Collective composite-rotating consensus of multi-agent systems*

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This paper investigates a distributed composite-rotating consensus problem of second-order multi-agent systems, where all agents move in a nested circular orbit. A distributed control law is proposed which contains two parts: the local state feedback that guarantees the circular motion and the distributed relative state feedback that guarantees the consensus of all agents. A sufficient condition is derived to drive all agents as well as ensure their circle centers make circular motion in a distributed manner. Finally, a numerical simulation is included to demonstrate our theoretical results.

Keywords: composite-rotating consensus, second-order dynamics, complex systems

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1. Introduction

Consensus problems have received considerable attention from many fields including physics, robotics, computer science, and control engineering. "Consensus" means to reach an agreement regarding a certain quantity of interest. Numerous studies have been conducted on consensus problems.[1–15] For example, in Ref. [2], the authors used graph theory and matrix theory to study first-order multi-agent consensus problems, and in Ref. [4], the authors presented some second-order consensus algorithms to guarantee state consensus of systems subjected to the saturation of the actuator and limited available information. In nature, there exists a kind of collective consensus motion such as the motion of celestial bodies where some stars nestedly surround other stars, e.g., the movement of the moon with respect to the sun. It is meaningful to study such motions because they can find important potential applications in flyaround satellites, spacecraft docking, the spacecraft fault detection, space exploration, and so on. For example, in the satellite flight with a circular formation, the following satellites need to move around not only the earth but also the leader-satellite. However, currently, there are rare works concerned about these collective motions. Though in Refs. [5]–[7] the authors have studied collective rotating motions, the circle centers of all agents are assumed to converge to a fixed point, and none of them has paid attention to the composite case, where the circle centers have some certain movement.

In this paper, our work will focus on this composite-rotating consensus problem, where each agent and its circle center move in a circle simultaneously. Here, it should be pointed out that the trajectories of the circle centers of all agents are not prescribed in advance but self-formed. To solve the composite-rotating consensus problem, we introduce a consensus control law which contains two parts: one is the local state feedback that guarantees the final rotating motion and the other is the communication state feedback that guarantees the system consensus. By a model transformation, it is proved that all agents as well as their centers eventually reach a consensus and move in a circle as long as the initial conditions of all agents satisfy a certain condition.

The following notation will be used throughout this paper. $\mathbb{R}^m$ and $\mathbb{C}^m$ represent the set of all m-dimensional real and complex column vectors; I_m denotes the m-dimensional unit matrix; j represents the imaginary unit; $\otimes$ denotes the kronecker product; $\mathbf{1}$ represents $[1,1,\ldots,1]^{\mathrm{T}}$ with compatible dimensions (sometimes, we use $\mathbf{1}_n$ to denote $\mathbf{1}$ with dimension n); 0 denotes zero value or zero matrix with appropriate dimensions.

2. Graph theory

Let $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ be an undirected graph of order n, where $\mathcal{V}=\{\mathrm{s}_1,\ldots,\mathrm{s}_n\}$ is the set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A}=[a_{ik}]$ is a weighted adjacency matrix. The node indexes belong to a finite index set $\mathcal{I}=\{1,2,\ldots,n\}$. An edge of $\mathcal{G}$ is denoted by $e_{ik}=(\mathrm{s}_i,\mathrm{s}_k)$. The adjacency matrix $\mathcal{A}$ is defined as $a_{ii}=0$ and $a_{ik}\geq 0$. $a_{ik}=a_{ki}>0$ if and only if $e_{ik}\in\mathcal{E}$. Thus, the adjacency matrix $\mathcal{A}$ is a symmetric nonnegative matrix. The set of neighbors of node s_i is denoted by $N_i=\{\mathrm{s}_k\in\mathcal{V}:(\mathrm{s}_i,\mathrm{s}_k)\in\mathcal{E}\}$. The out-degree of node s_i is defined as $d_o(\mathrm{s}_i)=\sum_{k=1}^{n}a_{ik}$. Then, the Laplacian corresponding to the undirected graph is defined as $L=[l_{ik}]$, where $l_{ii}=d_o(\mathrm{s}_i)$ and $l_{ik}=-a_{ik}$, $i\neq k$. Obviously, the Laplacian of

