Structure identification of an uncertain network coupled with complex-variable chaotic systems via adaptive impulsive control*

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In this paper, structure identification of an uncertain network coupled with complex-variable chaotic systems is investigated. Both the topological structure and the system parameters can be unknown and need to be identified. Based on impulsive stability theory and the Lyapunov function method, an impulsive control scheme combined with an adaptive strategy is adopted to design effective and universal network estimators. The restriction on the impulsive interval is relaxed by adopting an adaptive strategy. Further, the proposed method can monitor the online switching topology effectively. Several numerical simulations are provided to illustrate the effectiveness of the theoretical results.

Keywords: structure identification, network, complex-variable chaotic system, impulsive control

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1. Introduction

Dynamical networks are ubiquitous and have been widely used to model the large-scale physical and social systems consisting of interactive individuals or agents, such as neuronal networks, power grids, social relationship networks, and multi-agent systems.[1–4] The node dynamics in a network denote the dynamical behavior of the individuals, which are described by dynamical systems. Due to the complexity of physical and social systems, the node dynamics can be identical or nonidentical, which shows the node-dynamic complexity.[5] The edges in a network denote the interactions between any pair of nodes and can be directed or weighted, which shows the structural complexity.[5] As is well known, the system parameters of node dynamics and the topological structure play key roles in the dynamical evolution of a network. For complex dynamical networks with exact system parameters and a topological structure, lots of interesting and important results have been obtained.[6–14] Duan *et al.* investigated the synchronization of weighted networks and complex synchronized regions.[6] In Refs. [7]–[9], authors studied the pinning control of dynamical networks and provided some optimal pinning control strategies with respect to their topological structure. Clearly, all the above discussion is based on the assumption that the system parameters of node dynamics and the topological structure are known in advance.

However, in some cases, the system parameters and topological structure of a network cannot be known or exactly certain. Therefore, for better studying the dynamical properties of a network, the identification of the unknown system parameters and topological structure are necessary and important. Up to now, many methods based on synchronization are developed to design proper and effective network estimators for achieving this goal.[15–23] Yu *et al.* estimated the topology of a network based on the dynamical evolution for the first time.[15] Liu *et al.* investigated structure identification of dynamical networks with both topological structure and node dynamics via adaptive feedback control.[18] Xu *et al.* studied topology identification and adaptive synchronization of uncertain complex networks with adaptive double scaling functions.[21]

On the other hand, impulsive control, as a typical discontinuous control scheme, has been widely used to design proper controllers for achieving synchronization of coupled dynamical systems.[12,24–31] It has been shown that impulsive control is an effective, robust, and low-cost form of the view of control. Chen *et al.* studied the adaptive impulsive synchronization of uncertain real-variable chaotic systems, in which an adaptive strategy was adopted to relax the restrictions on the impulsive interval.[24] To the best of our knowledge, the structure identification of dynamical networks based on impulsive synchronization has seldom been investigated, which deserves to be studied.

Recently, many complex-variable chaotic systems are introduced to model physical systems involving complex variables, such as complex-variable Lorenz, Chen, and dynamo systems.[32–34] Further, complex networks coupled with complex-variable chaotic systems are introduced and studied as well.[35,36] Motivated by the above discussion, this paper investigates the structure identification of a network coupled with a complex-variable chaotic system. Based on impulsive stability theory and the Lyapunov function method, an impulsive control scheme combined with an adaptive strategy are adopted to design effective and universal network estimators. Section 2 introduces the model of a network coupled with complex-variable chaotic systems and gives some useful preliminaries. Section 3 investigates the structure identification of a complex-variable network and designs proper network es-

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timators via adaptive impulsive control. Section 4 provides several numerical examples to verify the theoretical results. Section 5 concludes this paper.

Notations: Throughout this paper, for any complex number (or complex vector) x, $\bar{x}$ denotes the conjugate of x, x^{r} and x^{i} denote its real and imaginary parts, $\|x\| = x^{\mathrm{T}}\bar{x}$. For any symmetric matrix P, $\lambda_{\mathrm{m}}(P)$ and $\lambda_{\mathrm{M}}(P)$ denote the smallest and largest eigenvalues of the matrix.

2. Model and preliminaries

Consider a network consisting of N nonidentical nodes with linear coupling, which is described by

$$\dot{x}_k(t) = f_k(x_k(t),\Theta_k) + \varepsilon\sum_{l=1}^{N} a_{kl}\Gamma x_l(t),\ k = 1,2,\ldots,N, \quad (1)$$

where $x_k(t) = (x_{k1}(t),x_{k2}(t),\ldots,x_{kn}(t))^{\mathrm{T}} \in \mathbb{C}^n$ is an n-dimensional complex vector with $x_{kl}(t) = x_{kl}^{\mathrm{r}}(t) + \mathrm{j}x_{kl}^{\mathrm{i}}(t)$ and $\mathrm{j} = \sqrt{-1}$, $\Theta_k = (\theta_{k1},\theta_{k2},\ldots,\theta_{kp_k})^{\mathrm{T}} \in \mathbb{C}^{p_k}$ is a p_k-dimensional system parameter vector of the k-th node, $f_k : \mathbb{C}^n \times \mathbb{C}^{p_k} \to \mathbb{C}^n$ is an n-dimensional complex-valued vector function denoting the node dynamics, $\varepsilon > 0$ is the coupling strength, $\Gamma = \mathrm{diag}(\gamma_1,\ldots,\gamma_n) \in \mathbb{R}^{n\times n}$ is the inner coupling matrix, $A = (a_{kl}) \in \mathbb{R}^{N\times N}$ is the zero-row-sum outer coupling matrix determining the topology and coupling strength of the network, which is defined as: if node k is affected by node l $(l \neq k)$, then $a_{kl} \neq 0$; otherwise, $a_{kl} = 0$.

In what follows, the node dynamics $f_k(x_k(t))$ is always assumed to be linear with respect to the system parameters, i.e., it can be written in the following form

$$\dot{x}_k(t) = g_k(x_k(t)) + F_k(x_k(t))\Theta_k,\ k = 1,2,\ldots,N,$$

where $g_k : \mathbb{C}^n \to \mathbb{C}^n$ is an n-dimensional complex-valued vector function, $F_k : \mathbb{C}^n \to \mathbb{C}^{n\times p_k}$ is an $n \times p_k$ matrix function.

