Theory of noise in a kilo-Hz cascaded high-energy Yb-doped nanosecond pulsed fiber amplifier*

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(Received 3 February 2013; revised manuscript received 10 October 2013; published online 20 February 2014)

A theoretical analysis of noise in a high-power cascaded fiber amplifier is presented. Unlike the noise theory in low power communication, the noise of a high power system is redefined as the leaked output energy between pulses with coherent beat noise uncounted. This definition is more appropriate for high power usage in which the pulse energy receives more attention than the pulse shape integrity. Then the low power pre-amplifying stages are considered as linear amplification and analyzed by linear theory. In the high-power amplification stages, the inversion is assumed to recover linearly in the time interval between pulses. The time shape of the output pulse is different from that of the input signal because of different gains at the front and back ends of the pulse. Then, a criterion is provided to distinguish the nonlinear and linear amplifications based on the signal-to-noise ratio (SNR) analysis. Then, an experiment that shows that the output SNR actually drops off in nonlinear amplification is performed. The change in the noise factor can be well evaluated by pulse shape distortion.

Keywords: pulsed fiber amplifier, signal-to-noise ratio, cascaded amplifier, master oscillator power amplifier

PACS: 42.55.Wd, 42.60.Da, 42.55.Ah **DOI:** 10.1088/1674-1056/23/4/044214

1. Introduction

Ytterbium-doped pulsed fiber laser systems are widely used in commercial industries, replacing conventional bulk solid-state lasers. Compared with non-fiber Q-switched seeds, a direct modulated diode source provides compactness and robustness, as well as flexibility in terms of pulse parameters such as repetition rate, pulse duration, and shape. Considering that a modulated diode source can generally have an average power lower than 1 mW, cascaded amplifiers are typically required to obtain a high gain. Direct modulated diode seeded power amplifier cascades have been used to realize high peak power output with pulse durations from picoseconds to nanoseconds. However, more or less amplified self emission (ASE) output between output pulses are found, which is undesired as a background noise. A pulsed laser with too much ASE noise can cause a miscalculation of output peak power and pulse energy. The ASE noise of a high power pulsed amplifier can be measured with special techniques which will be presented in the paper.

With the development of pulsed fiber lasers, more and more extremely high records of output pulses are published including highest peak power, highest pulse energy, highest average power, etc.[1–10] However, as far as we know, the analysis on high power pulsed system back ground noise has seldom been made. And most noise theories are based on the application of fiber communications,[1,2] in which the noise is high frequency coherent interference signal that usually increases the bit error rate. However, for our high-power fiber lasers for the mechanical manufacture applications much attention has been paid to the integral energy in the pulse duration, other than the high frequency rise and fall uncertainty during pulse as for communications. The noise herein is considered as the direct current background, i.e., the ASE noise. Though the nonlinear effects of laser fiber exist, such as four-wave mixing,[3] stimulated Brillouin scattering,[4–6] and Raman scattering,[7] which makes the situation complicated. These nonlinear effects of fiber are all ignored for simplicity. This simplification is acceptable in the case where the peak power of output pulses is not too high. In Ref. [8] a numerical model analysis on low repetition rate multistage MOPA systems was made representatively. However, the ASE noise transfer between stages is not considered, and the pumping procedure is analyzed by steady state for a very low repetition rate. Therefore, this model does not hold true for a system with pulse repetition rates over several kHz.

In this paper, the ASE noise in a high-power cascaded fiber amplifier is analyzed, which is beneficial to the design and analysis of a cascaded high-power pulsed fiber laser.

*Project supported by the National High Technology Research and Development Program of China (Grant No. 2011AA8042032) and the National Natural Science Foundation of China (Grant Nos. 61077034 and 61301190).

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2. Noise theory

2.1. Noise factor definition

Prior to defining the noise factor, we define the signal-to-noise ratio (SNR) by dividing a whole working cycle into two periods. One is the seed-in period, in which a seed pulse goes through the fiber and it is amplified. The upper level population density rapidly drops off in this period. The other is the quiet period, in which the population density changes slowly and the stored energy gradually increases to its maximum by absorbing input pump power. For an output with low duty cycle ($\leq 1\%$), most ASE noise is produced in the quiet period. The ASE noise produced in the seed-in period is ignored.

The definition of high power system noise is based on the fact that we are usually interested in the actual pulse energy of a high power usage system, not the pulse shape integrity. On the other hand, the ASE noise of the seed-in period plays an important role in a low power communication system, mostly because the duty cycle of the communication system is much higher and the bit error rate of communication greatly depends on the integrity of pulse shapes. Then, the output SNR is defined as

$$\mathrm{SNR} = E_\mathrm{s}/E_\mathrm{n}, \tag{1}$$

where E_n is the output energy in the quiet period and considered as the noise energy, and E_s is the output energy in the seed-in period in one working cycle and considered as the signal energy.

