Further studies on stability analysis of nonlinear Roesser-type two-dimensional systems*

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This paper is concerned with further relaxations of the stability analysis of nonlinear Roesser-type two-dimensional (2D) systems in the Takagi–Sugeno fuzzy form. To achieve the goal, a novel slack matrix variable technique, which is homogenous polynomially parameter-dependent on the normalized fuzzy weighting functions with arbitrary degree, is developed and the algebraic properties of the normalized fuzzy weighting functions are collected into a set of augmented matrices. Consequently, more information about the normalized fuzzy weighting functions is involved and the relaxation quality of the stability analysis is significantly improved. Moreover, the obtained result is formulated in the form of linear matrix inequalities, which can be easily solved via standard numerical software. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed result.

Keywords: stability analysis, Roesser-type two-dimensional system, slack matrix variable, reducing conservatism

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1. Introduction

During the last several decades, two-dimensional (2D) systems[1,2] have attracted considerable attention due to their extensive applications of many practical systems,[3–5] e.g., image data processing and transmission, modeling of thermal processes, pattern recognition, digital signal filtering of complex systems, etc. The 2D system theory has been frequently addressed as an analysis tool to other technical problems, e.g., the problem of H_∞ control of 2D discrete state delay systems described by the second Fornasini–Marchesini (FM) state-space model has been studied in Ref. [6], and the problem of H_∞ filtering for 2D Markovian jump systems has been addressed in Ref. [7]. Moreover, stability analysis of 2D discrete systems described by the FM second model with state saturation has been given in Ref. [8]. However, it is worth noting that all the above investigations are focused on the linear 2D system. As is well known, most of the practical 2D systems in the realistic world are nonlinear systems and the above results fail to work in the nonlinear setting.

On the other hand, the Takagi–Sugeno (TS) fuzzy model has become an efficient tool for dealing with the problem of stability analysis and control synthesis of dynamical nonlinear systems,[9–15] essentially due to the fact that it could effectively approximate a wide class of nonlinear systems by a 'blending' of several local linear sub-system. The earlier results on stability analysis and control synthesis via the TS fuzzy models are implemented by applying the common Lyapunov function.[16,17] However, it applies a single Lyapunov matrix for all sub-systems and hence the obtained results are rather conservative.[18] To further reduce the conservatism, the fuzzy Lyapunov function has been proposed in Ref. [19], and its extension versions have been developed in Refs. [20]–[22]. More recently, the problem of stability analysis of nonlinear Roesser-type 2D systems has been well investigated in Ref. [23], where a homogeneous polynomially parameter-dependent Lyapunov function and several relaxed techniques have been proposed for obtaining less conservative stability criteria. Moreover, the obtained result in Ref. [23] is formulated in the form of linear matrix inequalities, which can be easily solved via standard numerical software. Therefore, it seems that the challenge nowadays is to improve the quality of the relaxations; for instance, in terms of efficiency, i.e. the ability to achieve the same results with lower computational complexity, or the easiness to extend a given stability condition to deal with similar problems.

Motivated by the above concerns, the problem of further relaxations of stability analysis of nonlinear Roesser-type 2D systems is investigated in this paper. A novel slack matrix variable technique, which is homogenous polynomially parameter-dependent on the normalized fuzzy weighting functions with arbitrary degree, is developed and the algebraic properties of the normalized fuzzy weighting functions are collected into a set of augmented matrices. Consequently, more information about the normalized fuzzy weighting functions is involved in the process of stability analysis and the relaxation quality of stability analysis is significantly improved.

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Moreover, the obtained result is formulated in the form of linear matrix inequalities, which can be easily solved via standard numerical software. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed result.

The rest of this paper is organized as follows: following the introduction, a system description and some preliminaries are given in Section 2. Stability analysis of the underlying nonlinear Roesser-type 2D systems is developed in Section 3. A numerical example is given to demonstrate the effectiveness of the given result in Section 4. Finally, some conclusions are drawn in Section 5.

For simplicity, the notations used are fully standard. For example, $X > 0$ (or $X \geq 0$) means the matrix X is symmetric and positive-definite (or symmetric and positive-semidefinite). For a square matrix E, $\mathrm{He}(E)$ is defined as $E + E^{\mathrm{T}}$. X^{T} denotes the transpose of X. The symbol I represents the identity matrix with an appropriate dimension. $\mathbb{Z}^+$ denotes the set of negative integers $\{0, 1, 2, \ldots\}$ and $n!$ represents factorial, i.e., $n! = n(n-1)(n-2)\cdots(2)(1)$ for $n \in \mathbb{Z}^+$ with $0! = 1$. A star symbol $*$ in a symmetric matrix denotes the transposed element in the symmetric position.

