Statistical properties of coherent photon-subtracted two-mode squeezed vacuum and its application in quantum teleportation*

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We introduce a kind of non-Gaussian entangled state, which can be obtained by operating a non-local coherent photon-subtraction operation on a two-mode squeezed vacuum. It is found that its normalization factor is only related to the Legendre polynomials, which is a compact expression. Its statistical properties are discussed by the negative region Wigner function with the analytical expression. As an application, the quantum teleportation for coherent states is considered by using the non-Gaussian state as an entangled channel. It is found that the teleportation fidelity can be enhanced by this non-Gaussian operation.

Keywords: coherent photon subtraction, two-mode squeezed vacuum, quantum teleportation, fidelity

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1. Introduction

Quantum entanglement is a central resource in quantum information processing.[1] In order to realize a long-distance quantum communication, the higher entanglement degree of quantum states is required. For the continuous variable (CV) quantum systems, the two-mode squeezed vacuum state is a usual entangled resource, and it is often used to realize quantum teleportation. It is shown that the fidelity (F) teleporting coherent state increases with the increasing squeezing parameter (r) and arrives at an ideal value $F = 1$ for an infinite squeezing case $r \to \infty$. In fact, the value of the squeezing parameter that we can obtain experimentally is only a small one. Thus, how to enhance the fidelity of quantum teleportation on the basis of the existing experimental condition is still problematic.

Recently, non-Gaussian operations, such as photon subtraction a and addition $a^\dagger$, have been proposed to probe quantum commutation rules.[2] Many studies have shown that these non-Gaussian operations can also be used to generate highly nonclassical quantum states and can be applied to improve entanglement between Gaussian states.[3,4] For example, by combining the photon-subtraction-then-photon-addition and photon-addition-then-photon-subtraction, one can arrive the entanglement distillation.[5,6] Except for these local operations, non-local operations have also attracted much attention.[7–12] For instance, the coherent photon-added $(xa^\dagger - \mathrm{i}yb^\dagger)$ are operated on two single-mode squeezed vacuum states, and it is found that the wave-function of output state possesses a vortex structure.[11] In addition, the phase-sensitive nonclassical properties of photon-added-then-subtracted coherent squeezed states are discussed.[13]

In a preceding paper,[14] the non-Gaussian entangled state generated by any order nonlocal coherent-photon-addition to the two-mode squeezed vacuum (TMSV) is introduced and the statistical properties are discussed according to, especially, the negativity of its Wigner function (WF). In addition, quantum teleportation is examined and this shows that asymmetric photon-added TMSV may be more useful for teleportation than the symmetric case. On the other hand, we notice that the resulting states from successive photon additions and successive photon subtractions are different from each other. For example, for an initial squeezed vacuum state, successive two-photon additions and successive two-photon subtractions will result in the same state produced by subtraction–addition ($a^\dagger a$) and addition–subtraction ($aa^\dagger$).[15,16] Thus, as a natural extension of Ref. [14], in this paper we shall introduce a generalized coherent photon-subtracted (CPS) entangled state, which can be obtained theoretically by operating the asymmetrical superposition $(a\cos\varphi + b\sin\varphi)^m$ on the TMSV. We derive a compact expression for the normalization factor of the CPS-TMSV, which is an m-order Legendre polynomial of squeezing parameter r. The statistical properties are discussed by the negative region Wigner function with the analytical expression. We will then investigate the quantum teleportation (QT) by using the CPS-TMSV as entangled resource to teleport the coherent state. It is shown that the fidelity can be enhanced by even number m and modulated by angle φ, which is different from the case of Ref. [14].

This paper is arranged as follows. In Section 2, the CPS-TMSV is introduced and its normalization is also derived, which turns out to be an m-order Legendre polynomial of

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squeezing parameter r. In Section 3, the statistical properties of the CPS-TMSV are discussed according to the negativity of Wigner function (WF), after analytically deriving the expression of WF. Section 3 is devoted to present an interesting application of the CPS-TMSV in quantum teleportation for teleporting coherent state. Our conclusions are involved in the last section.

2. Coherent photon-subtracted two-mode squeezed vacuum and its normalization

In this section, we introduce a kind of two-mode non-Gaussian state by repeatedly operating the coherent photon-subtracted operator $a\cos\varphi + b\sin\varphi$ on a two-mode squeezed vacuum (TMSV) (i.e., the coherent photon-subtracted CPS-TMSV), which is defined as

$$|\Psi\rangle = N_{m,\varphi}(a\cos\varphi + b\sin\varphi)^m S(r)\,|00\rangle\,, \tag{1}$$

where m is a non-negative integer, $N_{m,\varphi}$ is the normalization constant of the CPS-TMSV to be determined by $\mathrm{Tr}\,|\Psi\rangle\langle\Psi| = 1$, $S(r) = \exp[r(a^\dagger b^\dagger - ab)]$ is the two-mode squeezing operator with r being squeezing parameter, and $a^\dagger(a)$ and $b^\dagger(b)$ are photon creation (annihilation) operators for modes 1 and 2. In fact, photon-subtracted operations on only single-mode can be considered as two special cases in Eq. (1) by taking $\varphi = 0$ and $\varphi = \pi/2$. The coherent photon subtraction has been used to increase entanglement between Gaussian states.[17] More details can be found in Ref. [18].

