Consequent entanglement concentration of a less-entangled electronic cluster state with controlled-not gates*

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We present a highly efficient entanglement concentration protocol (ECP) for a four-electron system in a less-entangled cluster state. In this ECP, we only require one pair of less-entangled electron cluster states and one ancillary electron to complete the task. With the help of the controlled-not (CNOT) gate, the concentrated maximally entangled state can be retained for further application with some success probability. On the other hand, the discarded items can be reused to obtain a high success probability. All the features make this ECP useful in the current quantum information field.

Keywords: quantum communication, entanglement, entanglement concentration

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1. Introduction

Quantum entanglement is of vital importance in current quantum information processing. In many quantum communication protocols, such as quantum teleportation,[1–3] quantum secure direct communication,[4–8] quantum secret sharing,[9–11] and quantum key distribution,[12–17] the entanglement is required to set up the quantum channel. In quantum computation, the controlled-not (CNOT) gate or similar logic operation is required to create the entanglement.[18–22] For example, in the one-way quantum computation model, one should create the maximally entangled cluster state using the controlled-phase gate.[23] However, the entanglement is always fragile in a noisy environment. The noisy environment will reduce the entanglement. Generally speaking, the maximally entangled state will become a mixed state or a pure less-entangled state. In the former case, entanglement purification is required to distill the high quality mixed ensembles from the low quality mixed ensembles.[24–35] In the latter case, people resort to entanglement concentration to recover the maximally entangled state from the less-entangled state.[36]

Entanglement concentration is a powerful way to obtain the maximally entangled state from the less-entangled state with local operation and classical communication (LOCC). There are many entanglement concentration protocols (ECPs) in theory. Usually, all of the existing ECPs can be divided into three groups. The first group focuses on the two-particle Bell states or the multi-particle Greenberg–Horne–Zeilinger (GHZ) state, such as the ECP proposed by Bennett *et al.*,[36] the ECP with entanglement swapping,[37] the ECP with linear optics,[38] the ECP with nonlinear optics,[39–42] and so on.[43–49] The second group focuses on the multipartite W state,[50–60] such as the ECP with both linear and nonlinear optics,[50–52] the ECP with electronic spins,[58] and the ECP with quantum dots inside the microcavities.[59] In the third group, the ECPs focus on the cluster state,[61–63] such as the ECP with linear optics,[61] and the ECPs with cross-Kerr nonlinearity.[62,63] Although there have been some ECPs with the cluster state, such as the ECP proposed by Choudhury *et al.*,[61] they all focus on the less-entangled cluster states encoded in the polarized photons, no ECP has dealt with the electron-spin cluster state. So far, only some ECPs for the Bell state and the W state of electron spins were proposed. In 2012, Ren *et al.* proposed an effective ECP for the electron-spin GHZ state with the help of the CNOT gate. In their work, they only required a pair of less entangled N-electron GHZ states and an auxiliary electron, and could realize the repeatable concentration for the N-electron GHZ state.[64]

Inspired by the work of Ren *et al.*, in this paper, a novel ECP for nonlocal four-electron systems in a partially entangled pure cluster state is proposed, resorting to an ancillary electron and the CNOT gates. The electron system is one of the best candidates to realize quantum computation.[65] Some novel protocols focusing on the electrons have been proposed, including the implement of the CNOT gate,[65,66] the entanglement purification,[67–69] the entanglement concentration for both Bell state and W state,[49,56,64] and so on.[70] Our ECP is useful in current one-way quantum computation, which usually requires the maximally entangled cluster state. It has sev-

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eral advantages. First, only one copy of the less-entangled state is required. Second, in the protocol, only one ancillary single electron is used, which makes the ECP optimal. Moreover, our ECP can be extended to deal with the K-electron cluster state and only one of the parties needs to perform the operation to reconstruct the standard maximally entangled cluster state. It can also be repeated to obtain a high success probability. All these advantages make our ECP useful in current one-way quantum computation.