2. System description and preliminaries

2.1. Representation of nonlinear Roesser-type 2D systems by the TS fuzzy modeling approach

Consider a class of discrete-time nonlinear Roesser-type 2D systems described as follows:[23]

$$\boldsymbol{x}^+(s,l) = \mathscr{F}(\boldsymbol{x}(s,l)), \tag{1}$$

with

$$\boldsymbol{x}(s,l) = \begin{bmatrix} \boldsymbol{x}^{\mathrm{h}}(s,l) \\ \boldsymbol{x}^{\mathrm{v}}(s,l) \end{bmatrix}, \ \boldsymbol{x}^+(s,l) = \begin{bmatrix} \boldsymbol{x}^{\mathrm{h}}(s+1,l) \\ \boldsymbol{x}^{\mathrm{v}}(s,l+1) \end{bmatrix},$$

where $\boldsymbol{x}^{\mathrm{h}}(\cdot,\cdot)$ is the horizonal state in $\mathbf{R}^{n_1}$, $\boldsymbol{x}^{\mathrm{v}}(\cdot,\cdot)$ is the vertical state in $\mathbf{R}^{n_2}$, where n_1 and n_2 are dimensions of the horizonal state vector and the vertical state vector respectively. $\mathscr{F}(\cdot)$ is a general nonlinear function satisfying $\mathscr{F} \in C^1$. s, l are two integers in $\mathbb{Z}^+$. Moreover, the boundary conditions along two independent directions are defined as follows: $\boldsymbol{x}^{\mathrm{h}}(0,l) = f(l), \boldsymbol{x}^{\mathrm{v}}(s,0) = g(s)$, where $f(l)$ and $g(s)$ are corresponding boundary conditions along two independent directions.

Extending the usual one-dimensional (1D) TS fuzzy modeling method to the 2D case, a discrete-time Roesser-type 2D TS fuzzy model described by the following fuzzy rules could be used to represent the discrete-time nonlinear Roesser-type 2D system (1)

IF $z_1(s,l)$ is $M_{i1}, z_2(s,l)$ is M_{i2}, and, $\ldots$, and $z_L(s,l)$ is M_{iL},

THEN,

$$\begin{aligned} &\boldsymbol{x}^+(s,l) = A_i \boldsymbol{x}(s,l), \ i = 1, \ldots, r \\ &\boldsymbol{x}^{\mathrm{h}}(0,l) = f(l), \ \boldsymbol{x}^{\mathrm{v}}(s,0) = g(s), \end{aligned} \tag{2}$$

where $z_p(s,l)$, for $p = 1, \ldots, L$ are the premise variables, M_{ip} is the fuzzy set, r is the number of IF-THEN rules. Each subsystem matrix A_i belongs to the following structure

$$A_i = \begin{bmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{bmatrix},$$

with $A_i^{11} \in \mathbf{R}^{n_1 \times n_1}, A_i^{12} \in \mathbf{R}^{n_1 \times n_2}, A_i^{21} \in \mathbf{R}^{n_2 \times n_1}, A_i^{22} \in \mathbf{R}^{n_2 \times n_2}$, respectively.

By using the product of inference, a singleton fuzzifier, and a center-average defuzzifer, the overall discrete-time Roesser-type 2D TS fuzzy system can be expressed as follows:

$$\begin{aligned} &\boldsymbol{x}^+(s,l) = \sum_{i=1}^{r} h_i(z(s,l)) A_i \boldsymbol{x}(s,l), \\ &\boldsymbol{x}^{\mathrm{h}}(0,l) = f(l), \ \boldsymbol{x}^{\mathrm{v}}(s,0) = g(s), \end{aligned} \tag{3}$$

where

$$\begin{aligned} h_i(z(s,l)) &= \frac{\mu_i(z(s,l))}{\sum_{i=1}^{r} \mu_i(z(s,l))}, \\ \mu_j(z(s,l)) &= \Pi_{m=1}^{L} M_{mj}(z(s,l)). \end{aligned}$$

Denote

$$X_r = \sup\{\|\boldsymbol{x}(s,l)\| : r = s + l\},$$

and then we give the definition of asymptotical stability for the discrete-time Roesser-type 2D TS fuzzy system (3).

Definition 1[23] The discrete-time Roesser-type 2D TS fuzzy system (3) is asymptotically stable if $\lim\limits_{s \to \infty,\ l \to \infty} X_r = 0$ with the initial and boundary conditions $\boldsymbol{x}^{\mathrm{h}}(0,l) = f(l)$ and $\boldsymbol{x}^{\mathrm{v}}(s,0) = g(s)$.

In the following sequel of this paper, the notations will be adopted for simplicity:

$$h_i = h_i(z(s,l)), \ A_z = \sum_{i=1}^{r} h_i A_i. \tag{4}$$

2.2. Homogeneous matrix polynomials and some useful technical lemmas

The following definitions about homogeneous matrix polynomials are given as follows, which are consistent with those in Ref. [19].