Then, the network (1) can be rewritten as

$$\begin{aligned}\dot{x}_k(t) &= g_k(x_k(t)) + F_k(x_k(t))\Theta_k \\ &\quad + \varepsilon\sum_{l=1}^{N} a_{kl}\Gamma x_l(t),\ \ k = 1,2,\ldots,N. \end{aligned} \quad (2)$$

Assumption 1 Suppose that there exists a positive constant L such that the complex-valued vector function $f_k(x_k,\Theta_k)$ satisfies

$$\begin{aligned}&(y_k - x_k)^{\mathrm{T}}\overline{(f_k(y_k,\Theta_k) - f_k(x_k,\Theta_k))} \\ &+ \overline{(y_k - x_k)}^{\mathrm{T}}(f_k(y_k,\Theta_k) - f_k(x_k,\Theta_k)) \\ &\le L(y_k - x_k)^{\mathrm{T}}\overline{(y_k - x_k)},\end{aligned}$$

for all $x_k, y_k \in \mathbb{C}^n$, $k = 1,2,\ldots,N$.

Remark 1 It is easy to verify that some typical complex-variable chaotic systems, such as complex-variable Lorenz, Chen, and dynamos systems, all satisfy Assumption 1.

Assumption 2 Denote

$$F_k(x_k(t)) = (F_k^1(x_k(t)),\ldots,F_k^{p_k}(x_k(t)))$$

with $F_k^l(x_k(t)) \in \mathbb{C}^n$ for $l = 1,\ldots,p_k$. Suppose that

$$\{\{F_k^l(x_k(t))\}_{l=1}^{p_k}, \Gamma x_k(t)\}_{k=1}^{N}$$

are linearly independent of the orbits $\{x_k(t)\}_{k=1}^{N}$.

Lemma 1[29] For any two complex numbers α,β, and any real constant $\rho > 0$, the following inequality holds

$$\alpha\overline{\beta} + \overline{\alpha}\beta \le \rho\alpha\overline{\alpha} + \rho^{-1}\beta\overline{\beta}.$$

Lemma 2[31] Let $P \in \mathbb{R}^{n\times n}$ be a symmetric matrix, then

$$\lambda_m(P)x^{\mathrm{T}}\bar{x} \le x^{\mathrm{T}}P\bar{x} \le \lambda_M(P)x^{\mathrm{T}}\bar{x},$$

hold for any $x \in \mathbb{C}^n$.

3. Structure identification of uncertain networks

In this section, structure identification of an uncertain network (2) coupled with complex-variable chaotic systems is studied in two cases via adaptive impulsive control.

Firstly, assume that only the outer coupling matrix A of network (2) is unknown and to be identified. For identifying the unknown topological structure, the network estimator with impulsive controllers and adaptive laws is designed as follows:

$$\begin{aligned}\dot{y}_k(t) &= f_k(y_k(t),\Theta_k) + \varepsilon\sum_{l=1}^{N} d_{kl}(t)\Gamma y_l(t),\ \ t \neq t_\sigma, \\ \Delta y_k(t_\sigma) &= y_k(t_\sigma^+) - y_k(t_\sigma^-) = B_{k\sigma}(t_\sigma)y_k(t_\sigma),\ \ t = t_\sigma, \\ \dot{d}_{kl}(t) &= -\delta_{kl}(y_l^{\mathrm{T}}(t)\Gamma\overline{(y_k(t) - x_k(t))} \\ &\quad + (y_k(t) - x_k(t))^{\mathrm{T}}\Gamma\overline{y_l(t)}),\end{aligned} \quad (3)$$

where $k,l = 1,2,\ldots,N$, $\sigma = 1,2,3,\ldots$,

$$y_k(t) = (y_{k1}(t),y_{k2}(t),\ldots,y_{kn}(t))^{\mathrm{T}} \in \mathbb{C}^n$$

is an n-dimensional complex vector, d_{kl} is the estimated value of a_{kl}, and $\delta_{kl} > 0$ is the adaptive gain. $B_{k\sigma}(t) \in \mathbb{R}^{n\times n}$ is the impulsive gain matrix, defined as: if $t = t_\sigma$, then $B_{k\sigma}(t) \neq 0$; otherwise $B_{k\sigma}(t) = 0$. Discrete instant set $\{t_\sigma\}$ satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_\sigma < \cdots$, $t_\sigma \to \infty$ as $\sigma \to \infty$, $y_k(t_\sigma^+) = \lim_{t\to t_\sigma^+} y_k(t)$, $y_k(t_\sigma^-) = \lim_{t\to t_\sigma^-} y_k(t)$. Any solutions of Eq. (3) are assumed to be left continuous at each t_σ, i.e., $y_k(t_\sigma^-) = y_k(t_\sigma)$.

Denote λ and $\beta_\sigma(t)$ as the largest eigenvalues of matrices $\varepsilon(A + A^{\mathrm{T}}) \otimes \Gamma$ and $(I + B_{k\sigma}(t))^{\mathrm{T}}(I + B_{k\sigma}(t))$ for $k = 1,2,\ldots,N$, then, one has $\beta_\sigma(t) = 1$ when $t \neq t_\sigma$. Let $e_k(t) = y_k(t) - x_k(t)$ be the synchronization error, then one has the following error system:

$$\begin{aligned}\dot{e}_k(t) &= f_k(y_k(t),\Theta_k) - f_k(x_k(t),\Theta_k) \\ &\quad + \varepsilon\sum_{l=1}^{N}(d_{kl}(t) - a_{kl})\Gamma y_l(t) \\ &\quad + \varepsilon\sum_{l=1}^{N} a_{kl}\Gamma e_l(t),\ t \neq t_\sigma, \\ \Delta e_k(t_\sigma) &= e_k(t_\sigma^+) - e_k(t_\sigma^-) = B_{k\sigma}(t_\sigma)e_k(t_\sigma), \\ &\quad t = t_\sigma,\ \sigma = 1,2,\ldots,\end{aligned}$$

$$\dot{d}_{kl}(t) = -\delta_{kl}(y_l^{\mathrm{T}}(t)\Gamma\overline{e_k(t)} + e_k^{\mathrm{T}}(t)\Gamma\overline{y_l(t)}), \quad k,l = 1,2,\ldots,N. \tag{4}$$

Theorem 1 Suppose that Assumptions 1 and 2 hold. Then, the unknown coupling matrix A of the uncertain dynamical network (1) can be identified by the estimated matrix $D = (d_{kl})$ of the network estimator (3), if there exists a constant $\alpha > 0$, such that

$$\ln\beta_\sigma(t_\sigma) + \alpha + \hat{L}(t_\sigma)(t_\sigma - t_{\sigma-1}) < 0,\ \sigma = 1,2,\ldots, \tag{5}$$

where

$$\dot{\hat{L}}(t) = \eta\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\overline{e_i(t)},$$

and $\eta > 0$ is a positive constant.

