The determination of the relative permittivity of periodic stratified media based on the iterative time-reversal method*

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In this paper, we present a new method to determine the relative permittivity of periodic stratified media using the iterative time-reversal method. Based on transmission line theory, the focal peak value of iterative time-reversal electromagnetic waves, which contain information about the periodic stratified medium, is computed in pulse-echo mode. Using the relationship between the focal peak value and the relative permittivity of the periodic stratified medium, the relative permittivity can be obtained by measuring the focal peak value. Numerical simulations are conducted, and the results demonstrate the feasibility of the proposed approach to the measurement of the relative permittivity of a periodic stratified medium.

Keywords: iterative time reversal, periodic stratified medium, peak value detection, relative permittivity determination

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1. Introduction

The stratified medium, as an approximate model, has been used to analyze numerous types of electromagnetic scattering phenomena.[1,2] Environmental monitoring, remote sensing, geological exploration, underground object orientation, and microwave circuit design are application examples of using the stratified medium model to solve electromagnetic scattering problems.[3,4] Numerous measurement methods for one-layer media have been developed. Yet, due to unknown parameters and the complexity of propagation, measurements for multi-layer media are still challenging.[5,6]

Recently, because of its adaptive focusing property, the time-reversal technique has been used in wireless communication,[7–11] nondestructive testing,[12,13] object positioning,[14,15] object imaging,[16,17] and super-resolution focusing.[18–21] The focusing signals always show autocorrelation of the propagator in the time domain. This behavior allows the time-reversal waves to exhibit adaptive focusing.[22] As a rule, the more complex the environment, the better results the time-reversal technique produces, which is advantageous as problem complexity increases.

In certain situations, the iterative time-reversal technique, a refined and improved time-reversal technique, is introduced to selectively transmit energy or information to objects, overcome the difficulty in dealing with targets that are not well separated, further eliminate the spatiotemporal sidelobes of the reconstructed pulse, and enhance the focusing capability.[23–25] The amount of research on the iterative time-reversal technique has been steadily increasing, such as Lanczos iterated time-reversal and iterative time-reversal with tunable convergence, both of which are tailored to solve the above-mentioned applications.[26,27]

In this paper, we present a new approach to the measurement of the relative permittivity of periodic stratified media using the iterative time-reversal method. The advantage of this approach is that it can measure the relative permittivity of periodic stratified media simply by utilizing the relationship between the focal peak values of iterative time-reversal waves and the relative permittivity of the periodic stratified medium. In other words, once the focal peak values of iterative time-reversal waves are measured, the relative permittivity of the periodic stratified medium can be determined, as demonstrated by numerical simulations.

2. The time-reversal model in periodic stratified media

As shown in Fig. 1, in the area of $z = z_0 < 0$, a plane wave is introduced and propagates in the $+z$ direction. The plane wave is given as follows:

$$\varphi^i(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\varphi}^i(\omega)\, \mathrm{e}^{-\mathrm{j}k(z-z_0)}\, \mathrm{e}^{\mathrm{j}\omega t}\, \mathrm{d}\omega, \tag{1}$$

where

$$\tilde{\varphi}^i(\omega) = \int_{-\infty}^{\infty} \varphi^i(z=z_0,t)\, \mathrm{e}^{-\mathrm{j}\omega t}\, \mathrm{d}t$$

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is the Fourier transform of $\varphi^i\ (z=z_0,t)$.

When the wave passes through the periodic stratified medium, the back-scattered signal is recorded, and given by

$$\varphi^r(z_0,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{11}(\omega)\,\mathrm{e}^{\mathrm{j}2kz_0}\tilde{\varphi}^i(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}\,\mathrm{d}\omega, \tag{2}$$

where $S_{11}(\omega)$ is the reflection coefficient of the electromagnetic wave at $z=0$.

