Preparation of multi-photon Fock states and quantum entanglement properties in circuit QED*

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We demonstrate the controllable generation of multi-photon Fock states in circuit quantum electrodynamics (circuit QED). The external bias flux regulated by a counter can effectively adjust the bias time on each superconducting flux qubit so that each flux qubit can pass in turn through the circuit cavity and thereby avoid the effect of decoherence. We further investigate the quantum correlation dynamics of coupling superconducting qubits in a Fock state. The results reveal that the lower the photon number of the light field in the number state, the stronger the interaction between qubits is, then the more beneficial to maintaining entanglement between qubits it will be.

Keywords: circuit quantum electrodynamics, multi-photon Fock state, preparation of quantum state, concurrence

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1. Introduction

In the past few decades, one of the most important developments in quantum optics is constructing many optical nonclassical states in different quantum systems.[1–8] As the most fundamental and important state in quantum optics, the Fock state $|n\rangle$ obeys a sub-Poisson distribution and constitutes a complete set $\sum_n |n\rangle\langle n| = 1$ that the arbitrary photon state can spread accordingly and the spread is exclusive. People paid much attention to how to prepare a Fock state since it plays an important role in quantum computation and communications. By using the standard atom–cavity interaction, Fock states $|n\rangle$ were experimentally produced and probed, and superposition of coherent states with different phases were obtained.[9] However, the quantum electrodynamics (QED) system has no advantage in integration despite the satisfactory experimental platform it provides for quantum information processing.[10]

To meet the requirements for quantum information processing and achieve scale and integrated control, the solid quantum system should be considered. In the solid field, superconducting qubits can act as the artificial atoms, and the one-dimensional (1D) superconducting transmission line resonator can be used as the microwave cavity field. Recent experimental studies show that the coherent time for superconducting qubits coupled to a microwave cavity can be extended to 0.1 ms.[11] Most recently, the method of atom–cavity interactions has been used in solid-state circuit QEDs to prepare multi-photon Fock states and generalized binomial states.[12]

In the present work, using the high-Q 1D superconducting transmission line resonator (TLR) as a data bus, we propose a scheme to realize a multi-photon Fock state in circuit QEDs. In the proposed scheme, each qubit passes in turn through the circuit cavity via externally biasing flux, and then we implement quantum measurement on the ground state after the qubit has passed through the circuit cavity. It effectively avoids the influence of decoherence since each qubit is operated alone.[13] We also investigate the dynamic behaviors of quantum entanglement qubits in circuit QED.

The rest of this paper is organized as follows. In Section 2, we show the model and present the scheme of preparing Fock states in circuit QEDs. The dynamical behaviors of quantum entanglement for the coupled superconducting qubits in circuit QEDs are investigated in Section 3. Finally, in Section 4 the obtained results are summarized and discussed.

2. Model and preparation of Fock states

Based on the interaction between atoms and the quantum light field, the generalized binomial states are prepared in the optical cavity.[14] In this paper, we present a scheme to generate multi-photon Fock states in the circuit cavity by transplanting and improving this scheme. The specific scheme is shown in Fig. 1. The system consists of superconducting qubits and a transmission line resonator. We choose the superconducting flux qubit since it possesses a longer decoherence time. The flux qubit is located at the position of the maximum quantum current (When the superconducting charge qubit is chosen, the charge qubit is located at the position of maximum quantum voltage). The flux qubit couples to the transmission line through external flux which is controlled by the counter, thereby the bias time on each qubit can be effectively

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controlled.[15] We should note that in the process of the experiment, the coupling strength between the qubits and the transmission line is difficult to manipulate because such a strength is associated with the geometric parameters. Therefore, coupling between the qubits and the transmission line must be opened or closed to regulate the eigenfrequency of qubits to resonate or detune with that of the cavity. To effectively manipulate the flux qubit, we place a direct current superconducting quantum interference device (dc SQUID) inside and externally bias a flux passing through this SQUID loop. Obviously, we can adjust the eigenfrequency of the flux qubit via adjusting the external flux.

According to the electromagnetic theory, it can be described by several standing wave modes of different frequencies about quantized current and voltage in superconducting transmission lines through selecting appropriate boundary conditions. Therefore, the quantized current in quantum transmission lines generate a multimode quantized magnetic field around them. Near the resonant frequency, the ideal 1D nonloss quantum transmission line is equivalent to a microwave quantum field whose Hamiltonian is $H_l = \omega_0 a^\dagger a$. Under rotating wave approximation, the superconducting qubit opens and interacts with the quantum field, whose Hamiltonian for the flux qubit-quantum field reads

$$H = \omega_0 a^\dagger a + \frac{1}{2}\omega\sigma_z + g\left(a\sigma_+ + a^\dagger\sigma_-\right), \tag{1}$$

where ω_0 and ω represent the resonant frequency of the transmission line and the eigen-frequency of the superconducting qubits, respectively; g denotes the coupling coefficient between the superconducting qubits and the transmission line cavity field.