2. ECP of four-electron less-entangled cluster state with CNOT gate

In this section, we will describe the basic principle of our ECP. Suppose four parties, Alice, Bob, Charlie, and David, share a less-entangled state of the form

$$\begin{aligned}&|\Phi\rangle_{\mathrm{ABCD}}\\ &= \alpha(|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}+|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}})\\ &\quad+\beta(|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}-|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}),\end{aligned}\tag{1}$$

with $|\alpha|^2+|\beta|^2=1/2$. Here $|\uparrow\rangle$ represents the spin-up state and $|\downarrow\rangle$ represents the spin-down state. In Eq. (1), the four electrons belong to Alice, Bob, Charlie, and David, respectively. As shown in Fig. 1, David prepares an ancillary electron in the initial state $|\Phi\rangle_{\mathrm{a}}=|\uparrow\rangle_{\mathrm{a}}$. After passing through the CNOT gate, the whole system evolves as

$$\begin{aligned}&|\Phi\rangle_{\mathrm{ABCD}}\otimes|\Phi\rangle_{\mathrm{a}}\\ &= \alpha(|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}+|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}})|\uparrow\rangle_{\mathrm{a}}\\ &\quad+\beta(|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}-|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}})|\uparrow\rangle_{\mathrm{a}}\\ &\rightarrow \alpha(|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}+|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}})|\uparrow\rangle_{\mathrm{a}}\\ &\quad+\beta(|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}-|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}})|\downarrow\rangle_{\mathrm{a}}.\end{aligned}\tag{2}$$

Then, David performs a rotation operation R1 on the ancillary electron, which results in

$$\begin{aligned}|\uparrow\rangle_{\mathrm{a}} &\rightarrow \frac{1}{\sqrt{\alpha^2+\beta^2}}(\alpha|\uparrow\rangle+\beta|\downarrow\rangle),\\ |\downarrow\rangle_{\mathrm{a}} &\rightarrow \frac{1}{\sqrt{\alpha^2+\beta^2}}(-\beta|\uparrow\rangle+\alpha|\downarrow\rangle).\end{aligned}\tag{3}$$

After R1, the state in Eq. (2) becomes

$$\begin{aligned}&|\Phi\rangle_{\mathrm{ABCD}}\otimes|\Phi\rangle_{\mathrm{a}}\\ &\rightarrow \frac{\alpha^2}{\sqrt{\alpha^2+\beta^2}}|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}|\uparrow\rangle_{\mathrm{a}}\\ &\quad+\frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}|\downarrow\rangle_{\mathrm{a}}\\ &\quad+\frac{\alpha^2}{\sqrt{\alpha^2+\beta^2}}|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}|\uparrow\rangle_{\mathrm{a}}\\ &\quad+\frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}|\downarrow\rangle_{\mathrm{a}}\\ &\quad-\frac{\beta^2}{\sqrt{\alpha^2+\beta^2}}|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}|\uparrow\rangle_{\mathrm{a}}\\ &\quad+\frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}|\downarrow\rangle_{\mathrm{a}}\\ &\quad+\frac{\beta^2}{\sqrt{\alpha^2+\beta^2}}|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}|\uparrow\rangle_{\mathrm{a}}\\ &\quad-\frac{\alpha\beta}{\sqrt{\alpha^2+\beta^2}}|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}|\downarrow\rangle_{\mathrm{a}}.\end{aligned}\tag{4}$$

Interestingly, if the ancillary electron is in state $|\downarrow\rangle$, the state in Eq. (4) will collapse to the maximally entangled state as

$$\begin{aligned}&|\mathrm{cluster}_4\rangle_{\mathrm{ABCD}}\\ &= \frac{1}{2}(|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}+|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}\\ &\quad+|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}-|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}).\end{aligned}\tag{5}$$

In order to obtain the above state, they let the ancillary electron pass through the first polarization beam splitter (PBS), which transmits the spin-up state $|\uparrow\rangle$ and reflects the spin-down one $|\downarrow\rangle$.

