Collective surrounding control in multi-agent networks*

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We study the collective encirclement control problem of a multi-agent system in this paper, where the agents move collectively to encircle the multiple targets. An algorithm is proposed and a sufficient condition is derived to realize the collective encirclement motion. Some simulation results are provided to show the effectiveness of the obtained theoretical results.

Keywords: surrounding motion, networks, multiple targets

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1. Introduction

In recent years, the consensus problem of a multi-agent network has received much attention in various fields, including physics, computer science, and mathematics.[1–18] For instance, the consensus problems for directed networks of agents with external disturbances and model uncertainties on fixed and switching topologies were investigated in Ref. [1]. Some distributed controllers and observers for the autonomous agents were developed to keep updating the information of the leader in Ref. [5]. In Ref. [15], a distributed coordination scheme for all the agents flocking to the polytope region formed by the leader was used. An impulsive distributed consensus algorithm was proposed for the continuous-time second-order multi-agent networks in Refs. [6]–[8]. In a military situation, we usually need to surround the enemy targets for attack and reconnaissance missions. It is meaningful to study the collective surrounding motion in a distributed manner. However, currently, the work on this issue is rare, perhaps only in Refs. [13] and [14], they only considered the fixed targets. Specifically, in Ref. [13], a group of unmanned aerial vehicles (UAVs) surrounding the target by using decentralized nonlinear model predictive control (MPC) was studied, however the model they used had only one target. In Ref. [14], the author used the leader–follower framework to make the followers surround the stationary leaders with a fixed communication graph. There is no work on surrounding a group of dynamic targets with a time-vary communication graph so far.

In this paper, our main objective is to study the collective surrounding motion problem of the agents' networks in a distributed manner. We firstly introduce a surrounding algorithm with two parts: one is to calculate the average position of the targets by a distributed method, and the other is to surround the targets. Then a sufficient condition is derived for the surrounding motion.

2. Problem description

2.1. Graph preliminaries

Let $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ be an undirected graph of order n, where $\mathcal{V}=\{v_1,\ldots,v_n\}$ is the set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{p\times p}$ is the weighted adjacency matrix. An edge of $\mathcal{G}$ is denoted by $e_{ij}=(v_i,v_j)$. The adjacency matrix is defined as $a_{ii}=0$ and $a_{ij}=a_{ji}\geq 0$, $a_{ij}>0$ if and only if $e_{ij}\in\mathcal{E}$. Since the graph considered is undirected, once $e_{ij}\in\mathcal{E}$, then $e_{ji}\in\mathcal{E}$. The set of neighbors of node v_i is denoted by $\mathcal{N}_i=\{v_j\in\mathcal{V}:(v_i,v_j\in\mathcal{E})\}$. An undirected path is a sequence of edges in an undirected graph of the form $(v_{i_1},v_{i_2}),(v_{i_2},v_{i_3}),\ldots$, where $v_{i_j}\in\mathcal{V}$. An undirected graph is connected if there is an undirected path between every pair of distinct nodes.[19]

2.2. Model

Consider a network of n agents. The graph $\mathcal{G}(t)$ describes the network topology between the agents at time t, and $\mathcal{N}_i(t)$ denotes the neighbor set of the i-th agent in $\mathcal{G}(t)$ at time t. Suppose that the i-th agent has the following dynamics:

$$\dot{y}_i(t)=u_i(t), \tag{1}$$

where $y_i(t)$ denotes the position of the i-th agent, and $u_i(t)$ is the control input.

In this paper, we will propose a distributed control algorithm that enables the agents to surround the targets. Let $r_i(t)$ be the position of the i-th target.

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Definition 1 The agents in system (1) achieve a collective surrounding motion if

$$\lim_{t\to+\infty}\left\{[y_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)] -\gamma[r_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)]\right\}=0 \quad (2)$$

for all $i\in\{1,2,\ldots,n\}$, where $\gamma>1$.

