A new coupled map car-following model considering drivers' steady desired speed*

Zhou Tong(周 桐)[a)b)], Sun Di-Hua(孙棣华)[a)b)†], Li Hua-Min(李华民)[a)b)], and Liu Wei-Ning(刘卫宁)[b)c)]

[a)] *College of Automation, Chongqing University, Chongqing* 400030, *China*
[b)] *Key Laboratory of Dependable Service Computing in Cyber Physical Society of Ministry of Education, Chongqing University, Chongqing* 400044, *China*
[c)] *College of Computer Science, Chongqing University, Chongqing* 400030, *China*

(Received 11 September 2013; revised manuscript received 28 October 2013; published online 25 March 2014)

Based on the pioneering work of Konishi *et al.*, in consideration of the influence of drivers' steady desired speed effect on the traffic flow, we develop a new coupled map car-following model in the real world. By use of the control theory, the stability condition of our model is derived. The validity of the present theoretical scheme is verified via numerical simulation, confirming the correctness of our theoretical analysis.

Keywords: traffic flow, traffic congestion, coupled map car-following model, steady desired speed effect

PACS: 02.60.Cb, 05.20.Dd, 05.45.Ra, 89.04.−a **DOI:** 10.1088/1674-1056/23/5/050203

1. Introduction

Traffic congestion is an important issue for modern traffic, which has attracted considerable attention recently. In order to understand the mechanism and properties of traffic congestion, many traffic flow models have been developed. For instance, in the macroscopic models, the traffic is viewed as a compressible fluid formed by vehicles; in the microscopic models, each individual vehicle is represented by a particle and the vehicle traffic is treated as a system of interacting particles driven far from equilibrium.[1] Among these models, the current car-following model is an effective method to study the microscopic traffic flow. Hereafter, many achievements have been obtained based on the car-following theory.[2–19] It is worth pointing out in all those achievements that Konishi *et al.*[11] proposed a coupled map (CM) car-following model (the KKH model, for short) to describe the dynamical behavior of an open flow, and a decentralized delayed-feedback control scheme was introduced in the model for the suppression of traffic jams. In 2006, Zhao and Gao[12] presented a simple congestion control method in the system (known as the ZG model), which incorporated the effects of the velocity difference between two successive vehicles. In 2007, Han *et al.*[13] presented a modified CM car-following model, in which the control signal was proportional to the velocity difference between the successive vehicles of an arbitrary number of preceding cars. In 2010, Yu *et al.*[14] developed a new control signal in the CM car-following model, which was intended to consider the relative velocity between the considering car and its following one. In 2012, Ge *et al.*[15–17] improved the OV function in the CM car-following model by introducing the headway information of two successive vehicles ahead, and the effects of vehicles (or non-motor vehicles) on other lanes without isolation belts, respectively. In 2013, Zhou *et al.*[18,19] developed a new control method in the CM car-following model by considering the driver's safe headway effect on the traffic system.

The CM car-following models mentioned above can successfully enhance the stability of traffic flow to suppress traffic jams, but cannot be employed to study the stability performance of driver's steady desired speed effect (DSDSE) on the traffic system. In reality, when a driver adjusts his/her speed, he/she first observes traffic conditions by using transportation cyber physical systems (TCPS),[20] then controls the vehicle timely to avoid running into the leading car, and simultaneously drives the vehicle with a steady desired speed without frequent acceleration and deceleration. Therefore, the DSDSE can influence the stability of traffic flow. However, there are few studies on the DSDSE in the CM car-following models.

Based on the previous work, we propose a new CM car-following model in consideration of the DSDSE. This paper consists of five sections: Section 2 introduces a new CM car following model; Section 3 provides the stability analysis of the model; Section 4 presents simulation results to verify the theoretical analysis; and Section 5 summarizes the paper.

*Project supported by the Major Consulting Project of Chinese Academy of Engineering (Grant No. 2012-ZX-22), the National Natural Science Foundation of China (Grant No. 71201178), the Natural Science Foundation of Chongqing City, China (Grant No. cstc2012jjB40002), the Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20120191110047), the Engineering Center Research Program of Chongqing City, China (Grant No. cstc2011pt-gc30005), and the Key Technology R&D Project of Chongqing City, China (Grant Nos. cstc2011AB2052 and cstc2012gg-yyjsB30001).