Define $\mathscr{K}(g)$ as the set of r-tuples obtained as all possible combinations of $k_1 k_2 \cdots k_r$, $k_i \in \mathbb{Z}^+, i = 1, \ldots, r$ such that $k_1 + k_2 + \cdots + k_r = g$. $\alpha_1^{k_1} \alpha_2^{k_2} \cdots \alpha_r^{k_r}, \alpha \in \Delta_r, k_i \in \mathbb{Z}_+, i = 1, 2, \ldots, r$ are the monomials, $k = k_1 k_2 \cdots k_r$, and $P_k \in \mathbf{R}^{n \times n}, \forall k \in \mathscr{K}(g)$ are matrix-valued coefficients. Here, by definitions, $\mathscr{K}(g)$ is the set of r-tuples obtained as all possible combinations of nonnegative integers $k_i, i = 1, 2, \ldots, r$, such that $k_1 + k_2 + \cdots +$

$k_r = g$. Since the number of vertices is equal to r, the number of elements in $\mathscr{K}(g)$ is given by

$$J(g) = \frac{(r+g-1)!}{g!(r-1)!}. \tag{5}$$

To give an example, for homogeneous polynomials of degree $g = 3$ with $r = 2$ variables, the possible values of the partial degrees are $\mathscr{K}(3) = \{30, 21, 12, 03\}$, $J(3) = 4$, corresponding to the generic form

$$P_3(\alpha) = \alpha_1^3 P_{30} + \alpha_1^2 \alpha_2 P_{21} + \alpha_1 \alpha_2^2 P_{12} + \alpha_2^3 P_{03}.$$

By definition, for r-tuples k and k', one writes $k \geq k'$ if $k_i \geq k_i'$, $(i = 1, \ldots, r)$. The usual operations of summation, $k + k'$, and subtraction, $k - k'$ (whenever $k \geq k'$), are defined componentwise. Moreover, let us define some new mathematical notations as follows:

$$\begin{aligned}
&\pi(k) \triangleq (k_1!)(k_2!)\cdots(k_r!),\ k \in \mathscr{K}(d),\\
&e_i = 0\cdots 0\ \underbrace{1}_{i-\text{th}}\ 0\cdots 0,\\
&h^k = h_1^{k_1} h_2^{k_2} \cdots h_r^{k_r},\ R_g(h) = \sum_{k\in\mathscr{K}(g)} h^k R_k,\\
&h(z(s+1,l))^k = h_1(z(s+1,l))^{k_1} h_2(z(s+1,l))^{k_2} \cdots\\
&\qquad\qquad h_r(z(s+1,l))^{k_r},\\
&h(z(s,l+1))^k = h_1(z(s,l+1))^{k_1} h_2(z(s,l+1))^{k_2} \cdots\\
&\qquad\qquad h_r(z(s,l+1))^{k_r}.
\end{aligned} \tag{6}$$

Next, we end this section with a technical lemma.

Lemma 1[23] The discrete-time Roesser-type 2D TS fuzzy system (3) is asymptotically stable if there exist appropriately dimensional matrices $P_k > 0$, G_k $(k \in \mathscr{K}(g),\ k' \in \mathscr{K}(g))$, $g \in \mathbb{Z}^+$, with

$$P_k = \begin{bmatrix} P_k^1 & 0 \\ 0 & P_k^2 \end{bmatrix},\ P_k^1 \in \mathbf{R}^{n_1\times n_1},\ P_k^2 \in \mathbf{R}^{n_2\times n_2},$$
$$G_k = \begin{bmatrix} G_k^1 & 0 \\ 0 & G_k^2 \end{bmatrix},\ G_k^1 \in \mathbf{R}^{n_1\times n_1},\ G_k^2 \in \mathbf{R}^{n_2\times n_2},$$

such that the following matrix inequality holds:

$$\begin{bmatrix} P_g(h) & * \\ G_g(h)A_z & G_g(h) + G_g(h)^{\mathrm{T}} - P_g(h^+) \end{bmatrix} > 0, \tag{7}$$

where

$$\begin{aligned}
&P_g(h)\\
&= \sum_{k\in\mathscr{K}(g)} h^k P_k,\ G_g(h) = \sum_{k\in\mathscr{K}(g)} h^k G_k,\\
&P_g(h^+)\\
&= \begin{bmatrix} \sum_{k\in\mathscr{K}(g)} h(z(s+1,l))^k P_k^1 & 0 \\ 0 & \sum_{k\in\mathscr{K}(g)} h(z(s,l+1))^k P_k^2 \end{bmatrix}.
\end{aligned}$$

3. Main results

In this section, the relaxed stability conditions of a discrete-time Roesser-type 2D TS fuzzy system will be developed by means of a novel slack matrix variable technique.

Theorem 1 The discrete-time Roesser-type 2D TS fuzzy system (3) is asymptotically stable if there exist appropriately dimensional matrices $P_k > 0$, G_k, $P(kk')$, and symmetric matrices $E_{kk'p}^{ii}$, matrices $E_{kk'p}^{ij}$ for $1 \leq i < j \leq r$ and $k \in \mathscr{K}(g)$, $k' \in \mathscr{K}(g)$, $p \in \mathscr{K}(g-1)$, $g \in \mathbb{Z}^+$, with