In Definition 1, $\frac{1}{n}\sum_{k=1}^{n}r_k(t)$ denotes the average position of the targets, thus $r_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)$ is the vector pointing from the average position of the targets to the position of the i-th target, and $y_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)$ is the vector pointing from the average position of the targets to the position of the i-th agent. Since $\gamma>1$, it is clear that with Eq. (2), all agents finally form a formation which is the same shape but larger than that of the targets (Fig. 1).

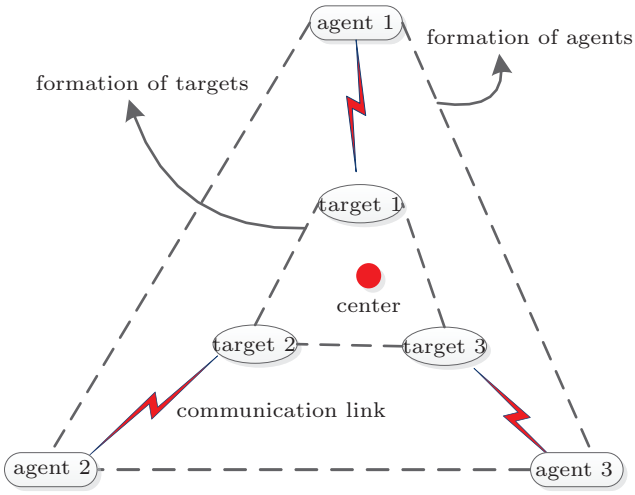


Fig. 1. (color online) Surrounding ketch. The center is obtained by the distributed method. Each agent can send the information of the target detected to its neighbors by a communication link. The dashed lines linking the three targets are the formation of the targets; those linking the agents is the formation of the agents. The agents are surrounding the targets.

3. Control design

To solve the collective surrounding motion problem, we propose the following algorithm:

$$\begin{aligned}u_i(t) &= -[y_i(t)-p_i(t)] \\ &\quad -k\alpha\int_0^t\sum_{j\in\mathcal{N}_i(t)}\frac{p_j(\tau)-p_i(\tau)}{\|p_j(\tau)-p_i(\tau)\|}\,\mathrm{d}\tau+\dot{p}_i(t), \quad (3)\end{aligned}$$

where $i=1,\ldots,n$, and $p_i(t)$ is the estimate for the average position of the targets by the i-th agent. To make it clear, we define $p_i(t)$ as the i-th average position of the targets. The $y_i(t)-p_i(t)$ is the relative position of the i-th agent to the i-th average position of the targets, and α and $k>1$ are positive constants. Here we define $(p_j(t)-p_i(t))/\|p_j(t)-p_i(t)\|=0$ when $|p_j(t)-p_i(t)|=0$.

To easily analyze the control algorithm above, Eq. (3) can be described as

$$\begin{aligned}\dot{\varphi}_i(t)&=\alpha\sum_{j\in\mathcal{N}_i(t)}\frac{p_j(t)-p_i(t)}{\|p_j(t)-p_i(t)\|},\\ p_i(t)&=\varphi_i(t)+r_i(t),\\ \dot{h}_i(t)&=-\{h_i(t)-k[r_i(t)-p_i(t)]\}, \quad (4)\end{aligned}$$

where φ_i is the internal state of the averaging estimate with $\varphi_i(0)=0$, and $h_i=y_i(t)-p_i(t)$. From Eq. (4), we can devise the surrounding algorithm in two steps. Firstly, the average position of the targets is gained by distributed estimates. Secondly, the vector $h_i(t)$ is approximated to the vector $k[r_i(t)-p_i(t)]$, which is the relative position of the i-th target to the i-th average position of the targets.

Theorem 1 Suppose that all the targets are static. If graph $\mathcal{G}(t)$ is connected, using algorithm (3) for system (1), we have $\lim_{t\to\infty}|h_i(t)-k[r_i-p_i(t)]|=0$ for all $i=1,\ldots,n$.