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any undirected graph is symmetric. A path is a sequence of ordered edges of the form $(s_{i_1}, s_{i_2}), (s_{i_2}, s_{i_3}), \ldots$, where $i_k \in \mathcal{I}$ and $s_{i_k} \in \mathcal{V}$. If there is a path from every node to every other node, the graph is said to be connected.

Lemma 1[16] If the undirected graph $\mathcal{G}$ is connected, then the Laplacian L of $\mathcal{G}$ has the following properties.

(i) Zero is a simple eigenvalue of L, and $\mathbf{1}$ is the corresponding eigenvector, i.e., $L\mathbf{1} = 0$.

(ii) The rest $n-1$ eigenvalues are all positive and real.

3. Model and problem statement

Consider a multi-agent system consisting of n agents. Each agent is regarded as a node in an undirected graph $\mathcal{G}$. Each edge $(s_i, s_k) \in \mathcal{E}$ corresponds to an available information channel between agents s_i and s_k. Suppose that the i-th agent s_i ($i \in \mathcal{I}$) has the following dynamics:

$$\begin{aligned} \dot{r}_i &= v_i, \\ \dot{v}_i &= u_i, \end{aligned} \tag{1}$$

where the vectors $r_i, v_i \in \mathbb{C}$ denote the position and velocity of the agent s_i and $u_i(t) \in \mathbb{C}$ is the control input.

In this paper, our objective is to propose a distributed control law using local information to enable multi-agent systems to move together in a circular orbit, where each agent has a circular motion while the trajectories of the circle center of each agent form a circle.

Definition 1 The multi-agent system (1) reaches a composite-rotating consensus if consensus is reached, i.e.,

$$\begin{aligned} &\lim_{t\to+\infty}[r_i(t) - r_k(t)] = 0, \\ &\lim_{t\to+\infty}[v_i(t) - v_k(t)] = 0, \end{aligned}$$

for all $i, k \in \mathcal{I}$, and each agent i surrounds the point $r_i(t) + \omega_1^{-1}\mathrm{j}v_i(t)$ whose trajectories form a circle, i.e., there exists a positive number ω_2 such that

$$\lim_{t\to+\infty}[\dot{r}_i(t) + \omega_1^{-1}\mathrm{j}\dot{v}_i(t) - \mathrm{j}\omega_2(r_i(t) + \omega_1^{-1}\mathrm{j}v_i(t))] = 0$$

for all $i \in \mathcal{I}$.

In Definition 1, the first two limits mean that all agents finally reach a consensus on position and velocity states. The term $\omega_1^{-1}\mathrm{j}v_i(t)$ denotes the radius of the i-th agent and the term $r_i(t) + \omega_1^{-1}\mathrm{j}v_i(t)$ denotes the circle center. By mechanics knowledge, it is easy to see that the limit $\lim_{t\to+\infty}[\dot{r}_i(t) + \omega_1^{-1}\mathrm{j}\dot{v}_i(t) - \mathrm{j}\omega_2(r_i(t) + \omega_1^{-1}\mathrm{j}v_i(t))] = 0$ means that the circle center $r_i(t) + \omega_1^{-1}\mathrm{j}v_i(t)$ eventually moves in a circle with angular velocity ω_2. In particular, when the term $r_i(t) + \omega_1^{-1}\mathrm{j}v_i(t)$ stays static without performing other motions including circular motion, i.e., $\omega_2 = 0$, the above composite-consensus problem would be degenerated into the general rotating consensus problem.

