A high order energy preserving scheme for the strongly coupled nonlinear Schrödinger system*

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A high order energy preserving scheme for a strongly coupled nonlinear Schrödinger system is proposed by using the average vector field method. The high order energy preserving scheme is applied to simulate the soliton evolution of the strongly coupled Schrödinger system. Numerical results show that the high order energy preserving scheme can well simulate the soliton evolution, moreover, it preserves the discrete energy of the strongly coupled nonlinear Schrödinger system exactly.

Keywords: average vector field method, strongly coupled nonlinear Schrödinger system, energy preserving scheme

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1. Introduction

In this paper, we study the following strongly coupled nonlinear Schrödinger (SCNLS) system:

$$\mathrm{i}u_t+\beta u_{xx}+[\alpha_1|u|^2+(\alpha_1+2\alpha_2)|v|^2]u+\gamma u+\Gamma v=0, \quad (1)$$

$$\mathrm{i}v_t+\beta v_{xx}+[\alpha_1|v|^2+(\alpha_1+2\alpha_2)|u|^2]v+\gamma v+\Gamma u=0, \quad (2)$$

where α_1, α_2, β, γ, and Γ are the parameters.[1,2] The SCNLS system has extensive applications in many fields of physics, including nonlinear optics and plasma physics. The SCNLS system has the following energy conservation property:[3]

$$\begin{aligned}E(t) &= \frac{1}{2}\int\Big[\frac{\alpha_1}{2}(|u|^4+|v|^4)+(\alpha_1+2\alpha_2)|u|^2|v|^2\\ &\quad+\gamma(|u|^2+|v|^2)+2\Gamma\Re(\bar{u}v)-\beta(u_x^2+v_x^2)\Big]\mathrm{d}x\\ &= E(0). \end{aligned} \quad (3)$$

Some numerical methods were proposed to solve the SCNLS system and gave some excellent results,[3–8] however few methods can preserve the energy of the system exactly.

In recent years, there has been an increased interest in constructing numerical methods to preserve the intrinsic properties of the original system as much as possible, which are called the structure-preserving algorithms. The structure-preserving algorithms have gained remarkable success in the numerical analysis of ordinary differential equations (ODEs) and partial differential equations (PDEs)[9–19] arising from a variety of application areas, including molecular dynamics, fluid dynamics, plasma physics, meteorology, etc. Quispel and McLaren proposed the average vector field (AVF) method,[14–18] which preserves the Hamiltonian energy exactly with high order accuracy

$$\frac{z^{j+1}-z^j}{\tau} = \big(\delta_n^i+\alpha\tau^2 f_k^i(\hat{z})f_n^k(\hat{z})\big)\times\int_0^1 f^n((1-\xi)z^j+\xi z^{j+1})\mathrm{d}\xi, \quad (4)$$

where subscripts denote the partial derivatives, i.e., $f_k^i=\partial f^i/\partial x^k$, $f^n=\mathrm{d}z^n/\mathrm{d}t$, α is an arbitrary constant, δ_n^i is the Kronecker delta function, and τ is the time step. For $\alpha=0$, the method is second order accuracy. For $\alpha=-1/12$, $\hat{z}=(z^j+z^{j+1})/2$, the method is fourth order accuracy.[14] The second order average vector field method has been applied to solve energy-conservation PDEs.[15] However, few people applied the high order average vector field method to solve energy-conservation PDEs. Here, we will construct a high order energy preserving scheme of the SCNLS system by using the average vector field method.

This paper is organized as follows. In Section 2, a high order energy preserving scheme of the SCNLS system is constructed by using the average vector field method. In Section 3, the soliton-evolution behaviors of the SCNLS system are simulated by using the new high order energy preserving scheme. Finally, we give the conclusion remarks in Section 4.

