Thermal quantum and total correlations in spin-1 bipartite system*

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(Received 1 August 2013; revised manuscript received 14 October 2013; published online 18 March 2014)

Thermal quantum and total correlations of a two spin-1 Ising model in the presence of an external homogeneous magnetic field and the Dzyaloshinski–Moriya (DM) interaction are investigated. The result indicates that the DM interaction plays a leading role in the quantum correlation measured by measurement-induced disturbance except for the region with small DM interaction and low temperature, while the DM interaction and the external magnetic field play competing roles in the negativity. The thermal total correlations measured by an alternative new measure defined in terms of the Wigner–Yanase skew information and the quantum mutual information display differences in the same region.

Keywords: two-qutrit system, measurement-induce disturbance, negativity

PACS: 03.67.Ud, 75.10.Jm, 65.90.+i **DOI:** 10.1088/1674-1056/23/5/050304

1. Introduction

Quantum entanglement and quantum correlation are two fascinating quantities in the quantum world. Entanglement, considered as the resource of quantum computation, quantum cryptography, and quantum information processing, has also been proved to be very useful in analyzing the behaviors of various condensed matter systems.[1,2] However, entanglement is not the only kind of meaningful correlation presented in quantum systems. The classification of correlations based on quantum measurements has arisen in recent years.[3–5] Quantum discord, initially introduced by Ollivier and Zurek[6] and by Henderson and Vedral,[7] has been demonstrated to be a novel resource for quantum computation.[8–10] Subsequently, several new quantifiers of correlations and nonlocality, which are more general than the entanglement, have been proposed.[11–14] In order to classify and quantify the statistical correlations in bipartite states, Luo[15] introduced a quantum–classical dichotomy and named it measurement-induced disturbance (MID). It is based on the idea that in the classical description of nature, measurements can be carried out without disturbance, while in the quantum description, generic measurements often disturb the system and the disturbance can be exploited to quantify the quantumness of correlations therein. Very recently, an alternative new measure defined in terms of the Wigner–Yanase skew information[13] (WYSIM) was proposed to quantify the total correlations in a quantum system. This measure for correlation has an appealing feature that its evaluation does not involve any optimization and can be straightforwardly calculated.

The physically realizable quantum mechanisms are not always at zero temperature but often in a thermal equilibrium, e.g., the initial state in the nuclear magnetic resonance based quantum computing is a thermal state.[16] The property of the state at a thermal equilibrium has attracted much attention, for example, the thermal entanglement has been extensively studied with various quantum spin chain models.[17–27] Dzyaloshinski[28] and Moriya[29] introduced an anisotropic antisymmetric exchange interaction, which is called the Dzyaloshinski–Moriya (DM) interaction, to explain the weak ferromagnetism of antiferromagnetic crystals about half a century ago. The thermal entanglement of quantum spin chains with the DM interaction has been investigated in recent years.[30] Recently, quantum correlation in spin chains has also attracted the attention of researchers.[31–33] For example, quantum correlation (quantified by quantum discord) of a two-qubit XYZ Heisenberg model in thermal equilibrium was investigated,[31] and it was shown that the behavior of the thermal quantum discord differs in many unexpected ways from the thermal entanglement. The thermal quantum discord, in contrast to the entanglement and other thermodynamic quantities, spotlights the critical points associated with quantum phase transitions for some spin chain models even at a finite temperature.[34–37] However, the previous studies about spin chain correlations are mostly for the spin 1/2 system and the thermal correlation of a qutrit system is seldom exploited. Higher spins, or qudits, provide certain benefits in quantum computation[37] and quantum communication.[38–40] By using hybrid qudit quantum gates,[41] more resourceful quantum information processing can be realized. Using qudits in quantum computation would be more fault tolerant than the qubit based error correction.[42] All of these motivate us to investigate the thermal quantum and total correlations in a spin-1 system.

In this paper, we will explore the thermal quantum correlation based on negativity and MID. The total correlations will

*Project supported by the Fundamental Research Funds for the Central Universities of China (Grant No. 2012QNA37).