Proof Consider the following Lyapunov function

$$V(t) = \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{\varepsilon\beta_\sigma(t)}{2\delta_{kl}}(d_{kl}(t) - a_{kl})^2 + \frac{\beta_\sigma(t)}{2\eta}(\hat{L}(t) - L - \lambda)^2,$$

for $t \in (t_{\sigma-1}, t_\sigma]$.

When $t \in (t_{\sigma-1}, t_\sigma)$, the function $V(t)$ can be written as

$$V(t) = \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{\varepsilon}{2\delta_{kl}}(d_{kl}(t) - a_{kl})^2 + \frac{1}{2\eta}(\hat{L}(t) - L - \lambda)^2,$$

and its derivative with respect to t along the trajectories of Eq. (4) can be calculated as

$$\begin{aligned}\dot{V}(t) &= \sum_{k=1}^{N}\left(\dot{e}_k^{\mathrm{T}}(t)\overline{e_k(t)} + e_k^{\mathrm{T}}(t)\overline{\dot{e}_k(t)}\right) + \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{\varepsilon}{\delta_{kl}}(d_{kl}(t) - a_{kl})\dot{d}_{kl}(t) + \frac{1}{\eta}(\hat{L}(t) - L - \lambda)\dot{\hat{L}}(t)\\ &= \sum_{i=1}^{N}(f_k(y_k(t),\Theta_k) - f_k(x_k(t),\Theta_k))^{\mathrm{T}}\overline{e_i(t)} + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\overline{(f_k(y_k(t),\Theta_k) - f_k(x_k(t),\Theta_k))}\\ &\quad + \varepsilon\sum_{k=1}^{N}\sum_{l=1}^{N} a_{kl}\left(e_l^{\mathrm{T}}(t)\Gamma\overline{e_i(t)} + e_k^{\mathrm{T}}(t)\Gamma\overline{e_l(t)}\right) + (\hat{L}(t) - L - \lambda)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\overline{e_i(t)}.\end{aligned}$$

Let $e(t) = (e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t))^{\mathrm{T}}$, according to Assumption 1, one has

$$\begin{aligned}\dot{V}(t) &\le \varepsilon\sum_{k=1}^{N}\sum_{l=1}^{N} a_{kl}\left(e_l^{\mathrm{T}}(t)\Gamma\overline{e_i(t)} + e_k^{\mathrm{T}}(t)\Gamma\overline{e_l(t)}\right) + (\hat{L}(t) - \lambda)\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\overline{e_i(t)}\\ &\le e^{\mathrm{T}}(t)\left(\varepsilon(A + A^{\mathrm{T}})\otimes\Gamma\right)\overline{e(t)} + (\hat{L}(t) - \lambda)e^{\mathrm{T}}(t)\overline{e(t)}\\ &\le \hat{L}(t)e^{\mathrm{T}}(t)\overline{e(t)}\\ &\le \hat{L}(t_\sigma)V(t),\end{aligned}$$

which gives

$$V(t) \le V(t_{\sigma-1}^+)\,\mathrm{e}^{\hat{L}(t_\sigma)(t - t_{\sigma-1})}. \tag{6}$$

When $t = t_\sigma$, one has

$$\begin{aligned}V(t_\sigma^+) &= \sum_{k=1}^{N} e_k^{\mathrm{T}}(t_\sigma^+)\overline{e_k(t_\sigma^+)} + \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{\varepsilon\beta_\sigma(t_\sigma)}{2\delta_{kl}}(d_{kl}(t_\sigma^+) - a_{kl})^2 + \frac{\beta_\sigma(t_\sigma)}{2\eta}(\hat{L}(t_\sigma^+) - L - \lambda)^2\\ &= \sum_{k=1}^{N} e_k^{\mathrm{T}}(t_\sigma^-)[(I + B_{k\sigma}(t_\sigma))^{\mathrm{T}}(I + B_{k\sigma}(t_\sigma))]\overline{e_k(t_\sigma^-)} + \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{\varepsilon\beta_\sigma(t_\sigma)}{2\delta_{kl}}(d_{kl}(t_\sigma^-) - a_{kl})^2\\ &\quad + \frac{\beta_\sigma(t_\sigma)}{2\eta}(\hat{L}(t_\sigma^-) - L - \lambda)^2\\ &\le \beta_\sigma(t_\sigma)V(t_\sigma^-).\end{aligned} \tag{7}$$

For $\sigma = 1$, from inequalities (6) and (7), one has

$$V(t_1^+) \le \beta_1(t_1)V(t_1^-) \le V(t_0^+)\beta_1(t_1)\,\mathrm{e}^{\hat{L}(t_1)(t_1 - t_0)}.$$

Similarly, for $k = 2$, one has

$$\begin{aligned}V(t_2^-) &\le V(t_1^+)\,\mathrm{e}^{\hat{L}(t_2)(t_2 - t_1)}\\ &\le V(t_0^+)\beta_1(t_1)\,\mathrm{e}^{\hat{L}(t_1)(t_1 - t_0)}\,\mathrm{e}^{\hat{L}(t_2)(t_2 - t_1)},\\ V(t_2^+) &\le \beta_2(t_2)V(t_2^-)\\ &\le V(t_0^+)\beta_1(t_1)\beta_2(t_2)\,\mathrm{e}^{\hat{L}(t_1)(t_1 - t_0)}\,\mathrm{e}^{\hat{L}(t_2)(t_2 - t_1)}.\end{aligned}$$

According to the mathematical induction, one can obtain the following inequality

$$V(t_\sigma^+) \le V(t_0^+)\prod_{\iota=1}^{\sigma}\beta_\iota(t_\iota)\,\mathrm{e}^{\hat{L}(t_\iota)(t_\iota - t_{\iota-1})},\quad \sigma = 1,2,\ldots.$$

From condition (5), one has $\beta_\iota(t_\iota)\,\mathrm{e}^{\hat{L}(t_\iota)(t_\iota - t_{\iota-1})} \le \mathrm{e}^{-\alpha}$ and $V(t_\sigma^+) \le V(t_0^+)\,\mathrm{e}^{-\sigma\alpha}$, which implies that $V(t_\sigma^+) \to 0$ as $\sigma \to \infty$.