As is well known, the coherent noise is the key component of the noise that causes the threshold judging errors of 0 and 1 codes in low power communication lasers. However, the coherent noise has little influence on the pulse integral in high power laser industry applications. Then, the noise definition is vastly different from that in fiber communication in which coherent noise plays a major part in noise generation. And the analysis in this paper is only for the high power kilo-Hz pulsed laser that cannot be directly used in the area of fiber communication because it can cause significant errors.

Then, the ASE noise factor of an amplifier is defined as

$$F = \mathrm{SNR}_\mathrm{in}/\mathrm{SNR}_\mathrm{out}, \tag{2}$$

where SNR_in and $\mathrm{SNR}_\mathrm{out}$ are the input and output SNRs of the amplifier, respectively.

As for a linear amplifier with a constant signal gain of G, the noise factor is

$$F_\mathrm{l} = \frac{E_\mathrm{si}}{E_\mathrm{ni}} \Big/ \left(\frac{GE_\mathrm{si}}{GE_\mathrm{ni}+E_\mathrm{na}}\right) = 1 + \frac{E_\mathrm{na}}{GE_\mathrm{ni}}, \tag{3}$$

where E_na is the noise energy added by the amplifier and E_ni is the input noise energy in one working cycle. For a nonlinear amplifier with different gains of noise and signal,

$$F_\mathrm{nl} = \frac{E_\mathrm{si}}{E_\mathrm{ni}} \Big/ \left(\frac{G_\mathrm{s}E_\mathrm{si}}{G_\mathrm{n}E_\mathrm{ni}+E_\mathrm{na}}\right) = \frac{G_\mathrm{n}}{G_\mathrm{s}}\left(1 + \frac{E_\mathrm{na}}{G_\mathrm{n}E_\mathrm{ni}}\right), \tag{4}$$

where G_n and G_s are the total gains of input noise and signal in one working cycle, respectively; G_n is defined as

$$G_\mathrm{n} = E_\mathrm{no}/E_\mathrm{ni} \tag{5}$$

with E_no being the output noise energy generated by the amplification of input noise. The total output noise is defined as

$$E_\mathrm{ntot} = E_\mathrm{no} + E_\mathrm{na}. \tag{6}$$

2.2. Noise factor of the nonlinear amplifier

For an amplifier with both high gain and high output pulse energy, the small signal gain of the amplifier tremendously fluctuates. We use the following rate equation to evaluate the signal and noise in the quiet period,[9]

$$\begin{aligned} &\frac{\partial P_\mathrm{ase}(z,t)}{\partial \mathrm{z}} + \frac{1}{V_\mathrm{s}}\frac{\partial P_\mathrm{ase}(z,t)}{\partial t} \\ &= \{\Gamma_\mathrm{s}[(\sigma_\mathrm{es}+\sigma_\mathrm{as})N_2(z,t) - N\sigma_\mathrm{as}] - \alpha\} P_\mathrm{ase}(z) \\ &\quad + 2M\sigma_\mathrm{es}N_2(z,t)\frac{hc^2}{\lambda_\mathrm{s}^3}\Delta\lambda, \end{aligned} \tag{7}$$

where P_s is the forward-propagating ASE power, Γ_s represents the power-filling factor for the signal, σ_es and σ_as represent the absorption and emission cross sections at the seed wavelength, respectively, N_2 and N represent the upper level population density and the total ion density, respectively, α is the transmitting attenuation per length unit, M is the number of modes propagating in the core, and h is Planck's constant, c is the light speed in vacuum, λ_s is the seed wavelength, and λ is the optical bandwidth of the amplifier.

Then, we assume that the stored energy increases slowly and stably. The time-dependent item is ignored. For simplicity, the fiber-transmitting attenuation and reabsorption are also ignored. Then, we obtain

$$\frac{\mathrm{d}P_\mathrm{s}(z)}{\mathrm{d}z} = \Gamma_\mathrm{s}\sigma N_2(z)P_\mathrm{s}(z) + 2M\sigma_\mathrm{es}N_2(z)\frac{hc^2}{\lambda_\mathrm{s}^3}\Delta\lambda, \tag{8}$$

where $\sigma = \sigma_\sigma + \sigma_\mathrm{e}$. This expression can be resolved as

$$\begin{aligned} P_L &= P_0 \exp\left(\frac{\Gamma_\mathrm{s}\sigma E_\mathrm{st}}{h\nu_\mathrm{s}A}\right) + \frac{2M\sigma_\mathrm{es}hc^2\Delta\lambda}{\Gamma_\mathrm{s}(\sigma_\mathrm{es}+\sigma_\mathrm{as})\lambda_\mathrm{s}^3} \\ &\quad \times \left(\exp\left(\frac{\Gamma_\mathrm{s}\sigma E_\mathrm{st}}{h\nu_\mathrm{s}A}\right) - 1\right), \end{aligned} \tag{9}$$

where

$$E_\mathrm{st} = h\nu_\mathrm{s}A\int_0^L N_2(z)\,\mathrm{d}z \tag{10}$$

is the stored energy in the fiber, P_0 is the input from the prior stage output, and P_L is the total output power of the stage. The small signal gain is defined as

$$G = \exp[(\Gamma_\mathrm{s}\sigma E_\mathrm{st})/(h\nu_\mathrm{s}A)]. \tag{11}$$

