Nonlocal non-Markovian effects in dephasing environments*

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(Received 29 May 2013; revised manuscript received 29 August 2013; published online 10 February 2014)

We study the nonlocal non-Markovian effects through local interactions between two subsystems and the corresponding two environments. It has been found that the initial correlations between two environments can turn a Markovian to a non-Markovian regime with extra control on the local interaction time. We further research the nonlocal non-Markovian effects from two situations: without extra control, the nonlocal non-Markovian effects only appear under the condition that two local dynamics are non-Markovian–non-Markovian (both of the two local dynamics are non-Markovian) or Markovian–non-Markovian, but not under the condition of Markovian–Markovian; with extra control, the nonlocal non-Markovian effects can occur under the condition of Markovian–Markovian. It shows that the function of correlations between two environments has an upper bound, which makes a flow of information from the environment back to the global system beginning finitely earlier than that back to one of the two local systems, not infinitely. Then, we proposed two special ways to distribute classical correlations between two environments without initial correlations. Finally, from numerical solutions in the spin star configuration, we found that the self-correlation (internal correlation) of each environment promotes the nonlocal non-Markovian effects.

Keywords: non-Markovian, Markovian, nonlocal, bound

PACS: 03.67.–a, 03.65.Yz, 42.50.–p, 03.65.Ta **DOI:** 10.1088/1674-1056/23/4/040302

1. Introduction

A realistic physical system inevitably interacts with the surrounding environment, leading to lose information from the system to the environment. If the environment can feed back the information to the system in the finite time, it signifies that the non-Markovian effects appear due to the environmental memory. And if the environment only feeds back the information in the infinite time, it means that the whole dynamics is Markovian. The dynamical process of Markovianity can also be treated as a limiting approximation of non-Markovianity.[1,2]

It is highly interesting to explore the non-Markovian effects, because there are many systems suffering from the strong back-action from the environment.[3–11] Hence, the non-Markovianity plays an important role in many respects. Up to now, there have been a lot of studies about this. Non-Markovianity can assist the formation of steady state entanglement;[12] non-Markovian coherent feedback control can also suppress the decoherence;[13] and non-Markovian effects are considered in a new theory of polymer reaction kinetics so that the dynamics of polymers can be controlled.[14] The direct observation of non-Markovian radiation dynamics has been completed in three-dimensional (3D) bulk photonic crystals;[15] and the observation of non-Markovian dynamics of a single quantum dot has also been completed in a micropillar cavity.[16]

It is worth noting that the authors in Ref. [17] found a new resource for quantum memory effects by the nonlocal non-Markovianity, where they utilized the correlations between two environments and controled the local interaction time to turn a Markovian to a non-Markovian regime. It will become more and more interesting in further research about the nonlocal non-Markovian effects, such as how to enhance and control the nonlocal non-Markovian effects.

In this article, we shall first discuss the nonlocal non-Markovian effects when the local dynamics are non-Markovian–non-Markovian, Markovian–non-Markovian, and Markovian–Markovian. Then, we find that without extra control, the nonlocal non-Markovian effects cannot appear under the condition of Markovian–Markovian. Besides taking control on the interaction time in Ref. [17], we find that reducing the strength of the interaction also turns a Markovian to a non-Markovian regime. Surprisingly, increasing the strength of the interaction can also do it. In both of the two examples given in Ref. [17] they considered that the initial correlations of two environments are nonlocal (having quantum correlations). We find that the classical correlations can perform very well without the quantum correlations. In many real situations, the classical correlations between two environments are easier to be formed as compared to the quantum correlations. And we advise two special ways to form the classical correlations, leading to the nonlocal non-Markovian effects. Finally, we investigate the non-Markovian effects in the spin star configuration under two different situations: with and without the self-correlation of each environment. We get the numerical so-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10975125 and 11375168).

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lution and obtain that the self-correlation of each environment can promote the nonlocal non-Markovian effects.

2. Theoretical model

We consider that the initial state of two subsystems is a pure state given by

$$|\Psi_S^{12}(0)\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle. \quad (1)$$

Under the local interactions with environments 1 and 2, a dephasing map for two qubits of general form[17] is

$$\begin{pmatrix} |a|^2 & ab^*\kappa_2(t) & ac^*\kappa_1(t) & ad^*\kappa_{12}(t) \\ ba^*\kappa_2^*(t) & |b|^2 & bc^*\Lambda_{12}(t) & bd^*\kappa_1(t) \\ ca^*\kappa_1^*(t) & cb^*\Lambda_{12}^*(t) & |c|^2 & cd^*\kappa_2(t) \\ da^*\kappa_{12}^*(t) & db^*\kappa_1^*(t) & dc^*\kappa_2^*(t) & |d|^2 \end{pmatrix},$$

where the initial value $\kappa_i(0) = 1$ for $i = 1, 2$, and 12; $\Lambda_{12}(0) = 1$.