Then, for $t \in (t_\sigma, t_{\sigma+1})$, one has

$$V(t) \le V(t_\sigma^+)\,\mathrm{e}^{\hat{L}(t_{\sigma+1})(t - t_\sigma)}$$

and $V(t) \to 0$ as $t \to \infty$, i.e., the error system (4) is globally exponentially stable about zero. Moreover, $y(t) = (y_1^{\mathrm{T}}(t), y_2^{\mathrm{T}}(t), \ldots, y_N^{\mathrm{T}}(t))^{\mathrm{T}}$ converges to the following largest invariant set

$$\mathscr{E} = \left\{y(t) : \sum_{l=1}^{N}(d_{kl}(t) - a_{kl})\Gamma y_l(t) = 0,\ k = 1,2,\ldots,N\right\},$$

as $t \to \infty$.

From Assumption 2, one has $d_{kl}(t) \to a_{kl}$ as $t \to \infty$, i.e., the unknown coupling matrix A is identified by the estimated matrix $D = (d_{kl})$. This completes the proof.

Secondly, assume that the coupling matrix A and system parameter vectors Θ_k are unknown and need to be identified. Choose the network estimator with impulsive controllers and updating law as

$$\begin{aligned}
\dot{y}_k(t) &= g_k(y_k(t)) + F_k(y_k(t))\Xi_k(t) \\
&\quad + \varepsilon \sum_{l=1}^{N} d_{kl}(t)\Gamma y_l(t), \ t \neq t_\sigma, \\
\Delta y_k(t_\sigma) &= y_k(t_\sigma^+) - y_k(t_\sigma^-) = B_{k\sigma}(t_\sigma) y_k(t_\sigma), \ t = t_\sigma, \\
\dot{\Xi}_k(t) &= -\rho_k \overline{F(y_k(t))}^{\mathrm{T}} e_k(t), \\
\dot{d}_{kl}(t) &= -\delta_{kl}(y_l^{\mathrm{T}}(t)\Gamma \overline{e_k(t)} + e_k^{\mathrm{T}}(t)\Gamma \overline{y_l(t)}),
\end{aligned} \tag{8}$$

where $k,l = 1,2,\ldots,N$, $\sigma = 1,2,\ldots,d_{kl}$, and $\Xi_k(t) = (\xi_{k1}(t),\xi_{k2}(t),\ldots,\xi_{kp_k}(t))^{\mathrm{T}}$ are the estimated values of a_{kl} and Θ_k, $\delta_{kl} > 0$ and $\rho_k > 0$ are the adaptive gains.

Then, the error system can be written as

$$\begin{aligned}
\dot{e}_k(t) &= f_k(y_k(t),\Theta_k) - f_k(x_k(t),\Theta_k) \\
&\quad + F_k(y_k(t))(\Xi_k(t) - \Theta_k) \\
&\quad + \varepsilon \sum_{l=1}^{N} (d_{kl}(t) - a_{kl})\Gamma y_l(t) \\
&\quad + \varepsilon \sum_{l=1}^{N} a_{kl}\Gamma e_l(t), \quad t \neq t_\sigma, \\
\Delta e_k(t_\sigma) &= e_k(t_\sigma^+) - e_k(t_\sigma^-) = B_{k\sigma}(t_\sigma) e_k(t_\sigma), \\
&\quad t = t_\sigma, \quad \sigma = 1,2,\ldots, \\
\dot{\Xi}_k(t) &= -\rho_k \overline{F(y_k(t))}^{\mathrm{T}} e_k(t), \\
\dot{d}_{kl}(t) &= -\delta_{kl}(y_l^{\mathrm{T}}(t)\Gamma \overline{e_k(t)} + e_k^{\mathrm{T}}(t)\Gamma \overline{y_l(t)}), \\
&\quad k,l = 1,2,\ldots,N.
\end{aligned} \tag{9}$$

Theorem 2 Suppose that Assumptions 1 and 2 hold. Then, the unknown coupling matrix A and system parameters Θ_k of the uncertain dynamical network (1) can be identified by $D = (d_{kl})$ and Ξ_k of the network estimator (8), if there exists a constant $\alpha > 0$, such that

$$\ln \beta_\sigma(t_\sigma) + \alpha + \hat{L}(t_\sigma)(t_\sigma - t_{\sigma-1}) < 0, \ \sigma = 1,2,\ldots, \tag{10}$$

where

$$\dot{\hat{L}}(t) = \eta \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\overline{e_i(t)},$$

and $\eta > 0$ is a positive constant.

Proof Consider the following Lyapunov function

$$\begin{aligned}
V(t) &= \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N} \frac{\beta_\sigma(t)}{2\rho_k}(\Xi_k(t) - \Theta_k)^{\mathrm{T}}\overline{(\Xi_k(t) - \Theta_k)} \\
&\quad + \sum_{k=1}^{N}\sum_{l=1}^{N} \frac{\varepsilon\beta_\sigma(t)}{2\delta_{kl}}(d_{kl}(t) - a_{kl})^2 \\
&\quad + \frac{\beta_\sigma(t)}{2\eta}(\hat{L}(t) - L - \lambda)^2,
\end{aligned}$$

for $t \in (t_{\sigma-1}, t_\sigma]$.

