Spin-star environment assisted entanglement generation in weakly coupled bipartite systems*

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We study the entanglement evolution in a weakly coupled bipartite system with a large energy level difference under the influence of spin-star environments. The subsystems can be coupled to a pure state or a thermal equilibrium state spin-star environment. Our results show that, in the case of the coupling strength being less than the energy level difference of the subsystems (weakly coupled), the spin-star environment can always be used to assist the entanglement generation of the bipartite system.

Keywords: entanglement, spin-star environment, weakly coupled system

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1. Introduction

Quantum entanglement is one of the most important resources in quantum information and quantum computation theory.[1] Preparing entangled states is an important task in realistic quantum systems;[1–3] however, in some cases, removing the entanglement in a quantum system is also required.[4–6] Almost all realistic quantum systems are unavoidably coupled to their environments, and in most cases, the environments play negative roles in the entanglement of the quantum systems.[7] However, in some special cases, the environments can also assist the generation of entanglement or other quantum properties of the system.[8–14]

In quantum systems, the transport between different energy levels is caused by the effective interactions between them. However, the interactions are not always strong enough to get the ideal transport probabilities needed, which affects the construction of coherence and entanglement in the quantum systems. The spin-star environment can cause the decoherence of a quantum system.[15] An interesting study about the spin-star environment assisted quantum energy transport in a dimer system was presented recently based on the exact dynamics of a two-qubit system under the spin-star environment,[16,17] and it was shown that the spin-star environment can also help the system's level transport, especially in the present of an interaction between different environments.[10,11] Since the energy transfers of quantum systems are always related with quantum state jumps, we can deduce that the spin-star environment may also be used to assist the entanglement generation of bipartite systems.

The value of the entanglement between two-level bipartite qubits measured by the concurrence ranges from 0 to 1,[18] just as we know, similar qubits can be entangled by coupling them together, and the concurrence can reach the maximal value 1 easily. Actually, this maximal concurrence 1 can only be easily realized when the coupling strength is much larger than the energy level difference between the two qubits. If the two qubits' energy level difference is very large, then the coupling between them will not always drive the system to the maximum entangled states with concurrence 1. Nevertheless, in this case, we can use an appropriate environment to obtain more large entangled states. In this paper, we will investigate this issue by studying the spin-star environment assisted entanglement generation in a bipartite system, with each subsystem coupled to its own environment. We will use the exactly solvable results of the system's quantum correlation to discuss the entanglement evolution, and show that the environment can also assist the entanglement generation of a quantum system.

The rest of this paper is arranged as follows. In Section 2, we will give the model and its solution. We will discuss the entanglement generation under the influence of the environment including the cases of pure state and thermal equilibrium state in Section 3. Finally, we will summarize our study in Section 4.

2. Model and solution

We consider an interacting bipartite system with each subsystem coupled to a spin-star environment; unlike the usual studies, we will focus on the case that the energy level difference $(\omega_1 - \omega_2)/2$ of two subsystems is large, and the coupling strength J between the subsystems is smaller than the energy level difference. We will show that their spin-star environments can assist the entanglement generation in this case.

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The system can be described by the following Hamiltonian ($\hbar = 1$):

$$\begin{aligned} H &= H_{\rm S} + H_{\rm B} + H_{\rm I}, \\ H_{\rm S} &= \frac{\omega_1}{2}\sigma_1^z + \frac{\omega_2}{2}\sigma_2^z + J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+), \\ H_{\rm B} &= \frac{\alpha_1}{2}\sum_{l=1}^{N_1}\sigma_{1,l}^z + \frac{\alpha_2}{2}\sum_{l=1}^{N_2}\sigma_{2,l}^z, \\ H_{\rm I} &= g_1\sigma_1^z \otimes \frac{1}{2}\sum_{l=1}^{N_1}\sigma_{1,l}^z + g_2\sigma_2^z \otimes \frac{1}{2}\sum_{l=1}^{N_2}\sigma_{2,l}^z. \end{aligned} \tag{1}$$

In the above equations, $H_{\rm S}$ is the Hamiltonian of the bipartite system, J is the coupling strength between the subsystems, and g_1 and g_2 are the coupling strengths of the interactions between the subsystems and their environments. The coupling between the qubits can drive the system to be entangled, while the environment may help or destroy this quantum correlation, we will discuss this problem in detail in the following studies.