The dephasing process of local systems $\rho_S^1(t)$ and $\rho_S^2(t)$ are fully determined by the functions $|\kappa_1(t)|$ and $|\kappa_2(t)|$. Besides $|\kappa_1(t)|$ and $|\kappa_2(t)|$, the dephasing process of global system $\rho_S^1(t)$ also depends on the functions $|\kappa_{12}(t)|$ and $|\Lambda_{12}^*(t)|$. It is possible that the functions $|\kappa_1(t)|$ and $|\kappa_2(t)|$ decrease, meanwhile, the function $|\kappa_{12}(t)|$ or $|\Lambda_{12}^*(t)|$ increases. In another words, it is possible that the local systems lose information by the local dephasing, but at the same time, the global system increases the information by the nonlocal non-Markovianity. The measure for the non-Markovianity of the dephasing process $\Phi(t)$[2] can be given by

$$\mathcal{N}(\Phi) = \max_{\rho_{1,2}(0)} \int_{\sigma>0} \mathrm{d}t\,\sigma(t, \rho_{1,2}(0)), \quad (2)$$

where ρ_1 and ρ_2 represent two different states of the same system,

$$\sigma(t, \rho_{1,2}(0) = \frac{\mathrm{d}}{\mathrm{d}t} D(\rho_1, \rho_2),$$

and the trace distance

$$D(\rho_1, \rho_2) = \frac{1}{2}\mathrm{tr}\sqrt{(\rho_1(t) - \rho_2(t))^\dagger(\rho_1 - \rho_2)}.$$

The function $\sigma(t, \rho_{1,2}(0) > 0$ represents that the information flows back to the system from the environment. Namely, the dynamics is non-Markovian.

For a two-level system, such as the local system 1, if the state of the system recovers coherence at a finite time t, i.e.,

$$\frac{\mathrm{d}}{\mathrm{d}t}|\kappa_1(t)| > 0,$$

the local dynamics is non-Markovian; otherwise, the local dynamics is Markovian. This can be proved as follows. Choose two arbitrary initial states

$$\begin{pmatrix} a_1 & b_1^* \\ b_1 & d_1 \end{pmatrix} \text{ and } \begin{pmatrix} a_2 & b_2^* \\ b_2 & d_2 \end{pmatrix}.$$

After the dephasing process, the two states are given by

$$\begin{pmatrix} a_1 & b_1^*\gamma(t) \\ b_1\gamma^*(t) & d_1 \end{pmatrix} \text{ and } \begin{pmatrix} a_2 & b_2^*\gamma(t) \\ b_2\gamma^*(t) & d_2 \end{pmatrix}.$$

Then, we obtain the function

$$\sigma(t, \rho_{1,2}(0)) = \frac{\frac{\mathrm{d}}{\mathrm{d}t}|\gamma(t)|}{\sqrt{|b_1 - b_2|^2|\gamma(t)|^2 + (a_1 - a_2 + d_2 - d_1)^2/4}}.$$

So $\sigma(t, \rho_{1,2}(0)) > 0$ is equivalent to

$$\frac{\mathrm{d}}{\mathrm{d}t}|\gamma(t)| > 0.$$

For a many-level system (global system), as long as the function

$$\frac{\mathrm{d}}{\mathrm{d}t}|\kappa_{12}(t)| > 0$$

or

$$\frac{\mathrm{d}}{\mathrm{d}t}|\Lambda_{12}(t)| > 0$$

at a finite time, the information of some systems must begin to increase, signifying that the global dynamics is non-Markovian, and *vice versa*. In other words, in the dephasing environment, whether the global dynamics is non-Markovian depends on $\frac{\mathrm{d}}{\mathrm{d}t}|\kappa_{12}(t)| > 0$ and $\frac{\mathrm{d}}{\mathrm{d}t}|\Lambda_{12}(t)| > 0$ (global system) at finite time t, besides $\frac{\mathrm{d}}{\mathrm{d}t}|\kappa_1(t)| > 0$ (local system 1) and $\frac{\mathrm{d}}{\mathrm{d}t}|\kappa_2(t)| > 0$ (local system 2).

