A fractal approach to low velocity non-Darcy flow in a low permeability porous medium*

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In this paper, the mechanism for fluid flow at low velocity in a porous medium is analyzed based on plastic flow of oil in a reservoir and the fractal approach. The analytical expressions for flow rate and velocity of non-Newtonian fluid flow in the low permeability porous medium are derived, and the threshold pressure gradient (TPG) is also obtained. It is notable that the TPG (J) and permeability (K) of the porous medium analytically exhibit the scaling behavior $J \sim K^{-D_T/(1+D_T)}$, where D_T is the fractal dimension for tortuous capillaries. The fractal characteristics of tortuosity for capillaries should be considered in analysis of non-Darcy flow in a low permeability porous medium. The model predictions of TPG show good agreement with those obtained by the available expression and experimental data. The proposed model may be conducible to a better understanding of the mechanism for nonlinear flow in the low permeability porous medium.

Keywords: fractal, porous media, non-Darcy flow, threshold pressure gradient, scaling law

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1. Introduction

The transport properties of porous media have received great attention in many fields since Darcy's law presented the basic theory for flow through porous media.[1–8] However, some experiments[9–13] on low velocity flow in low permeability porous media are not consistent with the classical Darcy's law in the presence of threshold pressure gradient (TPG). Prada and Civan[11] proposed a modified Darcy's law:

$$Q = \frac{KA}{\mu}\left(\frac{\Delta P}{L_0} - J\right) \quad \text{for} \quad \frac{\Delta P}{L_0} > J, \tag{1}$$

where J is the TPG, $\Delta P/L_0$ is the applied pressure gradient, K is the permeability, μ is the viscosity, L_0 is the straight-line length of a porous sample, Q is the flow rate, and A is the cross-sectional area. In Eq. (1), $Q = 0$ for $\Delta P/L_0 \leq J$.

Prada and Civan[11] also proposed an empirical correlation for calculating TPG

$$J = 16\,(K/\mu)^{-0.8}. \tag{2}$$

Equation (2) indicates that the TPG depends on the permeability and fluid viscosity. However, the constants 16 and -0.8 in Eq. (2) are empirical values, and the J does not have the dimension defined in Eq. (1). Wang *et al.*[14] derived an expression for TPG as follows:

$$J = \tau_0\sqrt{\phi/2K}. \tag{3}$$

where ϕ is the porosity, τ_0 is the yield stress. Equation (3) denotes that the TPG is proportional to yield stress, porosity, and inversely proportional to permeability.

The non-Darcy flow (NDF) mechanism at low velocity has always been a controversial subject; some investigators[14–18] explored the mechanisms and the influence factors on TPG, and other researchers[6,19,20] proposed many assumptions and models. Based on extensive simulations and experimental studies on NDF in reservoirs, most of researchers believed that this phenomenon is attributed to the anomalous boundary layers of fluid.[16,21] According to this viewpoint, the boundary water may be considered to be a crystalline substance formed by the action of the remaining force field on the solid surface. However, in the early 1970s this viewpoint was refused by some researchers including the advocators.[22] Based on colloid chemistry theory, the low velocity NDF of liquid in reservoirs may be attributed not to the anomalous boundary layers, but to the plastic flow caused by colloid substances in the reservoir rocks entering into the fluid.[15] Derjaguin *et al.*[22] also presented a similar conclusion based on their experimental analyses.

In recent years, the flows and displacements of different kinds of crude oils with non-Newtonian and yield-stress (plastic fluids) rheology properties have received considerable attention for their various industrial applications,[23–26] and some studies reported on the existence of the non-Newtonian behavior of crude oil.[14,27,28] Thus, for the heavy, gelled crude oil reservoir with low porosity and low permeability, it may be reasonably assumed that the crude oil flow in a reservoir is plastic flow.