Equation (9) can be written as

$$P_L = P_0 G + \left[\left(2M\sigma_{es}hc^2\Delta\lambda\right)/\left(\sigma\Gamma_s\lambda_s^3\right)\right](G-1). \quad (12)$$

As a single-stage amplifier, let $P_0 = 0$. We obtain the noise power added by the amplifier as

$$P_{na} = P_{nei}(G-1), \quad (13)$$

where the equivalent input noise is defined as

$$P_{nei} = \left(2M\sigma_{es}hc^2\Delta\lambda\right)/\left(\sigma\Gamma_s\lambda_s^3\right). \quad (14)$$

Then, by Eq. (3), the noise factor of a linear amplifier is obtained as

$$F = 1 + (P_{nei}/P_{ni})(1 - 1/G). \quad (15)$$

By Eq. (13), the output SNR reads

$$\mathrm{SNR}_{out} = P_{so}/P_{na} = P_{si}G/[P_{nei}(G-1)], \quad (16)$$

where P_{so} is the output signal power and P_{si} is the input signal power. In addition,

$$P_{so} = GP_{si}. \quad (17)$$

By Eq. (16), we obtain

$$\lim_{G\to+\infty} \mathrm{SNR}_{out} \approx P_{si}/P_{nei}. \quad (18)$$

As for a nonlinear system, we assume that the fiber stored energy recovers linearly in the time interval between pulses. This assumption is highly applicable for most pulsed lasers with repetition rates higher than approximately 10 kHz.[10] The intervals between pulses are much less than the ytterbium spontaneous lifetime (approximately 0.8 ms) and the pump power is not too high to saturate the fiber stored energy.

$$E_{st}(t) = [(E_{stmax} - E_{stmin})/T]t + E_{stmin}, \quad (19)$$

where T is the length of the working cycle, t is the time from the beginning of a quiet period, E_{stmax} and E_{stmin} are the maximum and minimum stored energy in a working cycle, respectively. Then, by Eq. (11), we have

$$G(t) = \exp\left(\frac{\Gamma_s\sigma}{h\nu_s A}\{[(E_{stmax} - E_{stmin})/T]t + E_{stmin}\}\right). \quad (20)$$

For a system with constant input noise, the noise energy in one working cycle is

$$\begin{aligned} E_{no} &= P_{ni}\int_0^T G(t)\,\mathrm{d}t \\ &= \{(G_{max} - G_{min})/[\ln(G_{max}) - \ln(G_{min})]\}P_{ni}T, \end{aligned} \quad (21)$$

where P_{ni} is the input noise power that remains unchanged with time. (In the case that the input noise is not a constant, the stage can be analyzed by combining it with its former stages. Then these stages can be regarded as one whole nonlinear stage with constant input noise from foregoing liner stages. Or if there is no foregoing linear stage, the equivalent input noise of the first stage can be taken as the constant noise from the foregoing stage. It is analyzable), the amplifier input noise is

$$E_{ni} = P_{ni}T, \quad (22)$$

G_{max} and G_{min} are the maximum and minimum small signal gains in one working cycle and given as

$$G_{max} = \exp[(\Gamma_s\sigma E_{stmax})/(h\nu_s A)], \quad (23)$$

$$G_{min} = \exp[(\Gamma_s\sigma E_{stmin})/(h\nu_s A)]. \quad (24)$$

In a linear amplification, G_{max} is equal to G_{min}. Then, we define the pulse narrowing factor as

$$\begin{aligned} G_{narrow} &= G_{max}/G_{min} \\ &= \exp[(\Gamma_s\sigma)/(h\nu_s A)(E_{stmax} - E_{stmin})]. \end{aligned} \quad (25)$$

The pulse shape distortion is evidently more severe for a larger value of G_{narrow}. Then, by Eqs. (5), (21), and (22), the noise gain is

$$G_n = E_{no}/E_{ni} = (G_{max} - G_{min})/[\ln(G_{max}) - In(G_{min})]. \quad (26)$$