3. Three dynamic processes

We consider that the environments have continuous energy levels w and the corresponding eigenstate $|w\rangle$ ($\hbar = 1$ throughout this article). The initial state of environment i is generally given by

$$\rho_{\mathrm{E}}^i(0) = Z_0^i \Big\{ \int_0^\infty \mathrm{d}w \exp[-(w - w_0^i)^2]|w + c_i\rangle\langle w + c_i| + \int_{-\infty}^0 \mathrm{d}w \exp[-(w + w_0^i)^2)]|w + c_i\rangle\langle w + c_i| \Big\}, \quad (3)$$

where for $i = 1$ and 2 (using this notation in the following sections), the energy level $w_0^i \geq 0$ and Z_0^i is the normalization coefficient. The energy level w_0^i and c_i can influence whether the local dynamics is non-Markovian or Markovian.

Firstly, we discuss that the local dynamics are non-Markovian–non-Markovian. The initial correlations between the two environments are classical correlations. So, without loss of generality, let the initial state of the two environments be

$$\rho_{\mathrm{E}}^{12}(0)$$

$$= Z_1\Big\{\int_0^{\infty} \mathrm{d}w \exp[-(w-1)^2]|w\rangle\langle w|\otimes|w\rangle\langle w| + \int_{-\infty}^{0} \mathrm{d}w \exp[-(w+1)^2)]|w\rangle\langle w|\otimes|w\rangle\langle w|\Big\}. \quad (4)$$

The interaction Hamiltonian is given by

$$H_{\mathrm{int}}^i = g_i \int_{-\infty}^{\infty} \mathrm{d}w \sigma_z^i \sigma_w^i, \quad (5)$$

where $\sigma_w = w|w\rangle\langle w|$ is the generalized operator of the environment; g_i is a coupling constant; and σ_z^i is the Pauli operator of system i. Then, we can obtain the dephasing factors

$$|\kappa_1(t)| = |\kappa_2(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-(w-1)^2]\cos(2gwt)\right|,$$
$$|\Lambda_{12}(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-(w-1)^2]\cos(4gwt)\right|,$$
$$|\kappa_{12}(t)| = 1, \quad (6)$$

where $g = g_i$ like g in the following Eq. (8) and Eq. (10). As shown in Fig. 1, at time $t = 0.36$, $|\Lambda_{12}|$ begins to increase when κ_1 and κ_2 keep on decreasing. It means that the flow of information from the environment back to the global system begins earlier than that back to both of the two local subsystems.

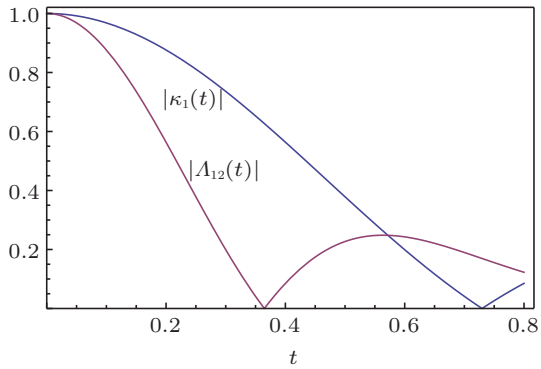


Fig. 1. (color online) The evolution of $|\kappa_1|$ changing with time t, in comparison with $|\Lambda_{12}|$. Here, the coupling constant $g = 1$. Both $|\kappa_1|$ and $|\Lambda_{12}|$ begin to increase after some time, signifying that the global and local dynamics are non-Markovian.

Secondly, we consider that the local dynamics are Markovian–non-Markovian. The initial state of two environments is given by

$$\rho_{\mathrm{E}}^{12}(0) = Z_1\Big\{\int_0^{\infty} \mathrm{d}w \exp[-w^2]|w\rangle\langle w|\otimes|w+1\rangle\langle w+1| + \int_{-\infty}^{0} \mathrm{d}w \exp[-w^2)]|w\rangle\langle w|\otimes|w+1\rangle\langle w+1|\Big\}. \quad (7)$$

The interaction Hamiltonian is the same as Eq. (5). Then we obtain

$$|\kappa_1(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-w^2]\cos(2gwt)\right|,$$
$$|\kappa_2(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-w^2]\cos[g(2w+2)t]\right|,$$
$$|\Lambda_{12}(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-w^2]\cos[g(4w+2)t]\right|,$$
$$|\kappa_{12}(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-w^2]\cos(2gt)\right|. \quad (8)$$

In Fig. 2, we show the change of $|k_1|$, $|k_2|$, $|k_{12}|$, and $|\Lambda_{12}|$ with time t. It is clearly seen that after some time, Λ_{12} begins to increase when κ_1 and κ_2 keep on decreasing, meaning that the flow of information from the environment back to the global system begins earlier than that back to the local subsystem 2. Where, $|\kappa_1|$ always reduces ($\frac{\mathrm{d}}{\mathrm{d}t}|\kappa_1| < 0$ for all the time), representing that the dynamics of subsystem 1 is Markovian.