$$\begin{aligned}
&P_k = \begin{bmatrix} P_k^1 & 0 \\ 0 & P_k^2 \end{bmatrix},\ P_k^1 \in \mathbf{R}^{n_1\times n_1},\\
&P_k^2 \in \mathbf{R}^{n_2\times n_2},\ G_k = \begin{bmatrix} G_k^1 & 0 \\ 0 & G_k^2 \end{bmatrix},\\
&G_k^1 \in \mathbf{R}^{n_1\times n_1},\ G_k^2 \in \mathbf{R}^{n_2\times n_2},\\
&P(kk') = \begin{bmatrix} \dfrac{g!}{\pi(k')} P_k^1 & 0 \\ 0 & \dfrac{g!}{\pi(k)} P_{k'}^2 \end{bmatrix},\\
&E_{kk'p}^{ii} \in \mathbf{R}^{(n_1+n_2)\times(n_1+n_2)},\\
&E_{kk'p}^{ij} \in \mathbf{R}^{(n_1+n_2)\times(n_1+n_2)},
\end{aligned}$$

such that the following linear matrix inequalities (LMIs) (8) and (9) hold:

$$\begin{aligned}
&\Pi(kk'k'') + \sum_{i=1}^{r} E_{kk'(k''-2e_i)}^{ii} + \mathrm{He}\left(\sum_{1\leq i<j\leq r} E_{kk'(k''-e_i-e_j)}^{ij}\right) > 0,\\
&\forall\, k \in \mathscr{K}(g),\ k' \in \mathscr{K}(g),\ k'' \in \mathscr{K}(g+1),
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\begin{bmatrix} E_{kk'p}^{11} & \cdots & E_{kk'p}^{1r} \\ \vdots & \ddots & \vdots \\ E_{kk'p}^{r1} & \cdots & E_{kk'p}^{rr} \end{bmatrix} < 0,\\
&k \in \mathscr{K}(g),\ k' \in \mathscr{K}(g), p \in \mathscr{K}(g-1),
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
\Pi(kk'k'') = &\sum_{i\in\{1,\ldots,r\},k''-e_i\geq 0} \frac{g!}{\pi(k)} \frac{g!}{\pi(k')}\\
&\times \begin{bmatrix} P_{k''-e_i} & * \\ G_{k''-e_i}A_i & G_{k''-e_i} + G_{k''-e_i}^{\mathrm{T}} \end{bmatrix}\\
&+ \frac{(g+1)!}{\pi(k'')} \begin{bmatrix} 0 & * \\ 0 & -P(kk') \end{bmatrix},
\end{aligned}$$

and with

$$\begin{cases} E_{kk'(k''-2e_i)}^{ii} = 0, \text{ for } k_i'' - 2 < 0, \\ E_{kk'(k''-e_i-e_j)}^{ij} = 0, \text{ for } k_i'' - 1 < 0 \text{ or } k_j'' - 1 < 0. \end{cases}$$

Proof Using Lemma 1, the discrete-time Roesser-type 2D TS fuzzy system (3) is asymptotically stable if the following matrix inequality holds

$$\begin{bmatrix} P_g(h) & * \\ G_g(h)A_z & G_g(h) + G_g(h)^{\mathrm{T}} - P_g(h^+) \end{bmatrix} > 0. \tag{10}$$

On the other hand, we have

$$\begin{aligned}
&\begin{bmatrix} P_g(h) & * \\ G_g(h)A_z & G_g(h)+G_g(h)^{\mathrm{T}}-P_g(h^+) \end{bmatrix} \\
&= \sum_{k\in\mathscr{K}(g)}\sum_{k'\in\mathscr{K}(g)} h(z(s+1,l))^k h(z(s,l+1))^{k'} \begin{bmatrix} \dfrac{g!}{\pi(k)}\dfrac{g!}{\pi(k')}P_g(h) & * \\ \dfrac{g!}{\pi(k)}\dfrac{g!}{\pi(k')}G_g(h)A_z & \dfrac{g!}{\pi(k)}\dfrac{g!}{\pi(k')}(G_g(h)+G_g(h)^{\mathrm{T}})-P(kk') \end{bmatrix} \\
&= \sum_{k\in\mathscr{K}(g)}\sum_{k'\in\mathscr{K}(g)}\sum_{k''\in\mathscr{K}(g+1)} h(z(s+1,l))^k h(z(s,l+1))^{k'} h^{k''} \times \Pi(kk'k''), \qquad (11)
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{\Pi}(kk'k'') = &\sum_{i\in\{1,\ldots,r\},k''-e_i\geq 0} \frac{g!}{\pi(k)}\frac{g!}{\pi(k')} \\
&\times \begin{bmatrix} P_{k''-e_i} & * \\ G_{k''-e_i}A_i & G_{k''-e_i}+G_{k''-e_i}^{\mathrm{T}} \end{bmatrix} \\
&+ \frac{(g+1)!}{\pi(k'')}\begin{bmatrix} 0 & * \\ 0 & -P(kk') \end{bmatrix}.
\end{aligned}$$

Thus, if $\boldsymbol{\Pi}(kk'k'')>0$ for all $k\in\mathscr{K}(g)$, $k'\in\mathscr{K}(g)$, $k''\in\mathscr{K}(g+1)$ hold, the inequality (10) evidently holds.