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be measured by WYSIM and quantum mutual information. The effects of the spin–spin coupling, temperature, the DM interaction, and an external magnetic field on the four quantities will be investigated. A detailed comparison between MID and negativity will be given. We will also probe into the distinct difference between WYSIM and quantum mutual information. This paper is organized as follows. In Section 2, the model is introduced. In the subsequent section, the definitions of MID, WYSIM, and negativity are given. The effects of the spin–spin coupling, temperature, the DM interaction, and an external magnetic field are investigated in Section 4. Finally, Section 5 is devoted to conclusions.

2. Model

The Hamiltonian describing a system of N qutrits in a quantum spin chain with Ising and DM interactions under an external magnetic field reads

$$H = J\sum_{i=1}^{N-1} S_z^i S_z^{i+1} + \boldsymbol{B}\cdot\sum_{i=1}^{N}\boldsymbol{S}^i + \boldsymbol{D}\cdot\sum_{i=1}^{N-1}(\boldsymbol{S}^i\times\boldsymbol{S}^{i+1}), \quad (1)$$

where $\boldsymbol{B}$ is the external homogeneous magnetic field, and J is the coupling parameter between the nearest neighbor spins. We have set $\hbar = 1$ and Bohr magneton $\mu_{\rm B} = 1$. The last term in the equation denotes the DM coupling arising from the spin–orbit coupling.[28,29] Here, S_α^i $(\alpha = x,y,z;\ i = 1,2)$ denotes a spin-1 operator, and in the basis of $|1\rangle = (1,0,0)^{\rm T}$, $|0\rangle = (0,1,0)^{\rm T}$, and $|-1\rangle = (0,0,1)^{\rm T}$, which are the eigenvectors of S_z with corresponding eigenvalues $1,0,-1$, its components take the form

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix},\ S_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -{\rm i} & 0\\ {\rm i} & 0 & -{\rm i}\\ 0 & {\rm i} & 0\end{pmatrix},$$
$$S_z = \begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}. \quad (2)$$

Focusing on the two-qutrit quantum Ising system with an external magnetic field and the DM interaction, the Hamiltonian can be simplified to

$$H = JS_z^1S_z^2 + \boldsymbol{B}\cdot(\boldsymbol{S}^1+\boldsymbol{S}^2) + \boldsymbol{D}\cdot(\boldsymbol{S}^1\times\boldsymbol{S}^2). \quad (3)$$

Specifically, if $\boldsymbol{B} = B\boldsymbol{z}$ and $\boldsymbol{D} = D\boldsymbol{z}$ are assumed, the Hamiltonian becomes

$$H = JS_z^1S_z^2 + B(S_z^1+S_z^2) + D(S_x^1S_y^2 - S_y^1S_x^2). \quad (4)$$

The first term decscribes a pure Ising interaction and it cannot establish pairwise entanglement between the qutrits in its eigenstates. The external magnetic field is along the z direction in the model and no pairwise entanglement between the qutrits in the ground or the excited states can be generated. The DM interaction in the third term is a source of nonlinear interactions and its presence would enforce a pairwise entanglement.[43]

Actually, the development of laser cooling and trapping provides us with more ways to control the atoms in traps. We can manipulate the atom–atom coupling and the atom number in each lattice well with a very good accuracy. For two spin-1 atoms, a Hamiltonian with linear and nonlinear interactions has been given.[44,45]

The eigenvalues of the Hamiltonian are found to be

$$E_1 = J+2B,\ E_2 = -J,\ E_3 = B-D,\ E_4 = -B+D,$$
$$E_5 = -B-D,\ E_6 = B+D,\ E_7 = J-2B,\ E_8 = -D\sigma,$$
$$E_9 = -D\gamma, \quad (5)$$

and the corresponding eigenvectors are

$$|\psi_1\rangle = |11\rangle,\ |\psi_2\rangle = \frac{1}{\sqrt{2}}(1,-1\rangle + |-1,1\rangle),$$
$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(1,0\rangle + {\rm i}|0,1\rangle),$$
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}({\rm i}|0,-1\rangle + |-1,0\rangle),$$
$$|\psi_5\rangle = \frac{1}{\sqrt{2}}(|0,-1\rangle + {\rm i}|-1,0\rangle),$$
$$|\psi_6\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle - {\rm i}|0,1\rangle),\ |\psi_7\rangle = |-1,-1\rangle,$$
$$|\psi_8\rangle = \frac{1}{\sqrt{\delta}}(-|1,-1\rangle + {\rm i}\gamma|0,0\rangle + |-1,1\rangle),$$
$$|\psi_9\rangle = \frac{1}{\sqrt{\alpha}}(-|1,-1\rangle + {\rm i}\sigma|0,0\rangle + |-1,1\rangle), \quad (6)$$