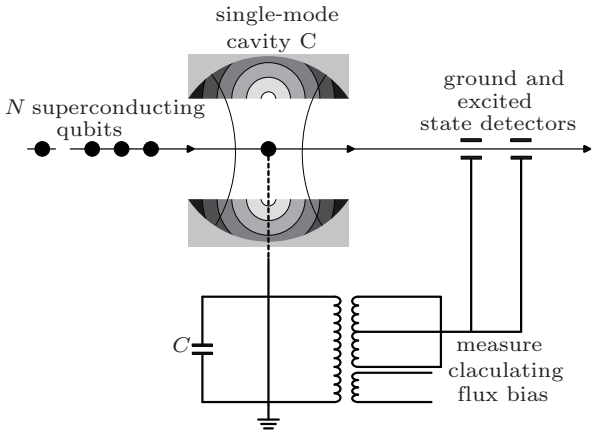


Fig. 1. Preparation and measurement of Fock states. The flux qubits containing a dc-SQUID are coupled to a transmission line. The flux qubits are located at the position of the maximum quantum current.

In this section, we present the scheme of preparing Fock states in a circuit QED. As shown in Fig. 1, the superconducting charge qubits are alternately located at the positions of the maximum quantized voltage. Therefore, we can realize the opening and closing of the qubits. Under resonance, the interaction between each qubit and the single mode field can be described by the typical *J*–*C* Hamiltonian. The typical evolution of the qubit state under this Hamiltonian is

$$|\uparrow n\rangle \to \cos\left(gt\sqrt{n+1}\right)|\uparrow n\rangle - \sin\left(gt\sqrt{n+1}\right)|\downarrow n+1\rangle,$$

where t is the qubit–cavity interaction time.

Initially, the quantum field is prepared in the vacuum state $|0\rangle$ and each qubit is prepared in the following state:

$$\left|\chi_j\right\rangle = |\uparrow\rangle, \quad j = 1,2,\ldots,n. \tag{2}$$

Through controlling the external magnetic field, we make each superconducting qubit interact in turn with the transmission line (quantum field) for an appropriate time. At the moment before each interaction completes, we implement measurement on the ground state of the superconducting qubit. After the action of n qubits, the quantum state of the circuit QED collapses to Fock state $|n\rangle$ via measuring the ground state of the qubits. Next, we briefly list the preparation procedures:

(i) The quantum field is in a vacuum state. Unlock the first charge qubit via externally adding a magnetic field to interact with the quantum field. The quantum state for the total system is given by

$$|\psi_t(t)\rangle \to \cos(gt)|\uparrow 0\rangle - \sin(gt)|\downarrow 1\rangle. \tag{3}$$

Measuring the ground state of the qubit at $t_1 = 3\pi/(2g)$, then the quantum state for the quantum field must collapse to the binomial state of $n = 1$, i.e.,

$$\left|\psi_f(1)\right\rangle \to |1\rangle. \tag{4}$$

(ii) Now, the quantum field is in the Fock state of $n = 1$. Unlock the first flux qubit via externally adding a magnetic field to interact with the quantum field. The quantum state for the total system is given by

$$|\psi_t(t)\rangle \to \cos\left(gt\sqrt{2}\right)|\uparrow 1\rangle - \sin\left(gt\sqrt{2}\right)|\downarrow 2\rangle. \tag{5}$$

Measuring the ground state of the qubit after $t_2 = 3\pi/(2g\sqrt{2})$, the quantum state for the quantum field must collapse to

$$\left|\psi_f(2)\right\rangle \to |2\rangle. \tag{6}$$

Evidently, $\left|\psi_f(2)\right\rangle$ is the Fock state of $n = 2$.