2. High order energy preserving scheme

Let $u(x,t)=p(x,t)+q(x,t)\mathrm{i}$, $v(x,t)=c(x,t)+d(x,t)\mathrm{i}$, equations (1) and (2) can be expressed in the Hamiltonian form

$$\frac{\mathrm{d}\boldsymbol{z}}{\mathrm{d}t}=\boldsymbol{J}^{-1}\frac{\delta H(\boldsymbol{z})}{\delta\boldsymbol{z}}, \quad \boldsymbol{J}=\begin{pmatrix}\boldsymbol{I} & \boldsymbol{O}_2\\ \boldsymbol{O}_2 & \boldsymbol{I}\end{pmatrix}, \quad \boldsymbol{I}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \quad (5)$$

where $\boldsymbol{z}=(p,q,c,d)\in\mathbf{R}^4$, $\boldsymbol{O}_2$ is the 2×2 zero matrix, and the Hamiltonian function is[4]

$$\begin{aligned}H(\boldsymbol{z}) &= \int\Big\{\frac{1}{4}\alpha_1\big[(p^2+q^2)^2+(c^2+d^2)^2\big]\\ &\quad+\frac{1}{2}(\alpha_1+2\alpha_2)(c^2+d^2)(p^2+q^2)\end{aligned}$$

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$$+\frac{1}{2}\gamma(p^2+q^2+c^2+d^2)+\Gamma(pc+qd)$$
$$-\frac{\beta}{2}(p_x^2+q_x^2+c_x^2+d_x^2)\Big\}\mathrm{d}x. \quad (6)$$

Let $x_i,\ i=0,1,2,\ldots,N_1$; $t_j,\ j=0,1,2,\ldots,N_2$ be the regular grids of the integral domain $[x_0,x_{N_1}]\times[t_0,t_{N_2}]$, z_i^j be an approximation to $z(x_i,t_j)$, and h be the space step, we can obtain the discrete Hamiltonian system of the SCNLS system

$$\frac{\mathrm{d}Z}{\mathrm{d}t}=\overline{\boldsymbol{J}}^{-1}\nabla H(\boldsymbol{Z}),\quad \overline{\boldsymbol{J}}=\begin{pmatrix}\boldsymbol{E}_{2n} & \boldsymbol{O}_{2n}\\ \boldsymbol{O}_{2n} & \boldsymbol{E}_{2n}\end{pmatrix},$$
$$\boldsymbol{E}_{2n}=\begin{pmatrix}\boldsymbol{O}_n & \boldsymbol{I}_n\\ -\boldsymbol{I}_n & \boldsymbol{O}_n\end{pmatrix}, \quad (7)$$

where $\boldsymbol{Z}=(p_1,p_2,\ldots,p_{N_1},q_1,q_2,\ldots,q_{N_1},c_1,c_2,\ldots,c_{N_1},d_1,d_2,\ldots,d_{N_1})$, and $\boldsymbol{O}_n$ is the $N_1\times N_1$ zero matrix,

$$\begin{aligned}H(\boldsymbol{Z})=&\sum_{i=1}^{N_1}\frac{\alpha_1}{4}\{[(p_i)^2+(q_i)^2]^2+[(c_i)^2+(d_i)^2]^2\}\\&+\frac{1}{2}(\alpha_1+2\alpha_2)[(c_i)^2+(d_i)^2][(p_i)^2+(q_i)^2]\\&+\frac{\gamma}{2}[(p_i)^2+(q_i)^2+(c_i)^2+(d_i)^2]+\Gamma(p_ic_i+q_id_i)\\&-\frac{\beta}{2}\Big[\Big(\frac{p_{i+1}-p_i}{h}\Big)^2+\Big(\frac{q_{i+1}-q_i}{h}\Big)^2+\Big(\frac{c_{i+1}-c_i}{h}\Big)^2\\&+\Big(\frac{d_{i+1}-d_i}{h}\Big)^2\Big]. \quad (8)\end{aligned}$$

