Complex solutions and novel complex wave localized excitations for the (2+1)-dimensional Boiti–Leon–Pempinelli system*

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With the help of the symbolic computation system Maple, the Riccati equation mapping approach and a linear variable separation approach, a new family of complex solutions for the (2+1)-dimensional Boiti–Leon–Pempinelli system (BLP) is derived. Based on the derived solitary wave solution, some novel complex wave localized excitations are obtained.

Keywords: Riccati mapping approach, Boiti–Leon–Pempinelli system, complex solutions, localized excitations

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1. Introduction

Mathematical modelling of physical systems often leads to nonlinear partial differential equations (NPDEs).[1–3] The exact solutions of such equations play an important role in nonlinear science, especially in nonlinear physics, since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to applications.[4–9] The exact solutions of NPDEs are an interesting and popular topic in nonlinear mathematical physics, and various methods for obtaining the exact solutions of nonlinear systems have been proposed, for example, the bilinear method, the standard Painlevé truncated expansion, the method of "coalescence of eigenvalue" or "wavenumbers", the homogenous balance method, the hyperbolic function method, the Jacobian elliptic method, the variable separation method, the (G'/G)-expansion method,[10–22] and the mapping method.[23–32] The mapping approach is a kind of classic, efficient, and well-developed method to solve nonlinear evolution equations. In Ref. [23], Mei and Zhang have obtained exact traveling wave solutions for a nonlinear evolution equation with the Riccari equation mapping method. In this paper, we will extend this method to construct complex solutions with $q = lx+my+nt+R$ ($R = \chi(x,t)+\varphi(y)$) for the famous (2+1)-dimensional Boiti–Leon–Pempinelli (BLP) system

$$\begin{aligned} u_{ty} &= (u^2 - u_x)_{xy} + 2v_{xxx}, \\ v_t &= v_{xx} + 2uv_x. \end{aligned} \tag{1}$$

This system has been studied by many scholars with different methods, for example, Dai and Wang obtained periodic structures based on the variable separation solution of the (2+1)-dimensional BLP equation.[33]

As is well known, to search for the solitary wave solutions for a nonlinear physical model, we can apply different approaches. One of the most efficient methods of finding the soliton excitations of a physical model is the so-called mapping approach with variable coefficients. The basic ideal of the algorithm is as follows. For a given nonlinear partial differential equation with independent variables $x = (x_0 = t, x_1, x_2, \ldots, x_m)$ and dependent variable u,

$$P(u, u_t, u_{x_i}, u_{x_i x_j}, \cdots) = 0, \tag{2}$$

where P is in general a polynomial function of its arguments, and the subscripts denote the partial derivatives, the solution can be assumed to be in the form

$$u = A(x) + \sum_{i=1}^{n} \{B_i(x)\phi^i[q(x)], \tag{3}$$

with

$$\phi' = a_0 + a_1\phi + a_2\phi^2, \tag{4}$$

$$q = lx + my + nt + R, \quad R = \chi(x,t) + \varphi(y), \tag{5}$$

where a_0, a_1, a_2, l, m, n are constants, $\chi \equiv \chi(x,t)$ and $\varphi \equiv \varphi(y)$ are two arbitrary functions of (x,t) and y, respectively. Substitute Eqs. (3)–(5) into the given NPDE and collect coefficients of the polynomials of ϕ, then eliminate each coefficient to obtain a set of equations of A, B_i, and q, and solve these equations to obtain A, B_i, and q, finally, as Eq. (4) is known to possess the solutions,[23] one obtains the complex solutions of the given NPDE.