When $t \in (t_{\sigma-1}, t_\sigma)$, the function $V(t)$ can be written as

$$\begin{aligned}
V(t) &= \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N} \frac{1}{\rho_k}(\Xi_k(t) - \Theta_k)^{\mathrm{T}}\overline{(\Xi_k(t) - \Theta_k)} \\
&\quad + \sum_{k=1}^{N}\sum_{l=1}^{N} \frac{\varepsilon}{2\delta_{kl}}(d_{kl}(t) - a_{kl})^2 + \frac{1}{2\eta}(\hat{L}(t) - L - \lambda)^2,
\end{aligned}$$

and its derivative with respect to t along the trajectories of Eq. (9) can be calculated as

$$\begin{aligned}
\dot{V}(t) &= \sum_{k=1}^{N} \left(\dot{e}_k^{\mathrm{T}}(t)\overline{e_k(t)} + e_k^{\mathrm{T}}(t)\overline{\dot{e}_k(t)}\right) \\
&\quad + \sum_{k=1}^{N} \frac{1}{\rho_k}\left(\dot{\Xi}_k^{\mathrm{T}}(t)\overline{(\Xi_k(t) - \Theta_k)} + (\Xi_k(t) - \Theta_k)^{\mathrm{T}}\overline{\dot{\Xi}_k(t)}\right) \\
&\quad + \sum_{k=1}^{N}\sum_{l=1}^{N} \frac{\varepsilon}{\delta_{kl}}(d_{kl}(t) - a_{kl})\dot{d}_{kl}(t) \\
&\quad + \frac{1}{\eta}(\hat{L}(t) - L - \lambda)\dot{\hat{L}}(t) \\
&\leq \hat{L}(t)e^{\mathrm{T}}(t)\overline{e(t)} \\
&\leq \hat{L}(t_\sigma)V(t),
\end{aligned}$$

which gives

$$V(t) \leq V(t_{\sigma-1}^+)\, \mathrm{e}^{\hat{L}(t_\sigma)(t - t_{\sigma-1})}. \tag{11}$$

When $t = t_\sigma$, one has

$$\begin{aligned}
V(t_\sigma^+) &= \sum_{k=1}^{N} e_k^{\mathrm{T}}(t_\sigma^+)\overline{e_k(t_\sigma^+)} \\
&\quad + \sum_{k=1}^{N} \frac{\beta_\sigma(t_\sigma)}{\rho_k}(\Xi_k(t_\sigma^+) - \Theta_k)^{\mathrm{T}}\overline{(\Xi_k(t_\sigma^+) - \Theta_k)} \\
&\quad + \sum_{k=1}^{N}\sum_{l=1}^{N} \frac{\varepsilon\beta_\sigma(t_\sigma)}{2\delta_{kl}}(d_{kl}(t_\sigma^+) - a_{kl})^2 \\
&\quad + \frac{\beta_\sigma(t_\sigma)}{2\eta}(\hat{L}(t_\sigma^+) - L - \lambda)^2 \\
&\leq \beta_\sigma(t_\sigma)V(t_\sigma^-).
\end{aligned} \tag{12}$$

Similar to the proof of Theorem 1, one can show that the error system (9) is globally exponentially stable about zero and $y(t) = (y_1^{\mathrm{T}}(t), y_2^{\mathrm{T}}(t), \ldots, y_N^{\mathrm{T}}(t))^{\mathrm{T}}$ converges to the following largest invariant set

$$\begin{aligned}
\mathscr{E} = \Big\{ & y(t) : F_k(y_k(t))(\Xi_k(t) - \Theta_k) \\
& + \sum_{l=1}^{N} (d_{kl}(t) - a_{kl})\Gamma y_l(t) = 0, \ k = 1,2,\ldots,N \Big\},
\end{aligned}$$

as $t \to \infty$.

From Assumption 2, one has $\Xi_k(t) \to \Theta_k$ and $d_{kl}(t) \to a_{kl}$ as $t \to \infty$, i.e., the unknown system parameters Θ_k and coupling matrix A are identified by $\Xi_k(t)$ and $D = (d_{kl})$. This completes the proof.

Remark 2 As is well known, different node dynamics may derive different constants L in Assumption 1. From the proofs of Theorems 1 and 2, by virtue of the adaptive strategy, the constant L and the largest eigenvalue λ of matrix $\varepsilon(A + A^{\mathrm{T}}) \otimes \Gamma$ do not need to be known beforehand. Therefore, the proposed methods are universal for different dynamical networks to some extent. Moreover, the proposed methods can monitor the online evolution of topological structure.

Remark 3 In most of the existing results about impulsive control, the largest impulsive interval is estimated on certain conditions, e.g., the constant L and impulsive gain matrices $B_{k\sigma}$ need to be known. Due to the fact that the estimated value of constant L is usually much larger than its actual value, the largest impulsive interval is much smaller than the value needed, which means that the control cost can be reduced. In other words, the constant L has a restriction on the largest impulsive interval. By combining the adaptive strategy and impulsive control, the proposed method can relax the restriction to a certain extent. For example, one can choose the impulsive gain matrices $B_{k\sigma}(t_\sigma) = b_\sigma(t_\sigma)I_n$, i.e., $\beta_\sigma(t_\sigma) = (1+b_\sigma(t_\sigma))^2$. Then, one can choose

$$\begin{aligned} &-\exp\left[-\frac{\alpha+\hat{L}(t_\sigma)\tau_\sigma}{2}\right]-1+\varepsilon \\ &\le b_\sigma(t_\sigma) \le \exp\left[-\frac{\alpha+\hat{L}(t_\sigma)\tau_\sigma}{2}\right]-1-\varepsilon, \end{aligned} \tag{13}$$

such that the conditions (5) and (10) hold, where $\tau_\sigma = t_\sigma - t_{\sigma-1}$ and ε is a small positive constant. That is to say, the impulsive intervals τ_σ can be chosen properly, regardless of the constant L, and the impulsive gain matrix can adjust itself to achieve the goals.

4. Numerical simulations

In this section, several numerical examples are illustrated to verify the effectiveness of the proposed method. For simplicity, in the following examples, choose the inner coupling matrix as $\Gamma = \mathrm{diag}\{1,1,1\}$.