where $\alpha = 2+\sigma^2$, $\delta = 2+\gamma^2$, $\sigma = (J-\Omega)/2D$, $\gamma = (J+\Omega)/2D$, and $\Omega = \sqrt{J^2+8D^2}$. From the expressions of the eigenvalues, if $2|B| \ll J$, the ground states cannot be the factorized ones $|\psi_{1,7}\rangle$, but always entangled; on the other hand, if $2|B| \gg J$, the DM interaction should fulfill the condition $D > B-J$ in order to avoid the state $|\psi_7\rangle$ being a factorized ground state.

The state of a system at thermal equilibrium is determined by the density matrix $\rho(T) = \exp(-\beta H)/Z$, where the partition function is $Z = {\rm tr}[\exp(-\beta H)]$ with $\beta = 1/k_{\rm B}T$; T is the temperature, and $k_{\rm B}$ is the Boltzmann constant. Hereafter, $k_{\rm B} = 1$ is set. The entanglement in the thermal state is called thermal entanglement. The explicit expression of the thermal density matrix can be obtained by using the equivalent formula

$$\rho(T) = \frac{1}{Z}\sum_{i=1}^{9}\exp(-\beta E_i)|\psi_i\rangle\langle\psi_i|. \quad (7)$$

With the thermal density matrix $\rho(T)$, we can evaluate quantum and total correlations.

3. MID, WYSIM, and negativity

First of all, the definitions of MID, WYSIM, and negativity should be given.

It is known to all that the total correlations in a bipartite

state ρ are well quantified by the quantum mutual information

$$I(\rho) = S(\rho^1) + S(\rho^2) - S(\rho). \tag{8}$$

In order to obtain the expression of the classical correlation, local measurements $\{\Pi_k\}$ ($\Pi_k\Pi_{k'} = \delta_{kk'}\Pi_k$ and $\sum_i \Pi_k = 1$) are applied to any bipartite state (of course, including the thermal state (7)). Here, the measurement $\Pi = \{\Pi_i^1 \otimes \Pi_j^2\}$ is induced by the spectral resolutions of the reduced states, $\rho^1 = \sum_i p_i^1 \Pi_i^1$ and $\rho^2 = \sum_j p_j^2 \Pi_j^2$. The $\Pi(\rho)$ is a classical state, thus $I(\Pi(\rho))$ quantifies the classical correlations in ρ. Then the quantum correlation can be measured by the discrepancy between $I(\rho)$ and $I(\Pi(\rho))$ and it is named measurement-induced disturbance

$$Q(\rho) = I(\rho) - I\Big(\Pi(\rho)\Big). \tag{9}$$

Here $S(\rho) = -\rho \log_2 \rho$ denotes the von Neumann entropy.

The MID is defined from the viewpoint of quantum measurement, and quantum mutual information $I(\rho)$ is used to measure the total correlations. In order to compare with quantum mutual information, another measure of the total correlations in terms of skew information is exploited, which is based on the notation of the Wigner–Yanase skew information[46] $I(\rho, X) = -\mathrm{tr}[\sqrt{\rho}, X]^2/2$. Here X is an observable (a Hermitian operator) and $[\cdot,\cdot]$ denotes the commutator. The skew information $I(\rho, X)$ depends on both the state ρ and the observable X. In order to obtain an intrinsic quantity capturing the information content of the state ρ without any other involvement, Luo introduces the average[47,48] $Q(\rho) := \sum_i I(\rho, X_i)$, where X_i is a family of observables constituting an orthonormal basis. The global information content of ρ^{12} with respect to the local observable of subsystem 1 is expressed as $Q_1(\rho^{12}) := \sum_i I(\rho^{12}, X_i^1 \otimes \mathbf{1}^2)$. Here $\mathbf{1}^2$ is the identity operator on subsystem 2. The $Q_1(\rho^{12})$ can be proved to be independent of the choice of the orthonormal basis X_i. Then the difference between the information content of ρ^{12} and $\rho^1 \otimes \rho^2$ with respect to the local observable of subsystem 1 can be used as a measure for the correlation in ρ^{12}, i.e.,

