Engineering steady-state entanglement for two atoms held in separate cavities through laser cooling*

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We propose a scheme to prepare the steady-state entanglement for two atoms, which are held in separate cavities that are coupled through a short optical fiber or optical resonator. The entangled steady-state with a high fidelity can be achieved even with a low cooperativity parameter, by making use of the driving laser fields. Such a cooling mechanism is based on a resonant laser pump of the unwanted ground states to the excited states, which finally decay to the desired steady-state.

Keywords: laser cooling, steady-state entanglement, optical fiber

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1. Introduction

A significant prerequisite for quantum information processing is to reliably control the quantum dynamics of physical objects. The most important resource for quantum technologies is the high-fidelity quantum entangled state, which can be obtained through the system's unitary evolution or irreversible dynamics assisted by dissipation.[1–5]

The challenge of this idea is to find ways to preserve the coherence of quantum entanglement against the system dissipation and to maintain the entanglement for an extended period of time. One of the fascinating methods therein is based on laser cooling,[5] i.e., to continuously produce coherent coupling between the specific atomic transitions and the electromagnetic fields until the system reaches a state (say, a steady state) with almost no excitations. Exploiting the approach of laser cooling to prepare entangled states has attracted great interest.[6–9] Busch *et al.*[7] presented a laser cooling method, analogous to the laser sideband cooling,[10] to engineer entanglement for two atoms inside an optical cavity with a rather low cooperative parameter. We have also showed the possibility for generating steady state entanglement for the coupled-cavity system through laser cooling.[9] Like the other methods assisted by dissipation, the laser cooling method possesses the advantages of not necessarily preparing the system's initial states and accurately controlling the interaction time, as compared with the protocols based on coherent unitary dynamics.[11–21] Besides, by employing the flexibility of the external optical pumping in such a cooling mechanism, the required final states with a much higher fidelity than those through other dissipation-assisted processes can be achieved.

To solve the difficulty associated with the exponential coherence decay of distributed quantum nodes,[22–27] the idea of laser cooling can be adopted. Recently, we illustrated a scheme with analytic results allowing engineering steady-state entanglement for two atoms distributed over a long dissipative bosonic medium.[4] In the scheme,[4] the classical microwaves are applied to generate the coherent shuffling among the three ground states, one of which decays to the desired entangled ground state. The effective decay of the target entangled state induced by losses of the dispersive bosonic modes is suppressed during the irreversible process. In the present work, we present an alternative scheme for engineering such a distributed steady entanglement by using the laser cooling mechanism. Due to the coherent cavity–fiber coupling, the energy levels are shifted in the dressed-state picture. Our present scheme uses the laser fields to drive the resonant transitions between the unwanted ground states and the excited states, whose population is mostly transferred to the desired entangled ground state due to dissipation, with the target entangled state being not affected by the laser driving due to large detunings and becoming the unique steady entanglement eventually. Our results predict that a high-fidelity two-atom entangled state can be obtained with a rather low cooperativity parameter, as compared to that of Ref. [4]. However, our present scheme requires several lasers to simultaneously drive two atoms trapped in two distant cavities and accurately choose the detunings, which may increase the complexity of the real experiment operation. The scheme can in principle be generalized to engineer steady entanglement for multiple atoms distributed across a quantum network.

2. The physical model

Consider two three-level atoms separately held in two cavities connected by a third optical resonator, which is visu-