†Corresponding author. E-mail: d3sun@163.com

© 2014 Chinese Physical Society and IOP Publishing Ltd

2. Models

In 1999, Konishi *et al.* proposed the KKH model[11] to suppress the traffic jam. The motion equation is given as follows:

$$\begin{aligned} v_i(n+1) &= \alpha_i[V_i^{\mathrm{OP}}(y_i(n)) - v_i(n)]T + v_i(n) + u_i(n), \\ y_i(n+1) &= v_i T - v_i(n)T + y_i(n), \end{aligned} \tag{1}$$

where $v_i(n)$ and $y_i(n)$ are respectively the velocity and headway of the i-th car at time $t = nT$; T is the sampling interval; α_i is the sensitivity of a driver; $V_i^{\mathrm{OP}}(\cdot)$ is the optimal velocity function; and $u_i(n)$ is the delayed feedback control scheme in the KKH model

$$\begin{aligned} u_i(n) &= k_i^{\mathrm{c}} w_i(n) + k_i^{\mathrm{d}}[v_i(n) - v_i(n-1)], \\ w_i(n+1) &= k_i^{\mathrm{a}} w_i(n) + k_i^{\mathrm{b}}[v_i(n) - v_i(n-1)], \end{aligned} \tag{2}$$

where k_i^{a}, k_i^{b}, k_i^{c}, $k_i^{\mathrm{d}} \in \mathbb{R}$ are the feedback gains. The control scheme in the KKH model has a complex form, and the feedback gains are difficult to design.

In order to overcome the deficiency of the KKH model, Zhao and Gao[12] proposed an improved model, i.e.,

$$\begin{aligned} v_i(n+1) &= \alpha_i[V_i^{\mathrm{OP}}(y_i(n)) - v_i(n)]T + v_i(n) + k\Delta v_i(n), \\ y_i(n+1) &= v_i T - v_i(n)T + y_i(n), \end{aligned} \tag{3}$$

where k is the feedback gain, and $\Delta v_i(n) = v_{i-1}(n) - v_i(n)$ is the velocity difference between the n-th considering vehicle and the preceding vehicle. The results illustrate that the ZG model has a simpler form, and its suppression performance of traffic jam is better than that of the KKH model. When $k = 0.85$, the optimal congestion suppression performance of the ZG model can be achieved.

The aforementioned models can improve the stability of traffic flow. However, these models are unable to study the DSDSE, since they do not consider it at all. In a real traffic system, the driver adjusts his speed to reach the steady desired speed to keep his own car running in a steady state. Meanwhile, when the desired speed of the driver is close to the steady speed of traffic flow, the traffic flow can keep stable; otherwise, when there is a big difference between them, the traffic flow may be unstable. For the above reason, we develop a new CM car-following model which incorporates the DSDSE, whose dynamics equation reads

$$\begin{aligned} v_i(n+1) &= \alpha_i\left[V_i^{\mathrm{OP}}(y_i(n)) - v_i(n)\right]T + v_i(n) \\ &\quad + 0.85v_i(n) - k(v_i(n) - v_0), \\ y_i(n+1) &= v_i T - v_i(n)T + y_i(n), \end{aligned} \tag{4}$$

where k is the feedback gain, $v_i(n) - v_0$ represents the adjustment term of the steady desired speed, and v_0 is the steady speed of traffic flow. In real traffic flow, each vehicle has its own safe headway η, and adjusts its velocity so as to fit η. Once $y_i(n)$ reaches η, it fluctuates around η. For simplicity, when the current $y_i(n)$ is far from η, we assume there will be constant acceleration or constant deceleration. Therefore, we adopt the following optimal velocity function:[12]

$$V_i^{\mathrm{OP}}(y_i(n)) = \frac{v_i^{\max}}{2}\left[1 + \bar{H}_{\mathrm{sat}}\left(2\frac{y_i(n) - \eta}{\varsigma}\right)\right], \tag{5}$$

where $v_i^{\max}$ is the maximum speed, ζ is a parameter, and the saturation function $\bar{H}_{\mathrm{sat}}(\cdot)$ is described as

$$\bar{H}_{\mathrm{sat}} = \begin{cases} +1, & \text{if } \rho > +1; \\ \rho, & \text{if } -1 \le \rho \le +1; \\ -1, & \text{if } \rho < -1. \end{cases} \tag{6}$$

We can ascertain that the essential condition for the existence of the steady state is $v_0 < v_i^{\max}$ to guarantee the existence of traffic flow. To avoid collisions and backward motions, we further assume that the i-th vehicle stops suddenly when the headway $y_i(n)$ is less than a minimum headway $y^{\min}$.