Pre- and post-multiplying inequality (9) by $(h\bigotimes I)^{\mathrm{T}}$ and its transpose respectively, we obtain

$$\sum_{i=1}^{r} h_i E_{kk'p}^{ii} + \mathrm{He}\left(\sum_{1\leq i<j\leq r} h_i h_j E_{kk'p}^{ij}\right) < 0, \qquad (12)$$

where $k\in\mathscr{K}(g)$, $k'\in\mathscr{K}(g)$, $p\in\mathscr{K}(g-1)$.

Multiplying inequality (8) by $h(z(s+1,l))^k h(z(s,l+1))^{k'} h^{k''}$, it follows that

$$\begin{aligned}
&h(z(s+1,l))^k h(z(s,l+1))^{k'} h^{k''} \\
&\times \left\{\Pi(kk'k'') + \sum_{i=1}^{r} E_{kk'(k''-2e_i)}^{ii}\right. \\
&\left. + \mathrm{He}\left(\sum_{1\leq i<j\leq r} E_{kk'(k''-e_i-e_j)}^{ij}\right)\right\} > 0, \\
&\forall\, k\in\mathscr{K}(g),\ k'\in\mathscr{K}(g),\ k''\in\mathscr{K}(g+1). \qquad (13)
\end{aligned}$$

By summing all possible inequalities which satisfy inequality (13), we can obtain

$$\begin{aligned}
&\sum_{k\in\mathscr{K}(g)}\sum_{k'\in\mathscr{K}(g)}\sum_{k''\in\mathscr{K}(g+1)} h(z(s+1,l))^k \\
&\times h(z(s,l+1))^{k'} h^{k''} \times \Pi(kk'k'') \\
&+ \sum_{k\in\mathscr{K}(g)}\sum_{k'\in\mathscr{K}(g)}\sum_{p\in\mathscr{K}(g-1)} h(z(s+1,l))^k h(z(s,l+1))^{k'} \\
&\times h^p \left\{\sum_{i=1}^{r} h_i E_{kk'p}^{ii} + \mathrm{He}\left(\sum_{1\leq i<j\leq r} h_i h_j E_{kk'p}^{ij}\right)\right\} > 0. \qquad (14)
\end{aligned}$$

Thus, from inequalities (12) and (14), it could be concluded that $\Pi(kk'k'')>0$ for all $k\in\mathscr{K}(g)$, $k'\in\mathscr{K}(g)$, $k''\in\mathscr{K}(g+1)$ holds if the LMIs (8) and (9) hold. In other words, the inequality (10) holds if the LMIs (8) and (9) hold, which guarantee the asymptotical stability for the discrete-time Roesser-type 2D TS fuzzy system (3).

This completes the proof.

Remark 1 In the above theorem, relaxed LMI-based stability conditions for the discrete-time Roesser-type 2D TS fuzzy system (3) are developed by means of a novel slack matrix variable technique, which is homogenous polynomially parameter-dependent on the normalized fuzzy weighting functions with arbitrary degree. Because the algebraic properties of the normalized fuzzy weighting functions are collected into a set of augmented matrices, more information about the normalized fuzzy weighting functions is involved in the process of stability analysis and then the relaxation quality of stability analysis is significantly improved. Furthermore, this fact will be illustrated by numerical experiments.

Remark 2 During the past several decades, the well-known Roesser-type 2D model[1] has attracted considerable attention due to its extensive applications of practical systems,[3–5] e.g., image data processing and transmission, modeling of thermal process, pattern recognition, digital signal filtering of complex systems, etc. It is worth noting that stability analysis is one of the fundamental problems in implementing state estimations via Roesser-type 2D state-space digital filters. Therefore, there have been lots of studies on stability analysis of 2D state-space digital filters. In Ref. [24], stability analysis of 2D state-space digital filters has been firstly investigated by using the Lyapunov function. Recently, stability of 2D digital filters described by the Roesser model using any combination of quantization and overflow nonlinearities has been investigated in Refs. [25] and [26], respectively. However, it is worth noting that the above investigations are focused on the linear Roesser-type 2D system. As is well known, most of the practical Roesser-type 2D systems in the realistic physical world belong to nonlinear systems and the above results fail to work in the nonlinear setting. More recently, the problem of stability analysis of nonlinear Roesser-type 2D systems has been well investigated in Ref. [23] and the obtained result is formulated in the form of linear matrix inequalities, which can be easily solved via standard numerical software. However, the stability criteria obtained in Ref. [23] is rather conservative. As a result, the problem of seeking for

less conservative stability criteria of nonlinear Roesser-type 2D model needs to be further investigated, which motivates us to carry out this work.

4. Numerical examples

Consider a nonlinear differential equation borrowed from[23], which is described as follows:

$$\frac{\partial^2 q(x,t)}{\partial x \partial t} = a_1 \frac{\partial q(x,t)}{\partial t} + a_2 \frac{\partial q(x,t)}{\partial x} + a_0 \sin^2(q(x,t)) q(x,t),$$

where the initial and boundary conditions $q(x,0) = q_1(x)$ and $q(0,t) = q_2(t)$, $q(x,t)$ is the variable function, and a_0, a_1, a_2 are real coefficients.

