Roll stability catastrophe mechanism of a flooded ship on regular sea waves*

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(Received 9 October 2013; revised manuscript received 25 November 2013; published online 20 February 2014)

Based on a typical one-free-degree ship roll motion equation, the cusp catastrophe model is built including the bifurcation set equation, splitting factor 'u' and regular factor 'v', where both 'u' and 'v' are further expressed with typical flooded ship parameters. Then, the roll catastrophe mechanism is analyzed mainly by means of 'u', under the given parameters of a typical trawler boat. The aim of this research is to reveal the mutagenic mechanism of the roll stability and provide a reference for improving ship roll stability.

Keywords: flooded ship, roll stability, catastrophe mechanism, splitting factor

PACS: 42.20.Ky, 02.30.Oz **DOI:** 10.1088/1674-1056/23/4/044703

1. Introduction

Roll motion is closely connected to ship safety, which might be seriously threatened by a failure involved with ship stability.[1–4] This failure may be caused by excessive cargo loads, cargo shifting, water on deck, etc.[5] In the worst case, capsizing or structural failure may occur as a result of these factors. Compared with other types of motions such as surge, sway, yaw, and pitch, roll motion and stability research in regular waves is of great practical importance. In this paper, we use the catastrophe theory developed by Rene Thom[6] to study roll stability, with the aim of finding how to use catastrophe theory to reveal the mechanism of ship roll capsizing. To the best of our knowledge, research in this respect has been little reported. The rest of this paper is organized as follows. In Section 2 a typical one-free-degree ship roll motion equation is briefly discussed. The catastrophe theory is discussed briefly and a catastrophe model about a one-free-degree ship roll motion is established in Section 3. In Section 4 an analysis of transverse stability based on catastrophe theory is presented. Finally some roll stability conclusions are drawn from the present study in Section 5.

2. A typical one-free-degree ship motion equation

In this paper, the nonlinear equation of ship rolling on regular waves established by Nayfeh is used, where roll moment is assumed to be related to the movement of both the roll and the wave. In addition, the additional roll inertia moment is assumed to be constant and the water wavelength is much larger than the size of the ship. θ is the absolute roll angle, and η is the slope of wave surface. According to Newton's second law of motion, a nonlinear ship roll motion equation can be established as follows:[7,8]

$$J\cdot\ddot{\theta} = -J\cdot(\ddot{\theta}-\ddot{\eta}) - M_{\mathrm{B}}(\dot{\theta}-\dot{\eta}) - M_{\mathrm{C}}(\theta-\eta), \tag{1}$$

where $\ddot{\theta}$ is the second-order derivative of θ with respect to time t, J is the rotational moment of inertia, ΔJ is the additional rotational moment of inertia, M_{B} is the nonlinear damping torque, M_{C} is the nonlinear restoring moment.

Nonlinear restoring moment M_{C} can be expressed in the form of the odd-order polynomial of $(\theta$–$\eta)$, namely

$$M_{\mathrm{C}} = D\overline{GM_0}(\theta-\eta)C_3(\theta-\eta)^3 + C_5(\theta-\eta)^5 + \cdots. \tag{2}$$

Nonlinear damping torque M_{B} can be described with a linear term plus a cubic term of $\dot{\theta}$–$\dot{\eta}$ according to Dalzell's proposal,[9] namely

$$M_{\mathrm{B}} = D_1\cdot(\dot{\theta}-\dot{\eta}) + D_3(\dot{\theta}-\dot{\eta})^3. \tag{3}$$