4. Rotating consensus control of multi-agent systems

In this section, we study rotating consensus problems of the multi-agent system (1). The control law is given by

$$u_i = u_{i1} + u_{i2}, \tag{2}$$

where

$$u_{i1} = \omega_1 \mathrm{j}v_i - \omega_1\omega_2\left(r_i + \mathrm{j}\frac{v_i}{\omega_1}\right)$$

and

$$u_{i2} = -\sum_{s_k\in N_i} a_{ik}[v_i - v_k] - \sum_{s_k\in N_i} a_{ik}\left[\left(r_i + \mathrm{j}\frac{v_i}{\omega_1}\right) - \left(r_k + \mathrm{j}\frac{v_k}{\omega_1}\right)\right]$$

for any $i \in \mathcal{I}$.

In control law (2), the term u_{i1} is the local state feedback, which contains two parts: one is $\omega_1\mathrm{j}v_i$ that can be regarded as the centripetal acceleration to make each agent circle around the point $r_i + \mathrm{j}(v_i/\omega_1)$ and the other is $-\omega_1\omega_2(r_i + \mathrm{j}(v_i/\omega_1))$ that plays a role in making each agent move in a circle. The term u_{i2} is the distributed state feedback that enables all agents to eliminate the disagreement dynamics and make all agents reach a consensus.

Denote

$$c_i = r_i + \mathrm{j}\frac{v_i}{\omega_1}, \quad \forall i \in \mathcal{I}, \quad \xi = [r_1, c_1, r_2, c_2, \ldots, r_n, c_n]^{\mathrm{T}},$$

$$A = \begin{bmatrix} \omega_1\mathrm{j} & -\omega_1\mathrm{j} \\ 0 & -\omega_2\mathrm{j} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 \\ -1 & 1 + \mathrm{j}/\omega_1 \end{bmatrix}.$$

With the control law (2), the closed-loop network dynamics can be written as

$$\dot{\xi}(t) = (I_n \otimes A - L \otimes B)\xi(t), \tag{3}$$

where L is the Laplacian of the graph $\mathcal{G}$.

Lemma 2 Let

$$\begin{aligned} \alpha(t) &= \frac{1}{n}\sum_{i=1}^{n} r_i(t) \\ &= \mathrm{e}^{\mathrm{j}\omega_1 t}\frac{1}{n}\sum_{i=1}^{n} r_i(0) + \frac{\omega_1}{\omega_1 + \omega_2}[\mathrm{e}^{-\omega_2\mathrm{j}t} - \mathrm{e}^{\mathrm{j}\omega_1 t}]\frac{1}{n}\sum_{i=1}^{n} c_i(0), \\ \beta(t) &= \frac{1}{n}\sum_{i=1}^{n} c_i(t) = \frac{1}{n}\sum_{i=1}^{n} c_i(0)\,\mathrm{e}^{-\omega_2\mathrm{j}t}, \end{aligned}$$

and $\delta(t) = \xi(t) - \mathbf{1}_n \otimes \begin{bmatrix}\alpha(t) & \beta(t)\end{bmatrix}^{\mathrm{T}}$. Then, $(\mathbf{1}_n^{\mathrm{T}} \otimes I_2)\delta(t) = 0$ and system (3) is equivalent to

$$\dot{\delta}(t) = (I_n \otimes A - L \otimes B)\delta(t). \tag{4}$$

Moreover, if

$$3\omega_1\omega_2\|\frac{1}{n}\sum_{i=1}^{n} c_i(0)\| < (\omega_1 + \omega_2)\|\frac{1}{n}\sum_{i=1}^{n} v_i(0)\|,$$

the variable $\beta(t)$ is encircled by the curve of $\alpha(t)$.