The fourth order averaged vector field method is applied to Eq. (7), we can obtain the high order energy preserving scheme of the SCNLS system

$$\begin{aligned}\frac{\boldsymbol{Z}^{j+1}-\boldsymbol{Z}^j}{\tau}=&\int_0^1 f((1-\xi)\boldsymbol{Z}^j+\xi\boldsymbol{Z}^{j+1})\mathrm{d}\xi\\&-\frac{1}{12}\tau^2\Big[\frac{\partial f}{\partial\boldsymbol{Z}}\Big(\frac{\boldsymbol{Z}^j+\boldsymbol{Z}^{j+1}}{2}\Big)\Big]^2\\&\times\int_0^1 f((1-\xi)\boldsymbol{Z}^j+\xi\boldsymbol{Z}^{j+1})\mathrm{d}\xi, \quad (9)\end{aligned}$$

where

$$f(\boldsymbol{Z})=\overline{\boldsymbol{J}}^{-1}\nabla H(\boldsymbol{Z}),$$
$$\frac{\partial f}{\partial\boldsymbol{Z}}\Big(\frac{\boldsymbol{Z}_i^j+\boldsymbol{Z}_i^{j+1}}{2}\Big)=\begin{pmatrix}J_{11}&J_{12}&J_{13}&J_{14}\\J_{21}&J_{22}&J_{23}&J_{24}\\J_{31}&J_{32}&J_{33}&J_{34}\\J_{41}&J_{42}&J_{43}&J_{44}\end{pmatrix},$$
$$J_{11}=-2\alpha_1\frac{p_i^j+p_i^{j+1}}{2}\frac{q_i^j+q_i^{j+1}}{2},$$
$$J_{22}=2\alpha_1\frac{p_i^j+p_i^{j+1}}{2}\frac{q_i^j+q_i^{j+1}}{2},$$
$$J_{24}=2(\alpha_1+2\alpha_2)\frac{d_i^j+d_i^{j+1}}{2}\frac{p_i^j+p_i^{j+1}}{2},$$
$$J_{44}=2\alpha_1\frac{d_i^j+d_i^{j+1}}{2}\frac{c_i^j+c_i^{j+1}}{2},$$
$$J_{13}=-2(\alpha_1+2\alpha_2)\frac{c_i^j+c_i^{j+1}}{2}\frac{q_i^j+q_i^{j+1}}{2},$$
$$J_{33}=-2\alpha_1\frac{c_i^j+c_i^{j+1}}{2}\frac{d_i^j+d_i^{j+1}}{2},$$
$$J_{31}=-2(\alpha_1+2\alpha_2)\frac{d_i^j+d_i^{j+1}}{2}\frac{p_i^j+p_i^{j+1}}{2},$$
$$J_{42}=2(\alpha_1+2\alpha_2)\frac{q_i^j+q_i^{j+1}}{2}\frac{c_i^j+c_i^{j+1}}{2},$$
$$J_{14}=-2(\alpha_1+2\alpha_2)\frac{d_i^j+d_i^{j+1}}{2}\frac{q_i^j+q_i^{j+1}}{2}-\Gamma,$$
$$J_{23}=2(\alpha_1+2\alpha_2)\frac{c_i^j+c_i^{j+1}}{2}\frac{p_i^j+p_i^{j+1}}{2}+\Gamma,$$
$$J_{32}=-2(\alpha_1+2\alpha_2)\frac{q_i^j+q_i^{j+1}}{2}\frac{d_i^j+d_i^{j+1}}{2}-\Gamma,$$
$$J_{41}=2(\alpha_1+2\alpha_2)\frac{p_i^j+p_i^{j+1}}{2}\frac{c_i^j+c_i^{j+1}}{2}+\Gamma,$$
$$\begin{aligned}J_{12}=&-(\alpha_1+2\alpha_2)\Big[\Big(\frac{c_i^j+c_i^{j+1}}{2}\Big)^2+\Big(\frac{d_i^j+d_i^{j+1}}{2}\Big)^2\Big]\\&-\alpha_1\Big(\frac{p_i^j+p_i^{j+1}}{2}\Big)^2-3\alpha_1\Big(\frac{q_i^j+q_i^{j+1}}{2}\Big)^2-\gamma,\end{aligned}$$
$$\begin{aligned}J_{21}=&(\alpha_1+2\alpha_2)\Big[\Big(\frac{c_i^j+c_i^{j+1}}{2}\Big)^2+\Big(\frac{d_i^j+d_i^{j+1}}{2}\Big)^2\Big]\\&+3\alpha_1\Big(\frac{p_i^j+p_i^{j+1}}{2}\Big)^2+\alpha_1\Big(\frac{q_i^j+q_i^{j+1}}{2}\Big)^2+\gamma,\end{aligned}$$
$$\begin{aligned}J_{34}=&-(\alpha_1+2\alpha_2)\Big[\Big(\frac{p_i^j+p_i^{j+1}}{2}\Big)^2+\Big(\frac{q_i^j+q_i^{j+1}}{2}\Big)^2\Big]\\&-\alpha_1\Big(\frac{c_i^j+c_i^{j+1}}{2}\Big)^2-3\alpha_1\Big(\frac{d_i^j+d_i^{j+1}}{2}\Big)^2-\gamma,\end{aligned}$$
$$\begin{aligned}J_{43}=&(\alpha_1+2\alpha_2)\Big[\Big(\frac{p_i^j+p_i^{j+1}}{2}\Big)^2+\Big(\frac{q_i^j+q_i^{j+1}}{2}\Big)^2\Big]\\&+\alpha_1\Big(\frac{d_i^j+d_i^{j+1}}{2}\Big)^2+3\alpha_1\Big(\frac{c_i^j+c_i^{j+1}}{2}\Big)^2+\gamma.\end{aligned}$$