Usually $\varphi = \theta - \eta$ is defined as the relative roll angle. For s regular transverse wave, η can be expressed as

$$\eta = \alpha_{\mathrm{m}}\cos\Omega t, \tag{4}$$

where α_{m} is the maximum slope of the wave surface and Ω is the fluctuation frequency. Therefore, the dimensionless form of Eq. (1) can be expressed as follows:

$$\ddot{\varphi}_2\mu\dot{\varphi} + \mu_3\dot{\varphi}^3 + \omega_0^2\varphi + \alpha_3\varphi^3 = K_{\mathrm{e}}\cos\Omega t, \tag{5}$$

$$\mu = \frac{D_1}{2(J+\Delta J)},\quad \mu_3 = \frac{D_3}{2(J+\Delta J)},\quad \omega_0 = \sqrt{\frac{D\cdot\overline{GM_0}}{J+\Delta J}},$$
$$\alpha_3 = \frac{C_3}{J+\Delta J},\quad \text{and}\quad K_{\mathrm{e}} = \frac{J}{\Delta J}\alpha_{\mathrm{m}}\omega_0^2. \tag{6}$$

*Project supported by the Natural Science Foundation of Zhejiang Province of China (Grant Nos. LY12E09004 and LY13E090004) and the Science Research Foundation of Zhejiang Ocean University of China (Grant No. 201157).

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3. Outline of fold catastrophe and cusp catastrophe

Catastrophe theory, developed by French mathematician Rene Thom[6] in the 1960s, is a special branch of dynamical system theory about the phenomenon characterized by sudden shifts in behavior arising from small changes in circumstances. Catastrophes are bifurcations between different equilibria or fixed point attractors which can be classified based on the number of control parameters. As to the one-control-parameter case, the type is called "fold catastrophe" with potential function $V = x^3 + ux$. If there are two controls, then one can find "cusp catastrophe" with $V = x^4 + ux^2 + vx$. With the increase of the number of control parameters, we can find other kinds such as swallowtail catastrophe with $V = x^5 + ux^3 + vx^2 + wx$, butterfly catastrophe with $V = x^6 + ux^4 + vx^3 + wx^2 + dx$, etc. As to the most simple catastrophe, i.e., fold type, when the value of u is negative the potential function has two extrema, one is stable and the other is unstable. If the parameter u is slowly increased, the system can follow the stable minimum point. At $u = 0$ the stable and unstable extrema meet and annihilate – this is the so-called bifurcation point. At $u > 0$ a stable solution no longer exists. If a physical system is followed through a fold bifurcation, one can find that as u reaches 0, the stability at $u < 0$ is suddenly lost, and the system will make a sudden transition to a new, very different behavior. Fold catastrophe can be described in Fig. 1.

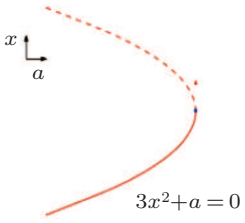


Fig. 1. (color online) Diagram of fold catastrophe.

From Fig. 2 we know that the cusp catastrophe is an expansion of fold catastrophe when the second parameter, v, is added. Varying the parameter value, one finds that there is now a dotted curve in (u, v) space where stability is lost and the stable solution will suddenly jump to an alternate outcome. The bifurcation curve loops back on itself, giving a second branch where this alternate solution itself loses stability and will make a jump back to the original solution set. By repeatedly increasing v and then reducing it, one can therefore observe hysteresis loops, as the system alternately follows one solution, jumps to the other, follows the other back, then jumps back to the first. However, this is only possible in the region of parameter space $u < 0$. As u increases, the hysteresis loops become smaller and smaller, until above $u = 0$ they disappear completely and there is only one stable solution. One can also consider what happens if one holds v constant and varies u. In the symmetrical case $v = 0$, one observes a pitchfork bifurcation as u is reduced, with one stable solution suddenly splitting into two stable solutions and one unstable solution as the physical system passes to $u < 0$ through the cusp point (0, 0).

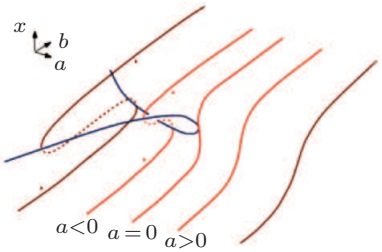


Fig. 2. (color online) Diagram of cusp catastrophe.