Proof Since $\mathcal{G}$ is undirected, it follows that

$$\frac{1}{n}\sum_{i=1}^{n}\dot{c}_i(t) = -\frac{1}{n}\omega_2 \mathrm{j}\sum_{i=1}^{n}c_i(t),$$

which implies

$$\frac{1}{n}\sum_{i=1}^{n}c_i(t) = \mathrm{e}^{-\omega_2 \mathrm{j}t}\frac{1}{n}\sum_{i=1}^{n}c_i(0).$$

Moreover, note that

$$\frac{1}{n}\sum_{i=1}^{n}\dot{r}_i(t) = \mathrm{j}\omega_1\frac{1}{n}\sum_{i=1}^{n}r_i(t) - \mathrm{j}\omega_1\frac{1}{n}\sum_{i=1}^{n}c_i(t).$$

By linear system theory, we have

$$\begin{aligned}
&\frac{1}{n}\sum_{i=1}^{n}r_i(t)\\
=\ & \mathrm{e}^{\mathrm{j}\omega_1 t}\frac{1}{n}\sum_{i=1}^{n}r_i(0) - \int_0^t \mathrm{e}^{\mathrm{j}\omega_1(t-s)}\mathrm{j}\omega_1\,\mathrm{e}^{-\omega_2 \mathrm{j}s}\frac{1}{n}\sum_{i=1}^{n}c_i(0)\mathrm{d}s\\
=\ & \mathrm{e}^{\mathrm{j}\omega_1 t}\frac{1}{n}\sum_{i=1}^{n}r_i(0) + \frac{\omega_1}{\omega_1+\omega_2}[\mathrm{e}^{-\omega_2 \mathrm{j}t} - \mathrm{e}^{\mathrm{j}\omega_1 t}]\frac{1}{n}\sum_{i=1}^{n}c_i(0).
\end{aligned}$$

It is clear that $(\mathbf{1}_n^{\mathrm{T}}\otimes I_2)\delta(t) = 0$. By simple calculations, we have

$$\begin{aligned}
\dot{\delta}(t) &= \dot{\xi}(t) - \mathbf{1}_n\otimes\left[\dot{\alpha}(t)\ \ \dot{\beta}(t)\right]^{\mathrm{T}}\\
&= (I_n\otimes A - L\otimes B)\xi(t)\\
&\quad -(I_n\otimes A - L\otimes B)(\mathbf{1}_n\otimes\left[\alpha(t)\ \ \beta(t)\right]^{\mathrm{T}})\\
&= (I_n\otimes A - L\otimes B)\delta(t),
\end{aligned}$$

where

$$(L\otimes B)(\mathbf{1}_n\otimes\left[\alpha(t)\ \ \beta(t)\right]^{\mathrm{T}}) = 0.$$

Also, it is easily obtained that

$$\begin{aligned}
&\frac{1}{n}\sum_{i=1}^{n}r_i(t) - \frac{1}{n}\sum_{i=1}^{n}c_i(t)\\
=\ & \mathrm{e}^{\mathrm{j}\omega_1 t}\frac{1}{n}\sum_{i=1}^{n}r_i(0) + \frac{\omega_1}{\omega_1+\omega_2}[\mathrm{e}^{-\omega_2 \mathrm{j}t} - \mathrm{e}^{\mathrm{j}\omega_1 t}]\\
&\times\frac{1}{n}\sum_{i=1}^{n}c_i(0) - \frac{1}{n}\sum_{i=1}^{n}c_i(0)\,\mathrm{e}^{-\omega_2 \mathrm{j}t}\\
=\ & \mathrm{e}^{\mathrm{j}\omega_1 t}\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0)\\
&-\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0)\,\mathrm{e}^{-\omega_2 \mathrm{j}t} - \mathrm{e}^{\mathrm{j}\omega_1 t}\mathrm{j}\frac{1}{\omega_1}\frac{1}{n}\sum_{i=1}^{n}v_i(0)\\
=\ & \mathrm{e}^{\mathrm{j}\omega_1 t}\left[\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0)\right.\\
&\left.-\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0)\,\mathrm{e}^{-\omega_2 \mathrm{j}t - \mathrm{j}\omega_1 t} - \mathrm{j}\frac{1}{\omega_1}\frac{1}{n}\sum_{i=1}^{n}v_i(0)\right].
\end{aligned}$$