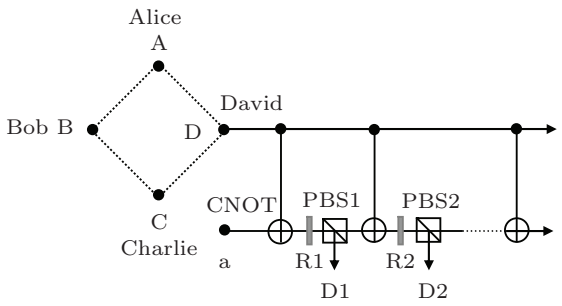


Fig. 1. Schematic diagram of the ECP for four-electron cluster state with an ancillary single electron. R1 and R2 represent different unitary rotation operations on the ancillary electron; PBSs are the electron polarization beam splitters, which transmit spin-up state $|\uparrow\rangle$ and reflect spin-down state $|\downarrow\rangle$.

Finally, if the single-electron detector registers the electron, the protocol is successful. The success probability is $P_{\mathrm{s1}}=8|\alpha\beta|^2$. Here, the subscript "s" means the successful case and the "1" means the first round.

Interestingly, as shown in Fig. 1, if the first step is a failure, the ancillary electron will pass through the PBS1, and it can be reused as the ancillary electron in the next round. In this way, from Eq. (4), the remained less-entangled state can be written as

$$\begin{aligned}|\Phi\rangle'_{\mathrm{ABCD}} &= \frac{\alpha^2}{\sqrt{\alpha^2+\beta^2}}|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}\\ &\quad+\frac{\alpha^2}{\sqrt{\alpha^2+\beta^2}}|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\uparrow\rangle_{\mathrm{C}}|\uparrow\rangle_{\mathrm{D}}\\ &\quad-\frac{\beta^2}{\sqrt{\alpha^2+\beta^2}}|\uparrow\rangle_{\mathrm{A}}|\uparrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}\\ &\quad+\frac{\beta^2}{\sqrt{\alpha^2+\beta^2}}|\downarrow\rangle_{\mathrm{A}}|\downarrow\rangle_{\mathrm{B}}|\downarrow\rangle_{\mathrm{C}}|\downarrow\rangle_{\mathrm{D}}.\end{aligned}\tag{6}$$

It can be rewritten as

$$\begin{aligned}&|\Phi_1\rangle_{\rm ABCD}\\ =\ &\alpha_1(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}+|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D})\\ &+\beta_1(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}-|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D})\end{aligned}\quad(7)$$

after performing a phase-flip operation on one of the electrons. Here

$$\begin{aligned}\alpha_1 &= \frac{\alpha^2}{\sqrt{2(\alpha^4+\beta^4)}},\\ \beta_1 &= \frac{\beta^2}{\sqrt{2(\alpha^4+\beta^4)}}.\end{aligned}\quad(8)$$

Therefore, as shown in Fig. 1, the ECP can be restarted in the second round. After passing through the CNOT gate, the initial less-entangled state in the form of Eq. (7) combined with the ancillary electron $|\Phi\rangle_{\rm a}$ can be written as

$$\begin{aligned}&|\Phi_1\rangle_{\rm ABCD}\otimes|\Phi\rangle_{\rm a}\\ =\ &\alpha_1(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}+|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D})|\uparrow\rangle_{\rm a}\\ &+\beta_1(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}-|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D})|\uparrow\rangle_{\rm a}\\ \to\ &\alpha_1(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}+|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D})|\uparrow\rangle_{\rm a}\\ &+\beta_1(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}-|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D})|\downarrow\rangle_{\rm a}.\end{aligned}\quad(9)$$

Then David performs a rotation operation R2 on the ancillary electron to complete the following transformation:

$$\begin{aligned}|\uparrow\rangle_{\rm a} &\to \frac{1}{\sqrt{\alpha_1^2+\beta_1^2}}(\alpha_1|\uparrow\rangle+\beta_1|\downarrow\rangle),\\ |\downarrow\rangle_{\rm a} &\to \frac{1}{\sqrt{\alpha_1^2+\beta_1^2}}(-\beta_1|\uparrow\rangle+\alpha_1|\downarrow\rangle).\end{aligned}\quad(10)$$