$$F(\rho) := Q_1(\rho) - Q_1(\rho^1 \otimes \rho^2). \tag{10}$$

We note that $Q_1(\rho^1 \otimes \rho^2)$ equals to $Q(\rho^1)$, which synthesizes the skew information of the local state ρ^1. From this point of view, $F(\rho)$ captures the correlations in ρ that can be probed by the local observable of the first subsystem. Despite the fact that the evaluation of most of the correlation measurements requires a potentially complex optimization process, an appealing feature of $F(\rho)$, which has been referred to as WYSIM, is that it can be calculated straightforwardly.[13]

The negativity is suitable for our system as it is computable for bipartite systems with arbitrary dimensional Hilbert spaces. It is also practically significant for quantum information processing as it provides bounds for the information capacity of the quantum channel as well as bounds for the distillable entanglement. The negativity[49,50] is defined by

$$N(\rho) = \frac{\|\rho^{\mathrm{T}_1}\| - 1}{2}. \tag{11}$$

The trace norm of ρ^{T_1} is equal to the sum of the absolute value of the eigenvalues of ρ^{T_1}. Here, T_1 denotes the partial transpose with respect to the first subsystem.

4. Results and discussion

It is difficult to obtain the analytical expressions of the total correlations (measured by $I(\rho)$ and $F(\rho)$) and quantum correlations (measured by $Q(\rho)$ and $N(\rho)$) of the thermal density matrix $\rho(T)$ of concern, so we perform numerical simulations in this presentation.

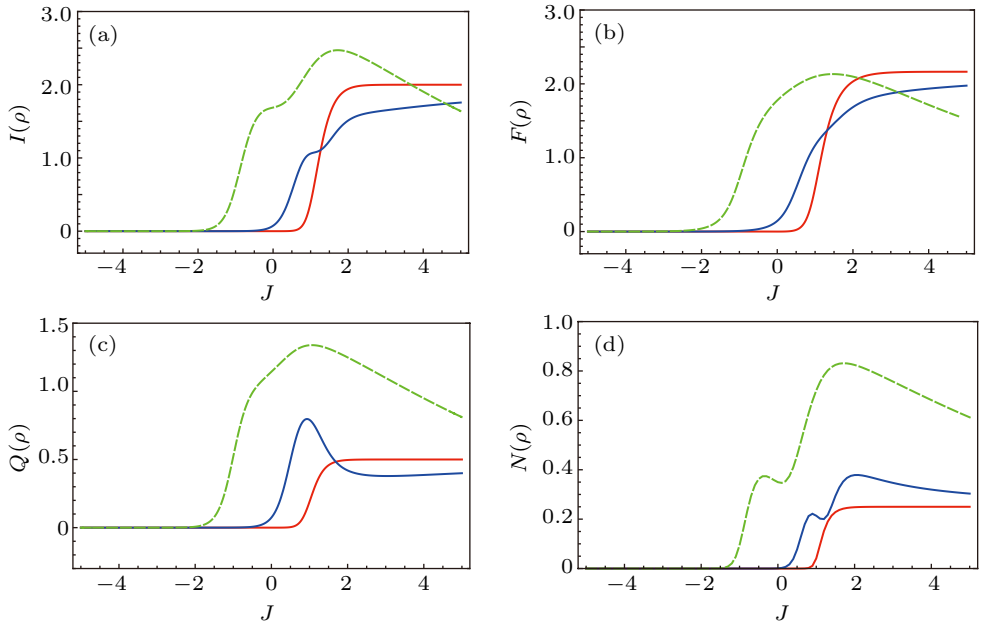


Fig. 1. (color online) (a) Quantum mutual information $I(\rho)$, (b) WYSIM $F(\rho)$, (c) MID $Q(\rho)$, and (d) negativity $N(\rho)$ as a function of the spin–spin coupling J for DM interaction $D = 0.0001$ (red line), $D = 0.5$ (blue line), and $D = 2$ (green dashed line). The other parameters are set as $B = 1$ and $T = 0.2$.