The change of entanglement can be measured by Wootters' concurrence,[18] which is defined as $C(\rho) = \max\{0, 2\lambda_{\max} - \sum_{i=1}^{4}\lambda_i\}$, and λ_i is the squire root of the density matrix $\rho\tilde{\rho} = \rho\cdot(\sigma^y\otimes\sigma^y)\rho^*(\sigma^y\otimes\sigma^y)$ in the standard basis.

Note that the Hamiltonian can be easily expressed in a matrix form in the standard basis of $\{|\uparrow\rangle_1|\uparrow\rangle_2, |\uparrow\rangle_1|\downarrow\rangle_2, |\downarrow\rangle_1|\uparrow\rangle_2, |\downarrow\rangle_1|\downarrow\rangle_2\}$, and we can rewrite the Hamiltonian as

$$H = I_{4\times4}\otimes H_{\rm B} + \begin{pmatrix} M & 0 & 0 & 0 \\ 0 & N & J & 0 \\ 0 & J & -N & 0 \\ 0 & 0 & 0 & -M \end{pmatrix}, \tag{2}$$

where the operators M and N in the matrix are defined as

$$\begin{aligned} M &= \frac{1}{2}(\omega_1 + \omega_2) + g_1 S_1^z + g_2 S_2^z, \\ N &= \frac{1}{2}(\omega_1 - \omega_2) + g_1 S_1^z - g_2 S_2^z, \end{aligned} \tag{3}$$

with collective operators $S_1^z = \frac{1}{2}\sum_{l=1}^{N_1}\sigma_{1,l}^z$ and $S_2^z = \frac{1}{2}\sum_{l=1}^{N_2}\sigma_{2,l}^z$.

Then the evolution operator of the system can be formally solved as

$$U(t) = \exp(-{\rm i}tH_{\rm B})\cdot \begin{pmatrix} \exp[-{\rm i}tM] & 0 & 0 & 0 \\ 0 & \cos\Omega t - {\rm i}\sin\Omega t\cos\theta & -{\rm i}\sin\Omega t\sin\theta & 0 \\ 0 & -{\rm i}\sin\Omega t\sin\theta & \cos\Omega t + {\rm i}\sin\Omega t\cos\theta & 0 \\ 0 & 0 & 0 & \exp[{\rm i}tM] \end{pmatrix}, \tag{4}$$

where we have defined

$$\tan\theta = \frac{J}{N} = \frac{J}{(\omega_1-\omega_2)/2 + g_1 S_1^z - g_2 S_2^z}, \tag{5}$$

$$\Omega^2 = J^2 + [(\omega_1-\omega_2)/2 + g_1 S_1^z - g_2 S_2^z]^2. \tag{6}$$

We will use this evolution operator to study the entanglement evolutions of the system in the following sections. We will find that the substances of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ of the system can be changed, in essential, the changes are in accord with the energy transport between the two qubits. In a bipartite system, such changes can change the composition of the state, which will change the entanglement and can be used to improve the entanglement of the system.

3. Entanglement evolutions under the influence of spin-star environments

3.1. Pure initial environment states

In this section, we will consider the simply case of pure initial environment state. Firstly, we will consider the entanglement evolution of the bipartite system with a pure initial state of the system, $|\psi(0)\rangle = |\uparrow\rangle_1|\downarrow\rangle_2$, and show that such a state can not always be driven to the maximum entangled state, while the environment can help it to evolve to a more entangled state. Then, we will also show that an entangled initial state will not always be driven to the non-entangled state without the environment's help.

Assume that the initial state of the environment is a pure eigenstate of the collective operators and can be described by the following function:

$$|\psi_{\rm B}(0)\rangle = \left|\frac{N_1}{2}, -\frac{N_1}{2}\right\rangle \otimes \left|\frac{N_2}{2}, -\frac{N_2}{2}\right\rangle, \tag{7}$$

where $|j,m\rangle$ stands for an eigenvector of the angular momentum operator, i.e.,

$$\begin{aligned} S^2|j,m\rangle &= j(j+1)|j,m\rangle, \\ S^z|j,m\rangle &= m|j,m\rangle, \end{aligned} \tag{8}$$

with $S^2 = S_x^2 + S_y^2 + S_z^2$, $j = 0,\ldots,N/2$, and $m = -j,\ldots,0,\ldots,j$.