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alized in this paper as a 'short' optical fiber, as shown in Fig. 1. Here, we assume the short fiber limit $(2l\bar{v})/(2\pi c) \ll 1$, where l is the length of the fiber and $\bar{v}$ is the decay rate of the cavity fields into a continuum of fiber modes, c is the speed of light in a vacuum and only the resonant mode b with frequency w_b of the fiber interacts with the cavities.[22] In a cavity quantum electrodynamics (QED) experiment, one operates our system in a regime where the frequencies of the atomic transition and the corresponding cavity field mode are close to each other and both far larger than the atom–cavity coupling strength, under which the counter-rotating interactions can be neglected in the whole system Hamiltonian. Under the rotating-wave approximation, the Hamiltonian of the whole system without optical laser drivings is (set $\hbar = 1$ and $w_0 = 0$):

$$H_{\rm NL} = \sum_{j=1}^{2}\left(\sum_{i=1}^{2} w_i|i\rangle_{jj}\langle i| + w_a a_j^{\dagger}a_j\right) + w_b b^{\dagger}b + \sum_{j=1}^{2}\left(g|2\rangle_{jj}\langle 1|a_j + v a_j b^{\dagger} + \text{H.c.}\right), \quad (1)$$

where w_i $(i = 0,1,2)$ denotes the energy of level $|i\rangle$ in each atom, and $a_j^{\dagger}$ (a_j) is the creation (annihilation) operator for the j-th cavity field mode with frequency w_a. The j-th cavity field mode is resonantly coupled to the $|1\rangle_j \leftrightarrow |2\rangle_j$ transition with coupling strength g, and v is the coupling constant between the cavity and the optical fiber. We obtain the corresponding eigenstate and eigenenergy of the Hamiltonian $H_{\rm NL}$ within the zero- and one-excitation subspace analytically, as shown in Tables 1 and 2. The excitation number of the total system is conserved during the state evolution since the excitation number operator $\hat{N} = \sum_{j=1}^{2}(|2\rangle_j\langle 2| + a_j^{\dagger}a_j) + b^{\dagger}b$ commutes with the system Hamiltonian $H_{\rm NL}$, which ensures the system evolves in the zero- and one-excitation subspaces. Here, $|AB,CDE\rangle$ represents that atom 1 (2) is in the state $|A\rangle$ $(|B\rangle)$ and there are C (E) photons in cavity 1 (2), and D photons in the optical fiber. $|\phi_n\rangle$ $(n = 1,2,...,16)$ is the corresponding eigenstate of the Hamiltonian in Eq. (1) within the one-excitation subspace, and the corresponding eigenenergy λ_n $(n = 1,2,...,16)$ is shifted by an amount depending on the coherent hopping strength and the atom–field coupling constant.

Table 1. The eigenstates and eigenenergies of the Hamiltonian in Eq. (1) within the zero-excitation subspace.

Eigenstate	Eigenenergy
$\|00,000\rangle$	0
$\|T\rangle = \frac{1}{\sqrt{2}}(\|01,000\rangle + \|10,000\rangle)$	w_1
$\|S\rangle = \frac{1}{\sqrt{2}}(\|01,000\rangle - \|10,000\rangle)$	w_1
$\|11,000\rangle$	$2w_1$

Table 2. The eigenstates and eigenenergies of the Hamiltonian in Eq. (1) within the one-excitation subspace.

Eigenstate	Eigenenergy
$\|\phi_1\rangle$	$\lambda_1 = w_2 - w_1 + \sqrt{2}v$
$\|\phi_2\rangle$	$\lambda_2 = w_2 - w_1 - \sqrt{2}v$
$\|\phi_3\rangle$	$\lambda_3 = w_2 + w_1$
$\|\phi_4\rangle$	$\lambda_4 = w_2 - w_1$
$\|\phi_5\rangle, \|\phi_6\rangle$	$\lambda_5 = \lambda_6 = w_2 + \sqrt{2\sqrt{4v^4+g^4}+2g^2+4v^2}/2$
$\|\phi_7\rangle, \|\phi_8\rangle$	$\lambda_7 = \lambda_8 = w_2 - \sqrt{2\sqrt{4v^4+g^4}+2g^2+4v^2}/2$
$\|\phi_9\rangle, \|\phi_{10}\rangle$	$\lambda_9 = \lambda_{10} = w_2 + \sqrt{-2\sqrt{4v^4+g^4}+2g^2+4v^2}/2$
$\|\phi_{11}\rangle, \|\phi_{12}\rangle$	$\lambda_{11} = \lambda_{12} = w_2 - \sqrt{-2\sqrt{4v^4+g^4}+2g^2+4v^2}/2$
$\|\phi_{13}\rangle$	$\lambda_{13} = w_2 + w_1 + \sqrt{2v^2+g^2}$
$\|\phi_{14}\rangle$	$\lambda_{14} = w_2 + w_1 - \sqrt{2v^2+g^2}$
$\|\phi_{15}\rangle$	$\lambda_{15} = w_2 + w_1 + g$
$\|\phi_{16}\rangle$	$\lambda_{16} = w_2 + w_2 - g$

