Mechanical properties of the thermal equilibrium Friedmann–Robertson–Walker universe model*

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The mechanical property of the thermal-equilibrium Friedmann–Robertson–Walker (TEFRW) universe is first studied. The equation of state and the scale factor of the TEFRW universe take the forms of $w = w(a; z_{\rm T})$ and $a = a(a; z_{\rm T}, H_0)$. For the universe consisting of the nonrelativistic matter and the dark energy, the behavior of the dark energy depends on the value of the present-day matter fraction. For the TEFRW universe consisting of N ingredients, the effective temperature is introduced. Lastly, a simple TEFRW universe model is analyzed.

Keywords: TEFRW universe, dark energy, effective temperature

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1. Introduction

The transition of the universe from the decelerating to accelerating expansion occurred at some past time.[1,2] The transition redshift $z_{\rm T}$ can be considered as an independent parameter describing the evolving universe.[1,3–10] The Friedmann–Robertson–Walker (FRW) universe, the expansion of which is accelerating, has both an apparent horizon (AH) and an event horizon. The system enclosed by the AH of the FRW universe satisfies both the first law[11] and the generalized second law of thermodynamics.[12]

The present-day universe contains mainly dark energy and matter. Weakly interacting massive particles are a possible candidate for being dark matter.[13–15] This means that matter can be approximately considered as a pressureless fluid. The thermal-equilibrium Friedmann–Robertson–Walker (TEFRW) universe model is an effective description for the evolving universe. By assuming that the dark matter can be considered as a pressureless fluid, the equation of state (EoS) of the dark energy in the TEFRW universe can be calculated.

In this paper, we mainly study the mechanical properties of the TEFRW universe. In Section 2, the forms of the scale factor and the EoS of the TEFRW universe are simplified. The energy density and the deceleration parameter are given in terms of the parameter $z_{\rm T}$. Section 3 is devoted to give an analysis of the properties of the dark energy under the assumption that the matter is non-relativistic. In Section 4, by introducing the effective temperature, the TEFRW universe model is generalized. Lastly, the TEFRW universe consisting of the three components is analyzed.

2. Mechanical properties of the TEFRW universe

The scale factor of the spatially flat FRW universe satisfying $T = bT_{\rm A}$, with b being a constant, T the temperature of the fluid, and $T_{\rm A}$ the AH temperature of the universe, takes the form[16]

$$a(t) = \left(\frac{3dA}{2b\sigma G}\,{\rm e}^{3dt_0} - 1\right)^{-1/3} \left(\frac{3dA}{2b\sigma G}\,{\rm e}^{3dt} - 1\right)^{1/3}, \quad (1)$$

where $A = {\rm e}^{3dt_*}$ with t_* and d being two constants, t_0 is the age of the universe, and σ denotes the co-moving entropy density of the universe. Under the scale transformation of $b \to kb$ and $\sigma \to k^{-1}\sigma$ with $k > 0$, the form of the scale factor (1) holds invariant. The EoS of the TEFRW universe is

$$w = -1 + \frac{4b\sigma G}{3dA}\,{\rm e}^{-3dt}, \quad (2)$$

which tends to -1 as $t \to \infty$. In what follows, we will first simplify the forms of the scale factor and the EoS of the TEFRW universe.

The universe transition from the decelerating to accelerating expansion occurred at time $t_{\rm T}$ (redshift $z_{\rm T}$), at which the EoS of the universe was $w_{\rm T} = -1/3$. Putting $w_{\rm T}$ in Eq. (2) yields

$$A\,{\rm e}^{3dt_{\rm T}} = \frac{2b\sigma G}{d}. \quad (3)$$

Substituting Eq. (3) into Eq. (1) gives the scale factor

$$a(t) = \left(\frac{3\,{\rm e}^{3d(t-t_{\rm T})} - 1}{3\,{\rm e}^{3d(t_0-t_{\rm T})} - 1}\right)^{1/3}. \quad (4)$$

