model is presented. Special emphasis on the microscopic structure for the region corresponding to synchronized traffic flow is given. Finally, Section 4 contains concluding remarks and a summary of findings.

2. The LAI model

In this section, a brief definition of the LAI model is presented (see Ref. [28] for a detailed explanation).

The LAI model is a CA model for the simulation of single-lane traffic flow.[33] The model takes into account normal drivers' spacing policies and transportation engineering practices to guarantee that the microscopic vehicle behavior is more in line with the vehicular movement in the real world. As a result, drivers' reactions are based on a safety analysis that captures the most appropriate action for a vehicle to take.

2.1. Some considerations for the model

The dynamics of the LAI model is based on the drivers' responses to the local traffic conditions and limited acceleration/deceleration capabilities of the vehicles, which considers the following aspects.

(i) At large distances between them, drivers tend to move with their desired maximum velocity $v_{\max}$.

(ii) At intermediate distances, drivers react to velocity changes of the vehicle in front, i.e., the setting rules for the new velocity depend not only on the relative velocity of neighbors vehicles, they now take into account their positive relative distance.

(iii) At small distances, however, drivers adjust their velocity such that safe driving is possible. For very small distances, drivers would perceive an unsafe situation and will react by applying emergency braking.

(iv) Thus, a driver decides the action to be performed at the current time, to accelerate, decelerate, or maintain his/her speed, according to the calculation of three following distances to guarantee safe driving.

(v) The safe following distances are calculated assuming the worst possible scenario after any basic maneuver is performed, such that the leader vehicle may slow down suddenly at any time-step.

(vi) In order to replicate the stochastic effects of individual driver behavior, the model considers random acceleration/deceleration capabilities based on the velocities of single vehicles.

2.2. Definition of the model

As an initial point of the definition of the LAI model, the stochastic parameter $R_{\rm a}$, which introduces an element of the human behavior in the acceleration process is given in Eq. (1). $R_{\rm a}$ is based on the fact that the driver's behavior may be biased depending on the local traffic situation (relatives distances between vehicles and their velocities) and traffic fluctuations resulting from the human factor in a stochastic way. Thus, $R_{\rm a}$ denotes the probability of vehicles to accelerate based on the velocity of the vehicle

$$R_{\rm a} = \min(R_{\rm d}, R_0 + v_n(t)(R_{\rm d} - R_0)/v_{\rm s}), \tag{1}$$

where the velocity $v_{\rm s}$ is a constant slightly above 0, $0 < R_{\rm a} \leq 1$, and $0 < R_0 < R_{\rm d} \leq 1$. Stochastic parameters R_0 and $R_{\rm d}$ control the velocity fluctuations of vehicles. Therefore, the stochastic parameter $R_{\rm a}$ (≤ 1) linearly interpolates between R_0 and $R_{\rm d}$ ($R_0 < R_{\rm d}$) if v_n is smaller than a slow velocity, $v_{\rm s}$. Note that for a given value for R_0, $R_{\rm d}$, and $v_{\rm s}$, the calculation involved in Eq. (1) for the acceleration probability $R_{\rm a}$ can be performed off-line and stored in a fixed table, that relates the value of $R_{\rm a}$ with the vehicle velocity v_n. In practice, this reduces the use of three parameters, R_0, $R_{\rm d}$, and $v_{\rm s}$, to only one $R_{\rm a}$.

Table 1. Variables and parameters of the LAI model.

	Variables		Parameters
x_n	position of the n-th vehicle	$v_{\max}$	maximum velocity
v_n	velocity of the n-th vehicle	$v_{\rm s}$	slow velocity
$d_{\rm acc}$	safe acceleration distance	$l_{\rm s}$	vehicle length (in cells)
$d_{\rm dec}$	safe deceleration distance	M	maximum decrease of the velocity in one time step
$d_{\rm keep}$	safe keep velocity distance	$R_{\rm s}$	slowing down probability
$R_{\rm a}$	acceleration probability	R_0	acceleration probability at low speed
$R_{\rm d}$	acceleration probability	Δv	acceleration/deceleration capability in normal conditions

Then, the dynamics of the LAI model consists of the following four consecutive steps, which are performed in parallel for all vehicles (see Table 1 for a summary of the parameters and variables of the model).

S1 Safe distances. Obtain the value for $d_{\mathrm{dec}_n} = d_{\mathrm{dec}}(t, v_n(t), v_{n+1}(t))$, $d_{\mathrm{acc}_n} = d_{\mathrm{acc}}(t, v_n(t), v_{n+1}(t))$, and $d_{\mathrm{keep}_n} = d_{\mathrm{keep}}(t, v_n(t), v_{n+1}(t))$.

S2 Determine the stochastic noise parameter R_{a}, dependent on the vehicle's speed v_n according to Eq. (1).

S3 The calculation of the velocity of vehicles along the road is carried out by simultaneously updating all of the lattice sites according to the following rules.

S3a Acceleration. If $d_n(t) \geq d_{\mathrm{acc}_n}$,

$$v_n(t+1) = \begin{cases} \min(v_n(t) + \Delta v, v_{\max}), & \text{if randf()} \leq (R_{\mathrm{a}}), \\ v_n(t), & \text{otherwise,} \end{cases}$$

where randf()$\in [0,1]$ denotes a uniform random number (specifically drawn for the vehicle n at time t).