To cool the atoms into the maximally entangled state $|S\rangle$, two lasers with Rabi frequencies Ω_m and frequencies $w_{{\rm L},m}$ $(m = 1,2)$ are applied to drive the transition $|0\rangle \leftrightarrow |2\rangle$, while another laser with Rabi frequency Ω_3 and frequency $w_{\rm L,3}$ drives the transition $|1\rangle \leftrightarrow |2\rangle$ in each cavity. The interaction Hamiltonian between the atoms and the lasers is:

$$H_{\rm AL} = \sum_{j=1}^{2}\left(\sum_{m=1}^{2}\Omega_m e^{{\rm i}w_{{\rm L},m}t}|0\rangle_{jj}\langle 2| + \Omega_3 e^{{\rm i}w_{\rm L,3}t}|1\rangle_{jj}\langle 2| + \text{H.c.}\right). \quad (2)$$

The probability that the system is excited to the subspaces with more than one excitation can be neglected when considering the weak excitation condition. In the interaction picture

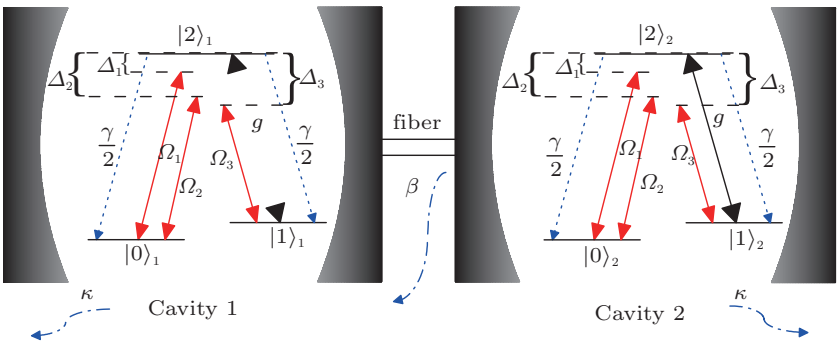


Fig. 1. (color online) Experimental setup and atomic level configuration for cooling two atoms into the steady entanglement via two separate cavities connected by a third optical resonator or short optical fiber. γ stands for the atomic spontaneous emission rate, while κ and β denote the cavity and fiber decay rates, respectively.

with respect to the Hamiltonian $H_{\rm NL}$, the laser–atom interaction Hamiltonian can be written as