By the conservation of energy, we have

$$E_{stmax} - E_{stmin} = E_{out} - E_{in}. \quad (27)$$

Then, equation (25) can be rewritten as

$$G_{narrow} = \exp[(\Gamma_s\sigma)/(h\nu_s A)(E_{out} - E_{in})]. \quad (28)$$

Given that $E_{out} \gg E_{in}$ in a high-gain amplifier, equation (28) can be approximated as

$$G_{narrow} \cong \exp[(\Gamma_s\sigma)/(h\nu_s A)E_{out}]. \quad (29)$$

Considering the seed-in period, pulse amplification can be analyzed by the following approximate method: the input pulse is discretized into elements with the same time length, and each element of the input pulse has its corresponding element of output pulse and has its special small signal gain. Then we have

$$\Delta E_{o(k)} = G_{(k)}\Delta E_{i(k)}, \quad (30)$$

$$G_{(k)} = \exp\left[\left(\Gamma_s\sigma E_{st(k)}\right)/(h\nu_s A)\right], \quad (31)$$

where $k = 1, \ldots N$. N is the number of elements. We can obtain

$$E_{in} = \sum_{k=1}^{N} \Delta E_{i(k)}, \quad (32)$$

$$E_{out} = \sum_{k=1}^{N} \Delta E_{o(k)}. \quad (33)$$

As $N \to +\infty$, equations (30) and (31) constitute

$$\mathrm{d}E_{\mathrm{o}} = \exp\left[(\Gamma_{\mathrm{s}}\sigma E_{\mathrm{st}})/(h\nu_{\mathrm{s}}A)\right]\mathrm{d}E_{\mathrm{i}}. \tag{34}$$

The decrease in the fiber stored energy is evidently equal to the increase in the pulse energy, indicating that

$$-\mathrm{d}E_{\mathrm{st}} = \mathrm{d}E_{\mathrm{o}} - \mathrm{d}E_{\mathrm{i}}. \tag{35}$$

Then, by using Eqs. (34) and (35), we obtain

$$\mathrm{d}E_{\mathrm{st}}/(1-\exp\left[(\Gamma_{\mathrm{s}}\sigma E_{\mathrm{st}})/(h\nu_{\mathrm{s}}A)\right]) = \mathrm{d}E_{\mathrm{i}}. \tag{36}$$

Integrating on both sides

$$\int_{E_{\mathrm{st}\max}}^{E_{\mathrm{st}\min}} \frac{\mathrm{d}E_{\mathrm{st}}}{1-\exp\left[(\Gamma_{\mathrm{s}}\sigma E_{\mathrm{st}})/(h\nu_{\mathrm{s}}A)\right]} = \int_{0}^{E_{\mathrm{in}}} \mathrm{d}E_{\mathrm{i}}, \tag{37}$$

we obtain

$$\ln\left(\exp\left(-\frac{\Gamma_{\mathrm{s}}\sigma E_{\mathrm{st}\max}}{h\nu_{\mathrm{s}}A}\right)-1\right) - \ln\left(\exp\left(-\frac{\Gamma_{\mathrm{s}}\sigma E_{\mathrm{st}\min}}{h\nu_{\mathrm{s}}A}\right)-1\right) = \frac{\Gamma_{\mathrm{s}}\sigma E_{\mathrm{in}}}{h\nu_{\mathrm{s}}A}. \tag{38}$$

Then, substituting Eqs. (23) and (24) into Eq. (38), we have

$$E_{\mathrm{in}} = (h\nu_{\mathrm{s}}A)/(\Gamma_{\mathrm{s}}\sigma)\ln\left[(1-1/G_{\max})/(1-1/G_{\min})\right]. \tag{39}$$

By Eq. (27), we have

$$\begin{aligned} E_{\mathrm{out}} &= E_{\mathrm{in}} + E_{\mathrm{st}\max} - E_{\mathrm{st}\min} \\ &= (h\nu_{\mathrm{s}}A/\Gamma_{\mathrm{s}}\sigma)\ln\left[(G_{\max}-1)/(G_{\min}-1)\right]. \end{aligned} \tag{40}$$

The signal gain of the entire pulse is obtained from Eq. (40)/Eq. (39), i.e.,

$$\begin{aligned} G_{\mathrm{s}} &= \frac{E_{\mathrm{out}}}{E_{\mathrm{in}}} \\ &= \ln\left(\frac{G_{\max}-1}{G_{\min}-1}\right)\Bigg/\left[\ln\left(\frac{G_{\max}-1}{G_{\min}-1}\right)-\ln\left(\frac{G_{\max}}{G_{\min}}\right)\right]. \end{aligned} \tag{41}$$

Alternatively, by Eq. (13), the noise added by the amplifier is obtained as

$$\begin{aligned} E_{\mathrm{na}} &= \int_0^T P_{\mathrm{na}}(t)\,\mathrm{d}t = P_{\mathrm{nei}}\left(\int_0^T G(t)\,\mathrm{d}t - T\right) \\ &= P_{\mathrm{nei}}T\left\{\frac{G_{\max}-G_{\min}}{\ln(G_{\max})-\ln(G_{\min})}-1\right\}. \end{aligned} \tag{42}$$