To clearly describe a quantum state, it is usually necessary to derive its normalization. Notice that the TMSV can be written as

$$S(r)\,|00\rangle = \mathrm{sech}r\exp(a^\dagger b^\dagger \tanh r)\,|00\rangle\,, \tag{2}$$

which is a useful entanglement resource in quantum information processing, such as quantum teleportation, quantum dense coding, and entanglement swapping. Thus, we can substitute Eq. (1) into the following form:

$$|\Psi\rangle = N_{m,\varphi}\mathrm{sech}r\,\frac{\partial^m}{\partial\tau^m}\,\mathrm{e}^{(a\cos\varphi+b\sin\varphi)\tau}\,\mathrm{e}^{a^\dagger b^\dagger\tanh r}\,|00\rangle\Big|_{\tau=0}. \tag{3}$$

By employing the operator formula $\mathrm{e}^{\lambda a}a^\dagger\,\mathrm{e}^{-\lambda a} = a^\dagger + \lambda$, one can rewrite Eq. (3) as

$$\begin{aligned}|\Psi\rangle &= N_{m,\varphi}\mathrm{sech}r\,\frac{\partial^m}{\partial\tau^m}\,\mathrm{e}^{(a^\dagger+\tau\cos\varphi)(b^\dagger+\tau\sin\varphi)\tanh r}\,|00\rangle\Big|_{\tau=0}\\ &= N_{m,\varphi}\mathrm{sech}r\frac{\partial^m}{\partial\tau^m}\exp\left[\frac{\tau^2}{2}\sin 2\varphi\tanh r\right]\\ &\quad\times\,\mathrm{e}^{(a^\dagger b^\dagger + a^\dagger\tau\sin\varphi + b^\dagger\tau\cos\varphi)\tanh r}\,|00\rangle_{\tau=0}\,.\end{aligned} \tag{4}$$

Next, let us examine the normalization of $|\Psi\rangle$. Using the normalization condition $\langle\Psi\,|\Psi\rangle = 1$, we have

$$\begin{aligned}1 &= N_{m,\varphi}^2\mathrm{sech}^2 r\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\,\mathrm{e}^{(\tau^2+s^2)\cos\varphi\sin\varphi\tanh r}\\ &\quad\times\,\langle 00|\,\mathrm{e}^{(ab+as\sin\varphi+bs\cos\varphi)\tanh r}\\ &\quad\times\,\mathrm{e}^{(a^\dagger b^\dagger + a^\dagger\tau\sin\varphi + b^\dagger\tau\cos\varphi)\tanh r}\,|00\rangle_{\tau,s=0}\,.\end{aligned} \tag{5}$$

Noticing the completeness relation of two-mode coherent states $1 = \int \mathrm{d}^2 z_1\mathrm{d}^2 z_2\,|z_1,z_2\rangle\langle z_1,z_2|/\pi^2$, ($|z_1,z_2\rangle = |z_1\rangle\otimes|z_2\rangle$ is the Glauber coherent state), and the integration formula

$$\int\frac{\mathrm{d}^2 z}{\pi}\,\mathrm{e}^{\zeta|z|^2+\xi z+\eta z^*} = -\frac{1}{\zeta}\,\mathrm{e}^{-\frac{\xi\eta}{\zeta}},\quad \mathrm{Re}\zeta < 0, \tag{6}$$

one can finally obtain

$$\begin{aligned}1 &= N_{m,\varphi}^2\mathrm{sech}^2 r\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\,\mathrm{e}^{(\tau^2+s^2)\cos\varphi\sin\varphi\tanh r}\\ &\quad\times\,\langle 00|\,\mathrm{e}^{(ab+as\sin\varphi+bs\cos\varphi)\tanh r}\\ &\quad\times\int\frac{\mathrm{d}^2 z_1\mathrm{d}z_2}{\pi^2}\,|z_1,z_2\rangle\langle z_1,z_2|\\ &\quad\times\,\mathrm{e}^{(a^\dagger b^\dagger + a^\dagger\tau\sin\varphi + b^\dagger\tau\cos\varphi)\tanh r}\,|00\rangle_{\tau,s=0}\\ &= N_{m,\varphi}^2\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\,\mathrm{e}^{(\tau^2+s^2)\cosh r\sinh r\cos\varphi\sin\varphi+s\tau\sinh^2 r}\Big|_{\tau,s=0}.\end{aligned} \tag{7}$$