Proof Theorem 1 can be viewed as a special case of Theorem 2, hence the proof is omitted.

Theorem 2 Suppose that all the targets are dynamic and $D^+r_i(t)\triangleq\limsup_{h\to0^+}\frac{1}{h}[r_i(t+h)-r_i(t)]\le\beta$. If graph $\mathcal{G}(t)$ is connected, and $\alpha>\beta(n-1)$, using algorithm (3) for system (1), we have $\lim_{t\to\infty}|h_i(t)-k[r_i(t)-p_i(t)]|=0$ for all $i=1,\ldots,n$.

Proof Consider the following Lyapunov function candidate:

$$V(t)=\frac{1}{2}\sum_{i=1}^{n}\left\{\varphi_i(t)+r_i(t)-\frac{1}{n}\sum_{k=1}^{n}[\varphi_k(t)+r_k(t)]\right\}^2.$$

Calculating $\dot{V}(t)$, we have

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{n}\left[\varphi_i(t)+r_i(t)-\frac{1}{n}\sum_{k=1}^{n}(\varphi_k(t)+r_k(t))\right]\\ &\quad\times\left\{\alpha\sum_{j\in\mathcal{N}_i(t)}\frac{p_j(t)-p_i(t)}{|p_j(t)-p_i(t)|}+\dot{r}_i(t)\right.\\ &\quad\left.-\frac{1}{n}\sum_{k=1}^{n}(\dot{\varphi}_k(t)+\dot{r}_k(t))\right\}. \quad (5)\end{aligned}$$

Noting that $(p_i(t)-p_j(t))/|p_i(t)-p_j(t)|$ is a noncontinuous odd function, we obtain

$$\begin{aligned}&\alpha\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}\left[\varphi_i(t)+r_i(t)\right.\\ &\left.-\frac{1}{n}\sum_{k=1}^{n}\varphi_k(t)+r_k(t)\right]\frac{p_j(t)-p_i(t)}{|p_j(t)-p_i(t)|}\\ &=\frac{\alpha}{2}\sum_{i=1}^{n}\left\{\sum_{j\in\mathcal{N}_i(t)}\left[\varphi_i(t)+r_i(t)-\sum_{k=1}^{n}(\varphi_k(t)+r_k(t))\right]\right.\\ &+\sum_{i\in\mathcal{N}_j(t)}\left[\varphi_j(t)+r_j(t)\right.\\ &\left.\left.-\sum_{k=1}^{n}(\varphi_k(t)+r_k(t))\right]\right\}\frac{p_j(t)-p_i(t)}{|p_j(t)-p_i(t)|}\\ &=\frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}\{\varphi_i(t)+r_i(t)\end{aligned}$$

$$-[\varphi_j(t)+r_j(t)]\}\frac{p_j(t)-p_i(t)}{|p_j(t)-p_i(t)|}. \quad (6)$$

Since $|D^+r_i(t)|\le\beta$, we obtain

$$\begin{aligned}
&\left[\varphi_i(t)+r_i(t)-\frac{1}{n}\sum_{k=1}^{n}(\varphi_k(t)+r_k(t))\right]\times\dot{r}_i(t)\\
&\le\beta\left|\sum_{i=1}^{n}\varphi_i(t)+r_i(t)-\frac{1}{n}\sum_{k=1}^{n}(\varphi_k(t)+r_k(t))\right|\\
&=\frac{\beta}{n}\sum_{i=1}^{n}\left|n[\varphi_i(t)+r_i(t)]-\sum_{k=1}^{n}(\varphi_k(t)+r_k(t))\right|\\
&\le\frac{\beta}{n}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\left|\varphi_i(t)+r_i(t)-[\varphi_j(t)+r_j(t)]\right|\\
&\le\beta\max_{i=1,\ldots,n}\sum_{j=1,j\neq i}^{n}\left|\varphi_i(t)+r_i(t)-[\varphi_j(t)+r_j(t)]\right|\\
&\le\beta(n-1)\max_{i=1,\ldots,n,i\neq j}\left|\varphi_i(t)+r_i(t)-[\varphi_j(t)+r_j(t)]\right|\\
&\le\frac{\beta(n-1)}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}\left|\varphi_i(t)+r_i(t)-[\varphi_j(t)+r_j(t)]\right|. \quad (7)
\end{aligned}$$