Equation (9) can be written as

$$\begin{aligned}\frac{p_i^{j+1}-p_i^j}{\tau}=&F_1-\frac{1}{12}\tau^2[(J_{11}J_{11}+J_{12}J_{21}+J_{13}J_{31}+J_{14}J_{41})F_1\\&+(J_{11}J_{13}+J_{12}J_{23}+J_{13}J_{33}+J_{14}J_{43})F_3\\&+(J_{11}J_{14}+J_{12}J_{24}+J_{13}J_{34}+J_{14}J_{44})F_4], \quad (10)\\-\frac{q_i^{j+1}-q_i^j}{\tau}=&F_2-\frac{1}{12}\tau^2[(J_{21}J_{12}+J_{22}J_{22}+J_{23}J_{32}+J_{24}J_{42})F_2\\&+(J_{21}J_{13}+J_{22}J_{23}+J_{23}J_{33}+J_{24}J_{43})F_3\\&+(J_{21}J_{14}+J_{22}J_{24}+J_{23}J_{34}+J_{24}J_{44})F_4], \quad (11)\\\frac{c_i^{j+1}-c_i^j}{\tau}=&F_3-\frac{1}{12}\tau^2[(J_{31}J_{11}+J_{32}J_{21}+J_{33}J_{31}+J_{34}J_{41})F_1\\&+(J_{31}J_{12}+J_{32}J_{22}+J_{33}J_{32}+J_{34}J_{42})F_2\\&+(J_{31}J_{13}+J_{32}J_{23}+J_{33}J_{33}+J_{34}J_{43})F_3], \quad (12)\\-\frac{d_i^{j+1}-d_i^j}{\tau}=&F_4-\frac{1}{12}\tau^2[(J_{41}J_{11}+J_{42}J_{21}+J_{43}J_{31}+J_{44}J_{41})F_1\\&+(J_{41}J_{12}+J_{42}J_{22}+J_{43}J_{32}+J_{44}J_{42})F_2\\&+(J_{41}J_{14}+J_{42}J_{24}+J_{43}J_{34}+J_{44}J_{44})F_4], \quad (13)\end{aligned}$$