By Eq. (4), the noise factor of a nonlinear amplifier with constant input noise is given as

$$\begin{aligned} F_{\mathrm{nl}} &= \left[\frac{1}{\ln\left(\frac{G_{\max}}{G_{\min}}\right)}-\frac{1}{\ln\left(\frac{G_{\max}-1}{G_{\min}-1}\right)}\right](G_{\max}-G_{\min}) \\ &\times\left\{1+\frac{P_{\mathrm{nei}}}{P_{\mathrm{ni}}}\left[1-\frac{\ln(G_{\max})-\ln(G_{\min})}{G_{\max}-G_{\min}}\right]\right\}. \end{aligned} \tag{43}$$

In general, the noise power added by the amplifier is much less than that generated by the input noise amplification for the stage after the first one.

This judgment is usually correct and we explain it by an example. Consider a cascade amplifier formed by 3 stages. The first stage is a single mode amplifier, of which the number of modes is 1. The mode number of the second stage is about 5. The third stage mode number is about 40. Each stage has a gain of 20 dB. Here, we assume that the fillers between stages have equal band width and no electrical optical switch is used. And the power filling factor of each stage is approximately equal.

First, if the output noise of the first stage is P_{noise1}, $P_{\mathrm{noise1}} \approx 100P_{\mathrm{noise0(1)}}$ according to Eq. (13). However, as Eq. (14) shows, equivalent input noise is proportional to the fiber mode number. Then the equivalent input noise of the second stage is $P_{\mathrm{noise0(2)}} \approx 5P_{\mathrm{noise0(1)}}$. It is obvious that $P_{\mathrm{noise1}} \gg P_{\mathrm{noise0(2)}}$, which means that the equivalent input noise of the second stage is far less than the output noise of the first stage. Therefore, the output noise of the second stage is mainly caused by amplification of the first stage output noise. The amplifier added noise of the second stage can be ignored.

Second, the output noise of the second stage is $P_{\mathrm{noise2}} = 100(P_{\mathrm{noise0(2)}} + 100P_{\mathrm{noise0(1)}}) \approx 500P_{\mathrm{noise0(1)}} + 10000P_{\mathrm{noise0(1)}} \approx 10000P_{\mathrm{noise0(1)}}$. The equivalent input noise of the third stage is $P_{\mathrm{noise0(3)}} \approx 40P_{\mathrm{noise0(1)}}$ at most. And again it is obvious that $P_{\mathrm{noise2}} \gg P_{\mathrm{noise0(3)}}$.

The key lies in the fact that the total gain of the foregoing stages is usually far greater than the mode number of the stage. Therefore the equivalent input noise of a stage is usually much less than the input noise from the foregoing stages. And the amplifier added noise can be ignored except that of the first stage. Then, by ignoring the amplifier-added noise caused by its equivalent input noise, we have $P_{\mathrm{nei}} \ll P_{\mathrm{ni}}$ and

$$\begin{aligned} F_{\mathrm{nl}} &\cong \frac{G_{\mathrm{n}}}{G_{\mathrm{s}}} \\ &= \left[\frac{1}{\ln\left(\frac{G_{\max}}{G_{\min}}\right)}-\frac{1}{\ln\left(\frac{G_{\max}-1}{G_{\min}-1}\right)}\right](G_{\max}-G_{\min}). \end{aligned} \tag{44}$$

Rewriting Eqs. (39) and (40), we obtain

$$\begin{cases} G_{\max} = (\exp(WE_{\mathrm{out}})-1)/(\exp(WE_{\mathrm{in}})-1), \\ G_{\min} = (\exp(-WE_{\mathrm{out}})-1)/(\exp(-WE_{\mathrm{in}})-1), \end{cases} \tag{45}$$

where

$$W = \Gamma_{\mathrm{s}}(\sigma_{\mathrm{es}}+\sigma_{\mathrm{as}})/(h\nu_{\mathrm{s}}A). \tag{46}$$

Equation (45) shows that the noise factor of a nonlinear pulse-narrowing amplifier can be controlled by regulating its input and output pulse energies. It should be noticed that as the

above is obtained under the assumption of linear stored energy recovering, this theory is only applicable for the amplifier with an output pulse at a frequency higher than kilo-Hz and it should not be directly used in the case that the output frequency is very low.

2.3. Noise factor of cascaded amplifier

For simplicity and accuracy, only the last few stages of a cascaded amplifier are treated as a nonlinear amplifier. The other prior stages are analyzed by traditional linear theory. Though the noise definition is different, the noise factor of the linear part is[6]

$$F_{\mathrm{L}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 \cdots G_{N-1}}, \tag{47}$$

where F_n and G_n are the noise factor and signal gain of the n stage, respectively. Given that the gain of each stage is usually more than 10 dB, the noise factor of the first stage or the two stages has a leading role in determining the noise factor of the linearly cascaded amplifier.