Setting $t = t_{\rm T}$ and putting it in Eq. (4) gives the scale factor of the TEFRW universe as

$$a = a(t; z_{\rm T}) = \frac{1}{1+z_{\rm T}}\left[\frac{3\,{\rm e}^{3d(t-t_{\rm T})} - 1}{2}\right]^{1/3}. \quad (5)$$

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At the late time ($t - t_{\rm T} \gg 1/3d$), the TEFRW universe will gradually evolve to an expanding de Sitter universe. Combining Eqs. (1) and (3) gives

$$A\,{\rm e}^{3dt} = \frac{2b\sigma G}{3d}\left[2(1+z_{\rm T})^3 a^3 + 1\right]$$

or $A\,{\rm e}^{3d(t-t_{\rm T})} = 2(1+z_{\rm T})^3 a^3 + 1$. Putting it in Eq. (2) yields

$$w = w(t; z_{\rm T}) = -1 + \frac{2}{2(1+z_{\rm T})^3 a^3 + 1}, \tag{6}$$

which approaches -1 as $a \to \infty$.

From $H = \dot{a}/a$ with $\dot{a} = {\rm d}a/{\rm d}t$, the Hubble parameter of the TEFRW universe is given as

$$H = \tilde{r}_{\rm A}^{-1} = \frac{2(1+z_{\rm T})^3 a^3 + 1}{2(1+z_{\rm T})^3 + 1} a^{-3} H_0, \tag{7}$$

where $H_0 = \frac{2(1+z_{\rm T})^3+1}{2(1+z_{\rm T})^3} d$, and $\tilde{r}_{\rm A}$ is the AH radius. From $\rho = 3H^2/8\pi G$, the total energy density of the TEFRW universe is obtained as

$$\rho = \rho_0 \left[\frac{2(1+z_{\rm T})^3 a^3 + 1}{2(1+z_{\rm T})^3 + 1}\right]^2 a^{-6}, \tag{8}$$

with $\rho_0 = 3H_0^2/8\pi G$. At the transition time, the energy density of the TEFRW universe is $\rho_{\rm T} = 9\rho_0(1+z_{\rm T})^6/[2(1+z_{\rm T})^3 + 1]^2$.

The dynamical feature of the universe is described by the second-order derivative of the scale factor. The deceleration parameter of the FRW universe is defined as

$$q(z(t)) = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{1}{H^2}\frac{{\rm d}H}{{\rm d}t}. \tag{9}$$

By assuming that the deceleration parameter described by a linear two-parameter, the universe transition is determined to occur at about $z_{\rm T} \simeq 0.46 \pm 0.13$.[3] Table 1 in Ref. [4] showed some other best-fit values of $z_{\rm T}$, which depend on the adopted form of $q(z)$. From Eqs. (7) and (9), the deceleration parameter of the TEFRW universe is obtained as

$$q(z) = \frac{2[(1+z)^3 - (1+z_{\rm T})^3]}{(1+z)^3 + 2(1+z_{\rm T})^3}, \tag{10}$$

which is approximately 2 for $z \gg z_{\rm T}$. It can be checked that q monotonously increases with increasing z. Around $z = 0$, Eq. (10) reduces approximately to $q = q_0 + q_1 z$ with $q_0 = [2 - 2(1+z_{\rm T})^3]/[1 + 2(1+z_{\rm T})^3]^2$ and $q_1 = 9/[1 + 2(1+z_{\rm T})^3]^2$.

3. Dark energy in the TEFRW universe

In this section, we consider a TEFRW universe filled with dark energy and non-relativistic matter, $\rho_{\rm m} = \rho_{\rm m0} a^{-3}$ (pressureless) with $\rho_{\rm m0}$ the present-day value of $\rho_{\rm m}$. From Eq. (8), the dark energy density is determined as

$$\rho_{\rm d} = \rho_0 \frac{[2(1+z_{\rm T})^3 a^3 + 1]^2 - \Omega_{\rm m0}[2(1+z_{\rm T})^3 + 1]^2 a^3}{[2(1+z_{\rm T})^3 + 1]^2 a^6}, \tag{11}$$

with $\Omega_{\rm m0} = \rho_{\rm m0}/\rho_0$. From $p_{\rm d} = p = w\rho$ with w given in Eq. (6) and $w_{\rm d} = p_{\rm d}/\rho_{\rm d}$, the EoS of the dark energy is obtained as

$$w_{\rm d} = -\frac{4(1+z_{\rm T})^6 a^6 - 1}{4(1+z_{\rm T})^6 a^6 + \{4(1+z_{\rm T})^3 - [2(1+z_{\rm T})^3 + 1]^2 \Omega_{\rm m0}\} a^3 + 1}. \tag{12}$$