Then the evolution of the total system can be easily worked out as

$$\begin{aligned} |\psi_{\rm tot}(t)\rangle &= U(t)|\uparrow\rangle_1|\downarrow\rangle_2 \otimes |\phi_{\rm B}(0)\rangle \\ &= (A_1|\uparrow\rangle_1|\downarrow\rangle_2 + B_1|\downarrow\rangle_1|\uparrow\rangle_2)\otimes|\phi_{\rm B}(0)\rangle, \end{aligned} \tag{9}$$

with

$$\begin{aligned} A_1 &= \cos\Omega_1 t + {\rm i}\sin\Omega_1 t\cos\theta_1, \\ B_1 &= -{\rm i}\sin\Omega_1 t\sin\theta_1, \end{aligned} \tag{10}$$

where

$$\Omega_1 = \sqrt{\left[\frac{1}{2}(\omega_1 - \omega_2) + \frac{1}{2}(g_2 N_2 - g_1 N_1)\right]^2 + J^2}.$$

The entanglement measured by the concurrence can be easily obtained as

$$\begin{aligned} C(t) &= \max\{0, 2|A_1 B_1|\} \\ &= 2\sqrt{\sin^2 \Omega_1 t \sin^2 \theta_1 (\cos^2 \Omega_1 t + \sin^2 \Omega_1 t \cos^2 \theta_1)}, \end{aligned} \tag{11}$$

which is closely related to the initial state of the bipartite system, and we can find that the concurrence can reach its maximum value $C(t) = 1$ in the case of

$$\cos 2\Omega_1 t = -\frac{\cos^2 \theta_1}{\sin^2 \theta_1} = -\frac{N_1'^2}{J^2}, \tag{12}$$

or $|A_1| = |B_1|$. Since the probability of transition from $|\uparrow\rangle$ to $|\downarrow\rangle$ can be obtained as

$$|B_1|^2 = \sin^2 \theta_1 \sin^2 \Omega_1 t = \frac{J^2}{\Omega_1^2} \sin^2 \Omega_1 t \leq \frac{J^2}{\Omega_1^2}, \tag{13}$$

thus, it is impossible to obtain the maximal concurrence 1 in the case of $J^2/\Omega_1^2 < 1/2$.

Actually, in the case of no effect of the environment, the above equation (12) can not always be satisfied. In the case of $J^2 < (\omega_1 - \omega_2)^2/4$, i.e., $J^2/\Omega^2 \leq 1/2$, the largest concurrence will not be obtained, since in this case, the mode $|B_1|$ is always less than $|A_1|$. While under the influence of the environment's evolution, Ω_1 will be changed, and if the following condition is satisfied:

$$J^2 \geq \left(\frac{\omega_1 - \omega_2}{2} + \frac{g_2 N_2 - g_1 N_1}{2}\right)^2 = N_1'^2, \tag{14}$$

then the system can be driven to have the largest concurrence 1. Note that the requirement of the largest concurrence is different from that of the largest transition probability,[10] since getting the largest entanglement does not need the largest transition probability of the energy level transition.

The entanglement evolution of the bipartite system without the influence of the environment is shown in Fig. 1(a), which tells us that without the influence of the environment, the concurrence of the bipartite system cannot reach the maximum value 1 in the case of $J^2/\Omega_1^2 \leq 1/2$. The results considering the influence of the environments are plotted in Figs. 1(b) and 1(c) and show that the bipartite system's concurrence can reach the maximum value 1 under the influence of the environment. Since the environment can assist the hopping between the lower and the higher states, it can also help the entanglement generation. However, it is different from the assisted energy transfer,[10] since a large entanglement does not need the entire hopping between the states.