After performing the rotation operation, the above state becomes

$$\begin{aligned}&|\Phi_1\rangle_{\rm ABCD}\otimes|\Phi\rangle_{\rm a}\\ \to\ &\frac{\alpha_1^2}{\sqrt{\alpha_1^2+\beta_1^2}}|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}|\uparrow\rangle_{\rm a}\\ &+\frac{\alpha_1\beta_1}{\sqrt{\alpha_1^2+\beta_1^2}}|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}|\downarrow\rangle_{\rm a}\\ &+\frac{\alpha_1^2}{\sqrt{\alpha_1^2+\beta^2}}|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}|\uparrow\rangle_{\rm a}\\ &+\frac{\alpha_1\beta_1}{\sqrt{\alpha^2+\beta^2}}|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}|\downarrow\rangle_{\rm a}\\ &-\frac{\beta_1^2}{\sqrt{\alpha_1^2+\beta_1^2}}|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}|\uparrow\rangle_{\rm a}\\ &+\frac{\alpha_1\beta_1}{\sqrt{\alpha_1^2+\beta_1^2}}|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}|\downarrow\rangle_{\rm a}\\ &+\frac{\beta_1^2}{\sqrt{\alpha_1^2+\beta_1^2}}|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}|\uparrow\rangle_{\rm a}\\ &-\frac{\alpha_1\beta_1}{\sqrt{\alpha_1^2+\beta_1^2}}|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}|\downarrow\rangle_{\rm a}.\end{aligned}\quad(11)$$

Interestingly, the state in Eq. (11) has the same form as the state in Eq. (4). Therefore, if D2 registers the electron, the above state will collapse to the maximally entangled cluster state shown in Eq. (5). The total success probability is

$$P_{\rm s2}=8|\alpha_1\beta_1|^2\cdot(1-P_{\rm s1})=\frac{8|\alpha\beta|^4}{|\alpha|^4+|\beta|^4}.\quad(12)$$

On the other hand, if D2 does not register the electron, after performing a phase-flip operation, the remained state can be written as

$$\begin{aligned}&|\Phi_2\rangle_{\rm ABCD}\\ =\ &\alpha_2(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D}+|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\uparrow\rangle_{\rm C}|\uparrow\rangle_{\rm D})\\ &+\beta_2(|\uparrow\rangle_{\rm A}|\uparrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}-|\downarrow\rangle_{\rm A}|\downarrow\rangle_{\rm B}|\downarrow\rangle_{\rm C}|\downarrow\rangle_{\rm D}),\end{aligned}\quad(13)$$

where

$$\begin{aligned}\alpha_2 &= \frac{\alpha_1^2}{\sqrt{2(\alpha_1^4+\beta_1^4)}},\\ \beta_2 &= \frac{\beta_1^2}{\sqrt{2(\alpha_1^4+\beta_1^4)}}.\end{aligned}\quad(14)$$

The state in Eq. (13) can also be reused as the initial state to restart the ECP. The success probability in the third round is

$$\begin{aligned}P_{\rm s3} &= 8|\alpha_2\beta_2|^2\cdot(1-P_{\rm s1})\cdot(1-P_{\rm s2})\\ &= \frac{8|\alpha\beta|^8}{(|\alpha|^8+|\beta|^8)(|\alpha|^4+|\beta|^4)}.\end{aligned}\quad(15)$$

In this way, it can be speculated that if the ancillary electron is detected in the n-th round, the success probability is

$$\begin{aligned}P_{sn} &= 8|\alpha_{n-1}\beta_{n-1}|^2\cdot(1-P_{\rm s1})\cdot(1-P_{\rm s2})\cdots\\ &= \frac{8|\alpha\beta|^{2^n}}{(|\alpha|^{2^n}+|\beta|^{2^n})\cdots(|\alpha|^8+|\beta|^8)(|\alpha|^4+|\beta|^4)}.\end{aligned}\quad(16)$$

The total success probability that Alice, Bob, Charlie, and David obtain a standard four-electron cluster state from a less-entangled state is

$$\begin{aligned}P_n &= P_{\rm s1}+P_{\rm s2}+\cdots+P_{sn}\\ &= 8|\alpha\beta|^2+\frac{8|\alpha\beta|^4}{|\alpha|^4+|\beta|^4}+\cdots\\ &\quad+\frac{8|\alpha\beta|^{2^n}}{(|\alpha|^{2^n}+|\beta|^{2^n})\cdots(|\alpha|^8+|\beta|^8)(|\alpha|^4+|\beta|^4)}\\ &= \sum_{k=1}^{n}\frac{4|\alpha\beta|^{2^k}}{\prod_{k=1}^{n}[|\alpha|^{2^k}+|\beta|^{2^k}]}.\end{aligned}\quad(17)$$