It is expected that $w_{\rm d}$ gives a satisfying description for the EoS of the dark energy at the present day. Putting $a = 1$ in Eq. (12) gives the present-day EoS of the dark energy as

$$w_{\rm d0} = -\frac{4(1+z_{\rm T})^6 - 1}{4(1+z_{\rm T})^6 + 4(1+z_{\rm T})^3 + 1 - [2(1+z_{\rm T})^3 + 1]^2 \Omega_{\rm m0}}, \tag{13}$$

which depends on the transition redshift and the present-day fraction of the matter. Given $w_{\rm d0}$ and $\Omega_{\rm m0}$, then from Eq. (13) the transition redshift can inversely be determined.

The matter fraction can be determined at a high precision. The value shown in Ref. [17] is $\Omega_{\rm m0} = 0.2735^{+0.0171}_{-0.0163}$. From the baryon and the dark matter fractions, $\Omega_{\rm b0}h^2 = 0.02267^{+0.00058}_{-0.00059}$ and $\Omega_{\rm dm0}h^2 = 0.1131 \pm 0.0034$ with $h = 0.705 \pm 0.013$,[18] we have $\Omega_{\rm b0} = 0.0456^{+0.00121}_{-0.00123}$, $\Omega_{\rm dm0} = 0.2276 \pm 0.0071$, and $\Omega_{\rm m0} = \Omega_{\rm b0} + \Omega_{\rm dm0} = 0.273 \pm 0.0083$, where the upper limit of h has been used in calculating $\Omega_{\rm b0}$ and $\Omega_{\rm dm0}$. The property of the dark energy depends on the relation between $z_{\rm T}$ and $\Omega_{\rm m0}$. Assuming that

$$\Omega_{\rm m0} = \tilde{\Omega}_{\rm m0} = \frac{2}{2(1+z_{\rm T})^3 + 1}, \tag{14}$$

then there is $w_{\rm d0} = -1$. For $\Omega_{\rm m0} > \tilde{\Omega}_{\rm m0}$, the present-day dark energy behaves as the phantom,[19–23] $w_{\rm d0} < -1$. The evolution law of the EoS of the dark energy for $\tilde{\Omega}_{\rm m0} = 0.27$ ($z_{\rm T} = 0.474$) is given in Fig. 1, which shows that the minimum of $w_{\rm d}$ is -1.0124 (at $a \simeq 1.257$).

Whether the dark energy has a crossing of the phantom divide relies on the value of $\Omega_{\rm m0}$. Inequality $w_{\rm d} < -1$ implies the following one:

$$\{4(1+z_{\rm T})^3 - [2(1+z_{\rm T})^3 + 1]^2 \Omega_{\rm m0}\} a^3 < -2, \tag{15}$$

which requires the condition

$$\Omega_{\rm m0} > \bar{\Omega}_{\rm m0} = \frac{4(1+z_{\rm T})^3}{[2(1+z_{\rm T})^3 + 1]^2}. \tag{16}$$

The scale factor, at which $w_{\rm d}$ changes from >-1 to <-1, is determined as

$$a_{\rm C}=\left\{\frac{2}{[2(1+z_{\rm T})^3+1]^2\Omega_{\rm m0}-4(1+z_{\rm T})^3}\right\}^{1/3}. \quad (17)$$

It can be seen that there is $\bar{\Omega}_{\rm m0}<\tilde{\Omega}_{\rm m0}$. For $\bar{\Omega}_{\rm m0}<\Omega_{\rm m0}<\tilde{\Omega}_{\rm m0}$, $a_{\rm C}>1$. Providing that $\Omega_{\rm m0}>\tilde{\Omega}_{\rm m0}$, then $a_{\rm C}<1$.