By comparison one knows that fold catastrophe is suitable for uncomplicated cases when only one control parameter exists. As to complex dynamic systems one can choose other kinds of catastrophes, among which the cusp catastrophe is widely chosen due to its concise equilibrium profile equation and bifurcation set equation. So far the cusp catastrophe has been successfully used to discuss the stability of nonlinear dynamic systems such as bridge collapse, fight-or-flight behavior of animals, prison riots, etc. But to the best of our knowledge, the research on stability of ship roll motion by cusp catastrophe theory is rarely reported.

4. Cusp catastrophe analysis of ship roll motion

4.1. Cusp catastrophe model

We seek a first-order uniform expansion of the solution of ship roll Eq. (6) with the multi-scale method[7] as follows:

$$\begin{aligned}\phi(t,\varepsilon) \;=\; & \phi_0(T_0,T_1,T_2)+\varepsilon\phi_1(T_0,T_1,T_2)\\ & +\varepsilon^2\phi_2(T_0,T_1,T_2)+\cdots,\end{aligned} \tag{7}$$

where $T_0 = t$ represents a fast scale characterizing motion with natural and excitation frequencies, and both $T_1 = \varepsilon t$ and $T_2 = \varepsilon^2 t$ represent the slow scale characterizing modulation and phase of the mode. The time derivatives are recast on a new time scale as

$$\begin{cases}\dfrac{\mathrm{d}\phi}{\mathrm{d}t} = D_0\phi_0+\varepsilon(D_0\phi_1+D_1\phi_0)\\ \qquad\qquad +\varepsilon^2(D_0\phi_2+D_1\phi_1+D_2\phi_0)+\mathrm{O}(\varepsilon^3),\\ \dfrac{\mathrm{d}^2\phi}{\mathrm{d}t^2} = D_0^2\phi_0+\varepsilon(D_0^2\phi_1+2D_0D_1\phi_0)+\varepsilon^2(D_0^2\phi_2\\ \qquad\qquad +2D_0D_1\phi_1+2D_0D_2\phi_0+D_1^2\phi_0)+\mathrm{O}(\varepsilon^3),\end{cases} \tag{8}$$

where $D_n = \partial/\partial T_n, n = 0,1,2$. Substituting Eqs. (7) and (8) into Eqs. (5) and (6) and equating the coefficients of similar power of ε, one can obtain the following ordinary differential equation set:

$$D_0^2\phi_0+\Omega^2\phi_0 \;=\; 0, \tag{9}$$

$$D_0^2\phi_1+\Omega^2\phi_1 = -2D_0D_1\phi_0-2\mu D_0\phi_0+\sigma\phi_0-\mu_3(D_0\phi_0)^3 -\alpha_3\phi_0^3+K_e\cos(\Omega T_0), \quad (10)$$

$$D_0^2\phi_2+\Omega^2\phi_2 = -2D_0D_2\phi_0-2D_0D_1\phi_1-D_1^2\phi_0+\sigma\phi_1 -2\mu D_0\phi_1-2\mu D_1\phi_0-3\mu_3(D_0\phi_0)^2D_0\phi_1 -3\mu_3(D_0\phi_0)^2D_1\phi_0-3\alpha_3\phi_0^2\phi_1. \quad (11)$$

The general solution of Eq. (9) can be written in the form of

$$\phi_0(T_0,T_1,T_2)=A(T_1,T_2)\,e^{i\Omega T_0}+\bar{A}(T_1,T_2)\,e^{-i\Omega T_0}, \quad (12)$$

where A is an undetermined complex function determined by imposing the solvable condition in the next approximation through eliminating the secular terms. Substituting Eq. (12) into Eq. (10), the following equation can be obtained:

$$\begin{aligned} &D_0^2\phi_1+\Omega^2\phi_1 \\ &= (\sigma a-2i\mu\Omega A-3\alpha_3A^2\bar{A}-3i\mu_3\Omega^3A^2\bar{A} \\ &\quad -2i\Omega D_1A+K_e/2)\,e^{i\omega T_0} \\ &\quad +(-\alpha_3A^3+i\mu_3\Omega^3A^3)\,e^{3i\omega T_0}+\mathrm{CC}, \end{aligned} \quad (13)$$

where CC is the complex conjugate of all the front terms. To satisfy the periodic solution condition, the coefficients of the secular terms including $\sin(\Omega T_0)$ or $\cos(\Omega T_0)$ in Eq. (13) must be zero, i.e.,

$$\sigma a-2i\mu\Omega A-3\alpha_3A^2\bar{A}-3i\mu_3\Omega^3A^2\bar{A} -2i\Omega D_1A+K_e/2=0. \quad (14)$$