Note that the vector

$$\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0) - \frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0)\,\mathrm{e}^{-\omega_2 \mathrm{j}t - \mathrm{j}\omega_1 t} - \mathrm{j}\frac{1}{\omega_1}\frac{1}{n}\sum_{i=1}^{n}v_i(0)$$

rotates about the point

$$\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0) - \mathrm{j}\frac{1}{\omega_1}\frac{1}{n}\sum_{i=1}^{n}v_i(0)$$

with radius

$$\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0)$$

and angular velocity $-w_1 - w_2$. Since

$$3\omega_1\omega_2\left\|\frac{1}{n}\sum_{i=1}^{n}c_i(0)\right\| < (\omega_1+\omega_2)\left\|\frac{1}{n}\sum_{i=1}^{n}v_i(0)\right\|,$$

then the term

$$\begin{aligned}
&\left\|\frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0) - \frac{\omega_2}{\omega_1+\omega_2}\frac{1}{n}\sum_{i=1}^{n}c_i(0)\,\mathrm{e}^{-\omega_2 \mathrm{j}t - \mathrm{j}\omega_1 t}\right\|\\
<\ & \left\|\mathrm{j}\frac{1}{\omega_1}\frac{1}{n}\sum_{i=1}^{n}v_i(0)\right\|.
\end{aligned}$$

In view of the periodical term $\mathrm{e}^{\mathrm{j}\omega_1 t}$, the variable $\frac{1}{n}\sum_{i=1}^{n}c_i(t)$ is encircled by the curve of the $\frac{1}{n}\sum_{i=1}^{n}r_i(t)$. This completes the proof.

Remark 1 When ω_2 is sufficiently smaller than ω_1, the term $\omega_2/(\omega_1+\omega_2)$ can become small enough to guarantee that

$$\frac{\omega_2}{\omega_1+\omega_2}\left\|\frac{1}{n}\sum_{i=1}^{n}c_i(0)\right\|$$

is sufficiently smaller than $\frac{1}{\omega_1}\left\|\frac{1}{n}\sum_{i=1}^{n}v_i(0)\right\|$ so that the distance between the variables $\frac{1}{n}\sum_{i=1}^{n}r_i(t)$ and $\frac{1}{n}\sum_{i=1}^{n}c_i(t)$ is mainly determined by $\frac{1}{\omega_1}\left\|\frac{1}{n}\sum_{i=1}^{n}v_i(0)\right\|$. Moreover, from Lemma 2, the average value of all circle centers $\frac{1}{n}\sum_{i=1}^{n}c_i(t)$ moves around a fixed point and the the average value of positions of all agents moves around $\frac{1}{n}\sum_{i=1}^{n}c_i(t)$ periodically.

Lemma 3 The roots of the equation

$$s^2 + s(a_1 + a_2\mathrm{j}) + a_3 = 0, \tag{5}$$

all have negative real parts, where $a_1, a_2, a_3 \in \mathbb{R}$, $a_1 > 0$, and $a_3 > 0$.

Proof Denote the roots of Eq. (5) by $s = b_1 + b_2\mathrm{j}$. Then Eq. (5) is equivalent to the following two equations:

$$b_1^2 - b_2^2 + a_1 b_1 - a_2 b_2 + a_3 = 0, \tag{6}$$

$$2b_1 b_2\mathrm{j} + a_1 b_2\mathrm{j} + b_1 a_2\mathrm{j} = 0. \tag{7}$$

Suppose that $b_1 \geq 0$. From Eq. (7), we have

$$b_2 = -b_1 a_2/(2b_1 + a_1).$$

Then substituting b_2 into Eq. (6), we have

$$\begin{aligned}&(2b_1+a_1)^2b_1^2-a_2^2b_1^2+(a_1b_1+a_3)\\&\times(2b_1+a_1)^2+a_2^2b_1(2b_1+a_1)\\=\;&(2b_1+a_1)^2b_1^2+(a_1b_1+a_3)(2b_1+a_1)^2\\&+a_2^2b_1(b_1+a_1)=0.\end{aligned}\tag{8}$$

Noting that $a_1>0$ and $a_3>0$, we have that Eq. (8) cannot hold if $b_1\geq 0$. Therefore, the real part of each root of Eq. (5) is negative.