$$
\begin{aligned}
H'_{\rm AL} = & \sum_{m=1}^{2}\Bigg\{\Bigg[\sum_{k=5}^{12} r_k\Omega_m \mathrm{e}^{\mathrm{i}(w_{\mathrm{L},m}-\lambda_k)t}|00,000\rangle\langle\phi_k| \\
& + \frac{\Omega_m}{\sqrt{4v^2+2g^2}}\Bigg(\sum_{k=13}^{14} g\,\mathrm{e}^{\mathrm{i}(w_{\mathrm{L},m}+w_1-\lambda_k)t}|T\rangle\langle\phi_k| \\
& + 2v\,\mathrm{e}^{\mathrm{i}(w_{\mathrm{L},m}+w_1-\lambda_3)t}|T\rangle\langle\phi_3|\Bigg) \\
& + \frac{\Omega_m}{\sqrt{2}}\sum_{k=15}^{16}\mathrm{e}^{\mathrm{i}(w_{\mathrm{L},m}+w_1-\lambda_k)t}|S\rangle\langle\phi_k|\Bigg] \\
& + \Omega_3\Bigg[\sum_{k=5}^{12} s_k\,\mathrm{e}^{\mathrm{i}(w_{\mathrm{L},3}+w_1-\lambda_k)t}(|T\rangle\langle\phi_k|+|S\rangle\langle\phi_k|)\Bigg] \\
& + \frac{\Omega_m}{\sqrt{2v^2+g^2}}\Bigg[\sum_{k=13}^{14} g\,\mathrm{e}^{\mathrm{i}(w_{\mathrm{L},m}+2w_1-\lambda_k)t}|11,000\rangle\langle\phi_k| \\
& + 2v\,\mathrm{e}^{\mathrm{i}(w_{\mathrm{L},m}+2w_1-\lambda_3)t}|11,000\rangle\langle\phi_3|\Bigg]\Bigg\}+\mathrm{H.c.}, \qquad (3)
\end{aligned}
$$

where the expressions of the 'Rabi frequencies' $r_k\Omega_m$ and $s_k\Omega_3$ can be analytically derived from the eigenstates in Tables 1 and 2. We solve the master equation of open dissipative quantum dynamics with the Lindblad form:

$$
\begin{aligned}
\dot{\rho} = & -\mathrm{i}[(H_{\rm NL}+H_{\rm AL}),\rho]+\frac{\kappa}{2}\sum_{j=1}^{2}(2a_j\rho a_j^{\dagger}-a_j^{\dagger}a_j\rho-\rho a_j^{\dagger}a_j) \\
& + \frac{\beta}{2}(2b\rho b^{\dagger}-b^{\dagger}b\rho-\rho b^{\dagger}b) \\
& + \frac{\gamma}{4}\sum_{j=1}^{2}\sum_{k=0}^{1}(2S_{jk}^{-}\rho S_{jk}^{+}-S_{jk}^{+}S_{jk}^{-}\rho-\rho S_{jk}^{+}S_{jk}^{-}), \qquad (4)
\end{aligned}
$$

where $S_{jk}^{+}=|2\rangle_j\langle k|$ and $S_{jk}^{-}=|k\rangle_j\langle 2|$. If we choose $w_{\mathrm{L},1}=\lambda_7$, $w_{\mathrm{L},2}=\lambda_{13}-w_1$, and $w_{\mathrm{L},3}=\lambda_3-2w_1$, the target state $|S\rangle$ is detuned to all the laser-driving transitions, while the other ground states are resonantly driven to at least one excited state, i.e., each of the resonant transitions $|00,000\rangle\leftrightarrow|\phi_7\rangle$, $|T\rangle\leftrightarrow|\phi_{13}\rangle$, and $|11,000\rangle\leftrightarrow|\phi_3\rangle$ is resonantly coupled to one laser field, as shown in Fig. 2. Due to the dissipation of the whole system and laser-driving transitions, if the detuning in every nonresonant laser-driving transition is far larger than the corresponding Rabi frequency, the excited states decay quickly and the atomic system is projected into the target state $|S\rangle$, which is the unique steady state.

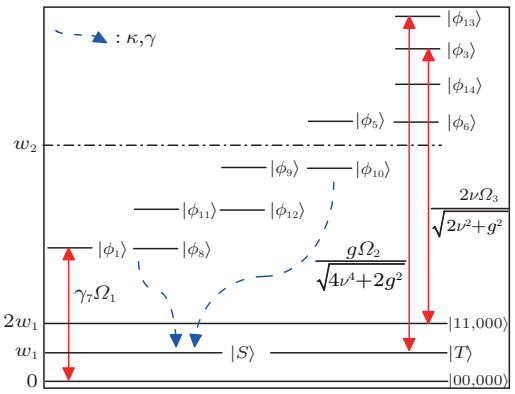


Fig. 2. (color online) The resonant laser-driving transitions for the ground states $|00,000\rangle$, $|T\rangle$, and $|11,000\rangle$ in the dressed state picture are plotted by red solid arrows. The inset shows that the dissipative process of the whole system is plotted by the blue dash arrows.