We know that $\sum_{i=1}^{n}\dot{\varphi}_i(t)=0$ and $\sum_{i=1}^{n}\varphi_i(0)=0$, because $\mathcal{G}(t)$ is undirected and connected and $\varphi_i(0)=0$. Thus $\sum_{i=1}^{n}\varphi_i(t)=0$, then

$$\begin{aligned}
&\frac{1}{n}\sum_{i=1}^{n}\left[\varphi_i(t)+r_i(t)-\frac{1}{n}\sum_{k=1}^{n}(\varphi_k(t)+r_k(t))\right]\\
&\times\sum_{j=1}^{n}[\dot{\varphi}_j(t)+\dot{r}_j(t)]\\
&=\frac{1}{n}\sum_{i=1}^{n}\left[r_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)\right]\times\sum_{j=1}^{n}\dot{r}_j(t)=0. \quad (8)
\end{aligned}$$

From Eqs. (6)–(8), we have

$$\begin{aligned}
\dot{V}(t)\le&\frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}\{\varphi_i(t)+r_i(t)-[\varphi_j(t)+r_j(t)]\}\\
&\times\frac{\varphi_j(t)+r_j(t)-[\varphi_i(t)+r_i(t)]}{|\varphi_j(t)+r_j(t)-[\varphi_i(t)+r_i(t)]|}\\
&+\frac{\beta(n-1)}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}\left|\varphi_j(t)+r_j(t)-[\varphi_i(t)+r_i(t)]\right|\\
=&\left(\frac{\beta(n-1)}{2}-\frac{\alpha}{2}\right)\\
&\times\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}\left|\varphi_j(t)+r_j(t)-[\varphi_i(t)+r_i(t)]\right|. \quad (9)
\end{aligned}$$

Since $\alpha>\beta(n-1)$, $\dot{V}(t)\le0$. By La Salle's invariant principle, we have $\lim_{t\to\infty}|p_i(t)-p_j(t)|=0$ for all $i=1,\ldots,n$. Thus we can obtain $\lim_{t\to\infty}|p_i(t)-\frac{1}{n}\sum_{k=1}^{n}r_k(t)|=0$ for all t.

We are now in a position to prove that all $p_i(t)$ will converge to the common point in finite time. It is clear that $p_i(t)-\frac{1}{n}\sum_{i=1}^{n}p_k(t)\le\frac{1}{n}\sum_{j=1}^{n}|p_i(t)-p_j(t)|$. Let $i_0,j_0\in\mathcal{I}=\{1,\ldots,n\}$, we define $|p_{i_0}(t)-p_{j_0}(t)|=\max_{i,j\in\mathcal{I}}|p_i(t)-p_j(t)|$ at time t. Noting that $\frac{1}{n}\sum_{j=1}^{n}|p_i(t)-p_j(t)|\le|p_{i_0}(t)-p_{j_0}(t)|\le\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}|p_i(t)-p_i(t)|$, we have

$$\begin{aligned}
p_i(t)-\frac{1}{n}\sum_{i=1}^{n}p_k(t)\le&\frac{1}{n}\sum_{j=1}^{n}|p_i(t)-p_j(t)|\\
\le&\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i(t)}|p_i(t)-p_i(t)|. \quad (10)
\end{aligned}$$