Theorem 1 Consider a network of second-order agents with fixed topology that is connected. The multi-agent system (1) with the control law (2) reaches a composite-rotating consensus, if

$$3\omega_1\omega_2\|\frac{1}{n}\sum_{i=1}^{n}c_i(0)\|<(\omega_1+\omega_2)\|\frac{1}{n}\sum_{i=1}^{n}v_i(0)\|.$$

Moreover,

$$\lim_{t\to+\infty}[r_i(t)-\alpha(t)]=0 \text{ and } \lim_{t\to+\infty}[c_i(t)-\beta(t)]=0$$

for any $i\in\mathcal{I}$, where $\alpha(t)$ and $\beta(t)$ have been defined in Lemma 2.

Proof By Lemma 1, there exists an orthogonal matrix $W\in\mathbb{R}^{n\times n}$ such that

$$W^{\mathrm{T}}LW=\mathrm{diag}\{0,\lambda_2,\ldots,\lambda_n\}$$

with $0<\lambda_2\leq\lambda_3\leq\cdots\leq\lambda_n$ and the first column of W is $\mathbf{1}_n/\sqrt{n}$. It follows that

$$\begin{aligned}&(W\otimes I_2)^{\mathrm{T}}\dot{\delta}(t)\\=\;&(W\otimes I_2)^{\mathrm{T}}(I_n\otimes A-L\otimes B)(W\otimes I_2)(W\otimes I_2)^{\mathrm{T}}\delta(t)\\=\;&\mathrm{diag}\{A,A-\lambda_2B,A-\lambda_3B,\ldots,\\&A-\lambda_nB\}(W\otimes I_2)^{\mathrm{T}}\delta(t).\end{aligned}\tag{9}$$

From Lemma 2, $(\mathbf{1}_n^{\mathrm{T}}\otimes I_2)\delta(t)=0$. Thus, we define $(W\otimes I_2)^{\mathrm{T}}\delta(t)=[0,\bar{\delta}^{\mathrm{T}}(t)]^{\mathrm{T}}$, where $\bar{\delta}(t)\in\mathbb{C}^{2n-2}$. Then system (4) can be transformed into the following system

$$\dot{\bar{\delta}}(t)=\mathrm{diag}\{A-\lambda_2B,A-\lambda_3B,\ldots,A-\lambda_nB\}\bar{\delta}(t).\tag{10}$$

And $\lim_{t\to+\infty}\delta(t)=0$ if $\lim_{t\to+\infty}\bar{\delta}(t)=0$. The characteristic polynomial of each $A-\lambda_iB$ is

$$\begin{aligned}&\det(sI-A+\lambda_iB)\\=\;&s^2+\left(\omega_2\mathrm{j}+\frac{\lambda_i}{\omega_1}\mathrm{j}+\lambda_i+\lambda_i\mathrm{j}-\omega_1\mathrm{j}\right)s+\lambda_i+\omega_1\omega_2=0,\\&i=2,3,\ldots,n.\end{aligned}$$

Then by Lemma 3, system (10) is hurwitz stable and $\lim_{t\to+\infty}\bar{\delta}(t)=0$. Hence, $\lim_{t\to+\infty}\delta(t)=0$. Then by Lemma 2 we have

$$\lim_{t\to+\infty}[r_i(t)-\alpha(t)]=0,\ \lim_{t\to+\infty}[c_i(t)-\beta(t)]=0,$$

and

$$\lim_{t\to+\infty}[r_i(t)+\mathrm{j}v_i(t)-\frac{1}{n}\sum_{i=1}^{n}r_i(0)-\mathrm{j}\frac{1}{n}\sum_{i=1}^{n}v_i(0)]=0$$

for any $i\in\mathcal{I}$. By simple calculations, we have $\lim_{t\to+\infty}[\dot{c}_i(t)-\mathrm{j}\omega_2c_i(t)]=0$ for any $i\in\mathcal{I}$. Thus, the multi-agent system (1) with the control law (2) reaches a composite-rotating consensus.