Using the newly found formula for the Legendre polynomial[21]

$$\begin{aligned}&\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\,\exp(-s^2-\tau^2+\frac{2xs\tau}{\sqrt{x^2-1}})\Big|_{s,\tau=0}\\ &= \frac{2^m m!}{(x^2-1)^{m/2}}\mathrm{P}_m(x),\end{aligned} \tag{8}$$

where $\mathrm{P}_m(x)$ is the Legendre polynomial. One finally obtain the normalization factor as

$$N_{m,\varphi}^{-2} = m!\chi^m\sinh^{2m} r\mathrm{P}_m(1/\chi), \tag{9}$$

where we have set $\chi = \{1-(\sin 2\varphi\coth r)^2\}^{1/2}$. From Eq. (9), one can see that the normalization factor is not only related to the angle φ but also to the Legendre polynomials, which are important for further studying analytically the statistical properties of the CPS-TMSV. In particular, when $\varphi = 0, \chi\to 1, N_{m,0}^{-2} = m!\sinh^{2m} r$, which just reduces to the case of single-mode photon-subtracted two-mode squeezed vacuum;[19] while for $m = 0$ corresponding to the two-mode squeezed vacuum state, $N_{0,\varphi}^{-2} = 1$ as expected. In addition, when $m = 1$, note $\mathrm{P}_1(\chi) = \chi$, one can find $\chi\mathrm{P}_1(1/\chi) = 1$ and $N_{1,\varphi}^{-2} = \sinh^2 r$ independent of parameter φ.

3. Wigner function of the CPS-TMSV

In this section, we consider the statistical properties of the CPS-TMSV by analytically deriving the expression of Wigner function (WF). For the two-mode system ρ, the WF can be calculated by[20]

$$W(\alpha,\beta) = \mathrm{e}^{2(|\alpha|^2+|\beta|^2)}\int\frac{\mathrm{d}^2 z_1\,\mathrm{d}^2 z_2}{\pi^4}\,\langle -z_1,-z_2|\,\rho\,|z_1,z_2\rangle$$

$$\times \exp\left[2(\alpha z_1^* - \alpha^* z_1) + 2(\beta z_2^* - \beta^* z_2)\right], \quad (10)$$

where $|z_1, z_2\rangle = |z_1\rangle \otimes |z_2\rangle$ is the two-mode coherent state.

Using the differential expression of $|\Psi\rangle$ (Eq. (4)) and the integration formula (6), one can finally obtain (see Appendix A)

$$W(\alpha,\beta) = W_0(\alpha,\beta) F_{m,\varphi}(\alpha,\beta), \quad (11)$$

where $W_0(\alpha,\beta)$ is the WF of TMSV,

$$W_0(\alpha,\beta) = \frac{1}{\pi^2}\exp\Big\{-2(|\alpha|^2+|\beta|^2)\cosh 2r + 2(\alpha\beta+\alpha^*\beta^*)\sinh 2r\Big\}, \quad (12)$$

and $F_{m,\varphi}(\alpha,\beta)$ is an item resulting from coherent photon-subtraction,

$$F_{m,\varphi}(\alpha,\beta) = N_{m,\varphi}^2 \frac{\partial^{2m}}{\partial\tau^m \partial s^m}\exp\{A(s^2+\tau^2) - s\tau\sinh^2 r + 2s(\bar{\beta}-\bar{\alpha}^*) + 2\tau(\bar{\beta}^*-\bar{\alpha})\}_{s,\tau=0}, \quad (13)$$

where we have set

$$\begin{aligned} A &= \cos\varphi\sin\varphi\cosh r\sinh r,\\ \bar{\alpha} &= (\alpha\cos\varphi+\beta\sin\varphi)\sinh^2 r,\\ \bar{\alpha}^* &= (\alpha^*\cos\varphi+\beta^*\sin\varphi)\sinh^2 r,\\ \bar{\beta} &= (\beta\cos\varphi+\alpha\sin\varphi)\cosh r\sinh r,\\ \bar{\beta}^* &= (\beta^*\cos\varphi+\alpha^*\sin\varphi)\cosh r\sinh r. \end{aligned} \quad (14)$$

In order to obtain the clear expression of the WF, expanding the item $\exp\{s\tau\sinh^2 r\}$ and using the single-variable Hermite polynomial formula [21]

$$H_n(x) = \frac{\partial^n}{\partial t^n}\exp(2xt-t^2)\Big|_{t=0}, \quad (15)$$

which leads to

$$\begin{aligned} &\frac{\partial^n}{\partial t^n}\exp(A_1 t + B_1 t^2)\Big|_{t=0}\\ &= (\mathrm{i}\sqrt{B_1})^n H_n\left[A_1/(2\mathrm{i}\sqrt{B_1})\right]\\ &= (-\mathrm{i}\sqrt{B_1})^n H_n\left[A_1/(-2\mathrm{i}\sqrt{B_1})\right], \end{aligned} \quad (16)$$

as well as the recurrence relation of $H_n(x)$,

$$\frac{\mathrm{d}}{\mathrm{d}x^l}H_m(x) = \frac{2^l m!}{(m-l)!}H_{m-l}(x), \quad (17)$$

one can finally derive the analytical expression of $F_{m,\varphi}(\alpha,\beta)$ as

$$F_{m,\varphi}(\alpha,\beta) = \sum_{l=0}^{m}\frac{(m!)^2(-\sinh^2 r)^l (A)^{m-l}}{N_{m,\varphi}^{-2} l!\left[(m-l)!\right]^2} \times \left|H_{m-l}\left[(\bar{\beta}-\bar{\alpha}^*)/(\mathrm{i}\sqrt{A})\right]\right|^2. \quad (18)$$

Thus, the analytical WF of the CPS-TMSV is determined by Eqs. (11), (12), and (18).