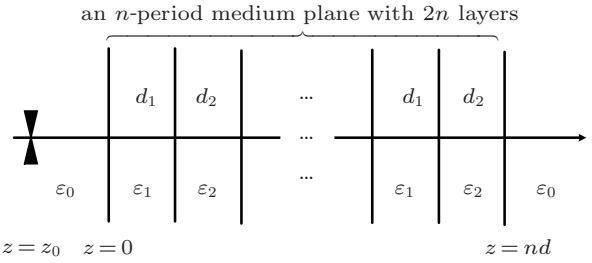


Fig. 1. The iterative time-reversal model in a periodic layered medium.

The received signal is processed with time reversal, and the expression used is

$$\begin{aligned}\varphi^i_{\mathrm{tr}}(z_0,t)&=\varphi^r(z_0,-t)\\&=\frac{1}{2\pi}\int_{-\infty}^{\infty}S^*_{11}(\omega)\,\mathrm{e}^{-\mathrm{j}2kz_0}\tilde{\varphi}^{i*}(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}\,\mathrm{d}\omega,\end{aligned} \tag{3}$$

where $S^*_{11}(\omega)$ and $\tilde{\varphi}^{i*}(\omega)$ are the phase-conjugations of $S_{11}(\omega)$ and $\tilde{\varphi}^i(\omega)$, respectively. The time-reversed electromagnetic wave then propagates through the periodic stratified medium, and the received signal is given by

$$\varphi^r_{\mathrm{tr}}(z_0,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}|S_{11}(\omega)|^2\,\tilde{\varphi}^i(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}\,\mathrm{d}\omega, \tag{4}$$

where the relationship $\tilde{\varphi}^{i*}(\omega)=\tilde{\varphi}^i(\omega)$ has been used, which means that the original signal is a time-reversal symmetric one. In fact, most signals are time-reversal-symmetric, such as sinusoidal signals, square signals, and triangular signals.

From Eq. (4), it is obvious that the time-reversed wave can eliminate the spatial phase shift due to spatial discontinuity and the dispersive property of the medium. Spatial focusing is achieved when different frequency components can arrive at $z=z_0$ with a maximum value. For a specific frequency component, the amplitude of the time-reversal wave is $|S_{11}(\omega)|^2$ times the amplitude of the wave emitted from the source, which can be easily derived by comparing Eq. (1) (note: $z=z_0$) with Eq. (4). In other words, the time-reversal technique explicitly shows that the original signal is modulated by a function that is equal to the energy-density spectrum function. Based on the Wiener–Khintchine theorem, the energy-density spectrum is the Fourier transform of the autocorrelation function. The autocorrelation function always exhibits a peak in the time domain, causing the time-reversal electromagnetic wave to feature adaptive self-focusing properties.

3. Computation of the iterative time-reversal wave peak

As discussed above, the focusing signal, after having used the time-reversal method, is a convolution of the original signal and the autocorrelation function, which is related to $|S_{11}(\omega)|^2$. In other words, research on the time-reversal focusing signal is equivalent to research on the autocorrelation function, which is usually called the time-reversal operator. The relationship between the autocorrelation function and $|S_{11}(\omega)|^2$ can be represented by

$$R(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}|S_{11}(\omega)|^2\,\mathrm{e}^{\mathrm{j}\omega\tau}\,\mathrm{d}\omega. \tag{5}$$

Based on Parseval's theorem, the maximum value of the correlation function is obtained as

$$R(0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}|S_{11}(\omega)|^2\,\mathrm{d}\omega. \tag{6}$$

This is the maximum value of the time-reversal wave when an impulse excitation is applied. Considering that $\varphi^i(z=z_0,t)$ is a finite energy signal, we define $|\omega|\le 2\pi\times10^{11}$ Hz, and its amplitude is 1 V, which satisfies our research requirements for the focusing characteristics of the time-reversal electromagnetic waves. Namely, a delta function, which is used as an excitation signal when studying the focusing characteristics of the time-reversal electromagnetic waves, is given by

$$\tilde{\varphi}^i(\omega)=\begin{cases}1, & \left(|\omega|\le 2\pi\times10^{11}\ \mathrm{Hz}\right),\\ 0, & \left(|\omega|>2\pi\times10^{11}\ \mathrm{Hz}\right).\end{cases} \tag{7}$$