For the following nonlinear stages, all are considered as nonlinear amplifiers. By Eq. (45), the nonlinear $G_{\max}$ and $G_{\min}$ can be obtained as

$$\begin{cases} G_{\max} = \prod_{k=1}^{n} (\exp(W_k E_{\mathrm{out}(k)}) - 1)/(\exp(W_k E_{\mathrm{in}(k)}) - 1), \\ G_{\min} = \prod_{k=1}^{n} (\exp(-W_k E_{\mathrm{out}(k)}) - 1)/(\exp(-W_k E_{\mathrm{in}(k)}) - 1), \end{cases} \tag{48}$$

where $E_{\mathrm{in}(k)}$ and $E_{\mathrm{out}(k)}$ are the input and output pulse energies of stage k, and W_k is the W of each stage. The relation between the output pulse energy of stage k and the input energy of the stage $k+1$ is

$$E_{\mathrm{in}(k)} = \alpha_{\mathrm{stage}(k)} E_{\mathrm{out}(k-1)}, \tag{49}$$

where $\alpha_{\mathrm{stage}(k)}$ consists of all attenuation, reabsorption, and input coupling losses of stage k. Although the attenuation and absorption are distributed all through the fiber for a real amplifier stage, we relocate them to the front of the stage as input coupling loss. The total signal transmitting loss caused by attenuation and reabsorption is usually not larger than -3 dB, which is much less than the signal gain. This approximation only leads to a slightly higher evaluated noise factor than the actual value. We accept this as a calculation error which does not obviously affect the results of the analysis. Then, substituting Eq. (48) into Eq. (44), the noise factor of the whole nonlinear part is obtained. The noise factor of the nonlinear amplifier is

$$F = F_{\mathrm{L}} F_{\mathrm{NL}}. \tag{50}$$

Then, to determine whether a stage can be considered to be linear, we calculate the reciprocal values of the noise factor with different pulse narrowing factors at different values of $G_{\min}$ by Eq. (44) (Fig. 1). This condition shows that the noise factors of the different values of $G_{\min}$ are close to each other. An amplifier stage is arbitrarily defined as the linear stage with a noise factor that obeys

$$F \geq 1.01. \tag{51}$$

In addition, the pulse-narrowing factor should not be larger than 1.4 (Fig. 1). Thus, the criterion whether an amplification is linear can be written as

$$G_{\mathrm{narrow}} \leq 1.4. \tag{52}$$

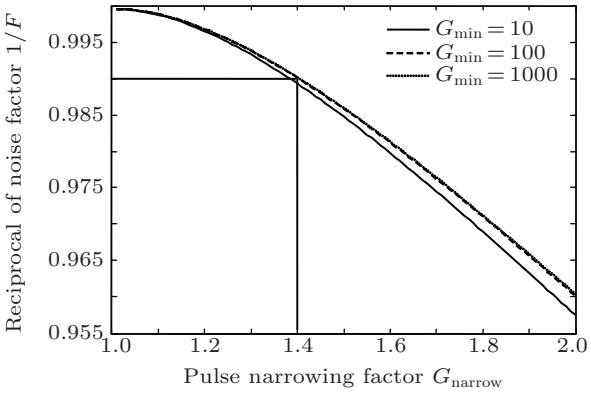


Fig. 1. Plots of reciprocal of noise factor $(1/F)$ versus pulse-narrowing factor G_{narrow} for different values of minimum gain $G_{\min}$.

3. Experiment

We perform an experiment to test the noise theory of nonlinear amplification. The experimental setup is shown in Fig. 2. A 1064-nm LD in a fiber-pigtailed package is driven directly with a current, resulting in the generation of 200-ns duration and 50-kHz square-shaped repetition rate pulses. The peak powers are approximately 1 W from the pigtail fiber with different pulse widths. These pulses are amplified with two stages of pre-amplifiers. For each stage, a 2-m long single mode (SM) ytterbium-doped-fiber (YDF) is core-pumped by a 450-mW 975-nm SM diode. The fiber *NA* is 0.14 with 6-m core diameter. The average power is amplified to 100 mW with 2-μJ pulse energy by the first stage. For the second stage, a 2-m long large mode area (LMA) double-clad YDF fiber is pumped by a 10-W multimode fiber 976-nm LD through a $(2+1)*1$ beam combiner. The fiber core size is 20 μm ($NA = 0.06$). The clad diameter is 125 μm ($NA = 0.37$). The average power is amplified to 2 W with 40-μJ pulse energy. For the third stage, a 6-m long LMA double clad YDF fiber is pumped by four 25-W multimode fiber 976-nm LDs through a $(6+1)*1$ beam combiner. The fiber core size is 20 μm ($NA = 0.06$). The clad diameter is 400 μm ($NA = 0.46$). The output power of the final stage is regulated by changing the pump power. The average output power can be as high as 55 W. Three pairs of isolators and filters are placed after the seed and between stages to suppress the back reflections and ASE. The fiber band width is 8 nm (at 0.5 dB).