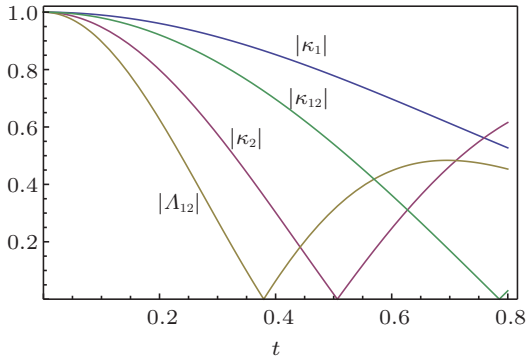


Fig. 2. (color online) The evolutions of $|\kappa_1|$, $|\kappa_2|$, $|\kappa_{12}|$, and $|\Lambda_{12}|$ changing with time t. Here, the coupling constant $g = 1$.

Finally, we consider the case that the local dynamics are Markovian–Markovian. A classical initial state of two environments is described by

$$\rho_{\mathrm{E}}^{12}(0) = Z_1\Big\{\int_0^{\infty} \mathrm{d}w \exp[-w^2]|w\rangle\langle w|\otimes|w\rangle\langle w| + \int_{-\infty}^{0} \mathrm{d}w \exp[-w^2]|w\rangle\langle w|\otimes|w\rangle\langle w|\Big\}. \quad (9)$$

Furthermore, by making use of the same interaction Hamiltonian (see Eq. (5)), we get

$$|\kappa_1(t)| = |\kappa_2(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-w^2]\cos(2gwt)\right|,$$
$$|\Lambda_{12}(t)| = \left|2Z_1\int_0^{\infty} \mathrm{d}w \exp[-w^2]\cos[4gwt]\right|,$$
$$|\kappa_{12}(t)| = 1. \quad (10)$$

In Fig. 3, we plot the evolution of $|\kappa_1|$ and $|\Lambda_{12}|$ with time t. It is shown that Λ_{12} always decreases. When the local dynamics are Markovian–Markovian, we cannot find an initial state of two environments to make the nonlocal non-Markovian effects appear, even the existence of quantum correlations between two environments.

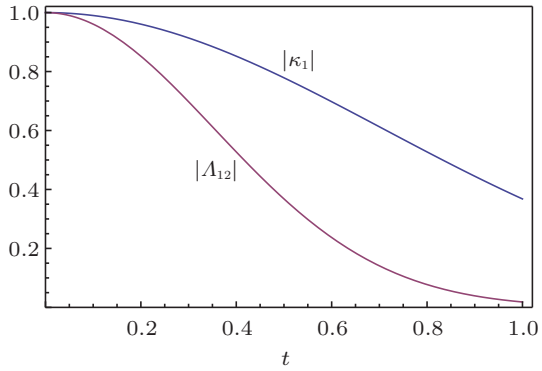


Fig. 3. (color online) The evolutions of $|\kappa_1|$ and $|\Lambda_{12}|$ changing with time t. Here, the coupling constant $g = 1$.

From the above three dynamic processes and corresponding figures, we gain the conclusion that the correlations between two environments only make the flow of information from the environment back to the global system begin finitely earlier than that back to one of two local subsystems. In other words, there is an upper bound on the function of the correlations between two environments. So if the local dynamics are Markovian–Markovian, the correlations cannot shorten the infinite starting time of the flow of information from the environment back to the global system to a finite time. Namely, the global dynamics is still Markovian when the local dynamics are Markovian–Markovian.

4. Extra control

In the foregoing investigations, we only consider the case without extra control. In other words, the whole Hamiltonian of the systems and the two environments is independent of time. So if there is any other control, it is possible to obtain the nonlocal non-Markovian effects under the condition that the local dynamics are Markovian–Markovian.

In Ref. [17], the authors controled the time of two local interactions to turn a Markovian to a non-Markovian regime. Here, we find that reducing the coupling strength g_i, i.e., reducing the rate of local dephasing, we can also achieve the goal.

Let the initial state of two environments be Eq. (9), and the interaction Hamiltonian be given by Eq. (5). Namely, the local dynamics are Markovian–Markovian. As discussed in Section 3, without extra control, the global dynamics is Markovian.

We now discuss the case with extra control to change the global dynamics from Markovianity to non-Markovianity. We first consider the case by reducing the coupling strength. At the beginning, we choose the coupling strength $g_1 = 3$ and $g_2 = 2$. At time $t = 1$, we reduce g_1 from 3 to 1 and get

$$|\kappa_1(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos(wt' + 3w)\right|,$$

$$|\kappa_2(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos[2wt' + 2w]\right|,$$

$$|\Lambda_{12}(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos[3wt' + 5w]\right|,$$

$$|\kappa_{12}(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos(w - wt')\right|, \quad (11)$$

where $t' = t - 1$. As shown in Fig. 4, it is obvious to see that $|\kappa_{12}|$ increases with time t'. This indicates that reducing the strength of coupling g_1 can turn a Markovian to a non-Markovian regime.