First, we establish the state space model for the above nonlinear differential equation. Let us define

$$x_{\mathrm{c}}^{\mathrm{h}}(x,t) = \frac{\partial q(x,t)}{\partial t} - a_2 q(x,t),$$
$$x_{\mathrm{c}}^{\mathrm{v}}(x,t) = q(x,t),$$

then the following 2D state space model can be easily obtained

$$\begin{bmatrix} \dfrac{\partial x_{\mathrm{c}}^{\mathrm{h}}(x,t)}{\partial x} \\ \dfrac{\partial x_{\mathrm{c}}^{\mathrm{v}}(x,t)}{\partial t} \end{bmatrix} = \begin{bmatrix} a_1 & a_1 a_2 + a_0 \sin^2(x_{\mathrm{c}}^{\mathrm{v}}(x,t)) \\ 1 & a_2 \end{bmatrix} \times \begin{bmatrix} x_{\mathrm{c}}^{\mathrm{h}}(x,t) \\ x_{\mathrm{c}}^{\mathrm{v}}(x,t) \end{bmatrix},$$

with boundary conditions: $x_{\mathrm{c}}^{\mathrm{h}}(0,t) = \dot{q}_2(t) - a_2 q_2(t)$, $x_{\mathrm{c}}^{\mathrm{v}}(x,t) = q_1(x)$.

To obtain a 2D TS fuzzy representation for this 2D nonlinear system, consider the two following rules obtained for $\sin^2(x_{\mathrm{c}}^{\mathrm{v}}(x,t))$

IF $\sin^2(x_{\mathrm{c}}^{\mathrm{v}}(x,t))$ is about 0, THEN

$$\begin{bmatrix} \dfrac{\partial x_{\mathrm{c}}^{\mathrm{h}}(x,t)}{\partial x} \\ \dfrac{\partial x_{\mathrm{c}}^{\mathrm{v}}(x,t)}{\partial t} \end{bmatrix} = A_1^{\mathrm{c}} \times \begin{bmatrix} x_{\mathrm{c}}^{\mathrm{h}}(x,t) \\ x_{v}^{\mathrm{h}}(x,t) \end{bmatrix};$$

IF $\sin^2(x_{\mathrm{c}}^{\mathrm{v}}(x,t))$ is about ∓ 1, THEN

$$\begin{bmatrix} \dfrac{\partial x_{\mathrm{c}}^{\mathrm{h}}(x,t)}{\partial x} \\ \dfrac{\partial x_{\mathrm{c}}^{\mathrm{v}}(x,t)}{\partial t} \end{bmatrix} = A_2^{\mathrm{c}} \times \begin{bmatrix} x_{\mathrm{c}}^{\mathrm{h}}(x,t) \\ x_{v}^{\mathrm{h}}(x,t) \end{bmatrix}.$$

Here,

$$A_1^{\mathrm{c}} = \begin{bmatrix} a_1 & a_1 a_2 \\ 1 & a_2 \end{bmatrix}, \quad A_2^{\mathrm{c}} = \begin{bmatrix} a_1 & a_1 a_2 + a_0 \\ 1 & a_2 \end{bmatrix}.$$

Without loss of generality, we choose the normalized fuzzy weighting functions as: $h_1(x,t) = 1 - \sin^2(x_{\mathrm{c}}^{\mathrm{v}}(x,t))$, $h_2(x,t) = \sin^2(x_{\mathrm{c}}^{\mathrm{v}}(x,t))$.

The above 2D TS fuzzy systems are discretized with sampling times T_1 and T_2 corresponding to variables x and t respectively. The obtained discrete-time Roesser-type 2D fuzzy system is given by:

IF $\sin^2(x^{\mathrm{v}}(k,l))$ is about 0, THEN

$$\begin{bmatrix} x^{\mathrm{h}}(k+1,l) \\ x^{\mathrm{v}}(k,l+1) \end{bmatrix} = A_1 \times \begin{bmatrix} x^{\mathrm{h}}(k,l) \\ x^{\mathrm{v}}(k,l) \end{bmatrix};$$

IF $\sin^2(x^{\mathrm{v}}(k,l))$ is about ∓ 1, THEN

$$\begin{bmatrix} x^{\mathrm{h}}(k+1,l) \\ x^{\mathrm{v}}(k,l+1) \end{bmatrix} = A_2 \times \begin{bmatrix} x^{\mathrm{h}}(k,l) \\ x^{\mathrm{v}}(k,l) \end{bmatrix}.$$

Here,

$$A_1 = \begin{bmatrix} 1 + a_1 T_1 & a_1 a_2 T_1 \\ T_2 & 1 + a_2 T_2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 + a_1 T_1 & (a_1 a_2 + a_0) T_1 \\ T_2 & 1 + a_2 T_2 \end{bmatrix}.$$

Then, the normalized fuzzy weighting functions of the above discrete-time Roesser-type 2D TS fuzzy system become $h_1(k,l) = 1 - \sin^2(x^{\mathrm{v}}(k,l))$, $h_2(k,l) = \sin^2(x^{\mathrm{v}}(k,l))$. Consider the following parameter values about a_0, a_1, T_1, and T_2: $a_0 = -2$, $a_1 = -3$, $T_1 = 0.4$, $T_2 = 0.7$, then we can calculate the feasible parameter intervals by evaluating the feasibility of the associated LMI problems with different methods for varying values of a_2.