In Fig. 1, the thermal quantum and the total correlations as a function of the spin–spin coupling are plotted with the DM interaction being fixed at three different values. At first glance, it is noted that the four quantities are not symmetrical to the spin–spin coupling and they begin to increase with the increase of the spin–spin coupling. When $J < B - D$, the state $|\psi_7\rangle = |-1,-1\rangle$ is the ground state, which is a separated state without correlation. As the temperature increases, both quantum and total correlations increase due to mixing of the excited states with the ground state. The dependences of quantum mutual information, WYSIM, MID, and negativity on DM anisotropic antisymmetric interaction D are in agreement with our predictions that the DM interaction is a source of nonlinear interactions between the qutrits and would enhance the pair correlations. The larger the value of D is, the earlier the four quantities begin to increase. This can be explained from the case of the ground state. Moreover, the most important point to notice is that quantum mutual information, WYSIM, and MID always exist even though they are very small for a small spin–spin coupling, however, the entanglement measured by negativity increases from zero.

The dependence of the entanglement measured by the negativity on the temperature, external magnetic field, and DM interaction for the same model considered in this paper has been studied in Ref. [43]. In the subsequent analysis, it will be compared with the MID. First of all, the influences of the temperature and the DM interaction on quantum mutual information, WYSIM, and MID are investigated. Through careful calculation, it is found that the effects of the temperature and the DM interaction on quantum mutual information, WYSIM, and MID show the same characteristics. Therefore, only the dependence of the MID on temperature versus the DM interaction is given in Fig. 2 as contour plots. Here, the external magnetic field B is fixed at three different values respectively. The figure indicates that the MID is increased by the DM interaction. The higher the external magnetic field is, the larger the amount of DM interaction is in order to induce quantum correlation, which agrees with the effect of the DM interaction on negativity. The amount of DM interaction should be increased to induce correlation as the the magnetic field gets higher, and the minimum value of D required to compensate the ordering effect of B is $D > B - J$. In Fig. 2(a), the ground state is $|\psi_9\rangle$, whose MID increases monotonously with the increase of the DM interaction, and $Q \sim 1.25$ at a low $B = 0.7$, $D = 1$. For a larger $B = 4$, if $D < B - J = 3$ (for the parameter in Fig. 2(c)), $|\psi_7\rangle$ with $Q = 0$ is the ground state; on the contrary, if $D > 3$, the ground state is $|\psi_5\rangle$, and $Q = 1$. For the intermediate value of $B = 2.5$, the ground states of the model are $|\psi_7\rangle$, $|\psi_5\rangle$, and $|\psi_9\rangle$ in turn, and the corresponding Q are 0, 1, and 1.46. Obviously, the MID increases monotonously. Comparing with Fig. 1(b) in Ref. [43], we find that quantum correlation increases monotonously by the DM interaction at relatively low temperatures when the external magnetic field is $B = 2.5$, while negativity behaves in a much more complicated way. The reason for this phenomenon maybe that with the increase of the DM interaction, the different ground states $|\psi_i\rangle$ become the main part of the thermal state and they have different negativities, however, the quantum correlation of these states increases monotonously.