As shown in Fig. 1, the periodic stratified medium is formed by n periods, and each period is composed of two simple media, whose permittivities are ε_1 and ε_2, respectively. The thickness values of the two media are d_1 and d_2, respectively. Therefore, the period of the stratified medium is $d=d_1+d_2$. As is well known, the reflection coefficient of the electromagnetic wave at $z=0$ is determined by

$$S_{11}(\omega)=\frac{Z_{\mathrm{in}}(z=0^-)-Z_0}{Z_{\mathrm{in}}(z=0^-)+Z_0}, \tag{8}$$

where $Z_0=\sqrt{\mu_0/\varepsilon_0}$ is the characteristic impedance of free space, and $Z_{\mathrm{in}}(z=0^-)$ is the input impedance at $z=0^-$. Based on the cascade theory, $Z_{\mathrm{in}}(z=0^-)$ is determined by

$$Z_{\mathrm{in}}\left(z=0^-\right)=\frac{A_{11}Z_0+A_{12}}{A_{21}Z_0+A_{22}}, \tag{9}$$

where A_{11}, A_{12}, A_{21}, and A_{22} are the transmission matrix elements, which can be written into a transfer matrix that consists of the matrix product of the transfer matrices in each period. That is,

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}=\begin{bmatrix}A'_{11} & A'_{12}\\ A'_{21} & A'_{22}\end{bmatrix}^n. \tag{10}$$

Using the transfer-matrix formalism, the transfer matrix in each period, which is a cascade of the transmission matrices in the two media, is given by

$$\begin{bmatrix} A'_{11} & A'_{12} \\ A'_{21} & A'_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta_1\cos\theta_2 - Y_2Z_1\sin\theta_1\sin\theta_2 & \mathrm{j}\,(Z_1\sin\theta_1\cos\theta_2 + Z_2\cos\theta_1\sin\theta_2) \\ \mathrm{j}\,(Y_1\sin\theta_1\cos\theta_2 + Y_2\cos\theta_1\sin\theta_2) & \cos\theta_1\cos\theta_2 - Y_1Z_2\sin\theta_1\sin\theta_2 \end{bmatrix}, \tag{11}$$

where $\theta_i = \beta_i d_i$ $(i = 1,2)$ are the electrical lengths of the equivalent transmission line of the two media, β_i $(i = 1,2)$ are the propagation constants, and $Z_i = 1/Y_i = \sqrt{\mu_0/\varepsilon_i}$ $(i = 1,2)$ are the characteristic impedances, respectively.

For convenience, we utilize Floquet's theorem to obtain the dispersion relation,

$$\cos\theta = \cos\beta d = \cos\theta_1\cos\theta_2 - \frac{1}{2}\left(Y_1Z_2\sin\theta_1\sin\theta_2 + Y_2Z_1\sin\theta_1\sin\theta_2\right), \tag{12}$$

where θ is the electrical length of each period and β is the propagation constant of the periodic structure. Using Eqs. (8)–(12), we have

$$|S_{11}(\omega)|^2 = 1 - \frac{4\sin^2\theta}{[2\sin\theta\cos(n\theta)]^2 + \sin^2(n\theta)\left[(1/\sqrt{\varepsilon_{\mathrm{r1}}} + \sqrt{\varepsilon_{\mathrm{r1}}})\sin\theta_1\cos\theta_2 + (1/\sqrt{\varepsilon_{\mathrm{r2}}} + \sqrt{\varepsilon_{\mathrm{r2}}})\cos\theta_1\sin\theta_2\right]^2}, \tag{13}$$

where $\varepsilon_{\mathrm{r1}}$ and $\varepsilon_{\mathrm{r2}}$ are the relative permittivities of the two mediums. If equation (13) is the spectrum of an excitation signal, then substituting Eq. (13) into Eq. (4) and using Parseval's theorem once more, the first iterative time-reversal focal peak value is obtained as

$$R_1(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |S_{11}(\omega)|^4\,\mathrm{d}\omega, \tag{14}$$

where $R_1(0)$ is the peak value after having performed an iterative time reversal. In this way, the peak value of the i-th iterative time-reversal wave is given by

$$R_i(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |S_{11}(\omega)|^{2(i+1)}\,\mathrm{d}\omega. \tag{15}$$