In this paper, we present an analytical model for predicting TPG for flow in a low porosity and low permeabil-

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ity porous medium based on plastic flow and fractal theory. As non-Darcy seepage behavior at high velocity is not covered in this work, interested readers may consult the recent references.[29–31]

2. Fractal characteristics of porous media

It has been shown that many natural porous media have fractal characteristics,[32–37] and the transport properties of porous media have been successfully analyzed by taking the fractal approach.[5,38–42] The total number N of pores in a fractal porous medium follows the relation[32]

$$N(\varepsilon \geq \lambda) = \left(\frac{\lambda_{\max}}{\lambda}\right)^{D_{\mathrm{f}}}, \tag{4}$$

where λ, $\lambda_{\max}$, and ε are the pore diameter, maximum pore diameter, and the length scale, the values of pore fractal dimension D_{f} are in ranges of $0 < D_{\mathrm{f}} < 2$ and $0 < D_{\mathrm{f}} < 3$ in two- and three-dimensional spaces respectively. Differentiating Eq. (4) with respect to λ results in the number of pores whose sizes are within an infinitesimal range from λ to $\lambda + \mathrm{d}\lambda$,

$$-\mathrm{d}N = D_{\mathrm{f}} \lambda_{\max}^{D_{\mathrm{f}}} \lambda^{-(D_{\mathrm{f}}+1)} \mathrm{d}\lambda, \tag{5}$$

where $-\mathrm{d}N > 0$, the negative sign in Eq. (5) implies that the pore number decreases with the increase of pore size. Yu and Li[43] pointed out that $(\lambda_{\min}/\lambda_{\max})^{D_{\mathrm{f}}} \cong 0$ should be followed for analyzing a porous medium by fractal geometry. In general, $\lambda_{\min}/\lambda_{\max} < 10^{-2}$ is considered as an approximate criterion in a porous medium.[38,43–48]

It is also assumed that a porous medium consists of a bundle of tortuous capillary tubes with different cross-sectional areas. The tortuous capillaries can be described by the fractal scaling law:[38]

$$L_{\mathrm{t}} = L_0^{D_{\mathrm{T}}} \lambda^{1-D_{\mathrm{T}}}, \tag{6}$$

where D_{T} is the fractal dimension for tortuous capillaries, with $1 < D_{\mathrm{T}} < 2$ in two dimensions and $1 < D_{\mathrm{T}} < 3$ in three dimensions, representing the extent of convolutedness of capillary pathway for fluid flow through a medium. Equations (4)–(6) provide a basic description of the fractal characteristics of a porous medium.

3. Flow in low permeability reservoirs

From the above brief review, heavy crude oil flow in a low permeability reservoir may be considered as a time-independent plastic fluid flow. The rheological equation of time-independent plastic fluid is given by[49]

$$\tau = \tau_0 + \mu_{\mathrm{d}}\gamma \ \text{ when } \ \tau > \tau_0; \ \ \gamma = 0 \ \text{ when } \ \tau < \tau_0, \tag{7}$$

where τ is the shear stress, τ_0 is the yield stress, γ is the shear rate, and μ_{d} is the plastic viscosity. After overcoming the yield stress, the shear stress and shear rate show a linear relationship between them. At low velocity, μ_{d} is assumed to be constant.

The flow rate q for capillary flow of a time-independent plastic fluid can be calculated by using the Buclingham–Reibe equation[49]

$$q = \frac{\pi r^4}{8\mu_{\mathrm{d}} L_{\mathrm{t}}} \left[\Delta P - \frac{4}{3}\frac{2L_{\mathrm{t}}\tau_0}{r} + \frac{1}{3}\left(\frac{2L_{\mathrm{t}}}{r}\right)^4 \frac{\tau_0^4}{\Delta P^3}\right], \tag{8}$$

when $\tau_0 = 0$, Eq. (8) is reduced to the Hagen–Poiseuille equation for Newtonian fluid flow in a circular tube. If the third term on the right-hand side of Eq. (8) is neglected, then we have

$$q(\lambda) = \frac{\pi}{128}\frac{\lambda^4}{\mu_{\mathrm{d}} L_{\mathrm{t}}}\left(\Delta P - \frac{16}{3}\frac{L_{\mathrm{t}}\tau_0}{\lambda}\right). \tag{9}$$