Now, the influences of the temperature and the external magnetic field on quantum mutual information, WYSIM, and MID are investigated and the contour plots are given in Figs. 3 and 4, respectively. As seen in the figures, all of the three quantities are decreased by the temperature and are symmetrical with respect to the magnetic field, which reflects the symmetry of the Hamiltonian under $B \to -B$. For all DM interactions, the WYSIM decreases monotonously with the increase of the absolute value of B and has the maximum in the region where the magnetic field has a low value. This is because the factorized state $|\psi_7\rangle$ becomes the dominant position in the thermal state when $|B|$ increases. When it comes to the quantum mutual information, for a very small DM interaction $D = 0.0001$, it behaves similarly as the WYSIM. On the other hand, for $D = 0.4$, there are two symmetric lobes in which the quantum mutual information is enhanced by the external magnetic field. This actually reflects the contributions from low-energy states $|\psi_3\rangle$ and $|\psi_5\rangle$ on the quantum mutual information. However, with the increase of the DM interaction to $D = 0.8$, the contribution of the two states to the quantum mutual information becomes inappreciable, and now the quantum mutual information has no lobes and decreases monotonously with increasing $|B|$. Obviously, the explicit difference between WYSIM and quantum mutual information is shown in Figs. 2(b) and 2(e), which indicate $|\psi_3\rangle$ and $|\psi_5\rangle$ have no obvious effect on the WYSIM at low temperatures. Quantum correlation quantified by the MID presents similar characteristics as the negativity when the DM interaction takes the values of 0.0001 and 0.4, however, it decreases much more slowly than the entanglement, as shown by comparing Fig. 4(a) with Fig. 2(a) in Ref. [43]. There are also two symmetric lobes next to the region with the external magnetic field for $D = 0.4$, but the lobes become inconspicuous or disappear with the increase of the DM interaction. These results reflect that the effect of the low-energy states $|\psi_3\rangle$ and $|\psi_5\rangle$ on the MID is similar to that on the quantum mutual information. Therefore, one can obtain large thermal quantum and total correlations just by using a large DM interaction. The previous research indicated that the DM interaction and the magnetic field always play competing roles in enhancing the thermal entanglement.[43,51] However, in this presentation, our results indicate that, for thermal quantum and total quantum correlations, the DM interaction almost plays a leading role, except for in some regions.

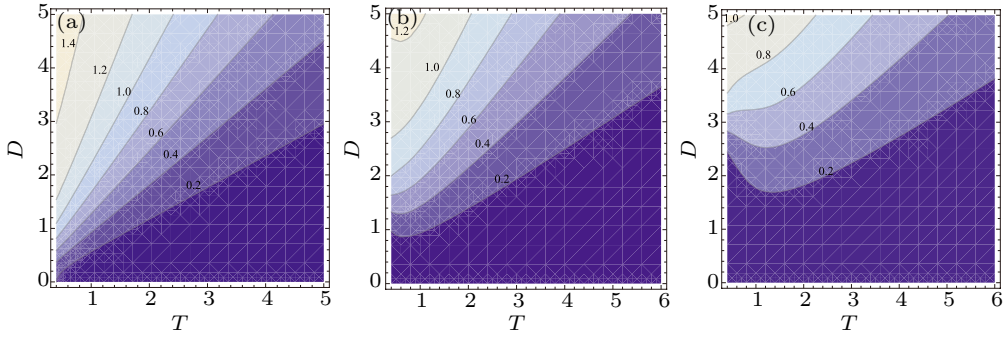


Fig. 2. (color online) The contour plot of MID $Q(\rho)$ as a function of temperature T and DM interaction D for magnetic fields: (a) $B = 0.7$, (b) $B = 2.5$, and (c) $B = 4.0$. Here, the spin–spin coupling is $J = 1$.