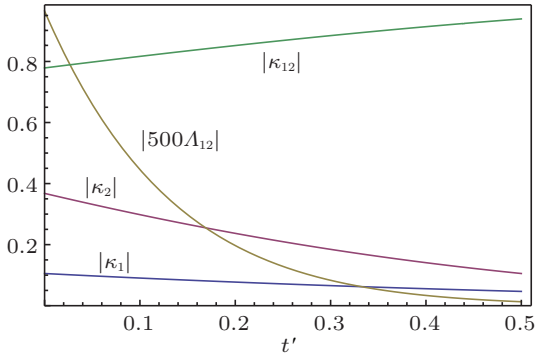


Fig. 4. (color online) The evolution of $|\kappa_1|$, $500|\Lambda_{12}|$, $|\kappa_2|$, and $|\kappa_{12}|$ with time t', where, in order to better compare them, we replace $|\Lambda_{12}|$ by $500|\Lambda_{12}|$.

Then, we discuss the case by increasing the coupling strength. We first set the coupling strength $g_1 = 2$ and $g_2 = 1$; when $t = 1$, we increase g_2 from 1 to 3, we can obtain

$$|\kappa_1(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos(wt' + 2w)\right|,$$

$$|\kappa_2(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos[2wt' + w]\right|,$$

$$|\Lambda_{12}(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos[5wt' + 3w]\right|,$$

$$|\kappa_{12}(t')| = \left|2Z_1 \int_0^\infty dw \exp[-w^2]\cos(w - wt')\right|, \quad (12)$$

where $t' = t - 1$. The evolutions of $|\kappa_1|$, $|\kappa_{12}|$, $|\kappa_2|$, and $|\kappa_{12}|$ with time t are shown in Fig. 5. It is clearly seen that $|\kappa_{12}|$ also increases, representing that the nonlocal non-Markovian effects appear when the coupling strength increases.

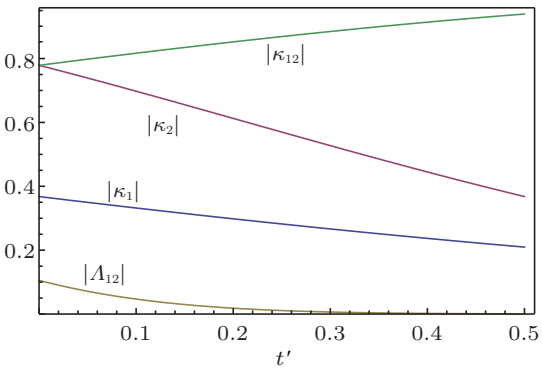


Fig. 5. (color online) The evolution of $|\kappa_1|$, $|\Lambda_{12}|$, $|\kappa_2|$, and $|\kappa_{12}|$ with time t'.

5. Classical correlation forming

Usually two nonlocal environments do not have any correlations. It is very hard to create quantum correlations between two macroscopical environments; however, classical correlations can be formed relatively easy. As discussed in the previous sections, classical correlations can perform very well for nonlocal non-Markovian effects. In this section, we propose two ways to form classical correlations between two environments.

We first consider that an entanglement system in Bell state $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$ disentangles by the local interaction with two uncorrelated environments, where the environments are composed of bosons. After some time, let the system interact with the same two environments. The initial uncorrelated environments will form classical correlations due to the interaction with the entanglement system, and the correlations will make the information flow back to the system.

The interaction Hamiltonian is given by

$$H^i_{\rm int}(t) = \chi_i(t)\sum_{k_i=1}^{n_i} g_{k_i}\sigma^i_z(b^\dagger_{k_i}+b_{k_i}) + \chi'_i(t)\sum_{k_i=1}^{n_i} g_{k_i}\sigma'^i_z(b^\dagger_{k_i}+b_{k_i}), \tag{13}$$

where the first term on the right-hand side is the interaction Hamiltonian between the system and the environments; the second one is the interaction Hamiltonian between the entanglement system ($\sigma^{\prime i}_Z$ is the Pauli operator of the corresponding subsystem i) and the same environments; the function $\chi'_i(t)=1$ for $0\le t\le t'$ and zero otherwise; $\chi_i(t)=1$ for $t^{\rm s}_i\le t\le t^{\rm f}_i$ and zero otherwise. Here, $t^{\rm s}_i$ and $t^{\rm f}_i$ denote the time when the interaction is switched on and switched off in system i, respectively; 0 and t' denote the time in the entanglement system. We denote that $t_i(t)=\int_0^t\chi_i(t')\mathrm{d}t'$ and $t'_i(t)=\int_0^t\chi'_i(t')\mathrm{d}t'$. The Hamiltonian of environments is described by

$$H^{12}_{\rm E} = \sum_{k_1=1}^{n_1} w_{k_1} b^\dagger_{k_1} b_{k_1} + \sum_{k_2=1}^{n_2} w_{k_2} b^\dagger_{k_2} b_{k_2}. \tag{14}$$