Firstly, verify Theorem 1 and choose the node dynamics of network (1) as the following complex-variable Lorenz systems[26]

$$\dot{x}_{k1} = -\theta_{k1}(x_{k1}-x_{k2}), \tag{14}$$

$$\dot{x}_{k2} = -x_{k1}x_{k3} + \theta_{k2}x_{k1} - \theta_{k3}x_{k2}, \tag{15}$$

$$\dot{x}_{k3} = -\theta_{k4}x_{k3} + (\overline{x_{k1}}x_{k2} + x_{k1}\overline{x_{k2}})/2, \tag{16}$$

where $k = 1,2,\ldots,6$, x_{k1} and x_{k2} are complex variables, x_{k3} is the real variable,

$$\Theta_k = (\theta_{k1}, \theta_{k2}, \theta_{k3}, \theta_{k4})^{\mathrm{T}}$$

are the system parameter vectors. They exhibit chaotic behavior when the system parameter vectors are chosen as $\Theta_k = (1.7+0.1k, 60+0.02\mathrm{j}, 1-0.06\mathrm{j}, 0.8)^{\mathrm{T}}$ with $\mathrm{j} = \sqrt{-1}$. Then, there exist three positive constants M_1, M_2, and M_3 such that $x_{k1} \le M_1$, $x_{k2} \le M_2$, and $x_{k3} \le M_3$. According to Lemma 1, one has

$$\begin{aligned} &(y_k - x_k)^{\mathrm{T}}\overline{(f_k(y_k,\Theta_k) - f_k(x_k,\Theta_k))} + \overline{(y_k - x_k)}^{\mathrm{T}}(f_k(y_k,\Theta_k) - f_k(x_k,\Theta_k)) \\ &= \begin{pmatrix} e_{k1} \\ e_{k2} \\ e_{k3} \end{pmatrix}^{\mathrm{T}} \overline{\begin{pmatrix} \theta_{k1}(e_{k2}-e_{k1}) \\ \theta_{k2}e_{k1} - \theta_{k3}e_{k2} - y_{k1}y_{k3} + x_{k1}x_{k3} \\ -\theta_{k4}e_{k3} + (\overline{y_{k1}}y_{k2} + y_{k1}\overline{y_{k2}} - \overline{x_{k1}}x_{k2} - x_{k1}\overline{x_{k2}})/2 \end{pmatrix}} \\ &\quad + \begin{pmatrix} \theta_{k1}(e_{k2}-e_{k1}) \\ \theta_{k2}e_{k1} - \theta_{k3}e_{k2} - y_{k1}y_{k3} + x_{k1}x_{k3} \\ -\theta_{k4}e_{k3} + (\overline{y_{k1}}y_{k2} + y_{k1}\overline{y_{k2}} - \overline{x_{k1}}x_{k2} - x_{k1}\overline{x_{k2}})/2 \end{pmatrix}^{\mathrm{T}} \overline{\begin{pmatrix} e_{k1} \\ e_{k2} \\ e_{k3} \end{pmatrix}} \\ &= -(\theta_{k1}+\overline{\theta_{k1}})e_{k1}\overline{e_{k1}} - (\theta_{k3}+\overline{\theta_{k3}})e_{k2}\overline{e_{k2}} - 2\theta_{k4}e_{k3}^2 + (\theta_{k1}+\overline{\theta_{k2}} - x_{k3})e_{k1}\overline{e_{k2}} \\ &\quad + (\overline{\theta_{k1}} + \theta_{k2} - x_{k3})\overline{e_{k1}}e_{k2} + \overline{e_{k1}}x_{k2}e_{k3} + e_{k1}\overline{x_{k2}}e_{k3} \\ &\le \left(\rho_1\theta_{k1}\overline{\theta_{k1}} + \rho_2\theta_{k2}\overline{\theta_{k2}} + \rho_3 M_3^2 + \rho_4 M_2^2 - \theta_{k1} - \overline{\theta_{k1}}\right)e_{k1}\overline{e_{k1}} \\ &\quad + \left(\rho_1^{-1}\theta_{k1}\overline{\theta_{k1}} + \rho_2^{-1}\theta_{k2}\overline{\theta_{k2}} + \rho_3^{-1}M_3^2 - \theta_{k3} - \overline{\theta_{k3}}\right)e_{k2}\overline{e_{k2}} + \left(\rho_4^{-1}M_2^2 - 2\theta_{k4}\right)e_{k3}^2, \end{aligned}$$

where ρ_1, ρ_2, ρ_3, and ρ_4 are positive constants. Then, one can choose

$$L = \max\{\rho_1\theta_{k1}\overline{\theta_{k1}} + \rho_2\theta_{k2}\overline{\theta_{k2}} + \rho_3 M_3^2 + \rho_4 M_2^2 - \theta_{k1} - \overline{\theta_{k1}}, \rho_1^{-1}\theta_{k1}\overline{\theta_{k1}} + \rho_2^{-1}\theta_{k2}\overline{\theta_{k2}} + \rho_3^{-1}M_3^2 - \theta_{k3} - \overline{\theta_{k3}}, \rho_4^{-1}M_2^2 - 2\theta_{k4}\}$$

with proper ρ_1, ρ_2, ρ_3, and ρ_4 such that Assumption 1 holds.

Suppose that the unknown coupling matrix A of network (1) is chosen as

$$A = \begin{bmatrix} 0 & -1 & 2 & 1 & 0 & -2 \\ 2 & -2 & 0 & -1 & 0 & 1 \\ 0 & -2 & 2 & 1 & -1 & 0 \\ 1 & -2 & 0 & -1 & 2 & 0 \\ 0 & 0 & 2 & 1 & -3 & 0 \\ 1 & 0 & 0 & 2 & -1 & -2 \end{bmatrix}.$$

In the numerical simulations, choose the coupled strength $\varepsilon = 0.01$, the initial values of $\hat{L}(t)$ and $d_{kl}(t)$ as $\hat{L}(0) = 2$ and $d_{kl}(0) = 0.5$, the adaptive gains $\eta = 0.02$ and $\delta_{kl} = 2$, the impulsive interval $\tau_\sigma = 0.2$, $\alpha = 0.4$, $b_\sigma(t_\sigma) = \exp(-(\alpha + \hat{L}(t_\sigma)\tau_\sigma)/2) - 1 - \varepsilon, \sigma = 1,2,\ldots$, and $\varepsilon = 0.001$ according to Remark 3. Moreover, choose the initial values of the state variables randomly. Figure 1 shows the structure identification of a network with fixed topology and Fig. 2 shows the impulsive gain b_σ versus σ.

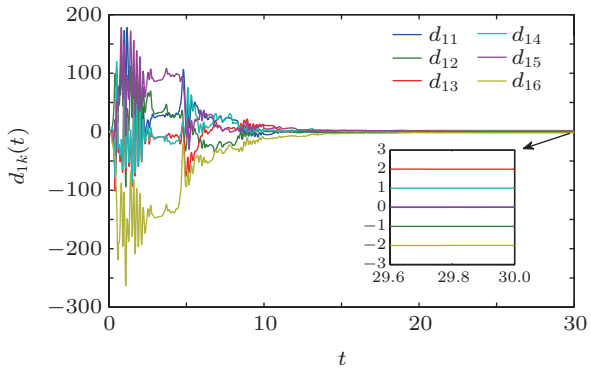


Fig. 1. (color online) Structure identification of a network with fixed topology.