3. ECP of less-entangled K-electron cluster state

It is straightforward to extend the ECP to the case of K-electron less-entangled cluster state. Here we take an even-number cluster state to explain the entanglement concentration. The K-electron maximally entangled cluster state can be written as

$$|\Phi_K\rangle = \frac{1}{2^{[K/2]}} \otimes_{e=1}^{K} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e). \tag{18}$$

Here $\sigma_z^{e+1}|\uparrow\rangle_{e+1} = |\uparrow\rangle_{e+1}$, $\sigma_z^{e+1}|\downarrow\rangle_{e+1} = -|\downarrow\rangle_{e+1}$, and $\sigma_z^{K+1} \equiv 1$. The $[K/2]$ means the maximal integer number which is less than or equal to $K/2$. The less-entangled cluster state can be written as

$$\begin{aligned} |\Phi_K\rangle_{\mathrm{ABC\cdots K}} = {} & \alpha \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\uparrow\rangle_{\mathrm{K}} \\ & + \beta \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\downarrow\rangle_{\mathrm{K}}. \end{aligned} \tag{19}$$

Here $|\alpha|^2 + |\beta|^2 = 2^{[K/2]-1}$. The K electrons are distributed by Alice, Bob, …, Kite, respectively. Then, one of the parties also needs to prepare an ancillary electron with the form $|\Phi\rangle_{\mathrm{a}} = |\uparrow\rangle_{\mathrm{a}}$. In this way, the state $|\Phi_K\rangle_{\mathrm{ABC\cdots K}}$ combined with the single electron can be written as

$$\begin{aligned} & |\Phi_K\rangle_{\mathrm{ABC\cdots K}} \otimes |\Phi\rangle_{\mathrm{a}} \\ = {} & \alpha \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\uparrow\rangle_{\mathrm{K}}|\uparrow\rangle_{\mathrm{a}} \\ & + \beta \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\downarrow\rangle_{\mathrm{K}}|\uparrow\rangle_{\mathrm{a}} \\ \rightarrow {} & \alpha \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\uparrow\rangle_{\mathrm{K}}|\uparrow\rangle_{\mathrm{a}} \\ & + \beta \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\downarrow\rangle_{\mathrm{K}}|\downarrow\rangle_{\mathrm{a}}. \end{aligned} \tag{20}$$

By rotating the ancillary electron with R1, it becomes

$$\begin{aligned} & |\Phi_K\rangle_{\mathrm{ABC\cdots K}} \otimes |\Phi\rangle_{\mathrm{a}} \\ \rightarrow {} & \frac{1}{\sqrt{\alpha^2+\beta^2}} \{\alpha\beta[\otimes_{e=1}^{K-1}(|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\uparrow\rangle_{\mathrm{K}}|\downarrow\rangle_e \\ & + \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\downarrow\rangle_{\mathrm{K}}|\downarrow\rangle_e] \\ & + \alpha^2 \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\uparrow\rangle_{\mathrm{K}}|\uparrow\rangle_e \\ & + \beta^2 \otimes_{e=1}^{K-1} (|\uparrow\rangle_e \sigma_z^{e+1} + |\downarrow\rangle_e)|\downarrow\rangle_{\mathrm{K}}|\uparrow\rangle_e\}. \end{aligned} \tag{21}$$

With the same principle, if the ancillary electron is in state $|\downarrow\rangle$, which will be detected by D1, the above state will become the maximally entangled cluster state, with the success probability

$$P_{\mathrm{s1}}^K = 2^{2[K/2]-1}|\alpha\beta|^2. \tag{22}$$

We can also obtain

$$\begin{aligned} P_{\mathrm{s2}}^K &= 2^{2[K/2]-1} \frac{|\alpha\beta|^4}{|\alpha|^4+|\beta|^4}, \\ P_{\mathrm{s3}}^K &= 2^{2[K/2]-1} \frac{|\alpha\beta|^8}{(|\alpha|^8+|\beta|^8)(|\alpha|^4+|\beta|^4)}, \\ &\cdots \\ P_{sn}^K &= 2^{2[K/2]-1} \\ &\times \frac{|\alpha\beta|^{2^n}}{(|\alpha|^{2^n}+|\beta|^{2^n})\cdots(|\alpha|^8+|\beta|^8)(|\alpha|^4+|\beta|^4)}. \end{aligned} \tag{23}$$