Table 1 shows the parameter feasible intervals of a_2 in which the discrete-time Roesser-type 2D TS fuzzy system is asymptotically stable could be ensured by applying those approaches provided by Theorem 1 and Theorem 2 of Ref. [23], and Theorem 1 of this paper, respectively, in details as given below. Method 1: Theorem 1 of Ref. [23] with $g = 2$; Method 2: Theorem 2 of Ref. [23] with $g = 2$ and $d = 4$; Method 3: Theorem 1 of this paper with $g = 2$. From Table 1, it can be seen that Theorem 1 of this paper provides the most relaxed results.

Table 1. Feasible stability parameter intervals of a_2 via several methods proposed in this paper.

Methods	Theorem 1 of Ref. [23]	Theorem 2 of Ref. [23]	Theorem 1 of this paper
intervals	$[-1.116, -0.429]$	$[-1.116, -0.426]$	$[-1.122, -0.426]$

Furthermore, figures 1–4 show the state trajectory of the system state variables with $a_2 = -1.7$ and $a_2 = -1.122$ respectively. From Table 1, one could find that $a_2 = -1.7$ is out of the calculated stability region by using all three approaches, and its state trajectories of the system state variables $x^{\mathrm{h}}(s,l)$ and $x^{\mathrm{v}}(s,l)$ are not asymptotically stable from Fig. 1 and Fig. 2. On the other hand, from Table 1, one could find that $a_2 = -1.122$ is only within the calculated stability region by using the approach proposed in this paper, and its state trajectories of the system state variables $x^{\mathrm{h}}(s,l)$ and $x^{\mathrm{v}}(s,l)$ are asymptotically stable from Fig. 3 and Fig. 4. Therefore, the effectiveness of the proposed approach has been illustrated in the underlying numerical example.

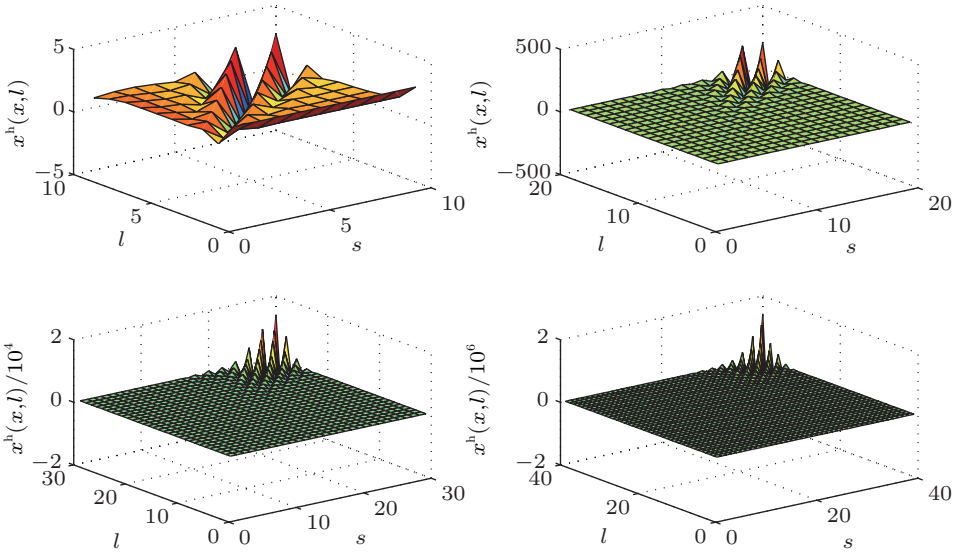


Fig. 1. (color online) State trajectory of the system state variable $x^{\mathrm{h}}(s,l)$ with $a_2 = -1.7$.

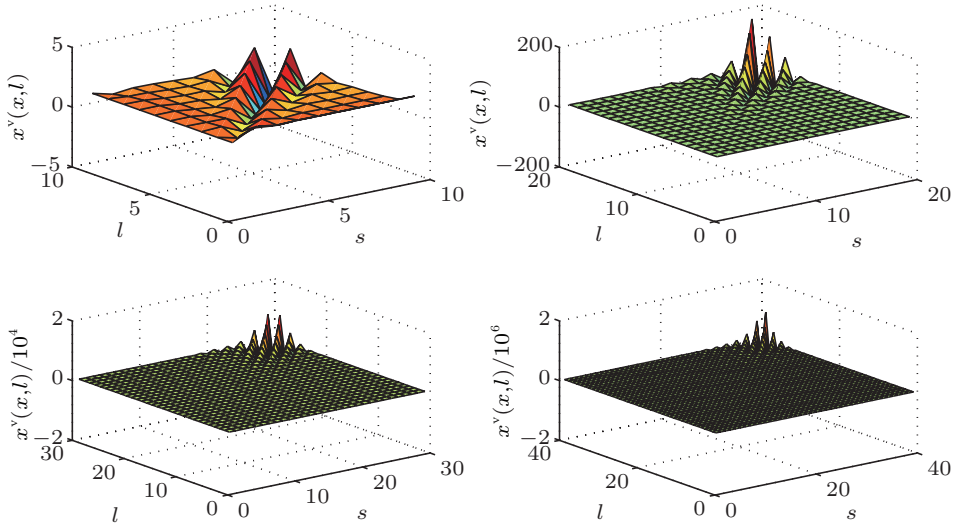


Fig. 2. (color online) State trajectory of the system state variable $x^{\mathrm{v}}(s,l)$ with $a_2 = -1.7$.