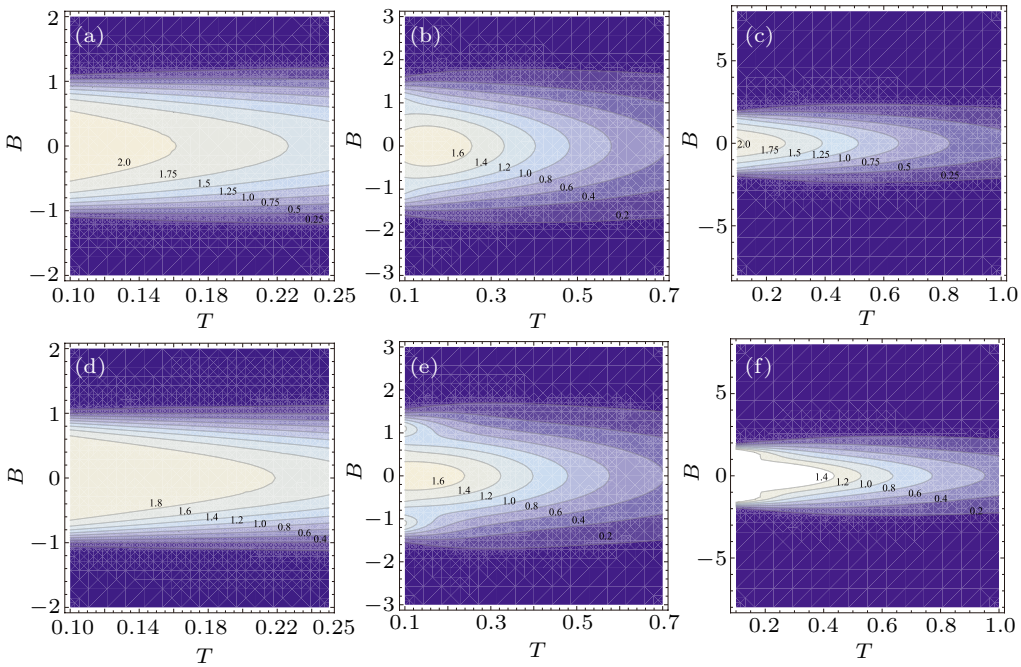


Fig. 3. (color online) Contour plots of (a)–(c) WYSIM $F(\rho)$ and (d)–(e) quantum mutual information $I(\rho)$ as functions of temperature T and external magnetic field B for different DM interactions, where (a) $D = 0.0001$, (b) $D = 0.4$, (c) $D = 0.8$, (d) $D = 0.0001$, (e) $D = 0.4$, and (f) $D = 0.8$. The spin–spin coupling is $J = 1$.

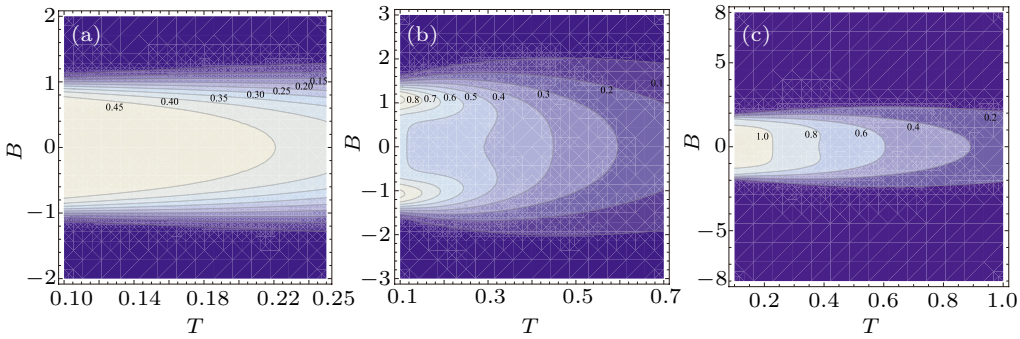


Fig. 4. (color online) Contour plots of MID $Q(\rho)$ as a function of temperature T and external magnetic field B for different DM interactions: (a) $D = 0.0001$, (b) $D = 0.4$, (c) $D = 0.8$. The spin–spin coupling is $J = 1$.

The contour plots of WYSIM, quantum mutual information, and MID as functions of the magnetic field and the DM interaction at three different temperatures are given in Fig. 5. It is found that the behaviors of WYSIM and quantum mutual information show the same characteristics. Therefore, only the contour plots of quantum mutual information are given. One should note that the quantum mutual information at an extremely low temperature is distinctly different from that at a relatively high temperature. At temperature $T = 0.6, 3.5$, it is increased by the DM interaction and decreased by the external magnetic field. However, at extremely low temperature $T = 0.05$, the quantum mutual information first decreases

slightly, increases subsequently, and in the end decays quickly with the increase of the external magnetic field for a small DM interaction. This can be explained by the variety of ground states dominating the thermal state. The ground states are $|\psi_9\rangle$, $|\psi_5\rangle$, and $|\psi_7\rangle$ in turn, and $I(|\psi_9\rangle) = 1.289$, $I(|\psi_5\rangle) = 2$, and $I(|\psi_7\rangle) = 0$ for the chosen parameters $J = 1$ and $D = 0.3$. At extremely low temperatures, the difference between MID and negativity is also obvious, which can be obtained by comparing Figs. 5(d) and 5(e) with Figs. 3(a) and 3(b) in Ref. [43]. At extremely low temperature $T = 0.05$, the MID firstly increases and then decays quickly with increasing the external magnetic field, however, the negativity decays firstly and then is increased by the external magnetic field. The results are also due to the variety of the ground state. At low temperature $T = 0.6$, the external magnetic field just causes the decay of the MID, while the negativity could be increased by the external magnetic field. For high temperature $T = 3.5$, the MID and the negativity behave similarly and both of them decrease monotonously with the increase of the external magnetic field.