3. Stability analysis

Following the stability analysis method proposed by Konishi *et al.*,[11] we assume that the leading vehicle runs constantly with speed v_0, and then the vehicles have the following steady states:

$$[v_i^*, y_i^*]^{\mathrm{T}} = \left[v_0, \frac{v_0}{r_i} - \frac{\zeta_i}{2} + \eta_i\right]^{\mathrm{T}}. \tag{7}$$

The traffic system (4) can be linearized at the steady state (7), that is,

$$\begin{aligned} \delta v_i(n+1) &= \alpha_i r_i T \delta y_i(n) - \alpha_i T \delta v_i(n) + \delta v_i(n) \\ &\quad + 0.85(\delta v_{i-1}(n) - \delta v_i(n)) - k\delta v_i(n), \\ \delta y_i(n+1) &= \delta v_{i-1}(n)T - \delta v_i(n)T + \delta y_i(n), \end{aligned} \tag{8}$$

where $\delta v_i(n) = v_i(n) - v_i^*$, and $\delta y_i(n) = y_i(n) - y_i^*$.

After Laplace transformation, we have

$$\begin{aligned} G_i(z) &= (1 \;\; 0)\begin{pmatrix} z - 0.15 + \alpha_i T + k & -\alpha_i r_i T \\ T & z - 1 \end{pmatrix}^{-1}\begin{pmatrix} 0.85 \\ T \end{pmatrix} \\ &= \frac{0.85(z-1) + \alpha_i r_i T^2}{P_i(z)}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} &P_i(z) = z^2 + a_i z + b_i, \quad a_i = \alpha_i T + k - 1.15, \\ &b_i = 0.15 - \alpha_i T - k + \alpha_i r_i T^2. \end{aligned}$$

According to our analysis, the traffic jam will never occur in the traffic system if $P_i(z)$ is stable and $||G_i(z)||_\infty \le 1$. Then we give a sufficient and necessary condition of no traffic congestion.

Theorem 1 There is no traffic jam in the controlled system (4) in the following five cases.

(i) If

$$0<\alpha_i r_i T^2<2,\quad \alpha_i^2T^2-\alpha_i^2 r_i T^3+1.7\alpha_i T>0, \tag{10}$$

then the feedback gain k is set as

$$\begin{aligned}&\max\{0.15-\alpha_i T+\alpha_i r_i T^2, R_{11}\}\\&<k<\min\left\{\frac{2.3-2\alpha_i T+\alpha_i r_i T^2}{2}, R_{12}\right\},\end{aligned} \tag{11}$$

where

$$\begin{aligned}R_{11}&=\frac{-1.7-2\alpha_i T+\alpha_i r_i T^2-\sqrt{\Delta_1}}{2},\\R_{12}&=\frac{-1.7-2\alpha_i T+\alpha_i r_i T^2+\sqrt{\Delta_1}}{2},\\\Delta_1&=(1.7+2\alpha_i T-\alpha_i r_i T^2)^2\\&\quad-4(\alpha_i^2T^2-\alpha_i^2 r_i T^3+1.7\alpha_i T).\end{aligned} \tag{12}$$

(ii) If

$$0<\alpha_i r_i T^2<2,\quad \alpha_i^2T^2-\alpha_i^2 r_i T^3+1.7\alpha_i T<0, \tag{13}$$

then the feedback gain k is set as

$$0.15-\alpha_i T+\alpha_i r_i T^2<k<\min\left\{\frac{2.3-2\alpha_i T+\alpha_i r_i T^2}{2}, R_{12}\right\}. \tag{14}$$

(iii) If

$$\begin{aligned}&0<\alpha_i r_i T^2<4,\\&0.6+\alpha_i^2T^2-\alpha_i^2 r_i T^3-2.3\alpha_i T+2\alpha_i r_i T^2>0,\end{aligned} \tag{15}$$

then the feedback gain k is set as

$$\begin{aligned}&\max\{-\alpha_i T+\alpha_i r_i T^2, R_{21}\}<k\\&<\min\left\{0.15-\alpha_i T+\alpha_i r_i T^2, \frac{2.3-2\alpha_i T+\alpha_i r_i T^2}{2}, R_{22}\right\},\end{aligned} \tag{16}$$

$$\begin{aligned}R_{21}&=\frac{2.3-2\alpha_i T+\alpha_i r_i T^2-\sqrt{\Delta_2}}{2},\\R_{22}&=\frac{2.3-2\alpha_i T+\alpha_i r_i T^2+\sqrt{\Delta_2}}{2},\\\Delta_2&=(-2.3+2\alpha_i T-\alpha_i r_i T^2)^2\\&\quad-4(0.6+\alpha_i^2T^2-\alpha_i^2 r_i T^3-2.3\alpha_i T+2\alpha_i r_i T^2).\end{aligned} \tag{17}$$