S3b Random slowing down. If $(d_{\mathrm{acc}_n} > d_n(t) \geq d_{\mathrm{keep}_n})$ or $(v_n == v_{\max})$,

$$v_n(t+1) = \begin{cases} \max(v_n(t) - \Delta v, 0), & \text{if randf()} \leq (R_{\mathrm{s}}), \\ v_n(t), & \text{otherwise.} \end{cases}$$

S3c Braking. If $d_{\mathrm{keep}_n} > d_n(t) \geq d_{\mathrm{dec}_n}$,

$$v_n(t+1) \to \max(v_n(t) - \Delta v, 0).$$

S3d Emergency braking. If $v_n(t) > 0$ and $d_n(t) < d_{\mathrm{dec}_n}(t)$,

$$v_n(t+1) \to \max(v_n - M, 0),$$

where M is the maximum decrease of velocity in one time-step.

S4 Vehicle movement.

$$x_n(t+1) \to x_n(t) + v_n(t+1).$$

Here, $x_n(t)$ and $v_n(t)$ respectively denote the position and velocity of vehicle n at time-step t (it is assumed that vehicle $n+1$ precedes vehicle n). Then, the space gap of the vehicle n is denoted as $d_n(t) = x_{n+1}(t) - x_n(t) - l_{\mathrm{s}}$, where l_{s} is the length (in cells) of a vehicle; it is assumed that the position of a vehicle is denoted by the cell that contains its rear bumper (see Table 1 for a summary of the parameters and variables of the LAI model), Δv denotes the magnitude to increase/decrease the velocity of a vehicle in one time-step under normal situations, and is determined in the following way:

$$\Delta v = 2.5m/\Delta x,$$

where Δx denotes the cell length (in meters) used for the system discretization. According to this definition, the magnitude of acceleration/deceleration is fixed to a given value, which shall be determined in accordance with the cell size utilized. Notice that because of the integer constraint on the values of Δv, the largest value of Δx is 2.5 m. Other acceptable values are $\Delta x = 2.5/k$, where k is an integer larger than 0.

2.3. Understanding the rules

Rules S3a–S3d are designed to update the velocity of vehicles; rule S4 updates the position. The starting point of the rules of the LAI model is the calculation of three safe following distances that the follower vehicle must have with its preceding vehicle (the leader vehicle) to accelerate (d_{acc}), keep its velocity (d_{keep}) or slow down (d_{dec}) in a safe way. The safe distances which allow safe movement are defined by the following equations:

$$d_{\mathrm{acc}} = \max(0, \sum_{i=0}^{(v_n(t)+\Delta v)_{\mathrm{div}M}} [(v_n(t)+\Delta v) - iM] - \sum_{i=0}^{(v_{n+1}(t)-M)_{\mathrm{div}M}} [(v_{n+1}(t)-M) - iM]), \quad (2)$$

$$d_{\mathrm{keep}} = \max(0, \sum_{i=0}^{(v_n)(t)_{\mathrm{div}M}} [v_n(t) - iM] - \sum_{i=0}^{(v_{n+1}(t)-M)_{\mathrm{div}M}} [(v_{n+1}(t)-M) - iM]), \quad (3)$$

$$d_{\mathrm{dec}} = \max(0, \sum_{i=0}^{(v_n(t)-\Delta v)_{\mathrm{div}M}} [(v_n(t)-\Delta v) - iM] - \sum_{i=0}^{(v_{n+1}(t)-M)_{\mathrm{div}M}} [(v_{n+1}(t)-M) - iM]), \quad (4)$$

where $X_{\mathrm{div}Y}$ denotes the integer division, that is, $X_{\mathrm{div}Y} = \lfloor X/Y \rfloor$, where "/" denotes normal division and $\lfloor z \rfloor$ is the floor function.

The first term on the r.h.s of Eqs. (2)–(4) determines the traveled distance by the follower vehicle if it accelerates ($v_n(t+1) = v_n(t) + \Delta v$), maintains its velocity ($v_n(t+1) = v_n(t)$) or slows down ($v_n(t+1) = v_n(t) - \Delta v$) respectively, at time-step $t+1$ and in the next time-step, it begins to slow down abruptly (with the maximum braking capability M) until it stops. The second term on the r.h.s. of Eqs. (2)–(4) counts for successive decelerations for the leading vehicle during time-steps $i = 0, 1, \ldots, (v_{n+1}(t) - M)_{\mathrm{div}M}$, by considering that it is going to slow down abruptly with maximum braking capacity.

Therefore, the subtractions on the r.h.s of Eqs. (2)–(4) correspond to the safe following distances required to stop a vehicle in an emergency situation, by considering that it will accelerate, keep its velocity or decelerate respectively, at time-step $t+1$. Note that these distances are always considered positive.

Thus, the description of the rules that define the dynamics of the LAI model is as follows. Rule S3a postulates that all the drivers strive to reach the maximum velocity whenever possible, and takes into account non uniform acceleration of vehicles by considering that drivers act differently. The stochastic factor considered in this rule by the parameter R_{a}, defined in

Eq. (1), involves a retarded acceleration as a function of the velocity of vehicles. Thus, the behavior of vehicles that come out of a traffic jam is also more in line with the real traffic behavior: a driver needs a moment to speed up the vehicle (see Ref. [28]).

On the other hand, rule S3b reflects the fact that drivers will try to keep their velocity (cruising) if they perceive the distance to the vehicle in front as safe. The random slowing down, controlled by the parameter R_s, contributes to the creation of observed spontaneous traffic jams and allows modeling disturbances caused by traffic. Thus, randomization is only applied to vehicles that are in cruising conditions and are not required to slow down, and as a result double braking is avoided.