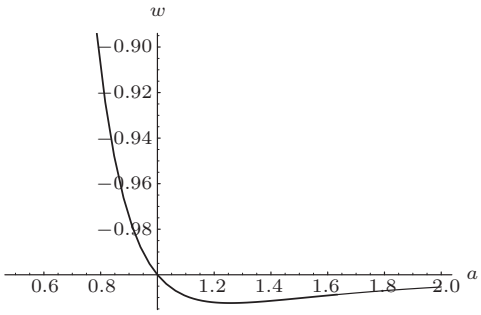


Fig. 1. The EoS of the dark energy for $\Omega_{\rm m0}=\tilde{\Omega}_{\rm m0}$ with $z_{\rm T}=0.474$. The horizonal and the longitudinal axes stand for the scale factor of the TEFRW universe and the EOS of the dark energy, respectively.

4. The TEFRW universe consisting of N ingredients

The matter and the dark energy in the universe have a very weak interaction. An introduction of the effective temperature is helpful for describing the thermal property of the universe. Consider the universe containing N ingredients (fluids) and assume that the particles making up the fluids have zero chemical potentials.[24,25] The i-th ingredient ($i=1,2,\ldots,N$) in the universe satisfies the first law of thermodynamics and the Euler relation

$$\mathrm{d}\rho_i=T_i\mathrm{d}s_i, \quad (18)$$

$$T_is_i=\rho_i+p_i, \quad (19)$$

where T_i, $s_i=\sigma_i a^{-3}$, ρ_i, and p_i are the temperature, the entropy density, the energy density, and the pressure of the i-th ingredient, respectively, with σ_i being the co-moving entropy density of the i-th ingredient. Making the sum for both sides of Eq. (19) gives

$$\left(\sum T_i\frac{s_i}{s}\right)s=\rho+p, \quad (20)$$

where $\rho=\sum\rho_i$, $p=\sum p_i$, and $s=\sum s_i$ are the total energy density, pressure, and entropy density of the universe. Define the effective temperature of the universe as

$$T=\sum T_is_i/s, \quad (21)$$

then Eq. (20) reduces to the Euler relation

$$Ts=\rho+p, \quad (22)$$

and Eq. (18) gives the differential form of the first law of thermodynamics

$$\mathrm{d}\rho=T\mathrm{d}s. \quad (23)$$

Clearly, the universe consisting of N different ingredients may be effectively considered as the one of a single fluid with the effective temperature T defined by Eq. (21).

5. Discussion

The energy density of the universe given in Eq. (8) may be expressed as

$$\rho=\rho_{\rm d}+\rho_{\rm m}+\rho_{\rm S}=\rho_\Lambda+\rho_{\rm m0}a^{-3}+\rho_{\rm S0}a^{-6}, \quad (24)$$

$$\rho_\Lambda=\frac{3(1+z_{\rm T})^6H_0^2}{2\pi[2(1+z_{\rm T})^3+1]^2}, \quad (25)$$

$$\rho_{\rm m0}=\frac{3(1+z_{\rm T})^3H_0^2}{2\pi[2(1+z_{\rm T})^3+1]^2}, \quad (26)$$

$$\rho_{\rm S0}=\frac{3H_0^2}{8\pi[2(1+z_{\rm T})^3+1]^2}. \quad (27)$$

Equations (25)–(27) give a simple decomposition to the energy density of the TEFRW universe. This indicates that the TEFRW universe can include Λ dark energy ($w_{\rm d}=-1$), non-relativistic matter, and stiff matter ($w_{\rm S}=1$) with energy density $\rho_{\rm S}$.[26] At the late time, the non-relativistic matter becomes a small fraction, the universe will be dominated by dark energy, and its effective temperature tends to a constant. The present-day universe is dominated by non-relativistic matter and Λ dark energy, the ratio between the fractions of the two energy components is $1/(1+z_{\rm T})^3$. At the early stage, the TEFRW universe is dominated by stiff matter.

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