(iv) If

$$\begin{aligned}&0<\alpha_i r_i T^2<4,\\&0.6+\alpha_i^2T^2-\alpha_i^2 r_i T^3-2.3\alpha_i T+2\alpha_i r_i T^2<0,\end{aligned} \tag{18}$$

then the feedback gain k is set as

$$\begin{aligned}&-\alpha_i T+\alpha_i r_i T^2<k\\&<\min\left\{0.15-\alpha_i T+\alpha_i r_i T^2, \frac{2.3-2\alpha_i T+\alpha_i r_i T^2}{2}, R_{22}\right\}.\end{aligned} \tag{19}$$

(v) If

$$0<\alpha_i r_i T^2<2,\quad 0.3+\alpha_i^2T^2-\alpha_i^2 r_i T^3-0.3\alpha_i T>0, \tag{20}$$

then the feedback gain k is set as

$$\begin{aligned}&k=0.15-\alpha_i T+\alpha_i r_i T^2,\\&-\alpha_i T+\alpha_i r_i T^2<k<\frac{2.3-2\alpha_i T+\alpha_i r_i T^2}{2}.\end{aligned} \tag{21}$$

This theorem provides us with the ability to design the feedback gain k to suppress the traffic jam in our model. From the theorem we can derive stability conditions to design the feedback gain, and then the traffic jam would never occur in the controlled vehicle group.

4. Simulation and comparison

In the simulations, the parameters of our model are set as[11]

$$\begin{aligned}&\eta=25.0\ \text{m},\ \ \zeta=23.3\ \text{m},\ \ v^{\max}=33.6\ \text{m/s},\ \ \alpha_i=2.0\ \text{s}^{-1},\\&v_0=20\ \text{m/s},\ \ T=0.1\ \text{s},\ \ y^{\min}=7.02\ \text{m},\\&r=v^{\max}/\zeta\approx 1.44\ \text{s}^{-1}.\end{aligned}$$

It is assumed that all vehicles have the same parameters. The initial positions and speeds are set as

$$x_i(0)=\sum_{j=i+1}^{N} y_j^*,\ \ y_j(0)=y_j^*,\ \ v_i(0)=v_i^*,\ \ i=1,2,\ldots,N.$$

We consider a situation where the leading vehicle suddenly stops twice within a short time, i.e., $v_0(n)=0$, $nT=100$–102 s, 106–108 s.[11]

The same external disturbance is used to examine the stability performance of three models on traffic flow. Let us set the feedback gain k in our model. In order to suppress a traffic jam, the feedback signal is designated as follows: it is found that these parameters fit three conditions (i), (iv), and (v). Substituting the parameters into Eqs. (11), (19), and (21), we obtain $0<k<0.181$. The feedback gain is fixed at $k=0.08$. In the KKH model, the procedure to design four feedback gains can be found in Ref. [11].

The simulation results of the KKH model, the ZG model, and our model are shown in Figs. 1–3, respectively. All the sub-figures (a) show the space-time plot of the traffic flow after $nT=90$ s. All the sub-figures (b) illustrate the variations of velocities of the first, 25-th, and 50-th vehicles. Figures 1(a), 2(a), and 3(a) illustrate that when the stability conditions are satisfied respectively for the three CM car-following models, the vehicle groups can adjust themselves to the steady state gradually even if the external disturbance appears in the beginning. However, it should be noted that the degree of headway fluctuation is smaller in Fig. 3(a) compared with that in Figs. 1(a) and 2(a). From Figs. 1(b), 2(b), and 3(b), it is obvious that as vehicle number i increases, the amplitude of speed fluctuation decreases gradually. However, the smoothness of vehicles' speeds and the amplitude of speed fluctuation in our

model are smaller than those of the other two controlled models.