In Figs. 1 and 2, by using Eqs. (11)–(18), we plot the graph of WF in the phase space for the squeezing parameter ($r = 0.3$) with different photon-subtracted numbers m and angle φ, (a) φ=0, (b) $\varphi = \pi/4$, (c) $\varphi = \pi/2$, and (d) $\varphi = 3\pi/4$, where $\alpha = (q_1 + \mathrm{i}p_1)/\sqrt{2}$, $\beta = (q_2 + \mathrm{i}p_2)/\sqrt{2}$ and $Q_\pm = (q_1 \pm q_2)/\sqrt{2}, \mathrm{P}_\pm = (p_1 \pm p_2)/\sqrt{2}$. From Figs. 1 and 2, we can see clearly that: (i) the WF of the CPS-MSV presents the x–y axis symmetry; (ii) there is some negative region of the WF in the phase space, which implies the nonclassicality of the CPS-TMSV; (iii) the negative region of the WF for odd number case of $m = 1$ is more obvious than that for the case of even number $m = 2$; (iv) the presence of negative region is modulated by angle φ, especially for the case of $m = 2$, and the WF presents a distortion distribution (non-Gaussian distribution) due to the present of the modulative φ; and, (v) it seems that there is a maximum negative region (a Gaussian-type distribution) when $\varphi = 0, \pi/4$ for $m = 1, 2,$, respectively. There is a rotation symmetry between the WFs in the (Q_+, P_+) and the (Q_-, P_-) phase space for the given same parameters.

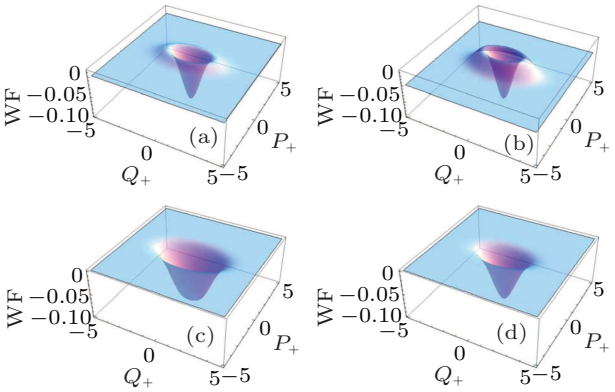


Fig. 1. (color online) The Wigner function $\mathrm{W}(\alpha,\beta)$ in phase space for several different parameter values φ with $r = 0.3$ and $m = 1$. (a) $\varphi = 0$; (b) $\varphi = \pi/4$; (c) $\varphi = \pi/2$, and (d) $\varphi = 3\pi/4$.

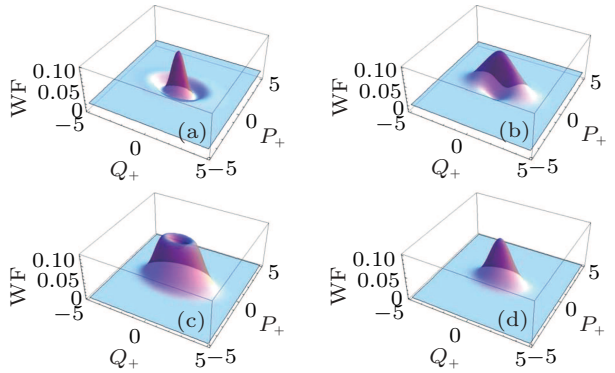


Fig. 2. (color online) The Wigner function $\mathrm{W}(\alpha,\beta)$ in phase space for several different parameter values φ with $r = 0.3$ and $m = 2$. (a) $\varphi = 0$, (b) $\varphi = \pi/4$, (c) $\varphi = \pi/2$, and (d) $\varphi = 3\pi/4$.

4. Application of the CPS-TMSV in quantum teleportation

In this section, we examine the application of the CPS-TMSV in quantum teleportation for teleporting the coherent state. Here we only consider the Braunstein and Kimble protocol[22] of quantum teleportation for single-mode coherent-input states $|z\rangle$. Note that the fidelity is independent of amplitude of the coherent state, so we can take $z=0$ for simplicity. Thus, we only need to calculate the fidelity of the vacuum input state with the CF $\chi_{\rm in}(\eta)=\exp[-|\eta|^2/2]$.