According to Eqs. (5) and (9), the total flow Q can be obtained by integral over a set of fractal pores as

$$\begin{aligned} Q &= -\int_{\lambda_{\min}}^{\lambda_{\max}} q(\lambda)\,\mathrm{d}N(\lambda) \\ &= \frac{KA}{\mu_{\mathrm{d}}}\left(\frac{\Delta P}{L_0} - \frac{16\tau_0}{3}\frac{3 + D_{\mathrm{T}} - D_{\mathrm{f}}}{3 - D_{\mathrm{f}}}\frac{\lambda_{\max}^{-D_{\mathrm{T}}}}{L_0^{1-D_{\mathrm{T}}}}\right), \end{aligned} \tag{10}$$

where

$$K = \frac{\pi}{128}\frac{L_0^{1-D_{\mathrm{T}}}}{A}\frac{D_{\mathrm{f}}}{3 + D_{\mathrm{T}} - D_{\mathrm{f}}}\lambda_{\max}^{3+D_{\mathrm{T}}}, \tag{11}$$

and the criterion $(\lambda_{\min}/\lambda_{\max})^{D_{\mathrm{f}}} \cong 0$[43] and the average tortuosity fractal dimension for fluid flow in a porous medium have been used for obtaining Eq. (10). Equation (10) is the total flow rate in a low permeability porous medium, and Eq. (11) is the fractal permeability of a porous medium.[38]

We determine the TPG

$$J = \frac{16\tau_0}{3}\frac{3 + D_{\mathrm{T}} - D_{\mathrm{f}}}{3 - D_{\mathrm{f}}}\frac{\lambda_{\max}^{-D_{\mathrm{T}}}}{L_0^{1-D_{\mathrm{T}}}} \tag{12}$$

from Eq. (11). Only when $\Delta P/L_0 > J$ can the fluid flow.

Combining the expression of permeability and TPG, and using Eq. (15) we have

$$\begin{aligned} J = {} & \frac{16\tau_0}{3}\frac{3 + D_{\mathrm{T}} - D_{\mathrm{f}}}{3 - D_{\mathrm{f}}} L_0^{(D_{\mathrm{T}}-1)/(1+D_{\mathrm{T}})} \\ & \times \left(\frac{1}{32}\frac{\phi}{1-\phi}\frac{2 - D_{\mathrm{f}}}{3 + D_{\mathrm{T}} - D_{\mathrm{f}}}\right)^{D_{\mathrm{T}}/(1+D_{\mathrm{T}})} \\ & \times K^{-D_{\mathrm{T}}/(1+D_{\mathrm{T}})}. \end{aligned} \tag{13}$$

From Eq. (13), a scaling law $J \sim K^{-D_{\mathrm{T}}/(1+D_{\mathrm{T}})}$ can be found clearly. For the $1 < D_{\mathrm{T}} < 3$ in three-dimensional space, the scaling exponent follows $0.5 < D_{\mathrm{T}}/(1 + D_{\mathrm{T}}) < 0.75$. In Wang *et al.*'s[14] results, the exponent is only 0.5.

For straight capillaries, $D_{\mathrm{T}} = 1$, we have

$$J = \frac{16\tau_0}{3}\frac{4 - D_{\mathrm{f}}}{3 - D_{\mathrm{f}}}\frac{1}{\lambda_{\max}}. \tag{14}$$

It can be seen from Eq. (12) that the TPG is proportional to the yield stress of fluid. The TPG is also related to the pore fractal dimension, tortuosity fractal dimension, and structural parameters, i.e., L_0 and $\lambda_{\max}$. When $\tau_0 = 0$, then $J = 0$ and Eq. (10) is reduced to Darcy's law for Newtonian fluid flow in a porous medium.

The seepage velocity $V = Q/A$, where A is[36]

$$A = \frac{\pi}{4}\frac{D_{\rm f}\lambda_{\max}^2}{2-D_{\rm f}}\frac{(1-\phi)}{\phi}. \tag{15}$$

So

$$V = \frac{1}{32\mu_{\rm d}}\frac{\Delta P}{L_0^{D_{\rm T}}}\frac{2-D_{\rm f}}{3+D_{\rm T}-D_{\rm f}}\frac{\phi\lambda_{\max}^{1+D_{\rm T}}}{1-\phi} - \frac{\tau_0}{6\mu}\frac{2-D_{\rm f}}{3-D_{\rm f}}\frac{\phi\lambda_{\max}}{1-\phi}. \tag{16}$$