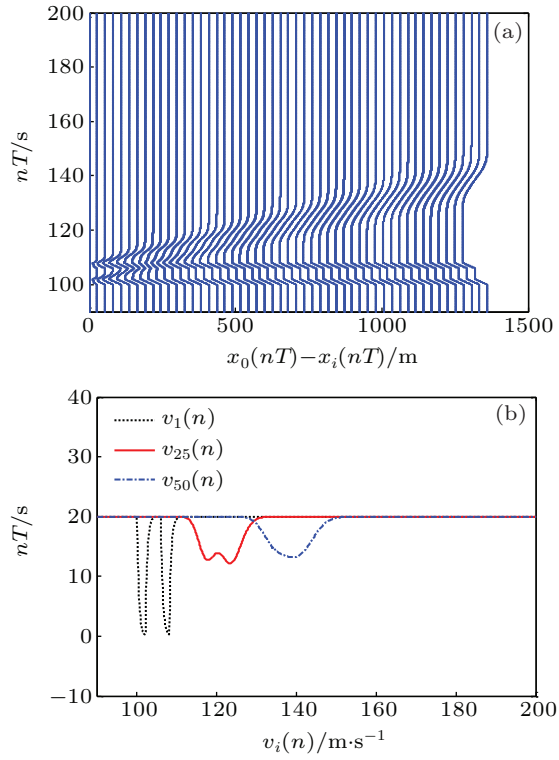


Fig. 1. (color online) Numerical simulations for the KKH model. (a) Space–time plot of the KKH model. (b) Temporal velocity behavior of the first, 25-th, and 50-th vehicles.

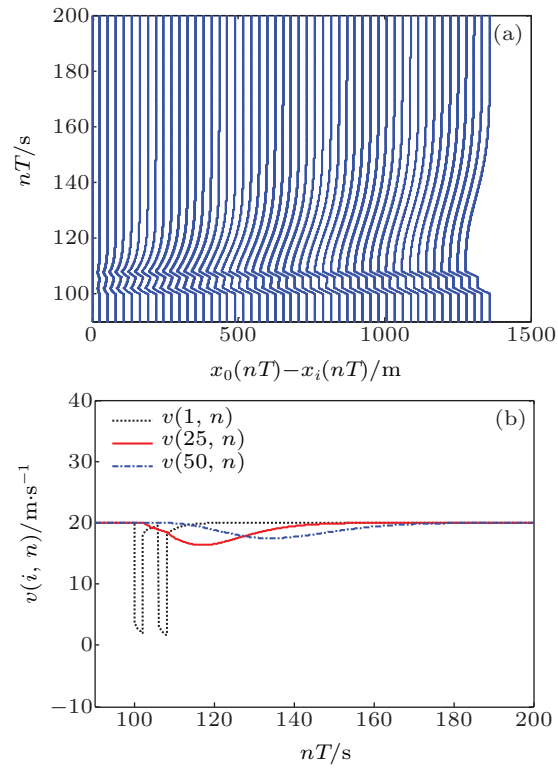


Fig. 2. (color online) Numerical simulations for the ZG model. (a) Space–time plot of the ZG model. (b) Temporal velocity behavior of the first, 25-th, and 50-th vehicles.

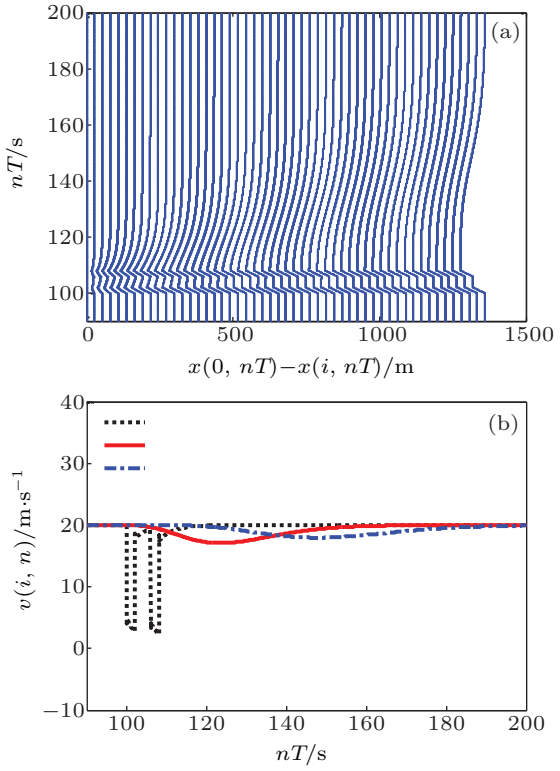


Fig. 3. (color online) Numerical simulations for our model. (a) Space–time plot of our model. (b) Temporal velocity behavior of the first, 25-th, and 50-th vehicles.