4.1. Characteristics function of $|\Psi\rangle$

First let us derive the characteristics function (CF) of $|\Psi\rangle$. For the two-mode system ρ, the CF is defined as $\chi(\alpha,\beta)={\rm Tr}[D_a(\alpha)D_b(\beta)\rho]$, where $D_{a,b}(\alpha)$ are the displacement operators. Using the displacement operator $D_a(\alpha)=\mathrm{e}^{|\alpha|^2/2}\mathrm{e}^{-\alpha^* a}\mathrm{e}^{\alpha a^\dagger}$ and in a similar way to deriving Eq. (11), one can obtain the CF of the CPS-TMSV (see Appendix B)

$$\chi(\alpha,\beta)=\chi_0(\alpha,\beta)\chi_m(\alpha,\beta), \tag{19}$$

where $\chi_0(\alpha,\beta)$ is just the CF of the TMSV,

$$\chi_0(\alpha,\beta)=\mathrm{e}^{-\frac{1}{2}(\alpha\alpha^*+\beta\beta^*)\cosh 2r+\frac{1}{2}(\alpha^*\beta^*+\alpha\beta)\sinh 2r}, \tag{20}$$

and $\chi_m(\alpha,\beta)$ is an item resulting from coherent photon-subtraction,

$$\begin{aligned}&\chi_m(\alpha,\beta)\\ &=N_{m,\varphi}^2\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\exp\left[A(s^2+\tau^2)+s\tau\sinh^2 r\right]\\ &\quad\times\exp\left\{\tau(\bar\alpha-\bar\beta^*)+s(\bar\beta-\bar\alpha^*)\right\}_{s=\tau=0}\\ &=N_{m,\varphi}^2\sum_{l=0}^{m}\frac{(m!)^2(-A)^{m-l}\sinh^{2l}r}{l!\left[(m-l)!\right]^2}\left|H_{m-l}\left[\gamma/(2\mathrm{i}\sqrt{A})\right]\right|^2,\end{aligned} \tag{21}$$

where $\bar\alpha,\bar\beta,A$ are defined in Eq. (14) and $\gamma=\bar\alpha-\bar\beta^*$. Equation (19) is just the CF of the CPS-TMSV, a non-Gaussian distribution due to the presence of Hermite polynomial in Eq. (21).

4.2. Fidelity of teleportation of coherent states

As mentioned above, photon subtraction from photon addition to bipartite Gaussian states can be used to improve the entanglement. Now, we consider the fidelity of teleportation of coherent states by using the CPS-TMSV as an entangled resource. For a continuous-variable (CV) system, a teleportation protocol has been given in terms of the CFs of the quantum states involved.[23] It is found that the output state CF $\chi_{\rm out}(\eta)$ has a remarkably factorized form $\chi_{\rm out}(\eta)=\chi_{\rm in}(\eta)\chi_{\rm E}(\eta^*,\eta)$, where $\chi_{\rm in}(\eta)$ and $\chi_{\rm E}(\eta^*,\eta)$ are the CFs of the input state and the entangled source, respectively. Then, the fidelity of quantum teleportation of CVs is given by[23]

$$\begin{aligned}\mathscr{F}&=\int\frac{\mathrm{d}^2\eta}{\pi}\chi_{\rm in}(\eta)\chi_{\rm out}(-\eta)\\ &=\int\frac{\mathrm{d}^2\eta}{\pi}|\chi_{\rm in}(\eta)|^2\chi_{\rm E}(-\eta^*,-\eta).\end{aligned} \tag{22}$$

Upon substituting these CFs $\chi_{\rm in}(\eta)=\exp[-|\eta|^2/2]$ and Eq. (19) into Eq. (22), we can derive the fidelity for teleporting a coherent state by using the CPS-TMSV as an entangled resource,

$$\mathscr{F}=\frac{N_{m,\varphi}^2}{\sigma}\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\left.\mathrm{e}^{2s\tau B_1+(s^2+\tau^2)B_2}\right|_{s=\tau=0}, \tag{23}$$

where we have set

$$\begin{aligned}\sigma&=2/[1+\tanh r],\\ B_1&=(\sinh r)^2/(2\sigma),\\ B_2&=\frac{\sinh 2r-\sinh^2 r}{2\sigma}\sin 2\varphi.\end{aligned} \tag{24}$$