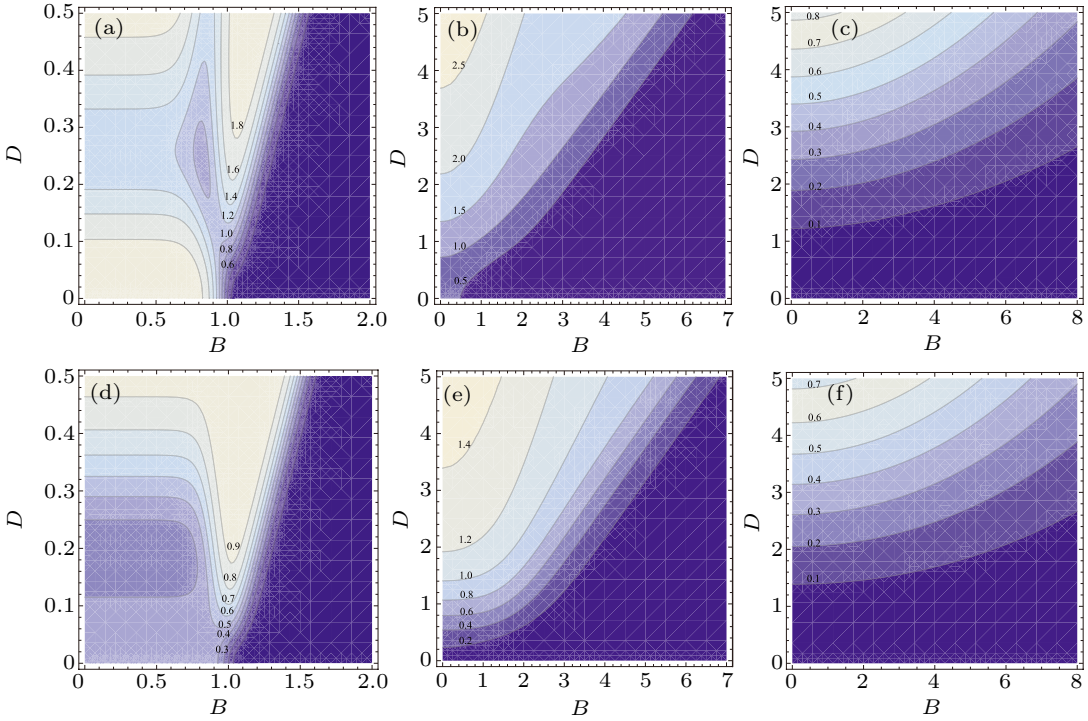


Fig. 5. (color online) Contour plots of (a)–(c) quantum mutual information $I(\rho)$ and (d)–(e) MID $Q(\rho)$ as functions of external magnetic field B and DM interaction, where (a) $T = 0.05$, (b) $T = 0.6$, (c) $T = 3.5$, (d) $T = 0.05$, (e) $T = 0.6$, and (f) $T = 3.5$. The spin–spin coupling is $J = 1$.

5. Conclusion

In conclusion, thermal quantum and total correlations in a two spin-1 Ising model with the DM interaction and an external homogeneous magnetic field are investigated. Here, the quantum correlations are measured by the measurement-induced disturbance and the negativity, while the total correlations are quantified by quantum mutual information and an alternative new measure defined in terms of the Wigner–Yanase skew information. The results indicate that for small DM interaction and low temperature, the quantum correlation measured by the MID and the entanglement quantified by the negativity present similar characteristics. However, for other cases, the MID and the entanglement show distinct differences. Unlike the entanglement on which the DM interaction and the external magnetic field play competing roles, the DM interaction plays a leading role on the MID. Therefore, in these cases, the external magnetic field should take a low value to obtain high correlations. The total thermal correlations measured by WYSIM and quantum mutual information always display the same characteristics except for the region with a small DM interaction and low temperature, where the quantum mutual information has two lobes. Because various systems, ranging from solid-state systems to optical lattices in the Mott regime, can be modeled after the Ising system, the results obtained here may contribute to realizing practical quantum information processing in these systems.

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