where

$$\begin{aligned}F_1=&-\int_0^1\frac{1}{h^2}\beta[((1-\xi)q_{i+1}^j+\xi q_{i+1}^{j+1})\\&-2((1-\xi)q_i^j+\xi q_i^{j+1})+((1-\xi)q_{i-1}^j+\xi q_{i-1}^{j+1})]\end{aligned}$$

$$
\begin{aligned}
&+[\alpha_1(((1-\xi)p_i^j+\xi p_i^{j+1})^2+((1-\xi)q_i^j+\xi q_i^{j+1})^2)\\
&+(\alpha_1+2\alpha_2)(((1-\xi)c_i^j+\xi c_i^{j+1})^2\\
&+((1-\xi)d_i^j+\xi d_i^{j+1})^2)](((1-\xi)q_i^j+\xi q_i^{j+1})\\
&+\gamma((1-\xi)q_i^j+\xi q_i^{j+1})+\Gamma((1-\xi)d_i^j+\xi d_i^{j+1})\,\mathrm{d}\xi,\\
F_2=&-\int_0^1\frac{1}{h^2}\beta[((1-\xi)p_{i+1}^j+\xi p_{i+1}^{j+1})\\
&-2((1-\xi)p_i^j+\xi p_i^{j+1})+((1-\xi)p_{i-1}^j+\xi p_{i-1}^{j+1})]\\
&+[\alpha_1(((1-\xi)p_i^j+\xi p_i^{j+1})^2+((1-\xi)q_i^j+\xi q_i^{j+1})^2)\\
&+(\alpha_1+2\alpha_2)(((1-\xi)c_i^j+\xi c_i^{j+1})^2\\
&+((1-\xi)d_i^j+\xi d_i^{j+1})^2)](((1-\xi)p_i^j+\xi p_i^{j+1})\\
&+\gamma((1-\xi)p_i^j+\xi p_i^{j+1})+\Gamma((1-\xi)c_i^j+\xi c_i^{j+1})\,\mathrm{d}\xi,\\
F_3=&-\int_0^1\frac{1}{h^2}\beta[((1-\xi)d_{i+1}^j+\xi d_{i+1}^{j+1})\\
&-2((1-\xi)d_i^j+\xi d_i^{j+1})+((1-\xi)d_{i-1}^j+\xi d_{i-1}^{j+1})]\\
&+[\alpha_1(((1-\xi)c_i^j+\xi c_i^{j+1})^2+((1-\xi)d_i^j+\xi d_i^{j+1})^2)\\
&+(\alpha_1+2\alpha_2)(((1-\xi)p_i^j+\xi p_i^{j+1})^2\\
&+((1-\xi)q_i^j+\xi q_i^{j+1})^2)](((1-\xi)d_i^j+\xi d_i^{j+1})\\
&+\gamma((1-\xi)d_i^j+\xi d_i^{j+1})+\Gamma((1-\xi)q_i^j+\xi q_i^{j+1})\,\mathrm{d}\xi,\\
F_4=&-\int_0^1\frac{1}{h^2}\beta[((1-\xi)c_{i+1}^j+\xi c_{i+1}^{j+1})\\
&-2((1-\xi)c_i^j+\xi c_i^{j+1})+((1-\xi)c_{i-1}^j+\xi c_{i-1}^{j+1})]\\
&+[\alpha_1(((1-\xi)c_i^j+\xi c_i^{j+1})^2+((1-\xi)d_i^j+\xi d_i^{j+1})^2)\\
&+(\alpha_1+2\alpha_2)(((1-\xi)p_i^j+\xi p_i^{j+1})^2\\
&+((1-\xi)q_i^j+\xi q_i^{j+1})^2)](((1-\xi)c_i^j+\xi c_i^{j+1})\\
&+\gamma((1-\xi)c_i^j+\xi c_i^{j+1})+\Gamma((1-\xi)p_i^j+\xi p_i^{j+1})\,\mathrm{d}\xi. \qquad (14)
\end{aligned}
$$