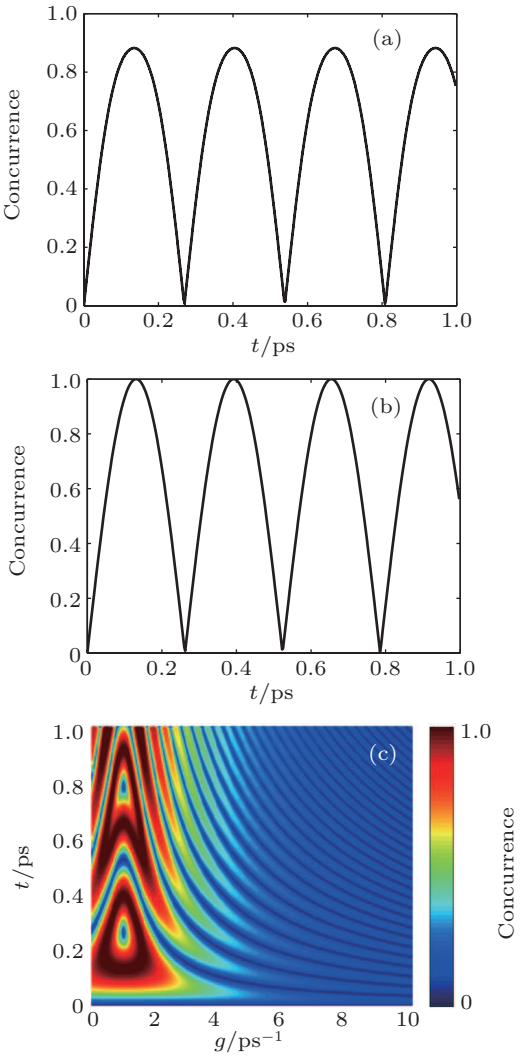


Fig. 1. (color online) The entanglement evolution of the bipartite system with initial state $|\uparrow\rangle_1|\downarrow\rangle_2$ as a function of time, and the coupling constants are (a) $g_1 = g_2 = 0$, (b) $g_1 = 0$, $g_2 = 1$, (c) $g_1 = 0$, $g_2 = g \in [0, 10]$. Other parameters are set as $N_1 = 0$, $N_2 = 20$, $J = 6$ ps^{-1}, and $\omega_2 - \omega_1 = 20$ ps^{-1}.

In an opposite manner, if the initial state is in its maximum Bell initial state, the entanglement cannot disappear completely to zero without the influence of the environment, see Fig. 2(a). However, under the influence of the environment, the entanglement can completely disappear during the evolution, see Figs. 2(b) and 2(c). That is to say, without the influence of the environment, the transition probability between the subsystems cannot be strong enough to reduce the whole entanglement with a weak coupling, and this is also closely related to the transition probability between different energy levels.

The former results tell us that in the case of the coupling strength between the two qubits less than the energy level difference of the subsystems, $J^2 < (\omega_1 - \omega_2)^2/4$, one can always enhance or reduce the entanglement by choosing a special environment. Note that only in the case of Eq. (14) being satisfied, the environment can help the entanglement generation. If the coupling strength g is large enough, the environment can also destroy the entanglement. For a special energy difference,

the coupling strength can help the entanglement generation in a special range determined by Eq. (14).