Form Eqs. (9) and (10), we have

$$\begin{aligned}
\frac{\dot{V}(t)}{\sqrt{V(t)}}\le&\frac{-\frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}|\varphi_i(t)+r_i-[\varphi_j(t)+r_j]|}{\frac{\sqrt{n}}{\sqrt{2}}|p_{i_0}(t)-p_{j_0}(t)|}, \quad (11)\\
\le&-\frac{\alpha}{\sqrt{2n}}<0. \quad (12)
\end{aligned}$$

By calculating the integrations in both sides of Eq. (11), we have

$$\sqrt{V(t)}-\sqrt{V(0)}\le-\frac{[\beta(n-1)-\alpha]}{\sqrt{2n}}t.$$

Thus $V(t)$ converges to 0 in finite time. So we can obtain the average position of the targets in finite time.

Now we will prove that the vector $h_i(t)$ is approximated to the vector $k[r_i(t)-p_i(t)]$. Define $e_i(t)=h_i(t)-k[r_i(t)-p_i(t)]$. Consider the following Lyapunov function candidate:

$$\widetilde{V}(e_i(t))=\frac{1}{2}\|e_i(t)\|_2=\frac{1}{2}\sum_{i=1}^{n}[h_i(t)-k(r_i(t)-p_i(t))]^2.$$

Note that $\widetilde{V}(e_i)\ge0$. To make it clear, we define $s_i(t)=r_i(t)-p_i(t)$. Calculating $\dot{V}(e_i)$, we have

$$\begin{aligned}
\dot{\widetilde{V}}(e_i(t))=&\sum_{i=1}^{n}(h_i(t)-ks_i(t))\times(\dot{h}_i(t)-k\dot{s}_i(t))\\
=&-\sum_{i=1}^{n}(h_i(t)-ks_i(t))^2\\
&+k\sum_{i=1}^{n}(h_i(t)-ks_i(t))\times\dot{\varphi}_i(t)\\
=&-\sum_{i=1}^{n}(h_i(t)-ks_i(t))^2\le0. \quad (13)
\end{aligned}$$

By La Salle's invariant principle, we have $\lim_{t\to\infty}|h_i(t)-k[r_i(t)-p_i]|=0$, for all $i=1,\ldots,n$. According to Theorem 2, we can obtain that algorithm (3) implements Definition 1.

Remark 1 In this paper, it is assumed that the numbers of the agents and the targets are the same. Our future work would be directed towards the case with different numbers.

4. Simulation results

In this section, we give some numerical simulation results to show the obtained theoretical results. In the simulation, four agents are cooperatively surrounding four targets.

The communication topology switches among the four graphs in Fig. 2.

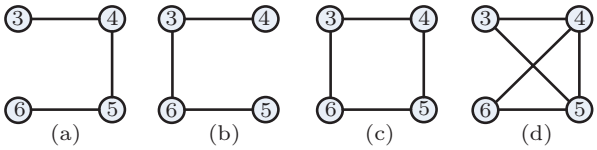


Fig. 2. Four undirected graphs: (a) $\mathcal{G}_a$, (b) $\mathcal{G}_b$, (c) $\mathcal{G}_c$, (d) $\mathcal{G}_d$.

Figure 3 shows the simulation results of the multi-agent system (1) using algorithm (3) with all the targets being static, where $\boldsymbol{r}_1 = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$, $\boldsymbol{r}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\boldsymbol{r}_3 = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$, and $\boldsymbol{r}_4 = \begin{bmatrix} 0.25 \\ -1 \end{bmatrix}$ are the positions of the four stationary targets.

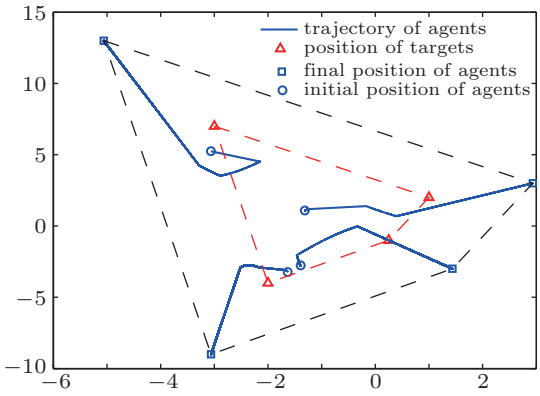


Fig. 3. (color online) Trajectories of agents using algorithm (3) for static targets.