The F_1 is equivalent to

$$
\begin{aligned}
G_1 =&-\beta\frac{(q_{i+1}^{j+1}-2q_i^{j+1}+q_{i-1}^{j+1})+(q_{i+1}^j-2q_i^j+q_{i-1}^j)}{2h^2}\\
&-\alpha_1\Big[((p_i^j)^2+\frac{1}{3}(p_i^{j+1}-p_i^j)^2+p_i^j(p_i^{j+1}-p_i^j))q_i^j\\
&+\Big(\frac{1}{2}(p_i^j)^2+\frac{1}{4}(p_i^{j+1}-p_i^j)^2+\frac{2}{3}p_i^j(p_i^{j+1}-p_i^j)\Big)\\
&\times(q_i^{j+1}-q_i^j)+\Big((q_i^j)^2+\frac{1}{3}(q_i^{j+1}-q_i^j)^2\\
&+q_i^j(q_i^{j+1}-q_i^j)\Big)q_i^j+\Big(\frac{1}{2}(q_i^j)^2+\frac{1}{4}(q_i^{j+1}-q_i^j)^2\\
&+\frac{2}{3}q_i^j(q_i^{j+1}-q_i^j)\Big)(q_i^{j+1}-q_i^j)\Big]-(\alpha_1+2\alpha_2)\\
&\times\Big[\Big((c_i^j)^2+\frac{1}{3}(c_i^{j+1}-c_i^j)^2+c_i^j(c_i^{j+1}-c_i^j)\Big)q_i^j\\
&+\Big(\frac{1}{2}(c_i^j)^2+\frac{1}{4}(c_i^{j+1}-c_i^j)^2+\frac{2}{3}c_i^j(c_i^{j+1}-c_i^j)\Big)\\
&\times(q_i^{j+1}-q_i^j)+\Big((d_i^j)^2+\frac{1}{3}(d_i^{j+1}-d_i^j)^2\\
&+d_i^j(d_i^{j+1}-d_i^j)\Big)q_i^j+\Big(\frac{1}{2}(d_i^j)^2+\frac{1}{4}(d_i^{j+1}-d_i^j)^2\\
&+\frac{2}{3}d_i^j(d_i^{j+1}-d_i^j)\Big)(q_i^{j+1}-q_i^j)\Big]\\
&-\gamma\Big(\frac{q_i^j+q_i^{j+1}}{2}\Big)-\Gamma\Big(\frac{d_i^j+d_i^{j+1}}{2}\Big). \qquad (15)
\end{aligned}
$$

The F_2, F_3, and F_4 can be obtained similarly. Theoretically, the new scheme (9) can preserve the discrete Hamiltonian function of the SCNLS system, which also preserve the discrete energy of the SCNLS system exactly.

3. Numerical simulations

In order to test the effectiveness of the new scheme (9), we will carry out some numerical experiments to simulate the soliton evolution of the SCNLS system by the new scheme (9). The relative discrete Hamiltonian energy error of the SCNLS system is defined as follows:

$$
E_{\mathrm{r}}(t)=\left|\frac{H(\boldsymbol{Z}^j)-H(\boldsymbol{Z}^0)}{H(\boldsymbol{Z}^0)}\right|, \qquad (16)
$$

where $H(\boldsymbol{Z}^0)$ is the initial discrete Hamiltonian energy, and $H(\boldsymbol{Z}^j)$ is the discrete Hamiltonian energy at $t=j\Delta t$. At the same time, we also analyze the accuracy of the average vector field method.

3.1. Accuracy comparison of the average vector field method

In order to show the accuracy and the rate of convergence in time for the average vector field method, we use the second and the fourth order AVF schemes to solve the ordinary differential equation

$$
\frac{\mathrm{d}u}{\mathrm{d}t}=-(u-1)^2,\quad u(0)=2. \qquad (17)
$$

Equation (17) has the exact solution $u=1/(1+t)+1$. To scale the error of the solution, we define $L_\infty(u)=\max\limits_i|u_i^j-u(x_i,t_j)|$. The rate of convergence in time can be obtained by using the formula $R=\ln(L_\infty(\tau_2)/L_\infty(\tau_1))/\ln(\tau_2/\tau_1)$.[8] Table 1 shows the error and the rate of convergence in time of the second and the fourth order AVF schemes. We can see that the fourth order AVF scheme can achieve the fourth order accuracy.