Specifically, the values of some parameters obtained in our simulations are given in Table 1 to compare the velocity change of the 50-th vehicle between the KKH model, the ZG model, and our model. In Table 1, $\overline{a}_{50}$ is the average acceleration of the 50-th vehicle, which denotes the rates of velocity change; S_{50} is the standard deviation of acceleration, which denotes the smoothness of the 50-th vehicle's speed; $\Delta v_{\max}$ is the maximum deviation of the 50-th vehicle's velocity. It can be clearly seen that the 50-th vehicle in our model runs more smoothly, and the amplitude of speed fluctuation is smaller than those of the two previous models.

Table 1. Comparison of the velocity fluctuation of the 50-th vehicle between three models.

Model	$\overline{a}_{50}$/m·s^{-2}	S_{50}/m·s^{-2}	$\Delta v_{\max}$/m·s^{-1}
KKH	0.6592	0.0191	5.1391
ZG	0.4310	0.0048	2.5484
Our model	0.3914	0.0034	2.0366

In summary, these simulations show that the stability of our model is better than the stability of the KKH model and the ZG model with the same parameters, which demonstrates that the DSDSE could be added into the CM car-following model to enhance the stability of traffic flow and suppress the traffic congestion more efficiently. Therefore, the simulation results are in agreement with the theoretical analysis.

5. Conclusions

In this paper, a new CM car-following model considering the DSDSE is proposed. The stability of our model is investigated, and non-jam criteria are derived. Through theoretical analysis and numerical simulations, we conclude that: (i) the stability of traffic flow is enhanced by considering the DSDSE; (ii) numerical simulations demonstrate that our model has a great ability to suppress a traffic jam, which is consistent with the theoretical analysis.

References

[1] Kerner B S and Rehborn H 1996 *Phys. Rev. E* **53** 1297

[2] Wang T, Gao Z Y and Zhao X M 2006 *Acta Phys. Sin.* **55** 634 (in Chinese)

[3] Jiang R, Wu Q S and Zhu Z J 2001 *Phys. Rev. E* **64** 017101

[4] Peng G H and Sun D H 2010 *Phys. Lett. A* **374** 1694

[5] Tang T Q, Li C Y, Wu Y H and Huang H J 2011 *Physica A* **390** 3362

[6] Bando M, Hasebe K, Nakanishi K and Nakayama A 1998 *Phys. Rev. E* **58** 5429

[7] Tang T Q, Wu Y H, Caccetta L and Huang H J 2011 *Phys. Lett. A* **375** 3845

[8] Jin S, Qu X B, Chen X and Wang D H 2011 *Physica A* **390** 1931

[9] Tang T Q, Huang H J, Wong S C and Jiang R 2007 *Acta Mech. Sin.* **23** 49

[10] Kurata S and Nagatani T 2003 *Physica A* **318** 537

[11] Konishi K, Kokame H and Hirata K 1999 *Phys. Rev. E* **60** 4000

[12] Zhao X M and Gao Z Y 2006 *Physica A* **366** 513

[13] Han X L, Jiang C Y, Ge H X and Dai S Q 2007 *Acta Phys. Sin.* **56** 4383 (in Chinese)

[14] Yu H M, Cheng R J and Ge H X 2010 *Commun. Theor. Phys.* **54** 117

[15] Ge H X, Cheng R J and Li Z P 2011 *Acta Phys. Sin.* **60** 080508 (in Chinese)

[16] Ge H X, Liu Y X and Cheng R J 2012 *Commun. Nonlinear Sci. Numer. Simulat.* **17** 4439

[17] Ge H X, Meng X P, Jian M and Lo S M 2012 *Phys. Lett. A* **377** 9

[18] Zhou T, Sun D H, Zhao M and Li H M 2013 *Chin. Phys. B* **22** 090205

[19] Sun D H, Zhou T, Liu W N and Zheng L J 2013 *Acta Phys. Sin.* **62** 170503 (in Chinese)

[20] Sun D H, Li Y F, Liu W N, Zhao M and Liao X Y 2013 *China Journal of Highway and Transport* **26** 144 (in Chinese)