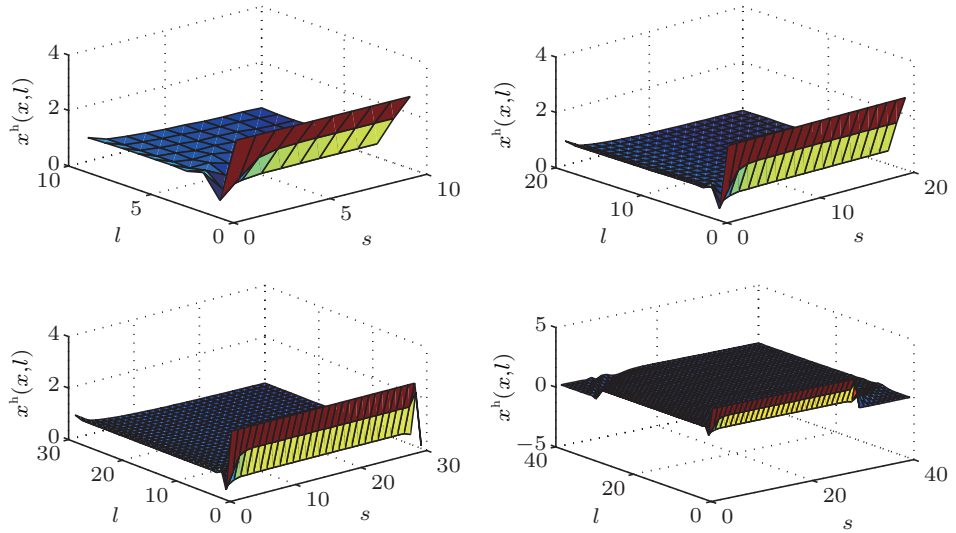


Fig. 3. (color online) State trajectory of the system state variable $x^{\mathrm{h}}(s,l)$ with $a_2 = -1.122$.

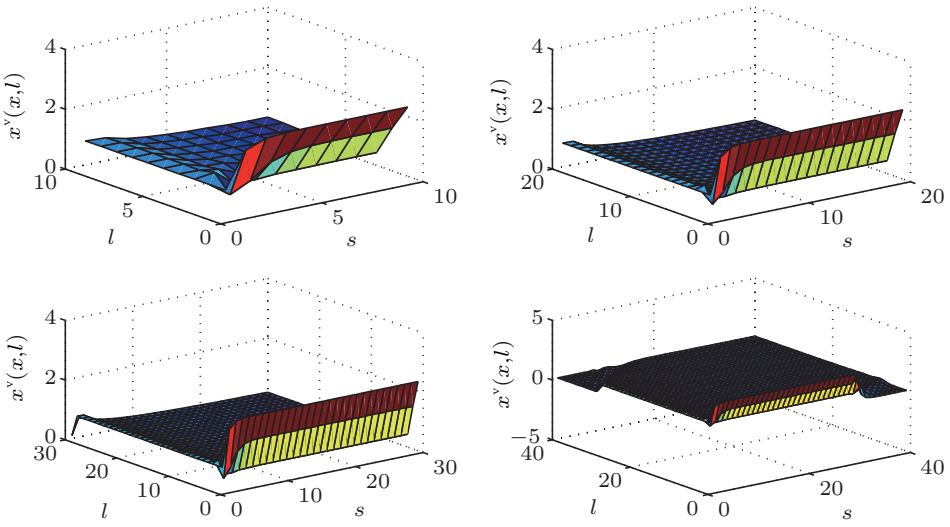


Fig. 4. (color online) State trajectory of the system state variable $x^{v}(s,l)$ with $a_2 = -1.122$.

5. Conclusions

The problem of further relaxations of the stability analysis of nonlinear Roesser-type 2D systems in the Takagi–Sugeno fuzzy form has been addressed in this paper. A novel slack matrix variable technique, which is homogenous polynomially parameter-dependent on the normalized fuzzy weighting functions with arbitrary degree, is developed and the algebraic properties of the normalized fuzzy weighting functions are collected into a set of augmented matrices. Consequently, more information about the normalized fuzzy weighting functions has been involved and the relaxation quality of stability analysis has been significantly improved. Moreover, the obtained result is formulated in the form of linear matrix inequalities, which can be easily solved via standard numerical software. The effectiveness of the proposed results has been illustrated by means of numerical results.

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