Table 1. The L_∞ error and convergence rate in time at $t=1$.

τ	0.1	0.05	0.01	0.005
2nd order scheme (L_∞)	4.383×10^{-4}	1.068×10^{-4}	4.188×10^{-6}	1.044×10^{-6}
R	–	2.037	2.020	2.016
4th order scheme (L_∞)	1.538×10^{-6}	9.224×10^{-8}	1.430×10^{-10}	8.817×10^{-12}
R	–	4.060	4.032	4.029

3.2. Evolution of single soliton

Now we simulate the evolution of a single soliton by choosing $\alpha_1 = 1.0$, $\alpha_2 = 0$, $\beta = 1.0$, $\gamma = 0$, $\Gamma = 0$ and the initial condition $u(x,0) = v(x,0) = \mathrm{sech}(x)\exp(\mathrm{i}x)$, $-30 \le x \le 30$ with the periodic boundary condition. We take $h = 0.2$ and $\tau = 0.02$. Figure 1 shows the evolution of the single soliton at $t \in [0,40]$. The soliton moves smoothly with a constant velocity. Figure 2 shows the relative energy error of the single soliton at $t \in [0,40]$. From the numerical results, we can conclude that the new scheme (9) can have excellent performance in simulating the evolution of a single soliton and preserve the discrete Hamiltonian energy of the SCNLS system exactly.

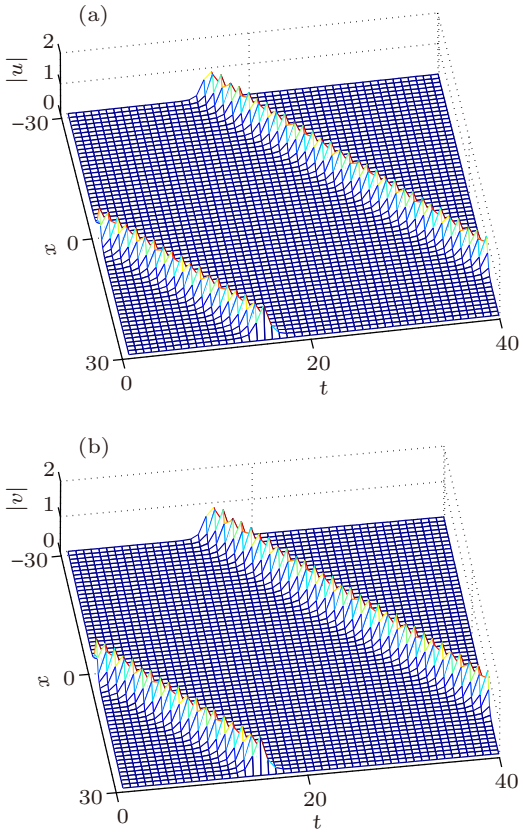


Fig. 1. (color online) Evolution of a single soliton: (a) $|u|$, (b) $|v|$.

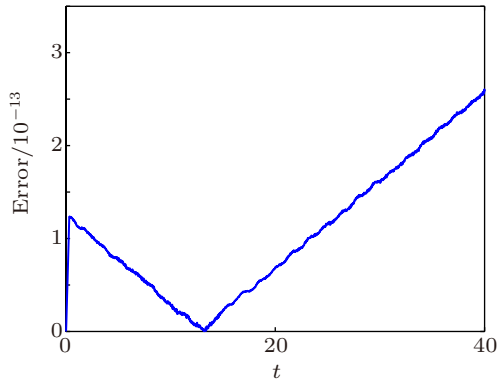


Fig. 2. (color online) Relative energy error of a single solition.