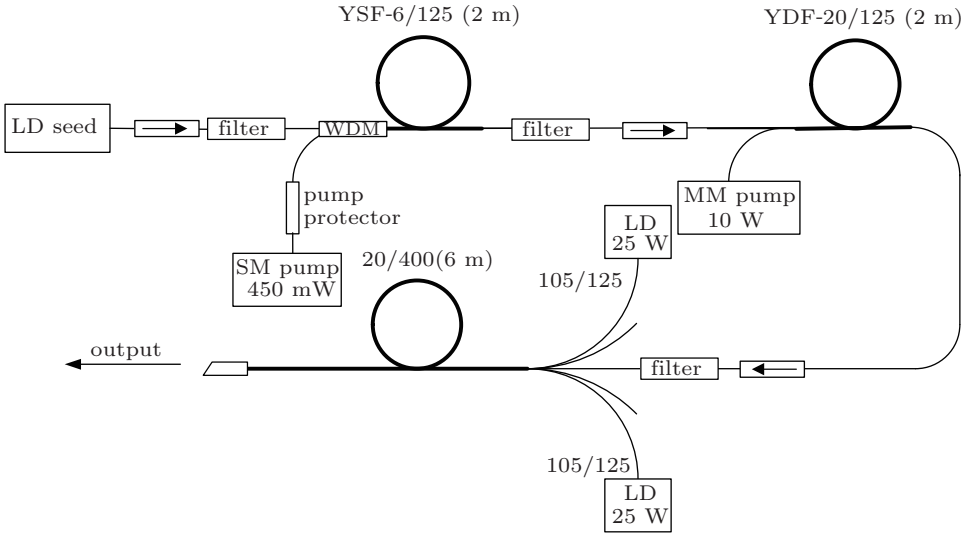


Fig. 2. Experiment on amplifier fiber noise.

In Eq. (29), h is 6.626×10^{-34} m^2·kg/s, ν_s is 282 THz for a 1064-nm signal wavelength, A is 28.3 μm^2 for a 6-μm fiber core, and σ is 3.0369×10^{-25} for Yb^{3+}. For the first stage, the pulse energy E_{out} is 2 μJ. Then, even if Γ_s is set to a maximum of 1, the calculated G_{narrow} of the first stage is approximately 1.19. Therefore, it is a linear stage according to the criterion of Eq. (52). For the second stage, the output pulse energy is 40 μJ. Even if Γ_s is set to be a maximum of 1, the calculated G_{narrow} is approximately 1.22. Then, the second stage is also a linear stage by Eq. (52). For the final third stage, considering that Γ_s is normally no less than 0.5, the calculated G_{narrow} is at least 16.4 for a maximum output pulse energy of 1.1 mJ. Thus, this stage should be treated as a nonlinear stage.

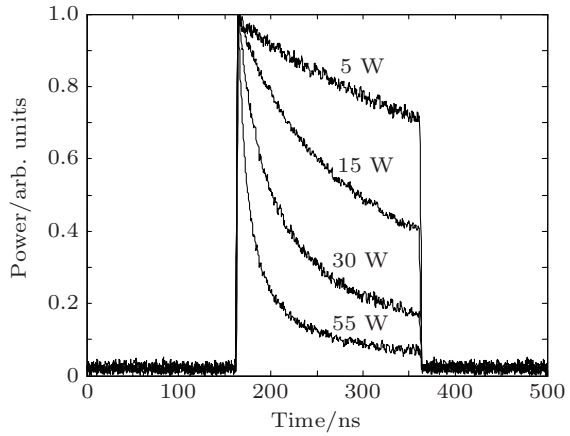


Fig. 3. Pulse shapes at different output powers.

Then, we measure the final output pulse shapes at different pump powers (Fig. 3).

Considering that the input pulse is square-shaped, the pulse narrowing factor can be easily evaluated.

Another experiment is performed to measure the output SNR (Fig. 4). The output power is coupled to an acoustic optical (AO) switch after proper attenuation and filtering. The filter is band-passed with 10 nm (at 3 dB) band width. Only the light power near the signal wavelength can pass through it. A pin hole is inserted to block the power of the AO switch zero order, and the first order is configured as the open state order.

We have previously used zero-order light as the output power. However, the AO switch could shut off all light power under its off-state, which affects the final SNR measurement result. Then, we use the first order light as output power. Results showed that the AO switch can shut off almost all light power under its off-state and has a transparency of approximately 40% under its on-state, which is sufficient for the measurement. The low transparency of the on-state can be considered as attenuation and does not affect the measurement of SNR theoretically.