4. Results and discussion

The iterative time-reversal focal peak values are shown in Fig. 2, with the parameters given as $d_1 = 5$ mm, $d_2 = 7.5$ mm, $n = 10$. From Fig. 2, it can be seen that the peak values of the iterative time-reversal waves decrease as the iteration increases.

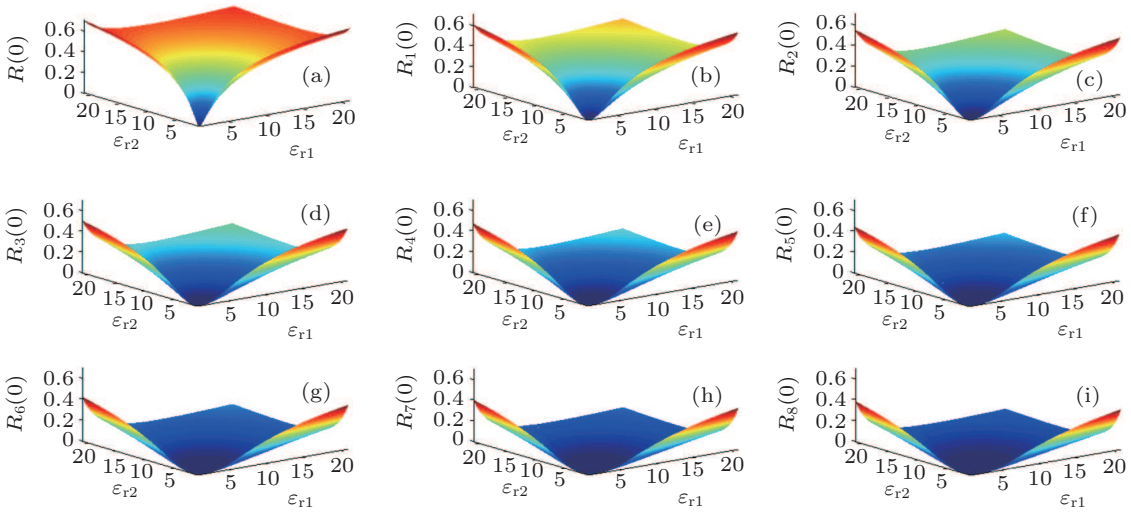


Fig. 2. (color online) The relationships between the peak value of the iterative time-reversal wave and the relative permittivity of the periodic stratified medium with time iterations. Panel (a) represents no iteration, and panels (b)–(i) show the relationships after iteration 1, 2, 3, 4, 5, 6, 7, and 8 times, respectively.

According to prior knowledge, the time-reversal iterations converge to an eigenvector corresponding to the largest eigenvalue. However, the received energy is lost after each iteration. Thus, the focal peak value decreases after each iteration. As shown in Fig. 2(a), as the values of two relative permittivities increase, the peak values of the time-reversal waves become larger. However, when the relative permittivity approaches one (the relative permittivity of the free space), then the peak values of the time-reversal waves lead to zero. The results can be easily explained by Eq. (13), which determines the peak values of the time-reversal waves. From Eq. (13), it is clear that $|S_{11}(\omega)|^2$ converges to one (the maximum value) when each of the relative permittivities tends to infinity, and $|S_{11}(\omega)|^2$ converges to zero (the minimum value) when both

of the relative permittivities are given as the relative permittivity of the free space. In fact, as a rule, in the free space a larger relative permittivity of the scatterer leads to a higher reflection energy.