2. Complex solutions of the BLP system

Now we apply the Riccati mapping approach to Eq. (1). By the balancing procedure, ansatz (3) becomes

$$u = A_1(x,y,t) + B_1(x,y,t)\phi(q(x,y,t)),$$

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$$v = A_2(x,y,t) + B_2(x,y,t)\phi(q(x,y,t)), \tag{6}$$

where A_1, B_1, A_2, B_2, and q are functions of (x,y,t) to be determined. Substituting Eqs. (6) and (4) into Eq. (1) and collecting the coefficients of the polynomials of ϕ, then setting each coefficient to zero, we have

$$A_1 = -\frac{1}{2}\frac{q_x^2 a_1 - q_t + q_{xx}}{q_x},$$
$$B_1 = -q_x a_2, \quad A_2 = 0, \quad B_2 = -q_y a_2, \tag{7}$$
$$q = lx + my + nt + R, \quad R = \chi(x,t) + \varphi(y). \tag{8}$$

Based on the solutions of Eq. (4), we can derive the following complex wave solutions of Eq. (1).

Case 1 Solitary wave solutions:

$$u_1 = \frac{1}{2}\frac{q_t - q_{xx} + 2q_x^2\tanh(q)}{q_x}, \tag{9}$$
$$v_1 = q_y\tanh(q), \tag{10}$$
$$u_2 = \frac{1}{2}\frac{q_t - q_{xx} + 2q_x^2\coth(q)}{q_x}, \tag{11}$$
$$v_2 = q_y\coth(q), \tag{12}$$
$$u_3 = \frac{1}{2}\frac{(q_t - q_{xx})\sinh(q) + q_x^2(1-\cosh(q))}{q_x}, \tag{13}$$
$$v_3 = \frac{1}{2}\frac{q_y(\cosh(q)+1)}{\sinh(q)}, \tag{14}$$
$$u_4 = \frac{1}{2}\frac{(q_t - q_{xx})(1+\tanh^2(q)) + 8q_x^2\tanh(q)}{q_x}, \tag{15}$$
$$v_4 = 4\frac{q_y\tanh(q)}{1+\tanh^2(q)}, \tag{16}$$

with $q = lx + my + nt + R$ and $R = \chi(x,t) + \varphi(y)$.

Case 2 Periodic wave solutions:

$$u_5 = \frac{1}{2}\frac{q_t - q_{xx} - 2\chi_x^2\tan(q)}{q_x}, \tag{17}$$
$$v_5 = -q_y\tan(q), \tag{18}$$
$$u_6 = -\frac{1}{2}\frac{-q_t + q_{xx} - 2q_x^2\cot(q)}{q_x}, \tag{19}$$
$$v_6 = q_y\cot(q), \tag{20}$$
$$u_7 = \frac{1}{2}\frac{(q_t - q_{xx})(1+\sin(q)) + q_x^2\cos(q)}{(1+\sin(q))q_x}, \tag{21}$$
$$v_7 = \frac{1}{2}\frac{q_y\cos(q)}{1+\sin(q)}, \tag{22}$$
$$u_8 = \frac{1}{2}\frac{(q_t - q_{xx})(1+\cos(q)) + 2q_x^2(-1+\cos(q))}{(1+\cos(q))q_x}, \tag{23}$$
$$v_8 = \frac{1}{2}\frac{q_y\cot(q)}{1+\cot(q)}, \tag{24}$$

with $q = lx + my + nt + R$ and $R = \chi(x,t) + \varphi(y)$.

Case 3 Variable separated solution:

$$u_9 = -\frac{1}{2}\frac{-q_t + q_{xx} - 2aq_x^2/(aq+k)}{\chi_x}, \tag{25}$$
$$v_9 = \frac{aq_y}{aq+k}, \tag{26}$$

with $q = lx + my + nt + R$ and $R = \chi(x,t) + \varphi(y)$, here a and k are two constants.

3. Novel complex wave excitations in the BLP system

Due to the functions $q = lx + my + nt + R$ and $R = \chi(x,t) + \varphi(y)$ included in the above cases, the physical quantities u and v may possess rich local structures. In this section, we mainly discuss some complex wave localized coherent excitations from the potential of the solitary wave solutions determined by Eqs. (25) and (26) in Case 3 and rewrite it in a simple form as

$$U = u_{9y} = v_{9x} = -\frac{a^2(m+\varphi_y)(l+\chi_x)}{(alx + amy + ant + a\chi + a\varphi + k)^2}. \tag{27}$$

3.1. Complex wave excitations

According to solution (27), if we choose χ and φ as

$$\chi = 1 + \tanh(x-t), \quad \varphi = 1 + \tanh(y), \tag{28}$$

we can obtain a complex wave excitation for the physical quantity U under condition (28) as presented in Fig. 1(a) with fixed parameters $a = 0.1$, $k = 0.5$, $l = 0.5$, $m = 0.1$, $n = 0.5$ at time $t = 0$.