Then, the AO switch output power is measured by a power meter. The AO switch is controlled by the signal from the pulse-triggering source through the AO switch controller. With the tuning of the controller, the AO switch blocks pulse energy and allows the ASE noise between pulses to go through it in the measurement of the noise power P_{noise}. Then, the AO switch is switched on in measuring the total output. The SNR can be easily calculated by

$$\mathrm{SNR}_{out} = (P_{total} - P_{noise})/P_{noise}. \tag{53}$$

We measure the noise and total output powers after the AO switch at different pump powers and arbitrarily consider the SNR with no pump power as the input signal SNR. Then, the noise factor of the final stage is calculated. Table 1 summarizes the experimental results.

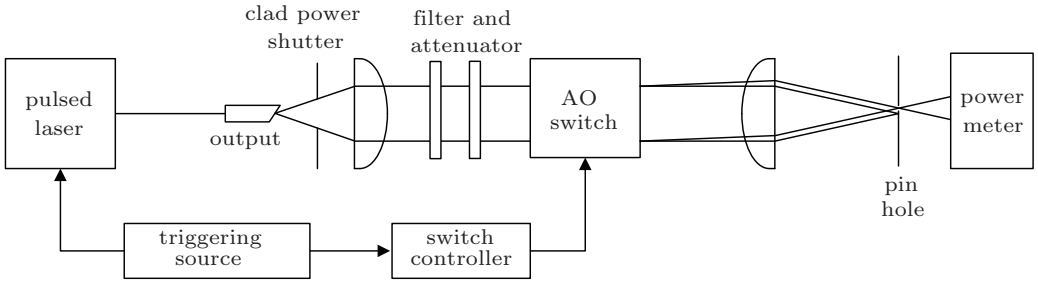


Fig. 4. Measurement setup of output SNR.

Table 1. Noise measurement results at different output powers.

Output power/W	0.7	5	15	30	55
Pulse narrowing factor	1	1.4	2.5	5.9	17
Total power after AO switch/W	0.2	0.8	0.9	0.6	0.9
Noise power after AO switch/mW	3	15	18	15	34
Output SNR/dB	17.0	17.2	16.9	15.9	14.1
1/Noise factor	1.000	0.990	0.934	0.746	0.488

The relation between the pulse-narrowing factor and noise factor as an extension of Fig.1 is shown in Fig. 5, in which the results in Table 1 are marked by "$*$". The experimental results accord with the theoretical values. However, the slightly larger error between the experimental data and the theoretical curve at a pulse-narrowing factor of 17 is most likely to be due to the shot noise and the limited frequency bandwidth of the photo-diode detector used in the pulse shape measurement. The pulse shapes in Fig. 4 are not very accurate, and the evaluated pulse-narrowing factor under high power is lower than its actual value.

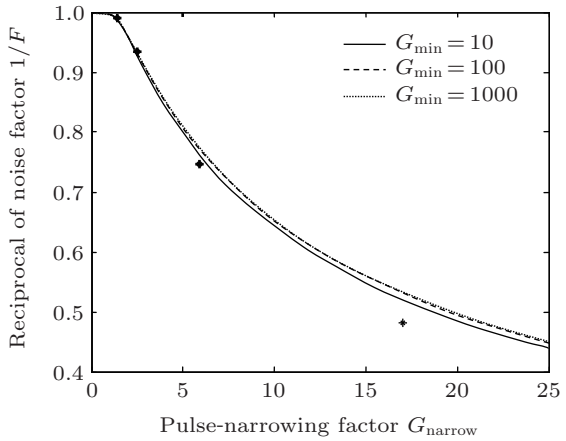


Fig. 5. Plots of reciprocal of noise factor $(1/F)$ versus pulse-narrowing factor $G_{\rm narrow}$ for different values of minimum gain $G_{\rm min}$ and actual noise measurement results (stars).

4. Conclusion

The noise characteristics of a cascaded high-power laser system are analyzed. As in a high power usage, the coherent noise is no longer the case of our consideration and only ASE noise between pulses is considered as noise. Based on this high power usage noise definition, the low-power amplification stage of a cascade is considered as linear amplification and analyzed by traditional linear theory, in contrast to the high-power amplification stages. As the nonlinear amplification that generates severe shape distortions of the seed pulse, the noise factor of a nonlinear amplifier is evaluated on the assumption that the inversion recovers linearly in time intervals between pulses. This condition indicates that the noise factor depends not only on the fiber gain, but also on the extent of the pulse shape distortion. Then, the pulse-narrowing factor is defined to represent the extent of the pulse shape distortion. A practical criterion of the pulse-narrowing factor is provided to ascertain whether an amplifier stage can be treated as a linear one. Then, an experiment is performed to study the relation between the amplifier noise and pulse-narrowing factors. The experiment results are in agreement with the theoretical values.

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