Then, the seepage velocity can be expressed as

$$V = a\left(\frac{\Delta P}{L_0} - J\right), \tag{17}$$

where

$$a = \frac{1}{32\mu_{\rm d}L_0^{D_{\rm T}-1}}\frac{2-D_{\rm f}}{3+D_{\rm T}-D_{\rm f}}\frac{\phi\lambda_{\max}^{1+D_{\rm T}}}{1-\phi}.$$

Equation (17) indicates that velocity in a porous medium is related to porosity, fractal dimensions, maximum pore size, and fluid property. Wang *et al.*[14] presented the general equation of motion for heavy oil flowing in a porous medium:

$$V = C\left(\frac{\Delta P}{L_0} - J\right)^m, \tag{18}$$

where C is the empirical coefficient and power index m is determined by rheological experiment.

4. Determination of the TPG

In order to determine the J from Eq. (12), it is crucial to correctly determine the fractal dimensions $D_{\rm f}$ and $D_{\rm T}$, and the structural parameters, L_0 and $\lambda_{\max}$. The maximum pore size $\lambda_{\max}$ can be given by[38]

$$\lambda_{\max} = \frac{R}{2}\left[\sqrt{\frac{2\phi}{1-\phi}} + \sqrt{\frac{2\pi}{\sqrt{3}(1-\phi)}} - 2\right], \tag{19}$$

where R is the characteristic radius of particles. In this work, we take $R = 34$ μm.[50] The fractal dimension $D_{\rm f}$ is determined by[43]

$$D_{\rm f} = 2 - \frac{\ln\phi}{\ln(\lambda_{\min}/\lambda_{\max})}, \tag{20}$$

where $\lambda_{\min}/\lambda_{\max} = 10^{-2}$ is used in the present work.

The value of $D_{\rm T}$ is determined by[51]

$$D_{\rm T} = 1 + \frac{\ln\tau_{\rm av}}{\ln(L_0/\lambda_{\rm av})}, \tag{21}$$

where $\lambda_{\rm av}$ and $\tau_{\rm av}$ are the average pore size and the averaged tortuosity, respectively, and they can be obtained by

$$\lambda_{\rm av} = \frac{D_{\rm f}\lambda_{\max}}{D_{\rm f}-1}\left[\frac{\lambda_{\min}}{\lambda_{\max}} - \left(\frac{\lambda_{\min}}{\lambda_{\max}}\right)^{D_{\rm f}}\right], \tag{22}$$

$$\tau_{\rm av} = \frac{1}{2}\left[1 + \frac{1}{2}\sqrt{1-\phi} + \frac{\sqrt{(\sqrt{1-\phi}-1)^2 + (1-\phi)/4}}{1-\sqrt{1-\phi}}\right]. \tag{23}$$

Other tortuosity–porosity relationships can be found by consulting recent papers.[52,53] In Eq. (21), L_0 is determined by extracting the root of Eq. (15).

5. Results and discussion

The TPG increases with the decrease of porosity, especially for low porosity, and the TPG increases drastically as porosity decreases (Fig. 1). However, when porosity is higher, the TPG decreases as porosity increases, and at higher porosity the TPG can be neglected. This reveals that the TPG is significant in a low porosity reservoir, and it should be considered in a porous medium with low porosity and low permeability. Besides the porosity, the TPG is also very sensitive to the tortuosity fractal dimension. The higher the tortuosity fractal dimension, the higher the TPG is. This is because a higher tortuosity fractal dimension implies the higher flow resistance, leading to the higher TPG. So, the fractal nature of tortuosity of capillaries should be taken into account for porous medium with low permeability.

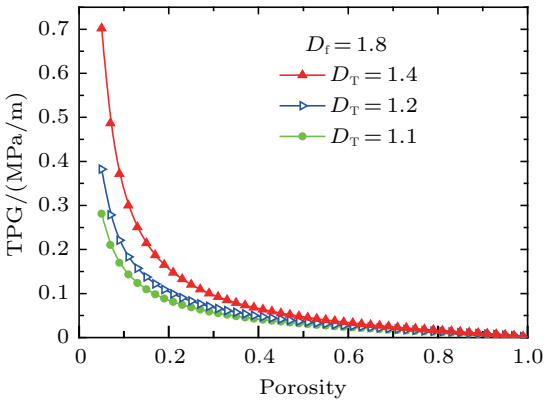


Fig. 1. (color online) Effects of fractal dimensions on TPG.