The initial state of environments is in the thermal equilibrium state $\rho^{12}_{\rm E}=(1/Z_{12})\exp(-\beta H^{12}_{\rm E})$, where Z_{12} is the partition function.

Then, using weak coupling approximation $[\exp(\alpha_k b^\dagger-\alpha^*_k b),\exp(-\mathrm{i}w_k b^\dagger_k b_k)]\approx 0$, and performing a continuum limit with Ohmic spectrum density $A_i w\exp(-w/\Omega_i)$,[18–20] where A_i is the coupling constant and Ω_i is the frequency cutoff, we get

$$|\kappa_1| = \left|\exp(-\Gamma_1)\cos\left[\int \mathrm{d}wA_1\exp(-w/\Omega_1)\xi_1\right]\right|,$$

$$|\Lambda_{12}| = \left|\exp(-\Gamma_1-\Gamma_2)\cos\left[\sum_{i=1}^{2}\int \mathrm{d}wA_i \times \exp(-w/\Omega_i)\xi_i\right]\right|, \tag{15}$$

where

$$\xi_i = 2\sin[w(t_i(t)-t'_i(t))]-2\sin[wt_i(t)]+2\sin[wt'_i(t)],$$

$$\Gamma_i = \int \mathrm{d}wA_i\exp(-w/\Omega_i)\coth(2w/\beta)(1-\cos[wt_i(t)],$$

$$\alpha_k = g_k\frac{1-\exp(\mathrm{i}w_k t)}{w_k}.$$

As shown in Fig. 6, the information begins to flow back to the global system when $|\Lambda_{12}|$ begins to increase, indicating that the correlations between the two environments are formed to make the nonlocal non-Markovian effects emerge. We find that the Bell state has the same function as the classical state $(1/2)(|00\rangle\langle 00|+|11\rangle\langle 11|)$ on forming the classical correlations of two environments, so it has robustness against some noise. If one wants to form stronger correlations, the number of Bell states should be increased.

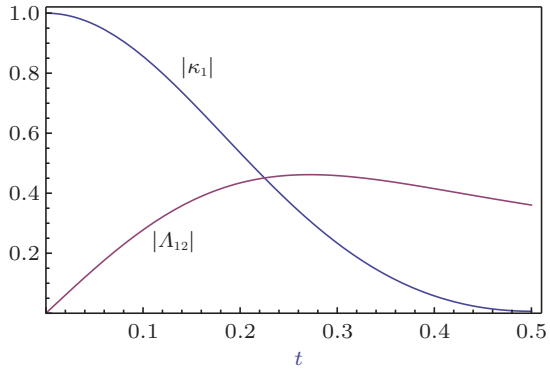


Fig. 6. (color online) The evolutions $|\kappa_1|$ and $|\Lambda_{12}|$ change with time t. Here, the parameter $\Gamma_2=0.5, A_1=1, \beta=0.2, \xi_2=\pi/2, t'_1(t)=t'_2(t)=1$ and $t_1(t)=t$.

The second way we proposed is that after two subsystems interact locally with the corresponding two environments without initial correlations for some time, by exchanging the locations of two subsystems, they interact locally with both environments. A similar calculation can be performed as before. It is easy to observe the nonlocal non-Markovian effects. The reason is that the system loses information to create correlations between two environments. It is necessary to exchange the locations of two subsystems because if the exchange does not happen, the correlations between the system and the two environments will keep the system losing information, and finally the appearance of nonlocal non-Markovian effects cannot occur. The benefit of this way is that the correlations between two environments can be formed without the needs of other correlation states. Obviously, very strong correlations cannot be formed like the first way, and furthermore, the strong non-Markovian effects cannot be created.