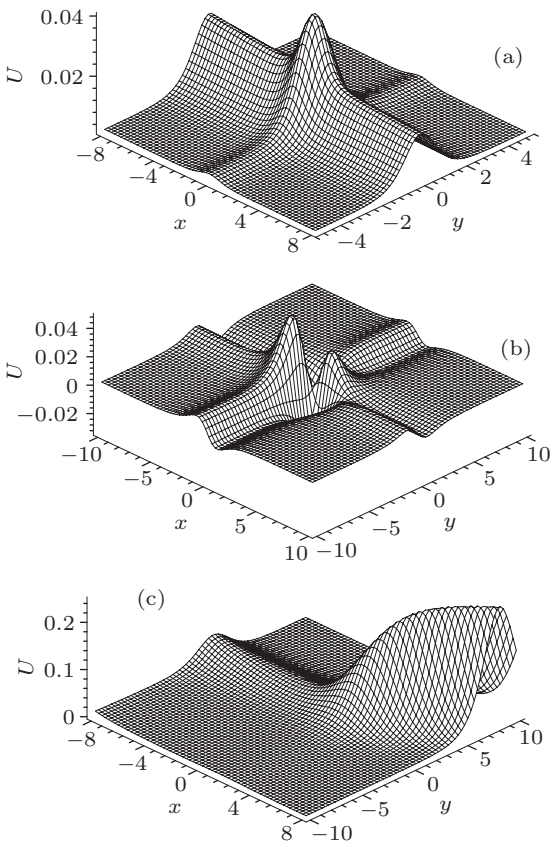


Fig. 1. Three types of complex wave localized excitations for field U defined by Eq. (27) under (a) condition (28), (b) condition (29), and (c) condition (30).

Furthermore, if we choose χ and φ as

$$\chi = 1 + \mathrm{sech}(x-t), \quad \varphi = 1 + \mathrm{sech}(y), \tag{29}$$

and

$$\chi = 1 + \exp(x-t), \quad \varphi = 1 + \exp(y), \tag{30}$$

we can obtain another two types of complex wave excitations for the physical quantity U under conditions (29) and (30) as presented in Figs. 1(b) and 1(c) with fixed parameters $a = 0.1$, $k = 0.5$, $l = 0.08$, $m = 0.08$, $n = 0.08$ and $a = 0.1$, $k = 0.8$, $l = 0.8$, $m = 0.1$, $n = 0.05$ at time $t = 0$.

3.2. The evolution of complex waves

Now we focus our attention on the intriguing evolution of the complex waves for the solution U. For instance, if we choose χ, φ and parameters a, k, l, m, n as those in Fig. 1(a), we can derive the time evolution of the complex wave for the physical quantity U, as presented in Fig. 2 for different times: (a) $t = -3$, (b) $t = 0$, (c) $t = 3$. From Fig. 2, we can see that the dromion soliton moves in a line solitary wave, and the amplitude of the dromion does not change with time.

Furthermore, if we choose χ and φ as

$$\chi = 1 + \tanh(x+t) + 0.5\tanh(x-t), \quad \varphi = 1 + \tanh(y), \tag{31}$$

we can derive the time evolution of two solitary complex waves, as presented in Fig. 3 with fixed parameters $a = 0.01$, $k = 0.5$, $l = 0.5$, $m = 0.1$, $n = 0.5$, for different time: (a) $t = -5$, (b) $t = -2$, (c) $t = 0$, (d) $t = 2$, (e) $t = 5$. From Fig. 3, one finds that the interactions of the two complex waves are completely elastic since their amplitudes, velocities, and wave shapes do not undergo any change after their collision.