A driver applies moderate braking, when the spacing that separates his/her vehicle to the vehicle in front becomes small, through rule S3c. Note that vehicles cannot change speeds instantly, that is, the maximum deceleration rate an individual driver is willing to use in non-emergency situations, as a comfortable deceleration, is bounded by Δv (in grid units). Thus, extreme decelerations are avoided.

On the other hand, since in the definition of the LAI model the most important driver decisions are related to safety, in cases of emergency braking (the leader brakes suddenly or the follower approaches a stopped vehicle), a collision avoidance sequence is required. Thus, the rule S3d allows drivers to react to an emergency braking of the leading vehicle producing perturbations in the other rules. In Ref. [28], emergency braking has a value of -5.00 m/s^2 (this is considered an acceptable value for this maneuver[29,34]) and will be reached in one time-step. Thus, the parameter M takes the value of $M = 5.00/\Delta x$.

It is important to note that the order for applying rules S3a to S3d is not relevant, since conditions for each one of them produce disjointed sets. As only rule S3b or S3c can be applied at a single time, double braking is avoided. The only probabilistic behavior to slow down is included in rule S3b which leads to similar velocity adjustments than rule S3c. This is consistent with the idea of treating random slowing down as a disturbance that should not occur very often.

According to the traffic engineering handbook,[35] the comfortable acceleration (deceleration) rate is about 2–3 m/s^2 (3.1 m/s^2) under normal conditions. Since a time-step corresponds to 1 s in the LAI model, an acceptable value for the cell size, which leads to a better agreement with a comfortable deceleration and acceleration is $\Delta x = 2.5$ m.

The parameters of the model are the following: maximum speed v_{max}, slow velocity v_s, vehicle length l_s (in cells), the maximum decrease of the velocity in one time step M, the random slowing down probability R_s, and the probabilities R_0 and R_d. However, under the consideration done for the parameter R_a in step S2, in practice, the effective number of parameters needed by the simulations in each time-step is five: v_{max}, l_s, R_d, R_a, and M. The additional parameters l_s and M are necessary to allow vehicles of different lengths and braking capabilities, although we only consider homogeneous vehicles in this work. In practice, the effective number of parameters needed by the simulations each time-step is five: v_{max}, l_s, R_d, R_a, and M.[28]

3. Simulation results and discussion

In this section, simulation results from the LAI model on a periodic single-lane system are presented. This simple system structure is in sharp contrast with realistic highway networks. It is, nevertheless, justified because it has been shown for a large class of models that different boundary conditions induce distinct steady states rather than changing their microscopic structure.[36] Therefore, if one tries to reproduce the spatial-temporal structure on a macroscopic level, the boundary conditions can be important. In comparison with local measures, an appropriate traffic model should be able to reproduce the empirical results regardless of the applied boundary conditions.

Thus, simulations are carried out on a road of $L = 20000$ cells. Each vehicle has a length of 5.0 m and each cell corresponds to 2.5 m. The time-step t is always taken to be 1 s, therefore, transitions are from $t \to t+1$. This time-step is on the order of human reaction time as pointed out in Ref. [29]. It can be easily modified. Velocities are changed according to the velocity updating rules S3a–S3d and then all cars are moved forward in step S4.

For each simulation, initially, N vehicles are randomly distributed on the lane around the loop with an initial speed taking a discrete random value between 0 and v_{max}. For simplicity, only one type of vehicle is considered, i.e., all vehicles have the same maximum velocity and length. Clearly, the global density remains constant with time. The global density is given by

$$\rho = \frac{N}{L}. \tag{5}$$

Each simulation is carried out for $T = 50000$ time-steps after the relaxation of 100000 time steps.

The parameters' values $\Delta x = 2.5$ m, $R_d = 1.0$, $R_0 = 0.8$, $R_s = 0.01$, $l_s = 2$, $M = 2$, and $v_s = 3$ are adopted for all results presented. These values lead to the best optimal agreement with the empirical findings.[28]

3.1. The fundamental diagram

The flow-density relation (fundamental diagram) characterizes the dependence of the vehicle flow on density and is one of the most important criteria in order to show that a model

reproduces the traffic flow behavior. Following that proposal, figure 1(a) shows the average range of flow during one-minute against the density for the LAI model. To construct the fundamental diagrams, the global density, ρ, varies between 0 and 196 veh/km, with a step of 2 veh/km. The local flow, $J_{\rm loc}$, and the average local velocity, $v_{\rm loc}$ are obtained by setting a virtual detector on a specific location. The $J_{\rm loc}$ is the number of vehicles passing at that detector per time step within 1 min and $v_{\rm loc}$ is the average velocity of the vehicles passing the detector within 1 min. Then, the spatial density is obtained via the hydrodynamic relation $J_{\rm loc} = \rho_{\rm loc} v_{\rm loc}$ and so, any given density is related to many points in the fundamental diagram obtained. As can be seen from Fig. 1(a), the data is widely scattered, and this is in agreement with empirical findings,[36] and suggests the existence of three different dynamical states of vehicular traffic: free flow, synchronized traffic, and jams. Therefore, the positive slope observed from the Fig. 1(a) corresponds to the free flow. The jammed vehicles produce the scattered points below, and the two-dimensional region in the middle of the diagram is the result from the synchronized states.[28] It is important to mark that because moving cars are only measured by the inductive loop, the average local velocity to wide moving jams is usually overestimated and thus, the density is underestimated. Besides, note that resulting simulation densities are less distributed than in empirical findings because of an artifact of the discretization of the velocities which determines the upper limit of detectable densities.