Using formula (8), we can re-express Eq. (23) as

$$\mathscr{F}=\frac{2^m m!}{\sigma N_{m,\varphi}^{-2}}D^{m/2}\mathrm{P}_m(B_1/\sqrt{D}), \tag{25}$$

where $D=B_1^2-B_2^2$, and $\mathrm{P}_m(-\omega)$ is the Legendre polynomials which are symmetric or antisymmetric; that is, $\mathrm{P}_m(-\omega)=(-1)^m\mathrm{P}_m(\omega)$. From Eq. (25) one can see that the fidelity of teleportation of coherent states by the CPS-TMSV is related to the Legendre polynomials, which is a concise result. It also can be seen that the fidelity is not only dependent on the parameter r, the coherent angle φ, but also on the order m. In particular, when $m=0$, note $\mathrm{P}_0(\omega)=1$, thus

$$\mathscr{F}_0=(1+\tanh r)/2,\ 1/2\leqslant\mathscr{F}_0<1, \tag{26}$$

which is just the fidelity of coherent states by the TMSV.[24] While for $m=1$, note $\mathrm{P}_1(\omega)=\omega$ and $N_{1,\varphi}^{-2}=\sinh^2 r$, then

$$\mathscr{F}_{1,\varphi}=\frac{2B_1}{\sigma\sinh^2 r}=(\mathscr{F}_0)^2, \tag{27}$$

which is independent of the angle φ. It is obvious that $\mathscr{F}_0<1$, thus $\mathscr{F}_{1,\varphi}<\mathscr{F}_0$, which indicates that the nonlinear process of subtracting correlated photons cannot improve the fidelity of the teleporting coherent states by the CPS-TMSV channel,[19] which is independent of the coherent angle φ. Generally, when $\varphi=0$, equation (25) is reduced to $\mathscr{F}_{m,0}=[(1+\tanh r)/2]^{m+1}$, which is in agreement with Eqs. (54) and (55) in Ref. [24]. In Fig. 3, for some different (m,r) values, the difference fidelity ($\Delta F=\mathscr{F}-\mathscr{F}_0$) of teleporting the coherent state is shown as a function of φ by using the CPS-TMSV as the entangled resource. It is found that: (i) for even number m, the fidelity with this resource can be greater than that with the TMSV with a changing φ, while for odd number m, it is impossible to obtain an enhanced fidelity; (ii) for the cases of $\varphi=0,\pi$, corresponding to single-mode photon-subtraction TMSV (i.e., an asymmetrical subtraction case), the fidelity also cannot be enhanced

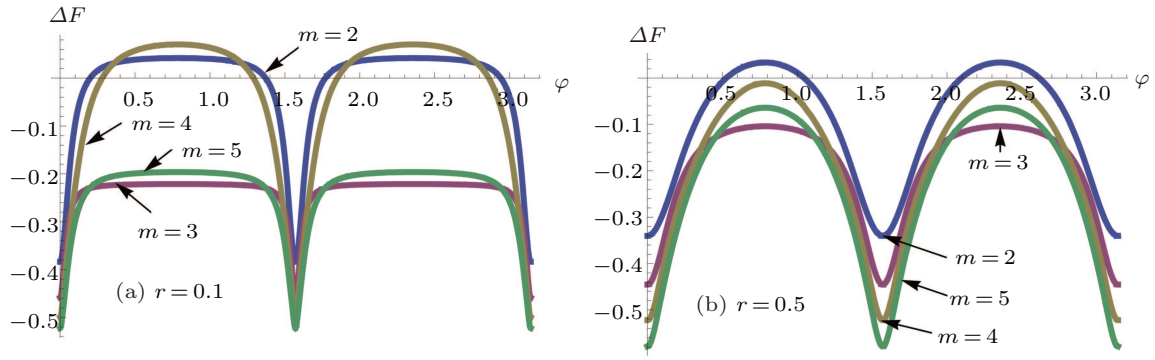


Fig. 3. (color online) The difference fidelity of teleportation of coherent states using the CPS-TMSV as an entanglement resource, with (a) and (b) corresponding to $r = 0.1, 0.5$, respectively.

for any squeezing parameter and any coherent-angle φ; (iii) the fidelity $\mathscr{F}$ has its maximum value at about $\varphi = \pi/4, 3\pi/4$; and, (iv) for a given value of r, the fidelity for $m = 5$ is bigger than that for $m = 3$, which increases with r (see Figs. 3(a) and 3(b)). In a small squeezing region, there is a greater maximum value for $m = 4$ than that for $m = 2$ (see Figs. 3(a) and 3(b)). However, the case is not true when the squeezing parameter r exceeds a threshold value that is modulated by angle φ and parameter m. In order to clearly see this point, we plot the difference fidelity between the case $m = 2$ and the case of $m = 4$ in Fig. 4.

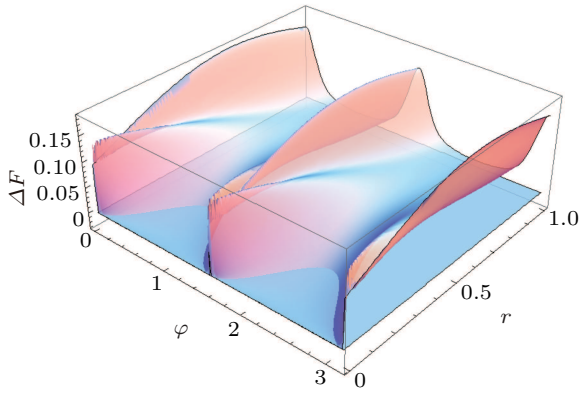


Fig. 4. (color online) Difference fidelity of teleportation of coherent states using the CPS-TMSV as an entanglement resource, with $m = 2, 4$ respectively.