5. A numerical simulation

A numerical simulation is given to show the theoretical results obtained in the previous sections. The graph $\mathcal{G}_a$ in Fig. 1 is the communication topology for the multi-agent system (1), where the weight of each edge is 1. Figure 2 shows the trajectories of all agents with the control law (2). It is obvious that all agents finally surround a common moving point whose trajectories form a circle.

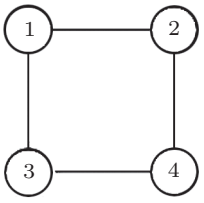


Fig. 1. The communication topology for the multi-agent system (1).

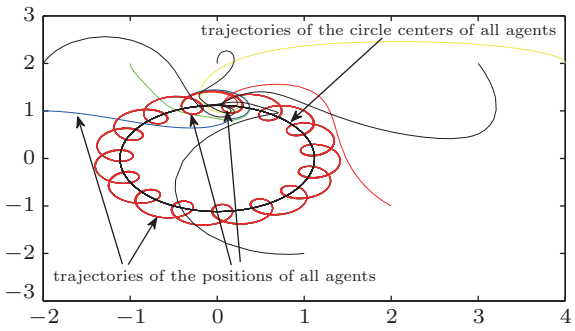


Fig. 2. (color online) Trajectories of the multi-agent system (1).

6. Conclusions

In this paper, we studied a composite-rotating consensus problem in a distributed manner, where all agents surround a moving point whose trajectories form a circle. A distributed consensus control law is introduced which contains two parts: one is the local state feedback that guarantees the final rotating states and the other is the communication state feedback that guarantees the system consensus. By model transformations, we gave a sufficient condition to enable all agents to reach a composite-rotating consensus. Finally, a numerical simulation is included to demonstrate our theoretical results.

References

[1] Jadbabaie A, Lin J and Morse A S 2003 *IEEE Trans. Autom. Control* **48** 988

[2] Olfati-Saber R and Murray R M 2004 *IEEE Trans. Autom. Control* **49** 1520

[3] Ren W and Beard R W 2005 *IEEE Trans. Autom. Control* **50** 655

[4] Ren W 2008 *IEEE Trans. Autom. Control* **53** 1503

[5] Sepulchre R, Paley D and Leonard N E 2008 *IEEE Trans. Autom. Control* **53** 706

[6] Pavone M and Frazzoli E 2007 *ASME Journal of Dynamic Systems, Measurement and Control* **129** 633

[7] Ren W 2009 *IEEE Trans. Autom. Control* **54** 1330

[8] Hong Y, Gao L, Cheng D and Hu J 2006 *Automatica* **42** 1177

[9] Lin P and Jia Y 2008 *Physica A* **387** 303

[10] Lin P and Jia Y 2009 *Automatica* **45** 2154

[11] Lin P and Jia Y 2010 *IEEE Trans. Automatic Control* **55** 778

[12] Hu J and Yuan H 2009 *Chin. Phys. B* **18** 3777

[13] Guan X and Li Y 2009 *Chin. Phys. B* **18** 3355

[14] Pei W, Chen Z and Yuan Z 2008 *Chin. Phys. B* **17** 337

[15] Li H, Lin P and Zhang C 2009 *Acta Phys. Sin.* **58** 158 (in Chinese)

[16] Godsil C and Royle G 2001 *Algebraic Graph Theory* (New York: Springer-Verlag)