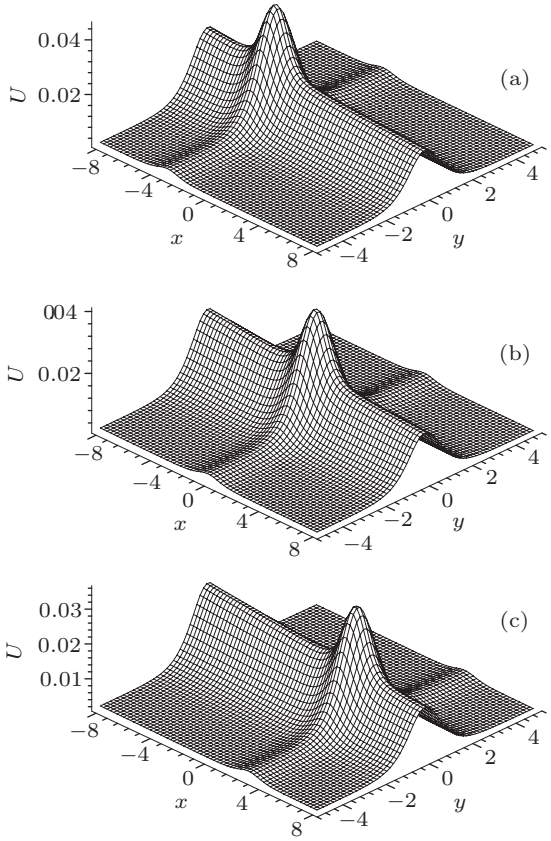


Fig. 2. Evolutional profile of single complex wave for solution U defined by Eq. (27) under condition (31) at different times: (a) $t = -3$, (b) $t = 0$, (c) $t = 3$.

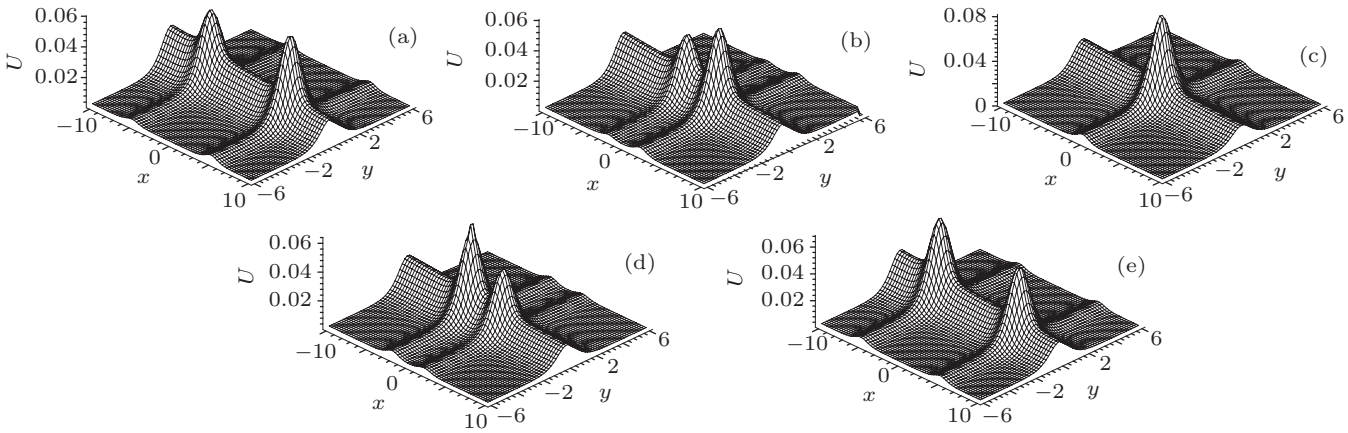


Fig. 3. Evolutional profile of two complex waves for solution U defined by Eq. (27) under condition (31) at different times: (a) $t = -5$, (b) $t = -2$, (c) $t = 0$, (d) $t = 2$, (e) $t = 5$.