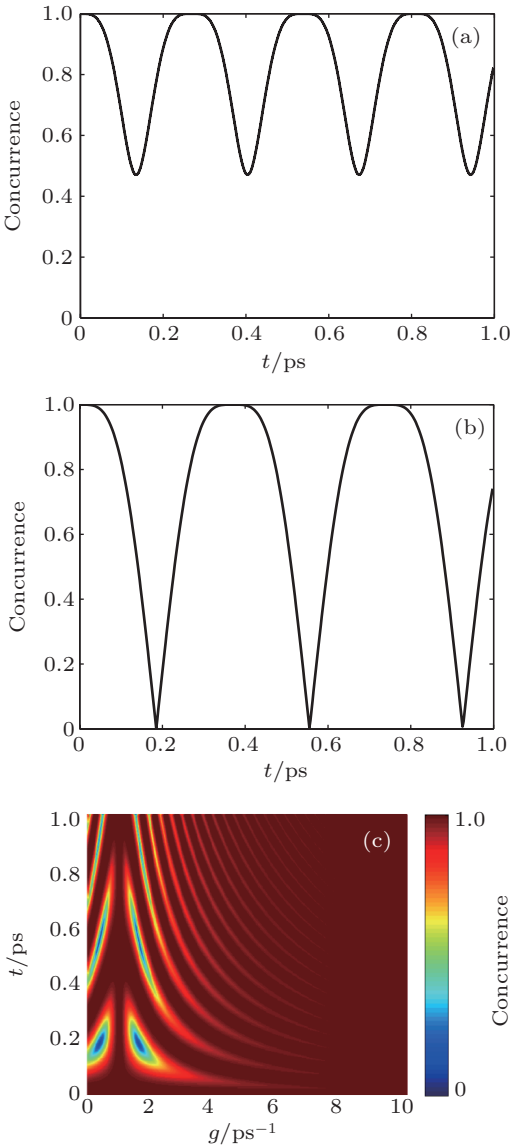


Fig. 2. (color online) The entanglement evolution of the bipartite system with initial state $(|\uparrow\rangle_1|\downarrow\rangle_2 \pm |\downarrow\rangle_1|\uparrow\rangle_2)/\sqrt{2}$ as a function of time, and the coupling constants are (a) $g_1 = g_2 = 0$, (b) $g_1 = 0$, $g_2 = 1$, (c) $g_1 = 0$, $g_2 = g \in [0, 10]$. Other parameters are set as N_1=0, $N_2 = 20$, $J = 6$ ps^{-1}, and $\omega_2 - \omega_1 = 20$ ps^{-1}.

Now, we consider a more general uncorrelated initial state

$$|\Psi_0\rangle = (a_1|\uparrow\rangle_1 + b_1|\downarrow\rangle_1) \otimes (a_2|\uparrow\rangle_2 + b_2|\downarrow\rangle_2). \tag{15}$$

It is known that the state will be entangled under the influence of the interaction, however the concurrence can not always reach the maximum value 1. We will show that the environment can also assist the entanglement generation in this case. The evolution of the system with this initial state can be described as

$$\begin{aligned} |\psi_{\rm tot}(t)\rangle &= U(t)|\Psi_0\rangle \otimes |\phi_{\rm B}(0)\rangle \\ &= \begin{pmatrix} a_1 a_2 \exp(-itM) \\ b_1 a_2 A_1^* + a_1 b_2 B_1 \\ b_1 a_2 B_1 + a_1 b_2 A_1 \\ b_1 b_2 \exp(itM) \end{pmatrix} \otimes |\phi_{\rm B}(0)\rangle, \end{aligned} \tag{16}$$

and the concurrence can also be easily found as

$$\begin{aligned} C &= 2|a_1 a_2 b_1 b_2 - (b_1 a_2 A_1^* + a_1 b_2 B_1) \\ &\quad \times (b_1 a_2 B_1 + a_1 b_2 A_1)| \\ &= \{\sin^2\theta_1 [4\sin^4 \Omega_1 t [(a_2^2 b_1^2 - a_1^2 b_2^2)\cos\theta_1 \\ &\quad + 2a_1 a_2 b_1 b_2 \sin\theta_1]^2 \\ &\quad + (a_2^2 b_1^2 + a_1^2 b_2^2)^2 \sin^2 2\Omega_1 t]\}^{1/2}. \end{aligned} \tag{17}$$

The state will be entangled under the evolution with the environment's influence. For the simple case of $a_1 = b_1 = a_2 = b_2 = 1/\sqrt{2}$, the corresponding concurrence can be reduced to

$$C = \sqrt{\sin^2 \Omega_1 t \sin^2\theta_1 (\cos^2 \Omega_1 t + \sin^2 \Omega_1 t \sin^2\theta_1)},$$

and the corresponding concurrence is shown in Fig. 3, where we also considered the weak coupling case.