The TPG is directly related to the yield stress. The larger the yield stress, the higher the TPG will be at the same permeability or for a given medium (Fig. 2). This is expected because the TPG is closely related to the rheological properties of fluid. Particularly, there is a clear linear law relation between the TPG and the permeability from the log–log plot of Fig. 2, and the slopes of the straight lines are both 0.61 (i.e., $J \sim K^{-0.61}$), where the value of exponent (0.61) is consistent with the available result (0.8[11] and 0.5[14]).

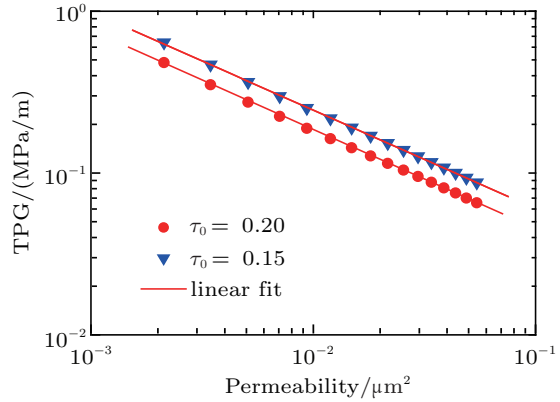


Fig. 2. (color online) The effects of yield stress on non-Darcy flow in porous media, in which the permeability is calculated by Eq. (11).

Comparisons between the predictions from the presented fractal model (Eq. (12)) and those from the available expression (Eq. (3))[14] and available experimental data[50,54,55] are shown in Figs. 3(a) and 3(b), respectively. The model predictions present good agreement with available expression and experimental data, and these verify the proposed fractal model.

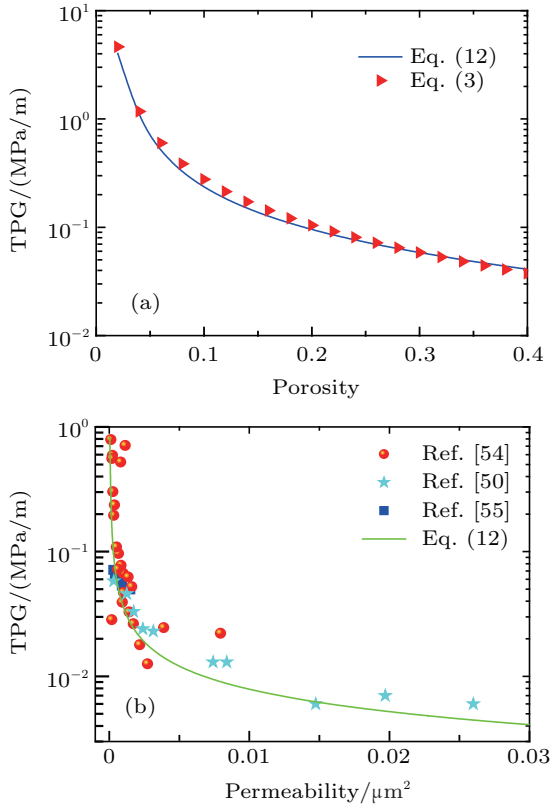


Fig. 3. (color online) Comparisons of the TPG between the present model and available expression (a) and experimental data (b), respectively.

Compared with previous fractal TPG model,[25] the model in this work does not consider capillarity, which has an important effect for enhancing the oil recovery.[36,56–60] However, we present a scaling law between the TPG and the permeability, in which the scale exponent is analytically related to the fractal dimension for tortuosity capillaries.

6. Conclusions

The conventional view on non-Darcy flow mechanisms, the studies on the different kinds of crude oil rheology properties and flow behavior for plastic fluids in porous medium are reviewed and summarized in this work. Based on the capillary bundle model and fractal geometry, a fractal model of the TPG for crude oil flow in a low permeability porous medium is established. The proposed TPG is given in terms of yield stress, pore fractal dimension, tortuosity fractal dimension, maximum pore diameter, and straight line length of porous media. A scaling law is found between the TPG and permeability of porous medium $J \sim K^{-D_T/(1+D_T)}$. The model predictions are compared with those obtained using the conventional method and available experimenalt data, and they are found to be in good agreement with each other.

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