To study the solution of Eq. (13), it is convenient to express A in the polar form:

$$A=\lambda(T_1,T_2)\,e^{i\beta((T_1,T_2))}/2. \quad (15)$$

Insert Eq. (15) into Eq. (14) and separate the real part from imaginary part, then we will have the following equations:

$$\begin{cases} K_e\cos\psi/2-3\alpha_3\lambda^3/8+\omega_0\lambda\dot{\beta}=0, \\ K_e\sin\psi/2-\mu\omega_0\lambda+3\mu_3\omega_0^3\lambda^3/8-\omega_0\dot{\lambda}=0. \end{cases} \quad (16)$$

Let $\psi=\sigma T_1-\beta$, and set $d\lambda/dT_1=0$ and $d\psi/dT_1=0$ in Eq. (16), we can obtain the equilibrium profile as follows:

$$\begin{cases} K_e\cos\psi/2=3\alpha_3\lambda^3/8-\omega_0\lambda\sigma, \\ K_e\sin\psi/2=\mu\omega_0\lambda-3\mu_3\omega_0^3\lambda^3/8. \end{cases} \quad (17)$$

Eliminating the phase ψ in Eq. (1), we can have the first-order approximate amplitude-frequency relation:

$$\frac{9}{16}(\alpha_3^2+\mu_3^2\omega_0^6)\lambda^6-3(\alpha_3\sigma+\mu\mu_3\omega_0^4)\lambda^4 +4\omega_0^2(\mu^2+\sigma^2)\lambda^2-K_e^2=0. \quad (18)$$

The equivalent equation to Eq. (18) is as follows:

$$\kappa_1\lambda^6+\kappa_2\lambda^4+\kappa_3\lambda^2-\frac{K_e^2}{4\Omega^2}=0, \quad (19)$$

where

$$\kappa_1=-\frac{16(\alpha_3+\mu\mu_3\omega_0^4)}{3(\alpha_3^2+\mu_3^2\omega_0^6)}, \quad \kappa_2=\frac{64\omega_0^2(\sigma^2+\mu^2)}{9(\alpha_3^2+\mu_3^2\omega_0^6)}, \quad \kappa_3=-\frac{16K_e^2}{9(\alpha_3^2+\mu_3^2\omega_0^6)}.$$

Letting $\lambda^2=s-k$ and taking Tschirnhaus transformation,[10] we can obtain the standard catastrophe manifold $s^3+2us+v=0$ from Eq. (19), where

$$v = -\frac{4}{27}\left(\frac{16(\alpha_3+\mu\mu_3\omega_0^4)}{3(\alpha_3^2+\mu_3^2\omega_0^6)}\right)^3 +\frac{1024\omega_0^2(\alpha_3+\mu\mu_3\omega_0^4)(\sigma^2+\mu^2)}{81(\alpha_3^2+\mu_3^2\omega_0^6)^2} -\frac{16K_e^2}{9(\alpha_3^2+\mu_3^2\omega_0^6)}$$

and

$$u=\frac{32\omega_0^2(\sigma^2+\mu^2)}{9(\alpha_3^2+\mu_3^2\omega_0^6)}-\frac{128(\alpha_3+\mu\mu_3\omega_0^4)^2}{27(\alpha_3^2+\mu_3^2\omega_0^6)^2}$$

are called regular factor and splitting factor, respectively.