4. Summary and discussion

The projective equation method is a kind of classic, efficient, and well-developed method to solve nonlinear evolution equations. In the past, Mei and Zhang have obtained the exact traveling wave solutions for the nonlinear evolution equations such as the Gross–Pitavskii equation with the Riccari equation mapping approach.[23] In this paper, we extend this approach and construct complex solutions of the (2+1)-dimensional Boiti–Leon–Pempinelli system. Based on the de-

rived solution U, we obtain three types of complex wave localized excitations and study the time evolution of the complex waves, which are different from the ones in the previous work.

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References

[1] Lou S Y 1996 *Commun. Theor. Phys.* **26** 487
[2] Lai D W C and Chow K W 2001 *J. Phys. Soc. Jpn.* **70** 666
[3] Tang X Y and Lou S Y 2003 *Commun. Theor. Phys.* **40** 62
[4] Vakhnenkl V O 1992 *J. Phys. A. Math. Gen.* **25** 4181
[5] Vakhnenkl V O and Parkes E J 1998 *Nonlinearity* **11** 1457
[6] Matsutani S 1995 *Mod. Phys. Lett. A* **10** 717
[7] Schleif M and Wunsch R 1998 *Eur. Phys. J. A* **1** 171
[8] Durovsky V G and Konopelchenko E G 1994 *J. Phys. A* **27** 4619
[9] Zhang J F 2002 *Commun. Theor. Phys.* **37** 277
[10] Zhang S L, Zhu X N, Wang Y M and Lou S Y 2008 *Commun. Theor. Phys.* **49** 829
[11] Zhang S L and Lou S Y 2007 *Commun. Theor. Phys.* **48** 385
[12] Taogetusang S 2009 *Acta Phys. Sin.* **58** 2121 (in Chinese)
[13] Taogetusang S 2009 *Acta Phys. Sin.* **58** 5887 (in Chinese)
[14] Ma S H, Fang J P and Zheng C L 2008 *Chin. Phys.B* **17** 2767
[15] Li B Q, Ma Y L and Xu M B 2010 *Acta Phys. Sin.* **59** 1409 (in Chinese)
[16] Ma Y L, Li B Q and Sun J Z 2009 *Acta Phys. Sin.* **58** 7042 (in Chinese)
[17] Ma S H, Li J B and Fang J P 2007 *Commun. Theor. Phys.* **48** 1063
[18] Ma S H and Fang J P 2006 *Acta Phys. Sin.* **55** 5611 (in Chinese)
[19] Ma S H, Wu X H, Fang J P and Zheng C L 2006 *Z. Naturforsch.* **61a** 249
[20] Ma S H, Fang J P and Zhu H P 2007 *Acta Phys. Sin.* **56** 4319 (in Chinese)
[21] Ma S H, Wu X H, Fang J P and Zheng C L 2008 *Acta Phys. Sin.* **57** 0011 (in Chinese)
[22] Ma S H, Qiang J Y and Fang J P 2007 *Acta Phys. Sin.* **56** 0620 (in Chinese)
[23] Mei J Q and Zhang H Q 2005 *Commun. Theor. Phys.* **44** 209
[24] Fang J P, Zheng C L and Zhu J M 2005 *Acta Phys. Sin.* **54** 2990 (in Chinese)
[25] Fang J P and Zheng C L 2005 *Acta Phys. Sin.* **54** 670 (in Chinese)
[26] Ma S H, Fang J P and Zheng C L 2009 *Chaos Soliton. Fract.* **40** 210
[27] Ma S H, Fang J P and Ren Q B 2010 *Acta Phys. Sin.* **59** 4420
[28] Yang Z, Ma S H and Fang J P 2011 *Chin. Phys. B* **20** 040301
[29] Ma S H, Fang J P, Ren Q B and Yang Z 2012 *Chin. Phys. B* **21** 050511
[30] Ma S H, Wu X H, Fang J P and Zheng C L 2008 *Acta Phys. Sin.* **57** 0011 (in Chinese)
[31] Ma S H, Fang J P, Hong B H and Zheng C L 2009 *Chaos Soliton. Fract.* **40** 1352
[32] Ma S H and Fang J P 2009 *Z. Naturforsch. A* **64a** 37
[33] Dai C Q and Ni Y Z 2006 *Physica Scripta* **74** 584