The total success probability by repeating the ECP for n times is

$$\begin{aligned} P_n^K &= P_{\mathrm{s1}}^K + P_{\mathrm{s2}}^K + \cdots + P_{sn}^K \\ &= 2^{[K/2]} \sum_{k=1}^{n} \frac{|\alpha\beta|^{2^k}}{\prod_{k=1}^{n}[|\alpha|^{2^k}+|\beta|^{2^k}]}. \end{aligned} \tag{24}$$

4. Discussion and summary

So far, we have fully described our ECP. It is interesting to calculate the total success probability. Figure 2 shows the total success probability P of our protocol as a function of the original entanglement efficient α^2. It can be found that when $\alpha^2 \subseteq (0, 0.25)$, P increases with the growth of α^2, while when $\alpha^2 \subseteq (0.25, 0.5)$, P reduces with the growth of α^2. When $\alpha^2 = 0.25$, P can reach the maximum. Actually, if $\alpha^2 = 0.25$, the initial less-entangled state is essentially the maximally entangled state. The entanglement concentration is essentially the entanglement transformation, it can not increase the entanglement of the original entanglement state. In this way, a higher original entanglement degree can lead to a higher success probability. Meanwhile, it is shown that by repeating the ECP, P can increase greatly. It indicates that we can greatly increase the success probability by repeating the ECP. As shown in Fig. 2, in a practical operation, repeating the ECP for three or five times is enough to obtain a high success probability.

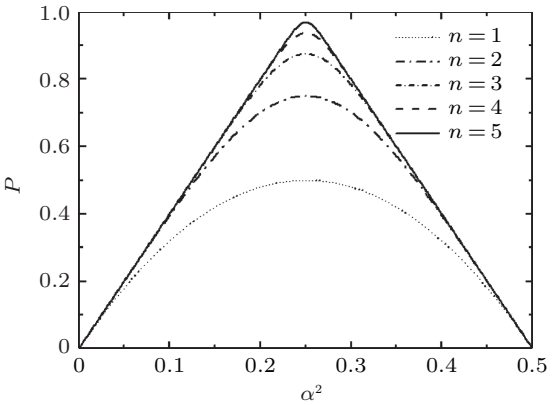


Fig. 2. The success probability as a function of the original coefficient α^2. The iteration number is $n = 1, 2, \ldots, 5$. It can be found that by repeating the ECP, we can increase the success probability greatly.

The key element for realizing the ECP is the CNOT gate. It is well known that the CNOT gate is hard to implement in a linear optical system. With only linear optical elements, the upper bound of the success probability for realizing the CNOT gate is only $3/4$.[72] Fortunately, in the electron system, as pointed out by Beenakker *et al.*, one can implement a deterministic CNOT gate combined with the charge detector and the PBS. In the early work of Ionicioiu and Amico, they designed a protocol for such a device using nondispersive phases

to separate spin-up and spin-down states, it is analogous to a Stern–Gerlach apparatus.[73] The charge detection has been realized in a two-dimensional electron gas. It was reported that the current achievable time resolution for charge detection is μs.[74] In a semiconductor system, it was shown that the resolution required for ballistic electrons is less than 5 ps.[75]

In summary, we have presented an ECP for nonlocal electron systems in a less-entangled cluster state with the help of one ancillary electron. Compared with the other ECPs for the cluster state, the present ECP has some advantages. First, it only needs one pair of less-entangled states. Second, the ancillary electron does not need to be destroyed and can be reused. Third, only one party should perform the ECP, which makes it feasible in practical experiments. Fourth, the ECP can be repeated to obtain a high success probability. All these advantages make our ECP useful in current quantum communication and computation.

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