Remark 1 As to the cusp catastrophe model described by Eq. (18), whether or not the catastrophe phenomenon arises is determined by the sign of splitting factor u. Concretely, the catastrophe phenomenon will occur as long as the sign of u becomes negative based on the cusp catastrophe mechanism described in Section 3.

4.2. Numerical example

In this subsection we investigate the case of a trawler boat, which was built in 1972 and crashed near west sea of Nordkinn Island in North Norway in 1974. The key parameters of such a boat are as follows, total length 66.01 m, distance between perpendiculars 56.85 m, ship beam 12.19 m, average draught 4.10 m, and displacement 1567 t.

Firstly we consider the case of an intact ship. The parameters corresponding to the formulas in Eq. (6) are $\mu=0.028$, μ_3=0.1659, α_3= −0.0694, and ω_0=0.4719, then we have $u=155.2593\times\sigma^2-845.1451$ and the curve of u versus σ is shown as the solid line in Fig. 3. It is shown that when $-2.3331<\sigma<2.3331$ the ship roll catastrophe phenomenon arises, which means that the roll motion becomes unstable.

Next we consider the case of a flooded ship, here only limited to the case where sea water pours into the cabin from the bottom. At this time a new free surface grows and many kinetic parameters will change such as the initial metacentric height decreasing and an extra heeling moment appearing as shown in Fig. 4; however, the most important change is the reduction of initial metacentric height, i.e. $\overline{GM_0}$, which directly influences the ship roll angular frequency $\omega_0=\sqrt{D\,\overline{GM_0}/(J+\Delta J)}$. As to the elementary stage of the

ship being damaged, it is acceptable to suppose that only the initial metacentric height changes and here we assume a decrease of 10% in $\overline{GM_0}$. With the analysis step above, we have $\omega_0 = 0.4247$ and the dotted curve shown in Fig. 3 means that the flooded ship's roll catastrophe phenomenon will come into being when $-2.2483 < \sigma < 2.2483$.

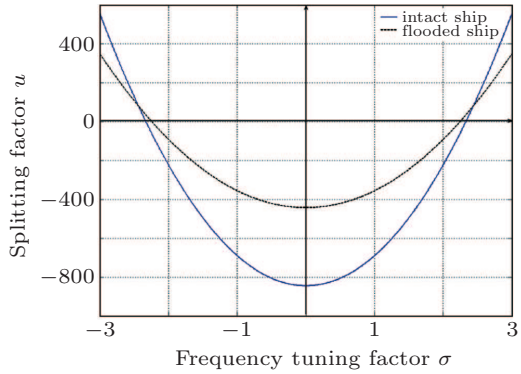


Fig. 3. (color online) Curves of splitting factor u versus frequency tuning factor σ.

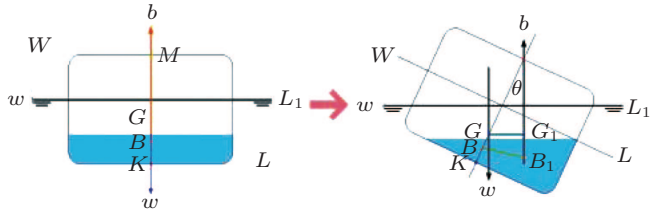


Fig. 4. (color online) Influence of free surface on a flooded ship's roll stability.

5. Conclusions

In this paper, a typical one-free-degree ship roll motion equation and cusp catastrophe theory are described, and then the roll catastrophe model is established, including theoretical derivations of both splitting factor u and regular factor v. Finally, we take a set of typical ship model parameters as an example and use splitting factor u expression to study the concrete influence of the frequency tuning factor σ on roll stability. This study aims at disclosing the concrete reasons why ship roll leads to capsizing from the viewpoint of catastrophe theory. Especially for the process of a ship being flooded, splitting factor u will change sharply along with ship hull deformation and increasing water, and we can judge the roll stability by means of splitting factor u. For a flooded ship, it is of great significance to seek effective methods against roll catastrophe by changing the sign of splitting factor u, where bifurcation control[11–20] should be considered too, which is our key task in the future.

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