Although the peak values of the time-reversal waves are related to the relative permittivities, it is not possible to determine the relative permittivities only by using the peak value of the time-reversal wave since there are many different relative permittivities corresponding to the same peak value. However, we realize that the decays in the peak values are different after having performed the time-reversal iteration each time, as is shown in Figs. 2(a)–2(i). With the time-reversal iterations, the peak values of the iterative time-reversal waves that correspond to the larger relative permittivities always decrease faster than those of the other peak values with smaller relative permittivities.

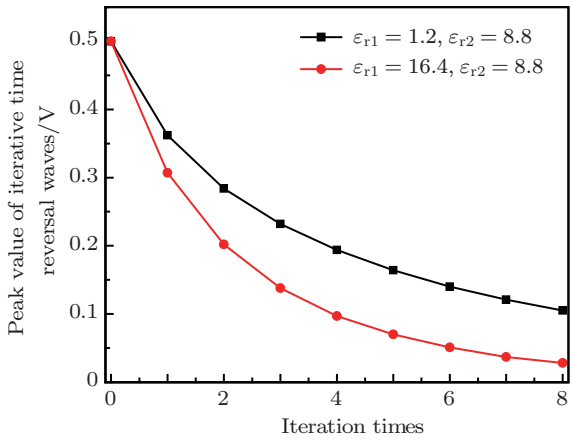


Fig. 3. (color online) Plots of the peak value of the iterative time-reversal wave versus the iteration times.

To further explain the relationship between the peak value of the iterative time-reversal waves and the iteration times, and to illustrate the iterative time-reversal method of determining the relative permittivity, the change in the peak value of the iterative time-reversal waves with iteration times is used and shown in Fig. 3, where two curves are deliberately chosen to correspond to the different relative permittivities. From Fig. 3, it is clear that the relative permittivities can be determined using the peak values of the first iterative time-reversal waves, although the relative permittivities correspond to the same peak value of the time-reversal waves, which are both shown to be 0.5 V. Furthermore, figure 3 also shows that the decay rate of the peak value of the iterative time-reversal wave varies with iteration time. The peak values corresponding to the relative permittivities $\varepsilon_{r1} = 1.2$ and $\varepsilon_{r2} = 8.8$ decrease more slowly than the peak values corresponding to the relative permittivities $\varepsilon_{r1} = 16.4$ and $\varepsilon_{r2} = 8.8$.

The above results are expected. When the electromagnetic wave is incident on an interface between air and the media whose relative permittivities are $\varepsilon_{r1} = 1.2$ and $\varepsilon_{r2} = 8.8$, there is less reflection on the interface and more energy penetrates the media, which means that more paths are received than the media whose relative permittivities are $\varepsilon_{r1} = 16.4$ and $\varepsilon_{r2} = 8.8$, as shown in Fig. 4. In most cases, the richer the multi-paths, the better the time-reversal focusing effects are, which leads to the conclusion that the peak values of the iterative time-reversal waves corresponding to the smaller relative permittivities always decrease more slowly than the other peak values of the waves with larger relative permittivities, as the iteration times are increased.