Although the analysis of local measurements is important, one of the main disadvantages of the fundamental diagram obtained from these types of measurements is that the detected values of the flow and the velocity strongly fluctuate; whereas density cannot be defined locally in a strict sense. In contrast, by means of traffic simulation, it is possible to get averaged quantities that are representative for a given density. Due to this fact, in Fig. 1(b), the fundamental diagram obtained from global measurements obtained from the LAI model is shown. Global density is obtained directly by Eq. (5), and the global average velocity $v_{\rm g}$ is defined as the average velocity of the vehicles. Again, the hydrodynamical relation allows the calculation of the global flow $J_{\rm g} = \rho v_{\rm g}$.

A fundamental diagram, in general, consists of two linear branches. The first positive branch corresponds to free flow, which intersects with a congested branch with negative slope at the density where the maximum flow occurs. As one can see in Fig. 1(b), the LAI model is able to reproduced these characteristics. The initial positive slope corresponds to a free-flow region. Here, the vehicles travel near the maximum speed (see the line plus square in the left axis), thus the flow increases as the density increases. The branch with a negative slope and small values for the flow represents wide moving jams, which implies that the higher the density on the jammed branch, the lower the flow. Therefore, even though there are more cars on the road, the number of cars passing a single point is less than if there were fewer cars on the road. However, after this free flow region, for density values between ρ_1 to ρ_2, traffic flow organizes in a so-called synchronized congested region, whose associated average speed is lower than the free flow speed but larger than the one corresponding to wide movings jam. Besides, there are no stopped cars.

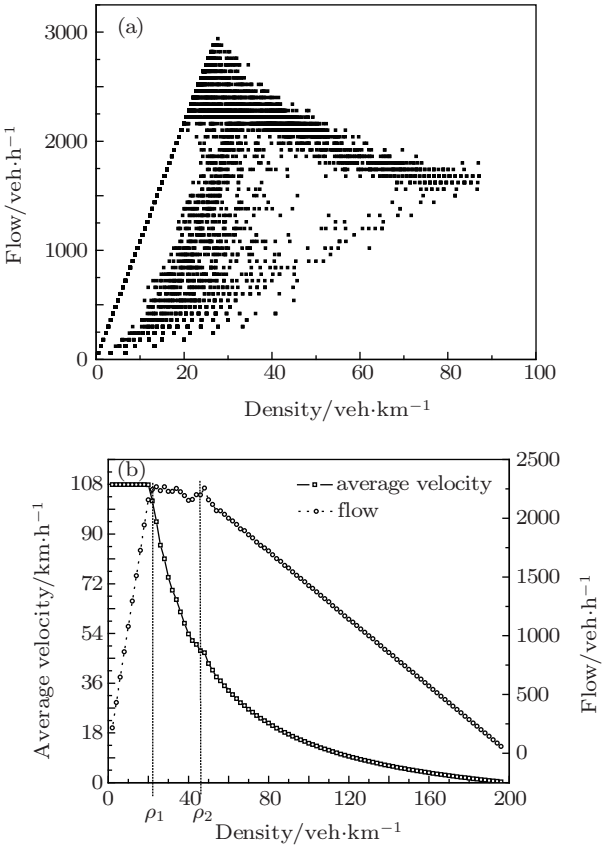


Fig. 1. Fundamental diagrams of the LAI model for $\Delta x = 2.5$ m, $R_{\rm d} = 1.0$, $R_0 = 0.8$, $R_{\rm s} = 0.01$, $v_{\rm s} = 3$, $M = 2$, and $l_{\rm s} = 2$. Panel (a) corresponds to the 1 min average flux against the density obtained from a virtual inductive loop. Panel (b) shows the fundamental diagram obtained from global measurements. That is the average velocity (left axis, square) and the average flow (right axis, circle) against the global density.

Small variations observed in the flow for the synchronized region are a consequence of discretization effects, the stochastic effects of individual driver behavior controlled by the parameter $R_{\rm s}$, and the special relation between vehicle headway and velocity that is embedded in the dynamics definition of the LAI model. In order to elucidate this behavior, figure 2(a) shows the fundamental diagram obtained for different values of $R_{\rm s}$ and $R_{\rm a} = 1.0$. It is important to stress that this is not a result of a transient process. Besides, randomization through the parameter $R_{\rm s}$ is applied only to vehicles that are in cruising conditions and do not require slowing down. As can be seen from Fig. 2(a), the random parameter $R_{\rm s}$ has a strong influence on the flow behavior in the LAI model: the larger the value of $R_{\rm s}$ (a higher variation of vehicles speeds), the lower

the average velocity (see Fig. 2(b)). Therefore, the average flow is lower as the value of the parameter R_s is increased. However, note that the density interval where the synchronized congested state occurs is not determined by the parameter R_s, that is, the values for the endpoints of the interval, ρ_1 and ρ_2, are the same for all values of $R_s > 0.0$.