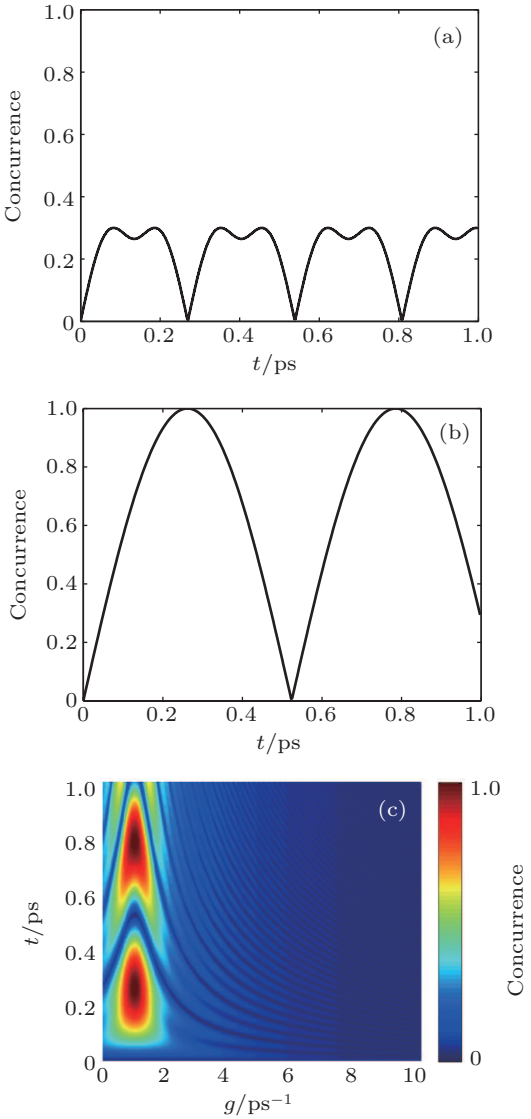


Fig. 3. (color online) The entanglement evolution of the bipartite system with initial state $|\psi_0\rangle = (a_1|\uparrow\rangle_1 + b_1|\downarrow\rangle_2) \otimes (a_2|\downarrow\rangle_1 + b_2|\uparrow\rangle_2)$ as a function of time, and the coupling constants are (a) $g_1 = g_2 = 0$; (b) $g_1 = 0$, $g_2 = 1$; (c) $g_1 = 0$, $g_2 = g \in [0, 10]$. Other parameters are set as $N_1 = 0$, $N_2 = 20$, $J = 6$ ps^{-1}, and $\omega_2 - \omega_1 = 20$ ps^{-1}.

Figure 3(a) shows the entanglement evolution without the influence of the environment, and Figs. 3(b) and 3(c) show the concurrence under the influence of the environment. The concurrence can also be enhanced and reach its maximum value under the influence of the environment.

From the above study with simple states of the system and the environment, we can find that the environment enhances the entanglement only by enhancing the transition between the substances $|\uparrow\rangle_1|\downarrow\rangle_2$ and $|\downarrow\rangle_1|\uparrow\rangle_2$ of the state, while the other substances of the state can also influence the entanglement of the system.

Although our study only considers the transition between states $|\uparrow\rangle_1|\downarrow\rangle_2$ and $|\downarrow\rangle_1|\uparrow\rangle_2$, the above study tell us that different environments of the qubits can help the system to obtain more entanglement. By comparing with the energy transition between the two qubits, it is clear that the entanglement generation of the bipartite system is closely related to the energy transition. If the environment can assist the energy transition, it can also assist the entanglement generation.

3.2. The case of nonzero temperature

In this section, we will extend the study of entanglement evolution to the case of a thermal equilibrium environment state. The initial state of the environment can be described by the canonical distribution

$$\rho_{\rm B}(0) = \frac{1}{Z_1 Z_2} {\rm e}^{-\beta(\alpha_1 S_1^z + \alpha_2 S_2^z)}, \tag{18}$$

where Z_1 and Z_2 are the partition functions of the environments, $\beta = 1/k_{\rm B}T$, and $k_{\rm B}$ is the Boltzmann constant.

We can expand the equilibrium state by the eigenstates $\sum_{j_i,m_i}|j_i,m_i\rangle\langle j_i,m_i|, (i=1,2)$ of the environments' operators S_1^z and S_2^z, i.e.,

$$\rho_{\rm B}(0) = \prod_{i=1,2}\frac{1}{Z_i}\sum_{j_i=0}^{N_i/2}\sum_{m_i=-j_i}^{j_i}\nu(N_i,j_i)|j_i,m_i\rangle\langle j_i,m_i|{\rm e}^{-\beta\alpha_i S_i^z}, \tag{19}$$

where the partition function of the corresponding bath can be obtained as

$$\begin{aligned} Z_i &= \sum_{j_i=0}^{N_i/2}\sum_{m_i=-j_i}^{j_i}\nu(N_i,j_i)\langle j_i,m_i|{\rm e}^{-\beta\alpha_i S_i^z}|j_i,m_i\rangle \\ &= \sum_{j_i=0}^{N_i/2}\nu(N_i,j_i)\frac{\sinh\beta\alpha_i(j_i+1/2)}{\sinh(\beta\alpha_i/2)}. \end{aligned} \tag{20}$$

In the above equations, $\nu(N_i,j_i)$ is the degeneracy of the environment,[16,17] which can be expressed as

$$\nu(N_i,j_i) = \begin{pmatrix} N_i \\ N_i/2-j_i \end{pmatrix} - \begin{pmatrix} N_i \\ N_i/2-j_i-1 \end{pmatrix}, \tag{21}$$

with $\sum_{j_i=0}^{N_i/2}\nu(N_i,j_i)(2j_i+1) = 2_i^N$.