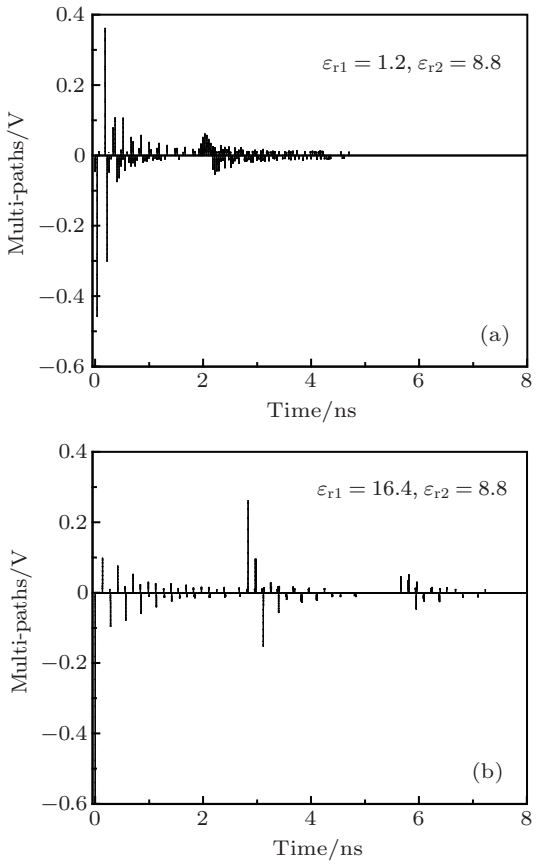


Fig. 4. Multi-paths with different relative permittivities. Panel (a) is for the case of relative permittivities $\varepsilon_{r1} = 1.2$, $\varepsilon_{r2} = 8.8$, and panel (b) is for the case of relative permittivities $\varepsilon_{r1} = 16.4$, $\varepsilon_{r2} = 8.8$. There are more paths in panel (a) than panel (b), which means that panel (a) has a better time-reversal focusing effect.

Typically, figure 5 shows the process of determining the relative permittivities based on the iterative time-reversal method. As mentioned above, firstly, if the peak value of the time-reversal waves is measured to be 0.5 V, which is shown in Fig. 5(a), it is clear that there are many (248) pairs of relative permittivities (a pair of relative permittivity means $(\varepsilon_{r1}, \varepsilon_{r2})$) corresponding to the same peak. In this case, the relative permittivities cannot be determined only by the peak value of the time-reversal wave. Therefore, after applying another time-reversal process, the peak values of the first iterative time-reversal waves, which correspond to the 248 pairs of relative permittivities, are different.