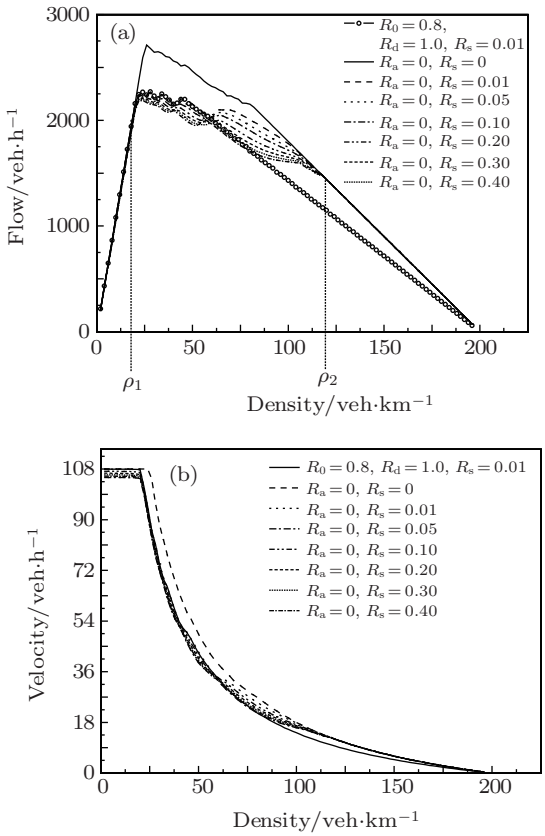


Fig. 2. Panel (a) corresponds to the global fundamental diagram for $R_a = 0$ and different values of R_s. Panel (b) shows the standard deviation of the velocity and the position of the vehicles on time.

In addition, more interesting is the fact that the observed variations in the fundamental flow diagram behave in a similar manner for all values for the parameter R_s greater than zero. The rationale behind of this behavior is as follows. After the free flow region, the iteration between vehicles is increased, thus average velocity decreases as density increases. However, the relation between vehicle headway and velocity considered in the definition of the LAI model produces values for their average velocity limited by the density. Thus, variations of flow become visible in the global fundamental diagram and different slopes are observed.

3.2. Analyzing the traffic states

Although the fundamental diagram is a good indicator to evaluate the behavior of a model, it is not sufficient to accurately identify the different types of traffic states (i.e., free flow, synchronized flow, and wide moving jams). Thus, in this subsection, an analysis of the LAI model based on the microscopic structure of the flow is presented.

Firstly, to exemplify the variations in the microscopic structure when changing the density value, the spatio-temporal diagrams of the LAI model for different densities are depicted in Fig. 3. Each vertical column of dots represents the instantaneous positions of the vehicles moving up-towards, while successive columns of dots represent the position of the same vehicles at successive time-steps (each vehicle is represented as a single colored dot and black dots represent stopped vehicles). Figures 3(a) and 3(b) show the spatial-temporal diagrams of free flow and synchronized flow, respectively. From Fig. 3(b), it can be noted that the vehicle's velocity is reduced in comparison with the one corresponding to a free-flow state. This behavior is different to the behaviour corresponding to a wide moving jam, where stop and go waves are observed and propagate in the opposite direction of traffic (see Fig. 3(c)).

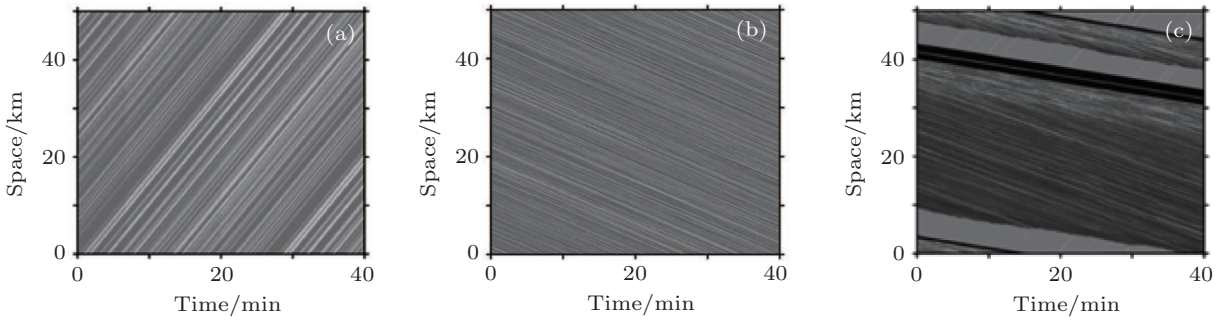


Fig. 3. The spatiotemporal shapes of the different traffic phases, free (a), synchronized (b), and jammed (c) are depicted for density values of 14, 36, and 54 veh/km respectively (0.035, 0.090, and 0.135 veh/cell, respectively).

For demonstrating the inner structure of the synchronized flow, a snapshot of a part of the road randomly chosen (zoomed at 60000 s), with an initial density of 36 veh/km is presented in Fig. 4. In Fig. 4(a), the solid circle represents vehicles moving from left to right, and their corresponding gap values are shown in Fig. 4(b). Note that this is not a transient process and exhibits a smooth velocity profile. Therefore, the points corresponding to the synchronized area in the

fundamental diagram shown in Fig. 2(a) are not attributed to the average effects of strong fluctuations; these points are a consequence of the special relation between vehicle headway and velocity that is embedded in the dynamics definition of the LAI model. In addition, figure 4(b) clearly shows a coordinated relation between the headway and the velocity in the synchronized congested region: an increase (decrease) in the speed of a vehicle is a consequence of an increase (decrease) of its spatial headway. Moreover, these obtained results also indicate that the dynamics defined by the LAI model to simulate the microscopic behavior for both the drivers and the vehicles is suitable.