6. The self-correlation promoting the nonlocal non-Markovian effects

In this section, we explore whether the self-correlation of two environments promotes the nonlocal non-Markovian effects. We consider a simple model: the spin star configuration.[21]

The interaction Hamiltonian is described by

$$H_{\rm int} = \sum_{i=1}^{2}\sum_{j=1}^{n_i} \eta_i(t) g_{ij} \sigma_{ij}^z \sigma_{iS}^z, \tag{16}$$

where $\eta_i(t) = 1$ for $t_i^{\rm s} \le t \le t_i^{\rm f}$ and zero otherwise; σ_{ij}^z and σ_{iS}^z are the Pauli operators of environment i and subsystem i, respectively. The initial two environments are in the thermal equilibrium state $\rho_{\rm E}^{12} = (1/Z_{12})\exp[-\beta H_{\rm E}^{12}]$, in which, the Hamiltonian of two environments $H_{\rm E}^{12} = \sum_{i=1}^{2} B_i S_i^z + \alpha S_1^z S_2^z$. We define operator $S_i^z = \sum_{j=1}^{n_i} \sigma_{ij}^z/2$ without self-correlation of each environment (the internal correlations among spins in each environment) and

$$S_i^z = \sum_{j=1}^{n_i} \left[\frac{1}{2}\sigma_{ij}^z + (J_i/B_i) \sum_{\langle mn \rangle} \sigma_{im}^z \sigma_{in}^z \right]$$

with self-correlation, where $\langle mn \rangle$ denotes $|m-n| = 1$ to show the nearest neighbor interaction in each environment. Then we obtain

$$\begin{aligned} |\kappa_1(t)| &= \left| {\rm Tr}\left[\exp\left[-2{\rm i} \sum_{j=1}^{n_1} \int_0^t {\rm d}t' \eta_1(t') g_{1j} \sigma_{1j}^z \right] \rho_{\rm E}^{12} \right] \right|, \\ |\Lambda_{12}(t)| &= \left| {\rm Tr}\left[\exp\left[-2{\rm i} \sum_{i=1}^{2}\sum_{j=1}^{n_i} \int_0^t {\rm d}t' \eta_i(t') g_{ij} \sigma_{ij}^z \right] \rho_{\rm E}^{12} \right] \right|. \end{aligned} \tag{17}$$

The numerical results of Eq. (17) are shown in Figs. 7 and 8, it is obvious that the nonlocal non-Markovian effects with the self-correlation is stronger, according to the growth of $\Lambda_{12}(t)$. In another words, the self-correlation promotes the non-local non-Markovian effects.

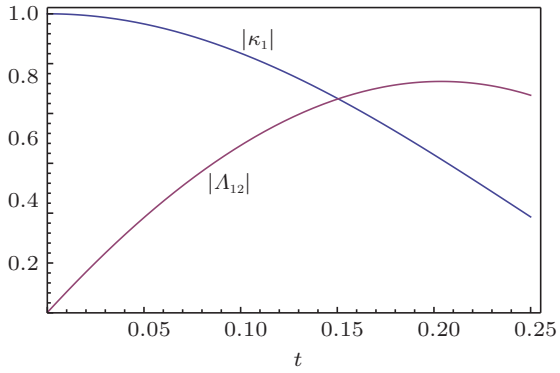


Fig. 7. (color online) The evolution of $|\kappa_1|$ and $|\Lambda_{12}|$ changing with time t with the self-correlation. The parameters are $n_i = 5$, $\alpha = 4$, $\beta = 0.01$, $B_i = 2$, $J_i = 10$, $\int_0^t {\rm d}t' \eta_2(t') = 0.2$, $\int_0^t {\rm d}t' \eta_1(t') = t$, and $g_{ij} = 1$ for $i = 1,2$; $j = 1,...,5$.

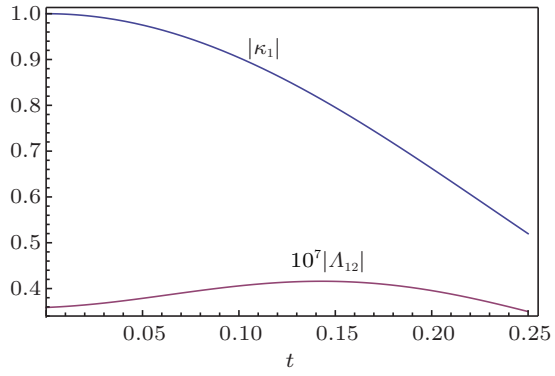


Fig. 8. (color online) The evolution of $|\kappa_1|$ and $10^7|\Lambda_{12}|$ changing with time t without the self-correlation. Here, the parameters are $n_i = 5$, $\alpha = 4$, $\beta = 0.01$, $B_i = 2$, $\int_0^t {\rm d}t' \eta_2(t') = 0.785$, $\int_0^t {\rm d}t' \eta_1(t') = t$, and $g_{ij} = 1$ for $i = 1,2$; $j = 1,...,5$.