Suppose that the quantum state for the quantum field collapses to the Fock state of $|n-1\rangle$ after interacting with $n-1$ qubits, and let the n-th qubit interact with the quantum field $|n-1\rangle$, then the quantum state for the total system will be

$$|\uparrow n-1\rangle \to \cos\left(gt\sqrt{n}\right)|\uparrow n-1\rangle - \sin\left(gt\sqrt{n}\right)|\downarrow n\rangle.$$

After the following time evolution

$$T_n = \frac{3\pi}{2g}\cdot\frac{1}{\sqrt{n}}, \tag{7}$$

measuring the ground state of the qubit, the quantum state for the quantum field must collapse to

$$|\psi_f(n)\rangle = |n\rangle . \tag{8}$$

In a word, we can measure the ground state of the qubits and prepare the quantum field of a multi-photon Fock state in the circuit cavity after n superconducting qubits have interacted in turn with the quantum field for some time through manipulating the external magnetic field. In our scheme, the charge qubit and flux qubit are respectively controlled by different external magnetic fields, which are exploited not only to adjust the eigenfrequencies of the qubits but also to manipulate the opening and closing of the qubits. According to previous deductions, we find that, in practice, the types of superconducting qubits are not specified in the proposed scheme, implying that it has excellent universality and is especially convenient to manipulate in experiment.

3. Quantum entanglement for two qubits

In this section, we explore the dynamical behaviors of quantum entanglement for the coupled superconducting qubits in circuit QED. Firstly, we simply introduce the experimental scheme in which coupling superconducting quantum bit is placed in the circuit cavity of the Fock state. The coupled circuit QED system is shown in Fig. 2.[16] The system consists of a charge qubit, a flux qubit, and a transmission line resonator. The charge qubit and the flux qubit are respectively placed at positions $l/2$ and $l/4$ of the transmission line, where the maximum quantization voltage and current can be achieved. The charge qubit couples to the transmission line through the gate capacity on which the coupling strength depends, whereas the flux qubit couples to the transmission line through the external flux. In graphic circuit systems, the eigenfrequency of charge qubits depends on external flux Φ_1, and the flux qubits depend on external flux Φ_3.

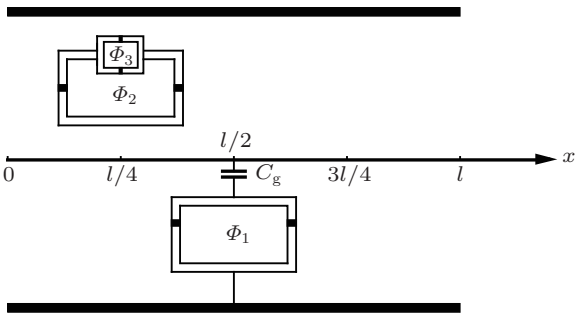


Fig. 2. Schematic diagram of coupling qubit circuit quantum electrodynamics. A superconducting quantum interference device (SQUID) charge qubit and a flux qubit containing a dc-SQUID are coupled to a transmission line of length l. The charge qubit couples to the transmission line through the gate capacity and the biased flux Φ_1, and the flux qubit couples to the transmission line through the biased flux Φ_3.

There are two steps in practical operation. At first, we completely shut down the external flux Φ_1 (that is, to shut down the charge qubit), as the experimental facility shows in Fig. 1, control the external flux Φ_3 by pulse counter, and realize the preparation of multi-photon Fock states according to the steps mentioned above. Then, after the preparation of a proper Fock state, $|n\rangle$, the interaction between qubits and a transmission line resonator can be achieved through the external fluxes Φ_1 and Φ_3 and the opening charge qubit and flux qubit.

In the interaction picture, with the resonant interaction considered, the effective Hamiltonian of the system is expressed as ($\hbar = 1$)

$$H_{\rm eff} = \sum_{j=A,B} g\left(a\sigma_j^{\dagger} + a^{\dagger}\sigma_j^{-}\right) + g_{\rm AB}\left(\sigma_1^{+}\sigma_2^{+}\sigma_1^{-}\sigma_2^{+}\right). \tag{9}$$

The detailed deduction of above-mentioned Hamiltonian is given in Ref. [17].

We suppose that the coupled superconducting qubits are in the Bell state, and given as follows:

$$|\psi_Q(0)\rangle_{\rm II} = \sin\alpha\,|\downarrow\downarrow\rangle + \cos\alpha\,|\uparrow\uparrow\rangle . \tag{10}$$