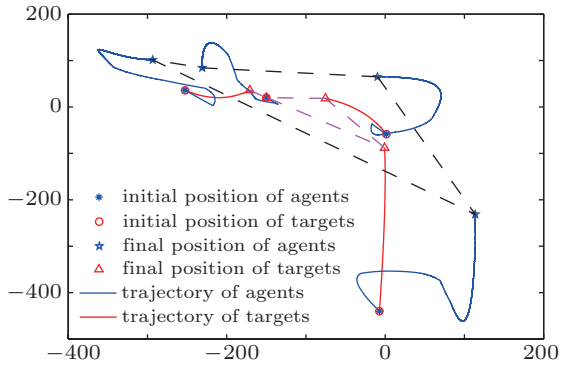


Fig. 4. (color online) Trajectories of the agents using algorithm (3) for dynamic targets.

Figures 4 and 5 show the simulation results of the multi-agent system (1) using algorithm (3) with dynamic targets, where

$$\boldsymbol{r}_1(t) = \begin{bmatrix} -0.4(t+8)^2+27 \\ -0.6(t-12)^2+28 \end{bmatrix},$$

$$\boldsymbol{r}_2(t) = \begin{bmatrix} -0.1\times(t-55)^2+50 \\ (t-4)^2+20 \end{bmatrix},$$

$$\boldsymbol{r}_3(t) = \begin{bmatrix} -0.2\times(t-6)^2 \\ -2\times(t-15)^2+10 \end{bmatrix}, \quad \boldsymbol{r}_4(t) = \begin{bmatrix} 150 \\ 20 \end{bmatrix}$$

are the positions of the four targets in Fig. 4. In Fig. 5, the trajectories of the targets on the same circumference are shown, and the radius is 50.

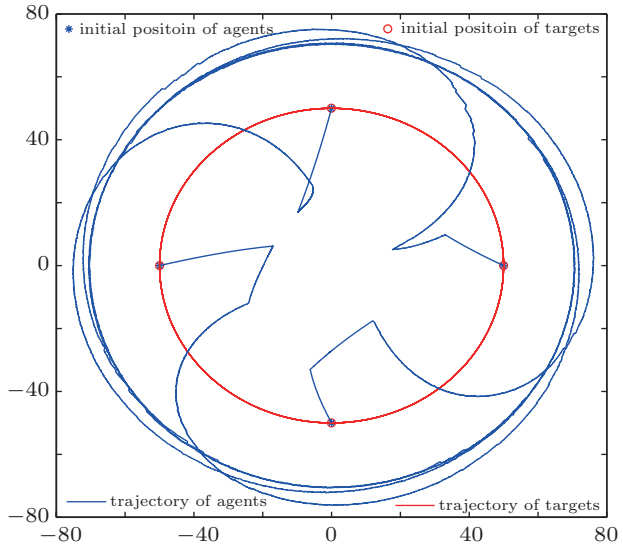


Fig. 5. (color online) Trajectories of the agents using algorithm (3) for dynamic targets with circular motion. The inner circle in Fig. 5 is the trajectories of the targets, and the outside one is the trajectories of the agents. It is clear that the agents can surround the targets by tracking the targets.

5. Conclusion

In this paper, we have studied a multi-agent cooperative control problem in which agents move collectively to surround multiple targets. To solve the problem, we used the distributed method to calculate the average position of the targets. By using the Lyapunov theory, the graph theory, and Laselle's invariance principle, we showed that the agents can surround the targets when our algorithm is applied. The proposed control law works not only for stationary targets but also for dynamic ones. The simulation results illustrate the effectiveness of our method.

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