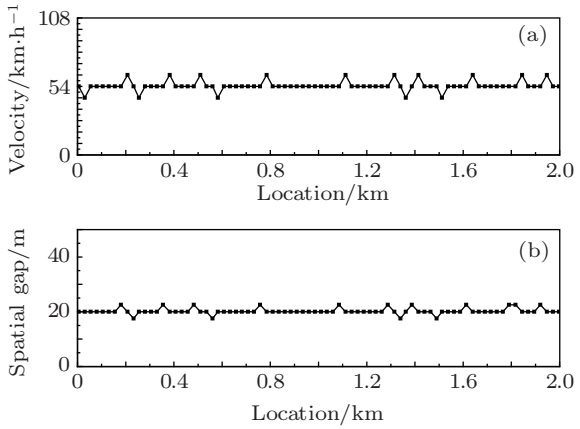


Fig. 4. Snapshot of synchronized flow with an initial density of 36 veh/km. (a) The solid circle represent vehicles moving from left to right. (b) The solid square represents the gaps associated to vehicles in panel (a).

On the other hand, in empirical observations, it is found that in synchronized congested flow, no long-range correlation exists in the autocorrelation of the time series of the local density, the local velocity, and the local flow rate, and the cross-correlation function between density and flow rate vanishes in a large time scale. Thus, to identify more clearly the synchronized traffic, time series obtained through a fixed virtual loop detector are analyzed. Consider the following autocorrelation function

$$A_x(\tau) = \frac{\langle x(t)x(t+\tau)\rangle - \langle x(t)\rangle\langle x(t+\tau)\rangle}{\sqrt{\langle x(t)^2\rangle - \langle x(t)\rangle^2}\sqrt{\langle x(t)^2\rangle - \langle x(t)\rangle^2}}, \tag{6}$$

where the brackets $\langle\ \rangle$ indicate the average over the whole series of x. Then, the autocorrelations of the local flow, the local density, and the local velocity are calculated and shown in Figs. 5(a)–5(c). It can be seen in these figures that all the temporal autocorrelations are short-ranged, and although there are some fluctuations in the values of the autocorrelations, they are close to zero along with the increase of τ. Figure 5(d) shows the cross-correlation function

$$C_{\rho,J}(\tau) = \frac{\langle \rho(t)J(t+\tau)\rangle - \langle \rho(t)\rangle\langle J(t+\tau)\rangle}{\sqrt{\langle \rho(t)^2\rangle - \langle \rho(t)\rangle^2}\sqrt{\langle J(t)^2\rangle - \langle J(t)\rangle^2}} \tag{7}$$

of the aggregated quantities ρ and J, local density and local flow, respectively. As can be seen from Fig. 5(d), the cross-correlation vanishes in a large time scale. These properties are also consistent with observed characteristics of synchronized flow.[37] This analysis thus confirms the reproduction of the synchronized congested flow for the LAI model. However, it is also interesting to understand the inner structure responsible for the existence of this synchronized state.

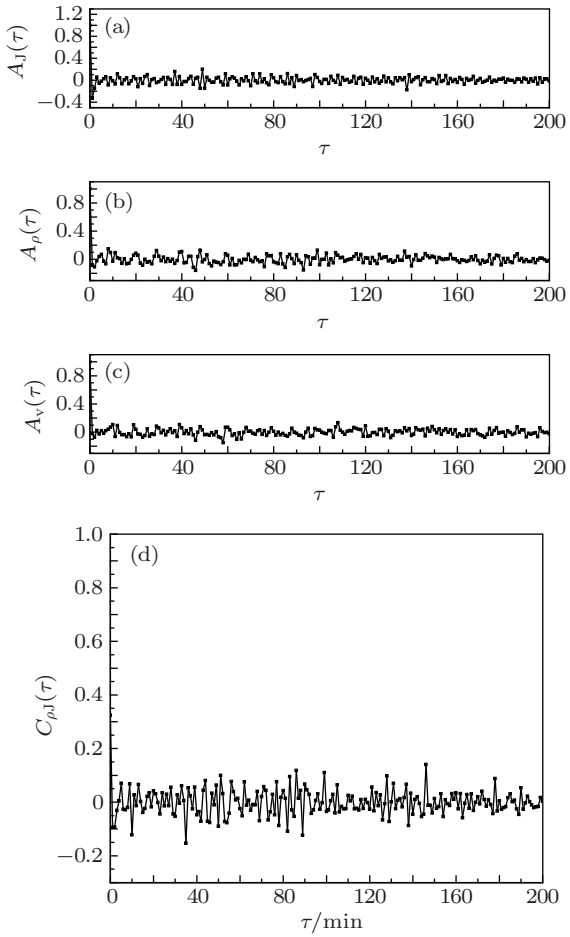


Fig. 5. Typical autocorrelation function resulting from the time series of the local flow rate (a), the local density (b), and the average local velocity (c). The cross-correlation function between density and flow (d). The global density is $\rho = 30$ veh/km.

3.3. Analyzing the inner structure based on platoons

In this subsection a deeper statistical analysis of the LAI model is presented. Particular emphasis is placed on analyzing platoons formation as a mechanism that makes the presence of synchronized flow possible.