By using the Schrödinger equation, the vector of the system can be

$$\begin{aligned} |\psi(t)\rangle \;=\; & c_1(t)\,|ee,n\rangle + c_2(t)\,|eg,n+1\rangle + c_3(t)\,|ge,n+1\rangle \\ & + c_4(t)\,|gg,n+2\rangle + c_5(t)\,|gg,n\rangle , \end{aligned} \tag{11}$$

where

$$\begin{aligned} c_1(n,t) \;=\; & \left[\frac{n+1}{2n+3}\,{\rm e}^{-{\rm i}\alpha t}\cos(\beta t)\right. \\ & \left. +{\rm i}\,\frac{(n+1)g_{\rm AB}}{2(2n+3)\beta}\,{\rm e}^{-{\rm i}\alpha t}\sin(\beta t) + \frac{n+2}{2n+3}\right]\cos\theta, \end{aligned} \tag{12}$$

$$c_2(n,t) \;=\; c_3(n,t) = {\rm i}\,\frac{g\sqrt{n+1}}{\beta}\,{\rm e}^{-{\rm i}\alpha t}\cos\theta\cdot\sin(\beta t), \tag{13}$$

$$\begin{aligned} c_4(n,t) \;=\; & \frac{\sqrt{(n+1)(n+2)}}{2(3+2n)\beta}\,{\rm e}^{-{\rm i}\alpha t}[2\beta\cos(\beta t) \\ & +{\rm i}\,g_{\rm AB}\sin(\beta t) - \beta\,{\rm e}^{{\rm i}\alpha t}]\cdot\cos\theta, \end{aligned} \tag{14}$$

$$c_5(n,t) \;=\; \sin\theta, \tag{15}$$

with

$$\alpha = \frac{g_{\rm AB}}{2}, \quad \beta = \frac{\sqrt{g_{\rm AB}^2 + 8(2n+3)g^2}}{2}.$$

According to Eqs. (11)–(15), we can easily obtain the density of the hybrid qubits:

$$\rho_{\rm AB} = \begin{pmatrix} |c_1(n,t)|^2 & 0 & 0 & c_1(n,t)\sin\theta \\ 0 & |c_2(n,t)|^2 & c_2(n,t)c_3^*(n,t) & 0 \\ 0 & c_2^*(n,t)c_3(n,t) & |c_3(n,t)|^2 & 0 \\ c_1^*(n,t)\sin\theta & 0 & 0 & |c_4(n,t)|^2+\sin^2\theta \end{pmatrix}. \quad (16)$$

We adopt the concurrence entanglement defined by Wootters to measure the system entanglement. The concurrence for the hybrid qubits can be calculated using[18]

$$C(t) = 2\max\{0, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}, |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}\}. \quad (17)$$

Figure 3 presents the effects of the photon number in the quantized field on the quantum entanglement. It shows that collapse and revival occur in the time evolution of the concurrency, and that more intense oscillations appear with the increasing of the photon number in the field of the photon number state. In Fig. 3, we can clearly see $C(t) = 0$ at some moments when $n \neq 0$, indicating that during the time oscillation, entanglement death appears. However, the entanglement death will not appear in the vacuum state. It may be explained as follows: n plays the role of coupling strength between the two qubits and the circuit cavity, large n is equivalent to strong coupling between the two subsystems. Once we only consider the two qubits by tracing out the circuit cavity as the environment, the large value of n means that the two qubits suffer strong dissipation. It indicates that reducing the average photons number will be beneficial to keeping the quantum correlation between qubits.

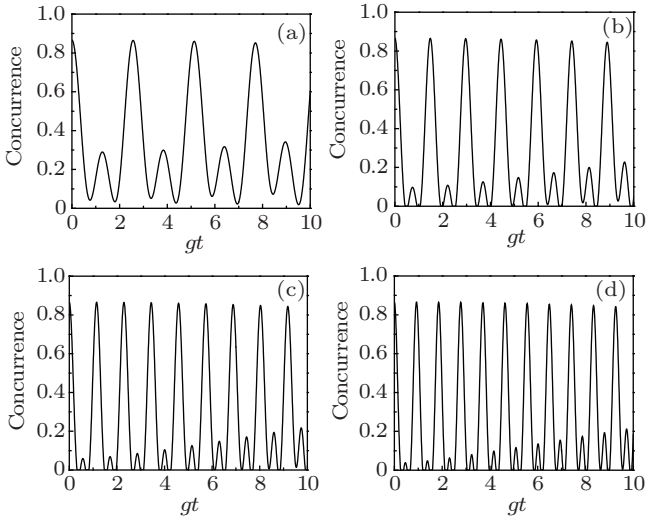


Fig. 3. Time evolutions of the concurrence versus the scaled time gt with $\theta = \pi/6, g_{\rm AB} = 0.1g$, and $n = 0$ (a), 3 (b), 6 (c), and 10 (d).

The initial entanglement degree of two coupling qubits is $C(0) = |\sin\theta|$. To describe the influence of initial entanglement degree on the system entanglement dynamics, we show the time evolution curves of concurrency under different values of θ in Fig. 4. Obviously, the point $\theta = \pi/4$ is critical for distinguishing an increased or decreased direction of quantum entanglement between the coupling qubits as time evolves. When $\theta < \pi/4$, the maximal value of the entanglement between two qubits decreases as θ decreases. Inversely, when $\theta > \pi/4$, the maximal value of the entanglement between two qubits decreases as θ increases.