For an initial state of the bipartite system $|\uparrow\rangle_1|\downarrow\rangle_2$, the final density operator under the influence of the environment can be worked out as

$$\begin{aligned} \rho_{\rm S}(t) &= {\rm Tr}_{\rm B}\Big[U(t)\rho_{\rm S}(0)\otimes\prod_{i=1,2}\frac{1}{Z_i}\sum_{j_i=0}^{N_i/2}\sum_{m_i=-j_i}^{j_i}\nu(N_i,j_i) \\ &\quad\times|j_i,m_i\rangle\langle j_i,m_i|{\rm e}^{-\beta\alpha_i S_i^z}U^{-1}(t)\Big] \\ &= \frac{1}{Z_1Z_2}\sum_{j_1=0}^{N_1/2}\sum_{m_1=-j_1}^{j_1}\sum_{j_2=0}^{N_2/2}\sum_{m_2=-j_2}^{j_2}{\rm e}^{-\beta\alpha_1 m_1-\beta\alpha_2 m_2} \\ &\quad\times\nu(N_1,j_1)\nu(N_2,j_2)U_{m_1,m_2}(t)\rho_{\rm S}(0)U^{-1}_{m_1,m_2}(t), \end{aligned} \tag{22}$$

where

$$U_{m_1,m_2}(t) = \langle j_1,m_1|U(t)|j_2,m_2\rangle. \tag{23}$$

Since our model only includes the transition between states $|\uparrow\rangle_1|\downarrow\rangle_2$ and $|\downarrow\rangle_1|\uparrow\rangle_2$, the final density operator of the system under evolution can be expressed as

$$\begin{aligned} \rho(t) &= \frac{1}{Z_1Z_2}\sum_{j_1=0}^{N_1/2}\sum_{m_1=-j_1}^{j_1}\sum_{j_2=0}^{N_2/2}\sum_{m_2=-j_2}^{j_2}{\rm e}^{-\beta\alpha_1 m_1-\beta\alpha_2 m_2} \\ &\quad\times\nu(N_1,j_1)\nu(N_2,j_2) \\ &\quad\times\Big[|A_{m_1,m_2}|^2|\uparrow_1\rangle|\downarrow_2\rangle\langle\uparrow_1|\langle\downarrow_2| \\ &\quad+A_{m_1,m_2}B^*_{m_1,m_2}|\uparrow_1\rangle|\downarrow_2\rangle\langle\downarrow_1|\langle\uparrow_2| \\ &\quad+A^*_{m_1,m_2}B_{m_1,m_2}|\downarrow_1\rangle|\uparrow_2\rangle\langle\uparrow_1|\langle\downarrow_2| \\ &\quad+|B_{m_1,m_2}|^2|\downarrow_1\rangle\uparrow_2\rangle\langle\downarrow_1|\langle\uparrow_2|\Big], \end{aligned} \tag{24}$$

with

$$\begin{aligned} A_{m_1,m_2} &= \cos\Omega_{m_1,m_2}t + {\rm i}\sin\Omega_{m_1,m_2}t\cos\theta_{m_1,m_2}, \\ B_{m_1,m_2} &= -{\rm i}\sin\Omega_{m_1,m_2}t\sin\theta_{m_1,m_2}, \\ \Omega_{m_1,m_2} &= \sqrt{[(\omega_1-\omega_2)/2+g_1m_1-g_2m_2]^2+J^2}. \end{aligned} \tag{25}$$

Then, by the above density operator, we can also work out the entanglement evolution of the bipartite system. The numerous results are shown in Fig. 4.

Comparing Fig. 1 with Fig. 4, we can find that in the case of the low temperature limit, the influence of the environment is similar to that in the case with the pure state environment. The entanglement can also be enhanced by the environment if the temperature is low enough. However, with the temperature increased, the enhancement of the environment to the entanglement generation will be limited, and only support the generation in a short time limit. When the temperature becomes high enough, it will no longer assist the entanglement generation. Similarly to the case with zero temperature, when the coupling strength g is large enough, the environment will also destroy the entanglement. The range of coupling strength g in which the environment can assist the generation of the entanglement is closely related to the energy level difference of the two qubits.