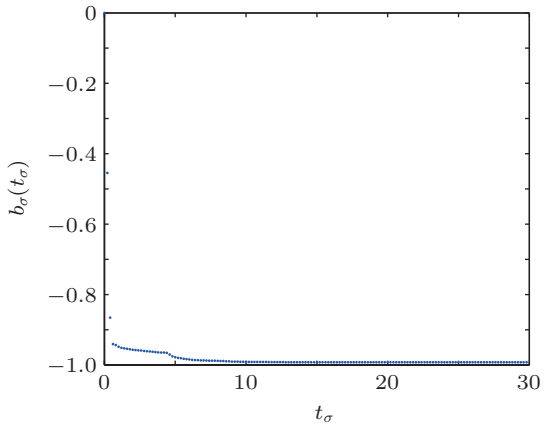


Fig. 2. (color online) The impulsive gain b_σ versus σ.

Secondly, consider the structure identification of network (1) with a switching topology, i.e., the outer coupling matrix switches from A to the following A' at time $t = 50$,

$$A' = \begin{bmatrix} 0 & -1 & 3 & 0 & 1 & -3 \\ 2 & -2 & 0 & -1 & 0 & 1 \\ 0 & -2 & 2 & 1 & -1 & 0 \\ 1 & -2 & 0 & -1 & 2 & 0 \\ 0 & 0 & 2 & 1 & -3 & 0 \\ 1 & 0 & 0 & 2 & -1 & -2 \end{bmatrix}.$$

In the numerical simulations, choose the coupled strength $\varepsilon = 0.01$, the initial values of $\hat{L}(t)$ and $d_{kl}(t)$ as $\hat{L}(0) = 2$ and $d_{kl}(0) = 0.5$, the adaptive gains $\eta = 2$ and $\delta_{kl} = 2$, the impulsive interval $\tau_\sigma = 0.1$, $\alpha = 4$, $b_\sigma(t_\sigma) = \exp(-(\alpha + \hat{L}(t_\sigma)\tau_\sigma)/2) - 1 - \varepsilon$, $\sigma = 1,2,\ldots,$ and $\varepsilon = 0.001$ according to Remark 3. Moreover, choose the initial values of the state variables randomly. Figure 3 shows the structure identification of a network with switching topology.

Thirdly, verify Theorem 2 and choose the node dynamics of network (2) as the following complex-variable Chen systems[27]

$$\dot{x}_{k1} = \theta_{k1}(x_{k2} - x_{k1}), \tag{17}$$

$$\dot{x}_{k2} = (\theta_{k2} - \theta_{k1})x_{k1} - x_{k1}x_{k3} + \theta_{k2}x_{k2}, \tag{18}$$

$$\dot{x}_{k3} = (\overline{x_{k1}}x_{k2} + x_{k1}\overline{x_{k2}})/2 - \theta_{k3}x_{k3}, \tag{19}$$

where $\Theta_k = (\theta_{k1}, \theta_{k2}, \theta_{k3})^{\mathrm{T}}$ are the system parameter vectors. According to Ref. [35], the complex-variable Chen system satisfies Assumption 1.

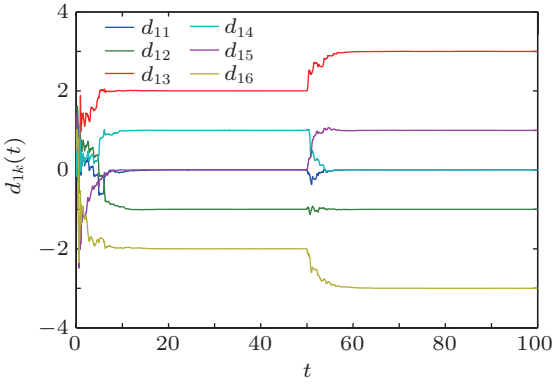


Fig. 3. (color online) Structure identification of a network with switching topology.

Suppose that the unknown coupling matrix is chosen the same as that in the first example and the unknown system parameter vectors are chosen as

$$\Theta_k = (\theta_{k1}, \theta_{k2}, \theta_{k3})^{\mathrm{T}} = (27, 22.2 + 0.2k, 1)^{\mathrm{T}}$$

for $k = 1,2,\ldots,6$.

In the numerical simulations, choose the coupled strength $\varepsilon = 0.01$, the initial values of $\hat{L}(t)$, $d_{kl}(t)$, and $\Xi_k(t)$ as $\hat{L}(0) = 10$, $d_{kl}(0) = 0.5$, and

$$\Xi_k(0) = (\xi_{k1}, \xi_{k2}, \xi_{k3})^{\mathrm{T}} = (25, 20, 0.5)^{\mathrm{T}},$$

the adaptive gain $\eta = 1$, $\delta_{kl} = 8$, and $\rho_k = 1$, the impulsive interval $\tau_\sigma = 0.05$, $\alpha = 80$, $b_\sigma(t_\sigma) = \exp(-(\alpha + \hat{L}(t_\sigma)\tau_\sigma)/2) - 1 - \varepsilon, \sigma = 1,2,\ldots,$ and $\varepsilon = 0.001$ according to Remark 3. Moreover, choose the initial values of the state variables randomly. Figures 4 and 5 show the identification of the unknown topological structure and system parameters respectively.

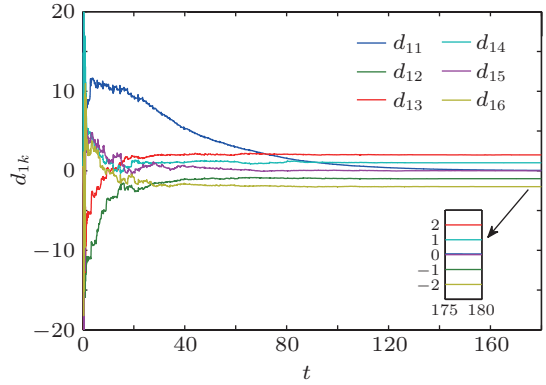


Fig. 4. (color online) Identification of the unknown topological structure.