A platoon is defined as a group of two or more consecutive vehicles traveling together with a similar value of velocity (v_p) and a reduced spatial gap between them. Henceforward, these will be called v_p-platoons. It is important to stress that according to the rules that define the dynamic of the LAI model, a vehicle moving with velocity v_p can change its velocity by $\pm\Delta v$ in any time-step. Therefore, in this paper, a v_p-platoon refers to a group of consecutive vehicles with ve-

locity v_p, v_{p+1}, or v_{p-1}, and a reduced spatial inter-vehicles gap whose value is determined in accordance with the safe following distances defined in Eqs. (2)–(4) to accelerate, decelerate or maintain the velocity of a vehicle. For example, let i and $i+1$ be indices to refer to two consecutive vehicles traveling with the same velocity v_p, these vehicles will belong to the same v_p-platoon if $d_{keep}(v_i, v_{i+1}) \leq \text{gap} \leq d_{acc}(v_i, v_{i+1})$, where v_i,v_{i+1} denote the velocities of vehicles i and $i+1$ respectively.

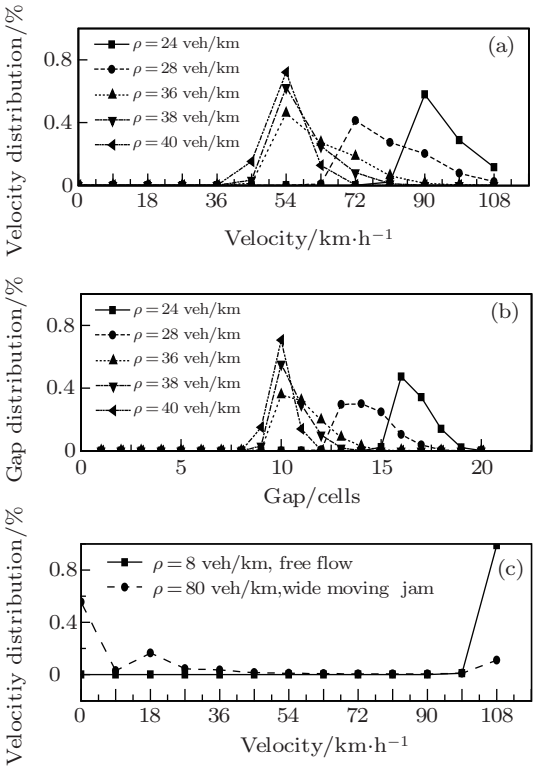


Fig. 6. Distributions of velocity for different density values. (a) Velocities distribution for the synchronized flow. (b) Gaps distribution for the synchronized flow. (c) Velocities distribution for free flow and wide moving jams.

As an initial point of the analysis, the distribution of velocity of individual vehicles for different density values and flow states was analyzed (see Fig. 6(a)). Note that the behavior of distribution for the synchronized congested flow (Fig. 6(a)) is close to a Gaussian distribution, which indicates that the vehicle speed in that region is around a certain synchronized value different from zero. This observed trend in the velocity distribution is in accordance with the distribution corresponding to the gap between vehicles shown in Fig. 6(b), which also exhibits the coordinated relation between the spatial gap and the velocity of the vehicles considered in the definition of the LAI model. It is important to mark that the behavior found for the synchronized congested flow is different to that observed for density values smaller than ρ_1 (free flow) and larger than ρ_2 (wide moving jams), as can be seen in Fig. 6(c). In fact, the behavior observed from Fig. 6 is due to the formation of vehicles platoons, as can be observed in Fig. 7, that shows the ratio between the number of vehicles moving in the platoons and the total number of vehicles existing as a function of the density value. As can be seen in this figure, in the free flow region, the proportion of vehicles traveling in platoons increases as the vehicles density increases. However, for $\rho \in [\rho_1, \rho_2]$, corresponding to the synchronized congested state, the percent of vehicles traveling in platoons is near to 100%. Therefore, the presence of synchronized flow in the LAI model is a result of the formation of platoons induced by the rules of the model, where vehicles move in a coherent form with similar velocities and a coordinated spatial gap, as has been observed in real traffic.

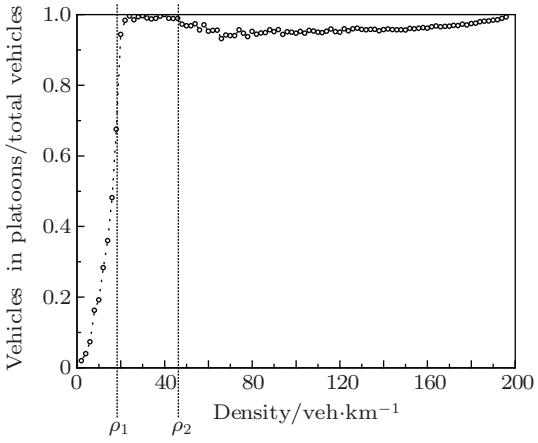


Fig. 7. The ratio between the number of vehicles moving in platoons and the total number of vehicles existing as a function of the density.