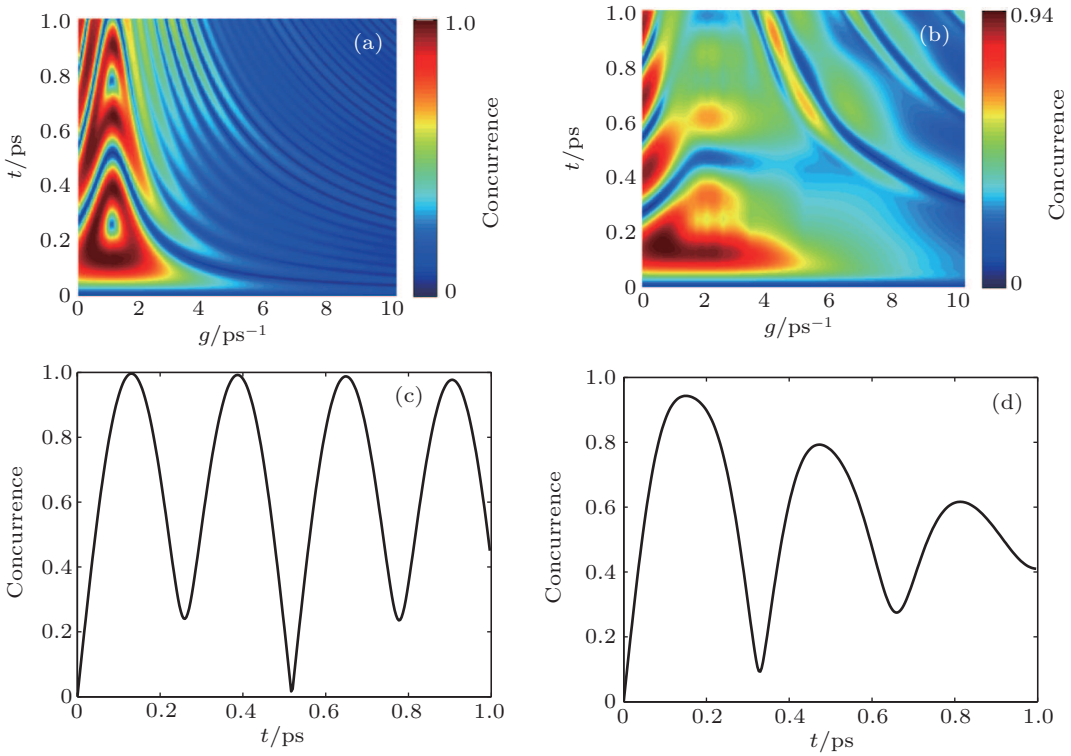


Fig. 4. (color online) The entanglement as a function of time and the coupling constant. The temperature of the spin-star environment is $T = 77$ K in panels (a) and (c) and $T = 300$ K in panels (b) and (d). The parameters are chosen to be $g_1 = 0$, $g_2 = g$, $N_2 = 20$, $J = 6$ ps^{-1}, $\alpha_2 = 250$ ps^{-1}, and $\omega_2 - \omega_1 = 20$ ps^{-1}.

We can also consider the case of a pure initial state of the bipartite system $|\psi\rangle = (a_1|\uparrow\rangle + b_1|\downarrow\rangle) \otimes (a_2|\uparrow\rangle + b_2|\downarrow\rangle)$, and the evolution of state $\rho(t)$ can also be easily worked out. However, since only similar results can be obtained, we will not them present here.

The above results tell us that the entanglement in a bipartite system with weak coupling can be enhanced by the spin-star environment not only at zero temperature, but also at non-zero temperature realistic cases.

4. Conclusion

For a bipartite quantum system with a large energy level difference, the quantum entanglement can not always reach the maximum concurrence 1 with a weak coupling strength. We have studied the influence of the spin-star environment on the generation of the entanglement and found that the environment can assist the entanglement generation in the bipartite system. Since the environment can assist the energy transfer of the subsystem, and the energy transfer between different levels is closely related to the other quantum properties, we can use the environment to do more useful work in quantum information theory.

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