7. Conclusion

In this article we explored a quantum system composed of two subsystems locally interacting with two environments. We found that the function of correlations between two environments has an upper bound, which makes a flow of information from the environment back to the global system starting finitely earlier than that back to one of the two local systems, not infinitely (when the whole Hamiltonian of the system and two environments is independent of time). It means that nonlocal non-Markovian effects cannot appear when both of the two local dynamics are Markovian. Of course, extra control can turn a Markovian to a non-Markovian regime. Besides the control of the local interaction time, reducing the coupling strength of the local interaction, i.e., reducing the rate of losing information of subsystems, can make the nonlocal non-Markovian effects appear. Surprisingly, enhancing the couping strength of the local interaction, i.e., increasing the rate of losing information of subsystems, can also achieve it. We proposed two ways, which are easy to be realized in experiment, to form the classical correlations between two environments without initial correlations. We also found that the self-correlation of two environments can promote the nonlocal non-Markovian effects.

Recently, the authors in Ref. [22] accessed the non-Markovianity based on ideas of divisibilities of channels; the authors in Ref. [23] proposed a way to quantify the memory effects in the spin-boson model; in Ref. [24], two different approaches to quantify non-Markovianity were proposed; the authors in Ref. [25] quantified non-Markovianity via correlations. It is very interesting to quantify the nonlocal non-Markovianity in different ways in order to better understand and characterize the nonlocal non-Markovian effects in more complex systems, which may be used to perform some quantum information process, such as quantum memory[26] and quantum error correction.[27] The results obtained in this paper will be helpful for how to utilize and control the nonlocal non-Markovian effects by controlling the local dynamics in future

works.

References

[1] Spohn H 1980 *Rev. Mod. Phys.* **52** 569
[2] Breuer H P, Laine E M and Piilo J 2009 *Phys. Rev. Lett.* **103** 210401
[3] Akihito I and Fleming G R 2009 *J. Chem. Phys.* **130** 234110
[4] Akihito I and Fleming G R 2009 *J. Chem. Phys.* **130** 234111
[5] Bellomo B, Franco R L and Compagno G 2007 *Phys. Rev. Lett.* **99** 160502
[6] Sarovar M, Ishizaki A, Fleming G R and Whaley K B 2010 *Nat. Phys.* **6** 462
[7] Dijkstra A G and Tanimura Y 2010 *Phys. Rev. Lett.* **104** 250401
[8] Liao J Q, Huang J F, Kuang L M and Sun C P 2010 *Phys. Rev. A* **82** 052109
[9] Imamoglu A 1994 *Phys. Rev. A* **50** 3650
[10] Wang X Y, Ding B F and Zhao H P 2013 *Chin. Phys. B* **22** 040308
[11] Ding B F, Wang X Y, Tang Y F, Mi X W and Zhao H P 2011 *Chin. Phys. B* **20** 060304
[12] Huelga S F, Rivas Á and Plenio M B 2012 *Phys. Rev. Lett.* **108** 160402
[13] Xue S B, Wu R B, Zhang W M, Zhang J, Li C W and Tarn T J 2012 *Phys. Rev. A* **86** 052304
[14] Guérin T, Bénichou O and Voituriez R 2012 *Nat. Chem.* **4** 568
[15] Hoeppe U, Wolff C, Kuchenmeister J, Niegemann J, Drescher M, Benner H and Busch K 2012 *Phys. Rev. Lett.* **108** 043603
[16] Madsen K H, Ates S, Lund-Hansen T, Löffler A, Reitzenstein S, Forchel A and Lodahl P 2011 *Phys. Rev. Lett.* **106** 233601
[17] Laine E M, Breuer H P, Piilo J, Li C F and Guo G C 2012 *Phys. Rev. Lett.* **108** 210402
[18] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (New York: Oxford University Press)
[19] Leggett A J, Chakravarty S, Dorsey A, Fisher M, Garg A and Zwerger W 1987 *Rev. Mod. Phys.* **59** 1
[20] Weiss U 2008 *Quantum Dissipative Systems* (Singapore: World Scientific)
[21] Breuer H P, Burgarth D and Petruccione F 2004 *Phys. Rev. B* **70** 045323
[22] Wolf M M, Eisert J, Cubitt T S and Cirac J I 2008 *Phys. Rev. Lett.* **101** 150402
[23] Clos G and Breuer H P 2012 *Phys. Rev. A* **86** 012115
[24] Rivas Á, Huelga S F and Plenio M B 2010 *Phys. Rev. Lett.* **105** 050403
[25] Luo S, Fu S and Song H 2012 *Phys. Rev. A* **86** 044101
[26] Sangouard N, Simon C, Riedmatten H D and Gisin N 2011 *Rev. Mod. Phys.* **83** 33
[27] Paz-Silva G A, Rezakhani A T, Dominy J M and Lidar D A 2012 *Phys. Rev. Lett.* **108** 080501