5. Conclusions and discussion

In summary, we have proposed a kind of non-Gaussian state, the coherent photon-subtracted two-mode squeezed vacuum (CPS-TMSV). We derived a compact expression for the normalization factor of the CPS-TMSV, which is an m-order Legendre polynomial of squeezing parameter r. Then, we discussed its statistical properties by deriving the analytical expression of Wigner function, whose negative region can be considered as an indicator of nonclassicality. It is shown that there is more obvious non-classicality for $m = 1$ than that for $m = 2$, which is modulated by the coherent angle φ, especially for $m = 2$. Furthermore, we have also presented its application in quantum teleportation by using the CPS-TMSV as the entanglement resource for transporting the coherent state. It is found that it is possible to enhance the fidelity for even number m while it is impossible for odd number m,, and the fidelity is modulated by the angle φ. In particular, for cases of $\varphi = 0, \pi$, corresponding to single-mode photon-subtraction TMSV, the fidelity cannot be enhanced for any squeezing parameter or any coherent-angle φ. In addition, there may be a bigger fidelity for $m = 4$ in a small squeezing region than that for $m = 2$. However, the case is not true when the squeezing parameter r exceeds a threshold value, which is modulated by angle φ and parameter m. By comparing the nonclassicality and the fidelity, one can see that there is obvious nonclassicality for $m = 1$, while the fidelity cannot be enhanced for $m = 1$. Thus, there is no necessary relation between nonclassicality and fidelity, like that between entanglement and fidelity.

Appendix A: the Wigner function of $|\Psi\rangle$

Using Eqs. (4) and (10), the Wigner function of $|\Psi\rangle$ can be calculated as

$$
\begin{aligned}
W(\alpha,\beta) &= N_{m,\varphi}^2 \mathrm{e}^{2(|\alpha|^2+|\beta|^2)} \mathrm{sech}^2 r \frac{\partial^m}{\partial \tau^m} \frac{\partial^m}{\partial s^m} \mathrm{e}^{(\frac{\tau^2}{2}+\frac{s^2}{2})\sin 2\varphi \tanh r} \int \frac{\mathrm{d}^2 z_1 \mathrm{d} z_2}{\pi^4} \mathrm{e}^{2(\alpha z_1^* - \alpha^* z_1)+2(\beta z_2^* - \beta^* z_2)} \\
&\times \langle -z_1, -z_2 | \, \mathrm{e}^{(a^\dagger b^\dagger + a^\dagger \tau \sin\varphi + b^\dagger \tau \cos\varphi)\tanh r} |00\rangle \langle 00| \, \mathrm{e}^{(ab + as\sin\varphi + bs\cos\varphi)\tanh r} | z_1, z_2 \rangle_{\tau,s=0} \\
&= \frac{N_{m,\varphi}^2}{\pi^2} \mathrm{e}^{-2(|\alpha|^2+|\beta|^2)\cosh 2r + 2(\alpha\beta + \alpha^*\beta^*)\sinh 2r} \frac{\partial^m}{\partial \tau^m} \frac{\partial^m}{\partial s^m} \mathrm{e}^{-s\tau \sinh^2 r + \frac{s^2+\tau^2}{4}\sin 2\varphi \sinh 2r} \\
&\times \mathrm{e}^{s\left[(2\alpha \sin\varphi + 2\beta\cos\varphi)\sinh r \cosh r - (2\alpha^* \cos\varphi + 2\beta^* \sin\varphi)\sinh^2 r\right]} \\
&\times \mathrm{e}^{\tau\left[(2\alpha^* \sin\varphi + 2\beta^* \cos\varphi)\sinh r \cosh r - (2\alpha \cos\varphi + 2\beta \sin\varphi)\sinh^2 r\right]} \Big|_{\tau,s=0}. \qquad \text{(A1)}
\end{aligned}
$$

Note Eq. (12), we can rewrite Eq. (A1) as Eq. (11).

Appendix B: the Characteristics function of $|\Psi\rangle$

Using Eq. (4), the CF of the CPS-TMSV can be calculated as

$$\begin{aligned}
&\chi(\alpha,\beta)\\
&= \mathrm{Tr}\left[D_a(\alpha)D_b(\beta)\left|\Psi\right\rangle\left\langle\Psi\right|\right]\\
&= \mathrm{e}^{(|\alpha|^2+|\beta|^2)/2}\left\langle\Psi\right|\mathrm{e}^{-\alpha^* a}\mathrm{e}^{-\beta^* b}\mathrm{e}^{\alpha a^\dagger}\mathrm{e}^{\beta b^\dagger}\left|\Psi\right\rangle\\
&= N_{m,\varphi}^2\,\mathrm{sech}^2 r\,\mathrm{e}^{(|\alpha|^2+|\beta|^2)/2}\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\mathrm{e}^{\frac{s^2+\tau^2}{2}\sin 2\varphi\tanh r}\\
&\quad\times\left\langle 00\right|\mathrm{e}^{(ab+as\sin\varphi+bs\cos\varphi)\tanh r}\mathrm{e}^{-\alpha^* a}\mathrm{e}^{-\beta^* b}\\
&\quad\times\mathrm{e}^{\alpha a^\dagger}\mathrm{e}^{\beta b^\dagger}\mathrm{e}^{(a^\dagger b^\dagger+a^\dagger\tau\sin\varphi+b^\dagger\tau\cos\varphi)\tanh r}\left|00\right\rangle_{\tau,s=0}.
\end{aligned}\tag{B1}$$