To draw conclusions about safety and order in the system as a consequence of platoon formation, the variance of the speed differences between individual cars (hereafter denoted as σ) against the density was also analyzed. High speed variance implies that vehicles in the system have widely varying speeds and that a vehicle would experience frequent speed changes per trip. Figure 8 illustrates the average flow and the speed variance of individual velocities from simulations of the LAI model. It is clear that for the free-flow region (when $\rho < \rho_1$), the speed variance is negligible since there are no slow vehicles and fluctuations are extremely rare. The global maximum for velocity variance is located after density ρ_2 and is followed by the sudden formation of large platoons traveling at zero speed, that smoothly reduces the variance. However, for density values varying from ρ_1 to ρ_2, there exists a smaller propagation of fluctuations that prevent variations of speeds by the formation of platoons or clusters of vehicles. This behavior can be clarified in Fig. 9 corresponding to the coefficient of variation (CV) for both the individual velocities and the individual gaps of vehicles. The CV is defined as the ratio of the standard deviation of velocities (gaps) to the mean. As can be seen in Fig. 9, the CV is low for both the velocities and the gaps of the individual vehicles in the density interval between

ρ_1 and ρ_2. This behavior clearly shows a low velocity variance due to the interaction of vehicles considered in the model that causes a transition to a highly correlated state in which all vehicles move with approximately the same speed and spatial gap. Therefore, in the density region corresponding to the synchronized congested state, the flow is high and stable. It is important to observe from Figs. 8 and 9 that this safe and coherent state disappears when the vehicle density exceeds ρ_2.

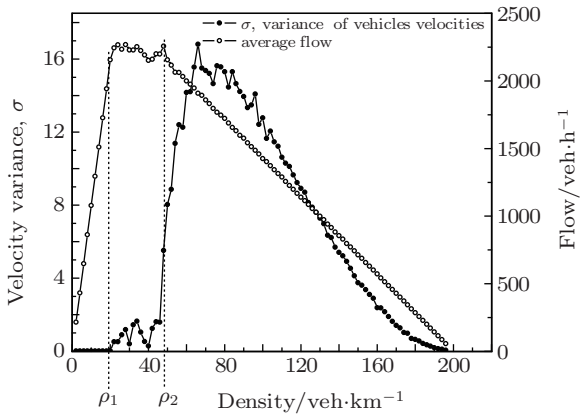


Fig. 8. The velocity variance (left axis) and the average flow (right axis) against density from simulations of the LAI model.

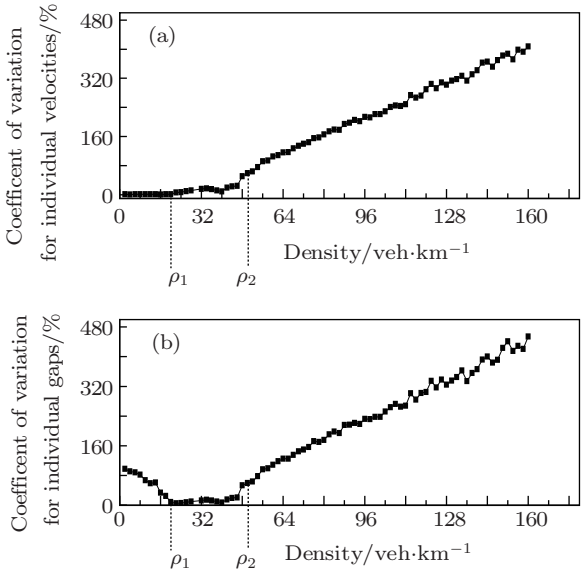


Fig. 9. Coefficient of variation for both the velocities (a) and the spatial gaps (b) against the global density.

Thus, the presented simulation results show that the formation and the dissolution of platoons as a function of the density observed at intermediate densities are the basic mechanisms to maintain a high synchronized flow. This formation of platoons is not induced by the discretization of the CA models. It is the result of a more complex pattern derived from an acceptable and coordinated microscopic behavior for both drivers and vehicles in the LAI model, which considers the relative distance and relative velocity of neighbor vehicles. This platoon formation plays an important role in automated highway systems, where the transition density ρ_2 can be increased because of smaller inter-vehicle gaps.

At a practical level, it is desirable to implement traffic rules and design highway controls that will lead to traffic moving in the coherent state observed in the LAI model. Close to the transition point, the formation of this state could be supported by variable speed limits or by automatic vehicle control systems.

4. Conclusions

In this paper, a statistical analysis of the inner microscopic structure of traffic phases reproduced by the CA model for traffic flow proposed in Ref. [28] (here referred to as the LAI model) was presented. The analysis focused on the formation of platoons or clusters as the mechanism that induces the presence of synchronized traffic flow.

Simulation results of the LAI model for density values associated with the synchronized congested state indicate the presence of several dense vehicle platoons which move in a coherent way. In these platoons, two or more vehicles move with similar velocity v_p and have a reduced headway induced by the safe following distances that a vehicle requires to accelerate, decelerate and maintain its velocity in a safe way. Therefore, the formation and dissolution of platoons as a function of the density observed at intermediate densities is the basic mechanism to maintain a high synchronized flow. Besides, it is important to stress that the obtained results indicate that the dynamics defined by the LAI model to simulate the microscopic behavior for both the drivers and the vehicles is suitable and reproduces empirical findings. In addition, it is shown that the formation of platoons is not induced by the discretization of the CA model, but it is as a consequence of a more complex behavior derived from the relation between the vehicle headway and the velocity considered in the definition of the LAI model. To our knowledge, the behavior of such a coherent structure has been seldom observed before in other CA models.

On the other hand, the statistical analysis carried out based on the velocities and gaps of individual vehicles shows that the density region corresponding to the synchronized congested state is also associated with a safe, high and stable flow. Therefore, the fact that all vehicles practically move with similar velocities in a coherent form means that the velocity variation is reduced.

From a practical point of view, the implementation of traffic rules and the design of highway controls that will lead to keeping traffic moving in a coherent state, as reproduced in the LAI model, are desirable. The formation of this coherent state could be supported by variable speed limits or by automatic vehicle control systems. The platoon formation reproduced

by the LAI model also plays an important role in automated highway systems (AHS).

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