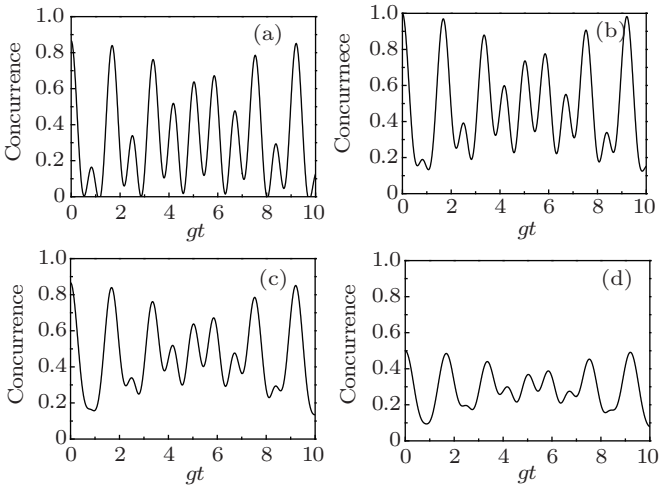


Fig. 4. Time evolutions of the concurrence versus the scaled time gt with $g_{\rm AB} = 0.6g$ and $\theta = \pi/6$ (a), $\pi/4$ (b), $\pi/3$ (c), and $\pi/2$ (d). The circuit QED is initially in state $|2\rangle$.

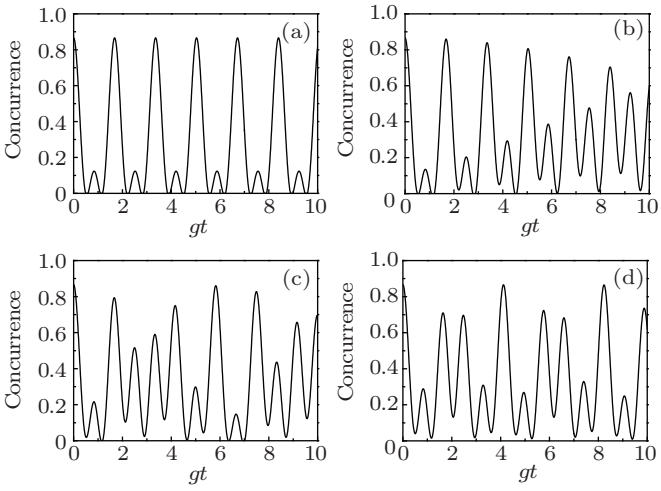


Fig. 5. Time evolutions of the concurrence versus the scaled time gt with $\theta = \pi/6$ and $g_{\rm AB}/g = 0$ (a), 0.3 (b), 1 (c), and 1.5 (d). The circuit QED is initially in state $|2\rangle$.

Figure 5 shows the effect of the interaction between qubits on entanglement dynamics. Evidently, the coupling coefficient $g_{\rm AB}$ influences not only the oscillation property of quantum entanglement but also the periodicity of quantum entanglements. We can see from Fig. 5 that collapse and revival

occur in the time evolution of the concurrency, and more intense oscillations will appear as the interaction between qubits strengthens; further, that the entanglement death will not appear when the interaction between qubits increases to a certain value. In the vacuum state, however, no entanglement death occurs. It indicates that in the scheme being investigated, the entanglement effect can be completely avoided through manipulating the coupling strength between qubits, thus benefitting the quantum information processing. In circuit QED, the coupling strength between superconducting qubits can be controlled via manipulating the external bias flux.

4. Conclusions

We proposed a scheme for preparing a multi-photon Fock state in circuit QED. The quantum system consists of a transmission line and several superconducting qubits, where the transmission line acts as a data bus. The effect of decoherence can be effectively avoided in this scheme since the manipulation is implemented by externally biasing the flux and each qubit is operated individually. The use of circuit QED makes the scheme more operable and integrated. However, the response speed of the detector needs improving with the increase of the photon number. Enhancing the response speed of the detector will be a main direction for improving this scheme.

In the optical field of number state, we also discuss the quantum entanglement dynamics of the coupling qubits. The results indicate that the time evolution of quantum entanglement is oscillating, and that collapse and revival periodically occur. It is also revealed that the lower the photon number of the light field in the number state (best in a vacuum state), the stronger the interaction between qubits is, then the more beneficial to maintaining entanglement between qubits it will be.

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