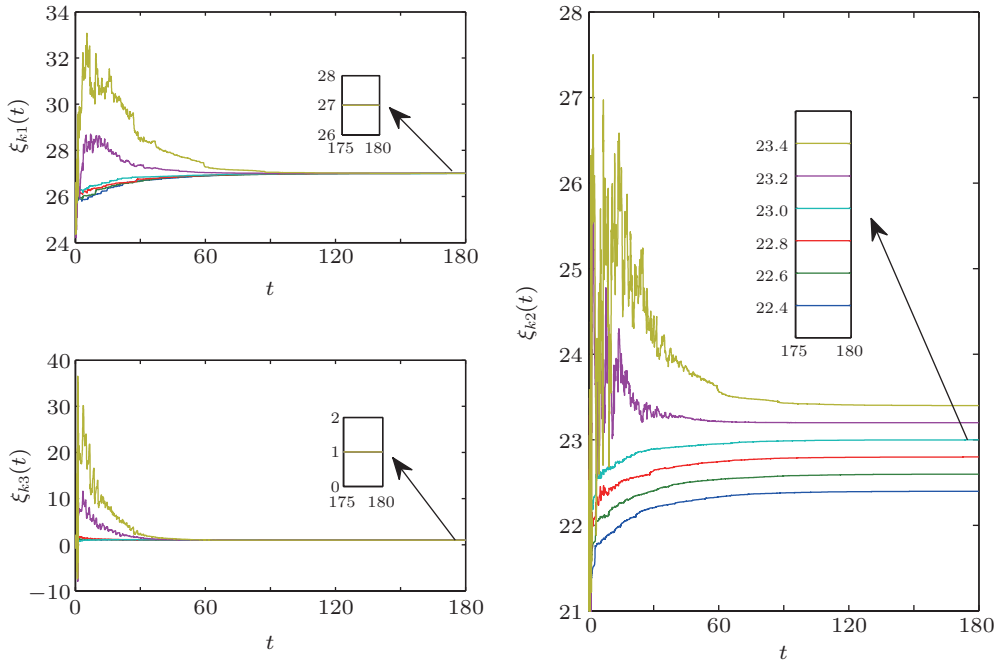


Fig. 5. (color online) Identification of unknown system parameters.

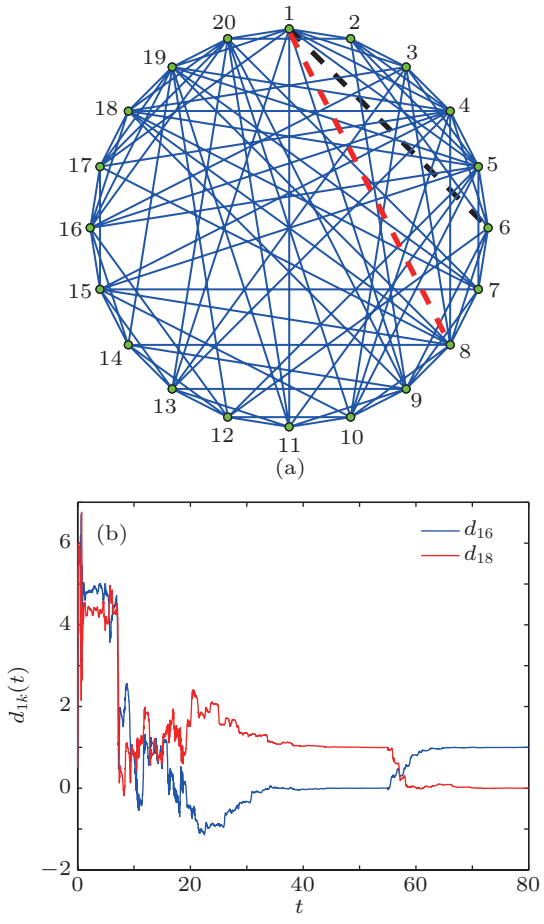


Fig. 6. (color online) (a) A network generated by the NW small-world network with $N = 20$, $K = 2$, $p = 0.2$. If there is an edge between nodes k and l, then $a_{kl} = 1$; otherwise, $a_{kl} = 0$. At time $t = 55$, the black dash line switches to the red dash line. (b) Identification of switching topological structure.

Finally, consider a small-world network consisting of ten Lorenz systems (14) and ten Chen systems (17), whose topological structure is generated by the NW small-world network[38] with $N = 20$, $K = 2$, $p = 0.2$, and shown in Fig. 6(a). Moreover, at time $t = 55$, the edge between nodes 1 and 6 switches to connect nodes 1 and 8, i.e., the black dashed line in Fig. 6(a) switches to the red dashed line. The system parameter vectors are chosen as $\Theta_k = (1.7+0.1k, 60+0.02\mathrm{j}, 1-0.06\mathrm{j}, 0.8)^{\mathrm{T}}$ for $k = 1,2,\ldots,10$ and $\Theta_k = (27, 21+0.1k, 1)^{\mathrm{T}}$ for $k = 11,12,\ldots,20$, and assumed to be known. In the numerical simulations, choose the coupled strength $\varepsilon = 0.01$, the initial values of $\hat{L}(t)$ and $d_{kl}(t)$ as $\hat{L}(0) = 2$ and $d_{kl}(0) = 0.5$, the adaptive gains $\eta = 0.01$ and $\delta_{kl} = 1$, the impulsive interval $\tau_\sigma = 0.2$, $\alpha = 4$, $b_\sigma(t_\sigma) = \exp(-(\alpha+\hat{L}(t_\sigma)\tau_\sigma)/2) - 1 - \varepsilon, \sigma = 1,2,\ldots,$ and $\varepsilon = 0.001$. Moreover, choose the initial values of the state variables randomly. Figure 6(b) shows the identification of a switching topological structure.

5. Conclusions

In this paper, the structure identification of an uncertain network coupled with complex-variable chaotic systems has been investigated. An impulsive control scheme is adopted to design network estimators for identifying the system parameters and the topological structure of a network. Further, an adaptive strategy is adopted to relax the restriction on the impulsive intervals. Based on impulsive stability theory and the Lyapunov function method, two network estimators are designed and verified to be effective and universal by several numerical examples. Moreover, the proposed methods can monitor the online evolution of network topological structure and be directly applied to the structure identification of networks coupled with real-variable chaotic systems.

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