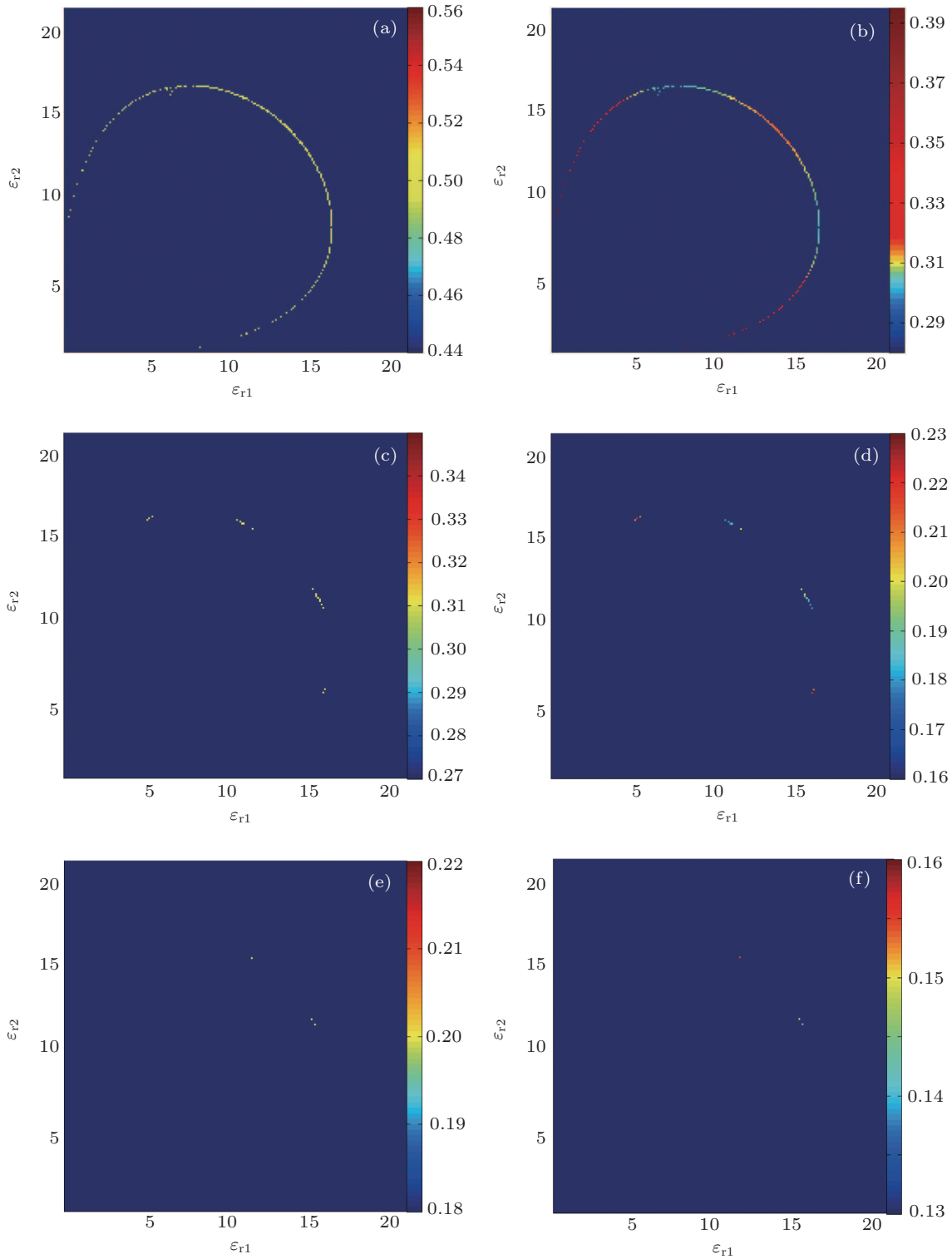


Fig. 5. (color online) The process of determining the relative permittivities. Panels (a), (c), and (e) are for the cases: the peak values of the iterative time-reversal waves are chosen to be 0.50 V, 0.31 V, and 0.20 V, respectively. Panels (b), (d), and (f) show the changes in the peak values of the iterative time-reversal waves after having performed iterative time-reversal 1, 2, and, 3 times, respectively.

As shown in Fig. 5(b), the peak values of the first iterative time-reversal waves, which correspond to the 248 pairs of relative permittivities, vary from 0.300 V to 0.395 V. Then, if the measured value is 0.31 V, we find only 18 pairs of relative permittivities corresponding to the same peak value of the first iterative time-reversal wave, which is shown in Fig. 5(c). Using the different decay rates, the peak values of the second iterative time-reversal waves, which correspond to the 18 pairs of relative permittivities, range from 0.18 V to 0.22 V, as presented in Fig. 5(d). In this case, if the measured peak value of the second iterative time-reversal wave is 0.20 V, then there are three pairs of relative permittivities that correspond to the

same value, as shown in Fig. 5(e).

Finally, when the third iterative time-reversal is carried out, it is clear that the peak values of the third iterative time-reversal wave are 0.145 V, 0.150 V, and 0.155 V, which is shown in Fig. 5(f). In this way, only one pair of relative permittivities corresponds to the peak value. Consequently, the relative permittivities are determined by using the iterative time-reversal method.

There are still two points that should be noted here. First, if the thickness values of the two media are chosen to be equal, namely, $d_1 = d_2$, then it is clear that equation (13) is symmetric, which means that we can determine the pair of relative permittivities, but cannot determine which one belongs to ε_{r1}. Second, if one of the relative permittivities is known, then the process of determination can be simplified; particularly on the assumption that if one of the media is air, then the other relative permittivity can be measured by utilizing the peak value of the time-reversal wave, and the iterative time-reversal method is not needed since there is a one-to-one relationship between the relative permittivity and the peak value of the time-reversal wave.

5. Conclusions

In this paper, an iterative time-reversal method is proposed to determine the relative permittivities of periodic stratified media. Using transmission line theory, a relationship between the peak value of the iterative time-reversal wave and the relative permittivities is obtained. Using the relationship, we demonstrate the process of determining the relative permittivities based on the iterative time-reversal method. As expected, utilizing the different decay rates of the peak value of the iterative time-reversal wave with its corresponding iteration times, the iterative time-reversal method can successfully determine the relative permittivities.

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