3.3. Evolution of two solitons

We take $\alpha_1 = 1.0$, $\alpha_2 = 1.0$, $\beta = 1.0$, $\gamma = 0$, $\Gamma = 0$ and the initial condition $u(x,0) = \sqrt{2}\mathrm{sech}(x + D_0/2)\exp(\mathrm{i}V_0 x/4)$, $v(x,0) = \sqrt{2}\mathrm{sech}(x - D_0/2)\exp(-\mathrm{i}V_0 x/4)$, $-40 \le x \le 40$, $D_0 = 20$, $V_0 = 1.0$ with the periodic boundary condition. We also set $h = 0.2$ and $\tau = 0.01$. Figure 3 shows the evolution of the solitons at $t \in [0,45]$, After the collision of the two solitons $u(x,t)$ and $v(x,t)$, the soliton $u(x,t)$ generates a new solition. The numerical result is consistent with the result obtained by the multi-symplectic method.[19] Figure 4 shows the relative energy error of the two solitons at $t \in [0,45]$. From the numerical results, we can also conclude that the new scheme (9) can have good performance in simulating the evolution of two solitons, moreover, it preserves the discrete Hamiltonian energy of the SCNLS system exactly.

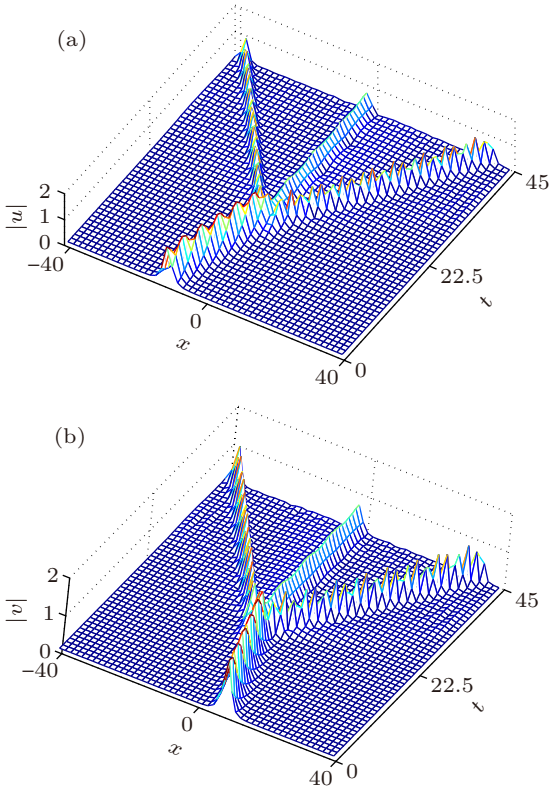


Fig. 3. (color online) Evolution of two solitons: (a) $|u|$, (b) $|v|$.

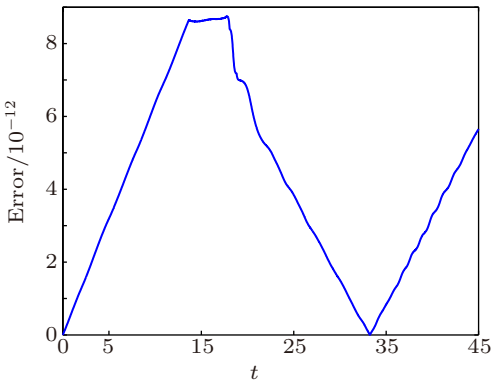


Fig. 4. (color online) Relative energy error of two solitons.

4. Conclusion

A new high order energy preserving scheme is proposed to simulate the soliton evolution of the SCNLS system. Numerical results show that the new high order energy preserving scheme has an excellent performance in simulating solitons evolution behaviors of the SCNLS system, moreover, preserve the corresponding discrete Hamiltonian energy. At the same time, the accuracy of the fourth order average vector field method is also verified. The energy preserving scheme is very important in simulating the energy conserving partial differential equation. It is obvious that the average vector field method can also be applied to other energy conserving partial differential equations, such as the KdV equation and the Maxwell equation.

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