3. Numerical verification and analysis

Based on the above cooling mechanism, we numerically simulate the master equation (4) in the zero- and one-excitation subspaces, as plotted in Fig. 3(a). The result shows that the desired steady entanglement can be obtained with a fidelity about 90% even when the cooperativity C is as low as 60. We note that the fidelity of the steady state entanglement in Ref. [4] with the same condition is just about 70%. The improved fidelity of the engineered steady state in the present scheme is due to optically pumping the population from unwanted ground states to the excited states that is finally transferred to the desired ground state due to dissipation, which is impossible for the previous scheme in Ref. [4]. For the typical experimental parameters $(g,v,\kappa,\gamma,\beta)/2\pi=(34,61.2,4.1,2.6,0.024)$ MHz,[28–30] we find that the fidelity for the steady entanglement is still higher than 87%. In Fig. 3(b), we plot the influence of different fluctuations in the atom–field coupling constants on the population of the target steady state $|S\rangle$, and the result shows that the population remains higher than 85% even when the relative fluctuations of the atom–field coupling constants are both 10%. This is

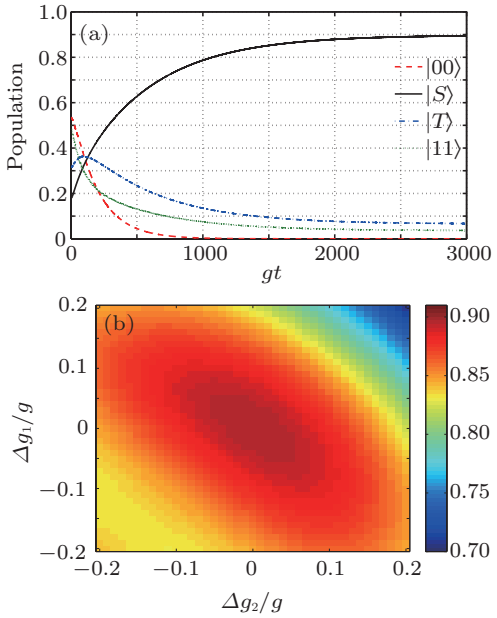


Fig. 3. (color online) (a) The populations of atomic states $|00\rangle$, $|S\rangle$, $|T\rangle$, and $|11\rangle$ against the dimensionless parameter gt for a random initial state. All curves are plotted with the set of optimized parameters $C=60$, $\kappa=0.04g$, $\gamma=0.4g$, $\Omega_1=\Omega_2=\Omega_3=0.03g$, $v=1.8g$, $\Delta_1=w_2-w_{\mathrm{L},1}=2.65g$, $\Delta_2=w_{\mathrm{L},2}-w_2=2.74g$, $\Delta_3=w_{\mathrm{L},3}-w_2+w_1=2.74g$, and $g_1=g_2=g$; (b) The population of the steady state $|S\rangle$ versus the relative fluctuations $\Delta g_1/g$ and $\Delta g_2/g$; the other parameters are the same as those in Fig. 3(a).

important from an experimental point of view. Another important feature of the scheme is that the distributed entanglement for two atoms can be engineered without preparing a specific initial state.

4. Conclusion

In conclusion, we proposed a cooling scheme for preparing the steady entanglement for two atoms held in two separated cavities. The high fidelity of the distributed steady entanglement can be achieved even when the cooperativity parameter is as low as 60. The cooling mechanism engineered in the present scheme allows a significant improvement in the fidelity of distributed steady entanglement as compared to the previous scheme.

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