Employing the completeness relation of coherent states $1=\int^2 \mathrm{d}z_1^2\mathrm{d}z_2\left|z_1,z_2\right\rangle\left\langle z_1,z_2\right|/\pi^2$, we can obtain

$$\begin{aligned}
&\chi(\alpha,\beta)\\
&= N_{m,\varphi}^2\,\mathrm{sech}^2 r\,\mathrm{e}^{(|\alpha|^2+|\beta|^2)/2}\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\mathrm{e}^{\frac{s^2+\tau^2}{2}\sin 2\varphi\tanh r}\\
&\quad\times\int\frac{\mathrm{d}^2 z_1^2\mathrm{d}z_2}{\pi^2}\mathrm{e}^{-|z_1|^2+z_1(s\sin\varphi\tanh r-\alpha^*+z_2\tanh r)}\\
&\quad\times\mathrm{e}^{z_1^*(\alpha+\tau\sin\varphi\tanh r+z_2^*\tanh r)}\\
&\quad\times\left.\mathrm{e}^{-|z_2|^2+z_2(s\cos\varphi\tanh r-\beta^*)+z_2^*(\tau\cos\varphi\tanh r+\beta)}\right|_{\tau,s=0}.
\end{aligned}\tag{B2}$$

Using the above integration again, we can obtain

$$\begin{aligned}
&\chi(\alpha,\beta)\\
&= N_{m,\varphi}^2\,\mathrm{e}^{(|\alpha|^2+|\beta|^2)/2}\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\mathrm{e}^{\frac{s^2+\tau^2}{2}\sin 2\varphi\tanh r}\\
&\quad\times\mathrm{e}^{(s\sin\varphi\tanh r-\alpha^*)(\alpha+\tau\sin\varphi\tanh r)}\\
&\quad\times\exp\{[s\cos\varphi\tanh r-\beta^*+(\alpha+\tau\sin\varphi\tanh r)\tanh r]\\
&\quad\times[\tau\cos\varphi\tanh r+\beta\\
&\quad+(s\sin\varphi\tanh r-\alpha^*)\tanh r]\cosh^2 r\}_{\tau,s=0}.
\end{aligned}\tag{B3}$$

Simplifying the integrated results, we see

$$\chi(\alpha,\beta)=\chi_0(\alpha,\beta)\chi_m(\alpha,\beta),\tag{B4}$$

where $\bar{\alpha}$, $\bar{\beta}$, and A are defined in Eq. (14), and $\chi_0(\alpha,\beta)$ is defined in Eq. (20).

Expanding the item $\exp(s\tau\sinh^2 r)$, we see

$$\begin{aligned}
\chi_m(\alpha,\beta) &= N_{m,\varphi}^2\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\sum_{l=0}^{\infty}\frac{\sinh^{2l} r}{l!}s^l\tau^l,\\
&\quad\exp\left\{A(s^2+\tau^2)+\tau\gamma-\gamma^* s\right\}\\
&= N_{m,\varphi}^2\sum_{l=0}^{\infty}\frac{\sinh^{2l} r}{l!}\frac{\partial^{2l}}{\partial\gamma^l\partial(-\gamma^*)^l}\\
&\quad\times\frac{\partial^{2m}}{\partial\tau^m\partial s^m}\exp\left\{A(s^2+\tau^2)+\tau\gamma-\gamma^* s\right\}_{s=\tau=0},
\end{aligned}\tag{B5}$$

where $\gamma=\bar{\alpha}-\bar{\beta}^*,-\gamma^*=\bar{\beta}-\bar{\alpha}^*$. Using the single-variable Hermite polynomial formula (15), we can have

$$\begin{aligned}
\chi_m(\alpha,\beta) &= N_{m,\varphi}^2\sum_{l=0}^{\infty}\frac{\sinh^{2l} r}{l!}\\
&\quad\times\frac{\partial^l}{\partial\gamma^l}(\mathrm{i}\sqrt{A})^m H_m\left[\gamma/(2\mathrm{i}\sqrt{A})\right]\\
&\quad\times\frac{\partial^l}{\partial(-\gamma^*)^l}(\mathrm{i}\sqrt{A})^m H_m\left[-\gamma^*/(2\mathrm{i}\sqrt{A})\right].
\end{aligned}\tag{B6}$$

Further using the recurrence relation of $H_n(x)$ (Eq. (17)), we can derive Eq. (21).

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